Automatic threshold resummation in MG5_aMC

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Outline

- Motivations
- Theoretical framework
- Implementation
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- Preliminary results
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Motivations

- Important to resum soft-gluon effects for precision phenomenology of a large class of processes.
- Automation of this resummation possible: achieving this in MG5_aMC could significantly extend the set of processes for which these threshold effects are known.
- A step towards including analytic resummation among the predictions offered by MG5_aMC.
- Could be a basis/template for the achievement of other kinds of resummation in the same framework and with similar methods.

Theoretical framework: setup

- Processes with no massless final-state QCD particles.
- Kinematics: $z = M^2/s$, $\tau = M^2/S$, M = Born-system invariant mass, S = hadronic, s = partonic CM energy squared.
- Resum dominant $\ln(1-z)$ in the threshold limit $z \to 1$: extra radiation soft with energy $\sim \sqrt{s}(1-z)$. So called 'pair'-invariant-mass (PIM) kinematics.
- Use SCET framework to perform the resummation in momentum space.

Theoretical framework: factorisation

- QCD factorisation theorem (with $\mu_r = \mu_f$):

$$d\sigma = \frac{1}{2S} \sum_{ij} \int_{\tau_{\min}}^{1} \frac{d\tau}{\tau} \int_{\tau}^{1} \frac{dz}{z} \mathcal{L}_{ij}(\frac{\tau}{z}, \mu_f) C_{ij}(z, M, \mu_f) d\Phi_n,$$

  $\mathcal{L}$ = luminosity, and partonic cross section $d\hat{\sigma}_{ij} \propto \int dM\, C_{ij}\, d\Phi_n$.

- At threshold, kernel C_{ij} factorises as

$$C(z, M, \mu_f) = \mathrm{Tr}[\mathbf{H}(M, \mu_f)\mathbf{S}(\sqrt{s}(1-z), \mu_f)] + \mathcal{O}(1-z).$$

- **H** and **S** matrices in colour space: $\mathbf{H}_{AB} = \frac{\langle c_A|\mathbf{H}|c_B\rangle}{\langle c_A|c_A\rangle\langle c_B|c_B\rangle}$.

- $|c_A\rangle$ = orthogonal basis of possible colour structures of the process. Example: in $q\bar{q} \to Q\bar{Q}$, $|c_1\rangle = \delta_{i_2 i_1}\delta_{i_3 i_4}$, $|c_2\rangle = t^a_{i_2 i_1} t^a_{i_3 i_4}$.

- **H** (**S**) includes virtual (real) contributions in the threshold limit.

Theoretical framework: resummation through RG evolution

- $\mathrm{Tr}[\mathbf{H}(\mu_f)\mathbf{S}(\mu_f)]$ = threshold approximation of fixed-order cross section (at lowest order = full Born since Born lives at $z = 1$).
- $\mathbf{H}$ satisfies RG equation $\frac{d}{d\log\mu}\mathbf{H}(\mu) = \mathbf{\Gamma}_H(\mu)\mathbf{H}(\mu) + \mathbf{H}(\mu)\mathbf{\Gamma}_H^\dagger(\mu)$.
- Analogous (but more complicated) equation for $\mathbf{S}$, deduced imposing RG invariance of total cross section and using DGLAP.
- All-order solutions: $\mathbf{H}(\mu) = \mathbf{U}_h(\mu_h,\mu)\mathbf{H}(\mu_h)\mathbf{U}_h^\dagger(\mu_h,\mu)$, $\tilde{\mathbf{s}}(\mu) = \mathbf{U}_s(\mu_s,\mu)\tilde{\mathbf{s}}(\mu_s)\mathbf{U}_s^\dagger(\mu_s,\mu)$.
- $\mathbf{U}_{h/s}(\mu_1,\mu_2) = \mathcal{P}\exp\int_{\mu_1}^{\mu_2}\frac{d\mu'}{\mu'}\mathbf{\Gamma}_{H/S}(\mu')$: diagonalise $\mathbf{\Gamma}_{H/S}$ to solve RG evolution.
- Evolution matrix $\mathbf{U}$ contains threshold logarithms.
  Resummation in direct space (SCET inspired): replace $\mathbf{H}^{\mathrm{fo}}(\mu_f)$ with $\mathbf{U}_h(\mu_h,\mu_f)\mathbf{H}^{\mathrm{fo}}(\mu_h)\mathbf{U}_h^\dagger(\mu_h,\mu_f)$, and similarly for $\mathbf{S}$.

Theoretical framework: resummation formula

- Perturbative accuracy of **H** and **S** determines log accuracy of the resummation: (N)LO **S** and **H** give (N)NLL resummation.
- NLL resummation formula

$$C_{ij}^{\mathrm{NLL}} = \exp[2(a_i^0 + a_j^0)]\mathrm{Tr}[\mathbf{U}^0(\mu_h,\mu_s)\mathbf{H}_{ij}^0(\mu_h)\mathbf{U}^{0\dagger}(\mu_h,\mu_s)\tilde{\mathbf{s}}^0(\mu_s)]\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\frac{z^{-\eta}}{(1-z)^{1-2\eta}}$$

- All quantities evaluated at lowest order, $\mathbf{H}^0$ obtained through Born information, $\tilde{\mathbf{s}}$ = Laplace transform of **S**:

$$(H_{ij}^0)_{AB} = \frac{1}{4}\frac{\langle c_A|\mathcal{M}_b\rangle\langle\mathcal{M}_b|c_B\rangle}{\langle c_A|c_A\rangle\langle c_B|c_B\rangle} \qquad \tilde{s}_{AB}^0 = \langle c_A|c_B\rangle$$

- a_k and η = colour factors times integrated anomalous dimensions

The method

- Process dependence encoded in **H**: can automate this resummation at the same level as automatic fixed-order code.
- Need to build a general structure to handle the process-independent parts.
- We chose the reweigthing way.
  Produce LO (for NLL resummation) events, reweight each through C_{ij}^{NLL} evaluated at the specific event kinematics, and obtain the resummed $d\sigma/dM$.
- Strengths: automatic, fast (if evaluation and integration of C_{ij}^{NLL} is), parallelisable, modular (different reweighting modules to achieve different resummations), accurate (accuracy given by size of original LO sample).
- Having kinematic configurations one could in principle obtain as many observables as wanted, as with a shower. Question: logarithmic accuracy for those?

Implementation: colour basis

$$(H^0_{ij})_{AB} = \frac{1}{4}\frac{\langle c_A|\mathcal{M}_b\rangle\langle\mathcal{M}_b|c_B\rangle}{\langle c_A|c_A\rangle\langle c_B|c_B\rangle} \qquad \tilde{s}^0_{AB} = \langle c_A|c_B\rangle$$

- Implemented all colour bases $|c_A\rangle$ for 2 coloured $\rightarrow$ 0,1,2 coloured + N colourless
- The interface automatically projects the Born matrix elements of `matrix.f` on the basis, to get $\mathbf{H}^0$.
- $\mathbf{H}^0$ validated against literature for $3 \otimes \bar{3} \rightarrow 1$ (Drell-Yan) and for all $2 \rightarrow 2$ colour combinations (but $8 \otimes 8 \rightarrow 8 \otimes 8$, still to fully test).
- $\tilde{\mathbf{s}}^0$ automatically follows from the implementation of the basis.

Implementation: RG evolution

$$\mathbf{U}(\mu_h, \mu) = \mathcal{P} \exp \int_{\mu_h}^{\mu} \frac{d\mu'}{\mu'} \mathbf{\Gamma}_H(\mu')$$

- Need to diagonalise $\mathbf{\Gamma}_H$ in evolution matrix $\mathbf{U}$ to solve RG.
- Efficient and fast numerical diagonalisation of complex squared matrices using dedicated routines from the `LAPACK` package.
- Excellent speed performances even for high-rank matrices.

Implementation: reweighting

- Born weight $\mathcal{P}_{\rm LO}$ for kinematics $\mathbf{X}$:

$$d\sigma_{\rm LO} = \frac{1}{2S}\sum_{ij}\int_{\tau_{\rm min}}^{1}\frac{d\tau}{\tau}\,d\Phi_n\mathcal{P}_{\rm LO}(\mathbf{X}) = \sum_{i=1}^{N} e_i(\mathbf{X}_i)$$

- Resummed weight for kinematics $\mathbf{X}$

$$d\sigma_{\rm NLL} = \frac{1}{2S}\sum_{ij}\int_{\tau_{\rm min}}^{1}\frac{d\tau}{\tau}\,d\Phi_n\mathcal{P}_{\rm NLL}(\mathbf{X}),$$

with $\mathcal{P}_{\rm NLL}(\mathbf{X}) = \int_{\tau}^{1}\frac{dz}{z}\,\mathcal{L}_{ij}(\frac{\tau}{z},\mu_f)\,C_{ij}^{\rm NLL}$.

- $d\sigma_{\rm NLL}$ obtained reweighting LO events by $\mathcal{P}_{\rm NLL}$:

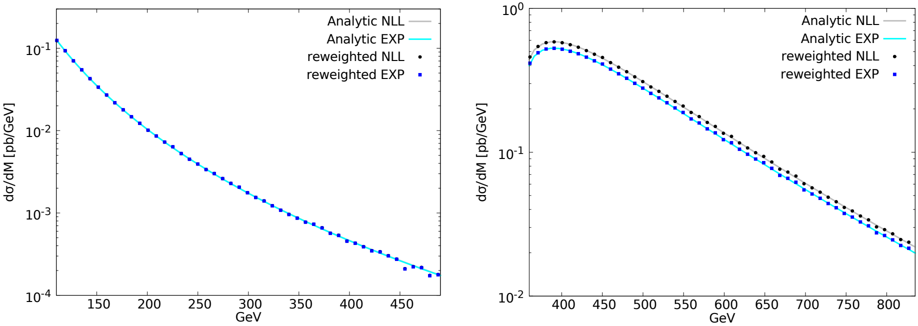
$$d\sigma_{\rm NLL} = \sum_{i=1}^{N} e_i(\mathbf{X}_i)\times\frac{\mathcal{P}_{\rm NLL}(\mathbf{X}_i)}{\mathcal{P}_{\rm LO}(\mathbf{X}_i)}.$$

- $\mathcal{P}_{\rm NLL}(\mathbf{X})$: 2D integration by quadrature methods, to maximise speed.

Implementation: general considerations

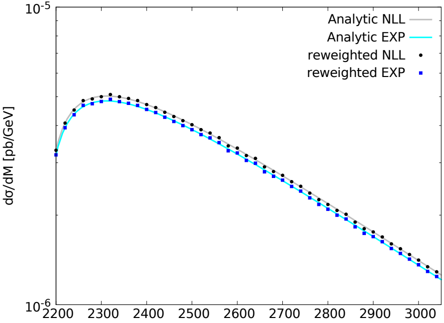
- Good speed and accuracy performance: one event typically reweighted in few to few tens of milliseconds.
- Possibility to include scale variations for all scales involved: μ_f, μ_s, and μ_h (but, yes, one run per variation).
- Highly parallelised: with 100 jobs, 1M events for a $2 \to 3$ process is reweighted in few minutes per scale variation.
- Parallelisation may be interesting for `MadSpin` developers/users: by implementing the same job organisation there, that tool as well could be parallelised, with huge performance improvements.

Validation: Drell-Yan and $t\bar{t}$



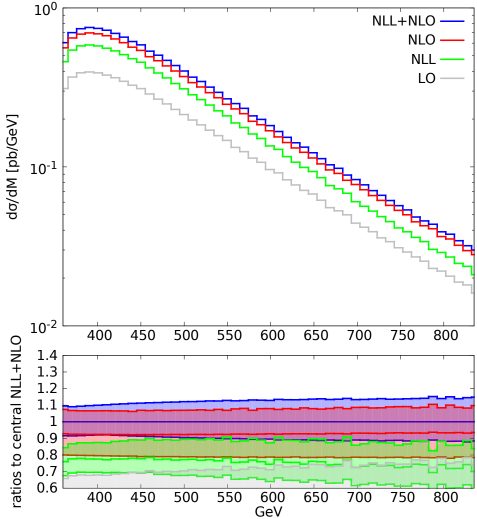
- Simplified DY (only γ^*, with $m_{ee} > 100\,\text{GeV}$ – left) and $t\bar{t}$ (right) @ LHC7.
- M = Born system invariant mass.
- Lines = analytic codes, dots = reweighting interface.

Validation: stop pair



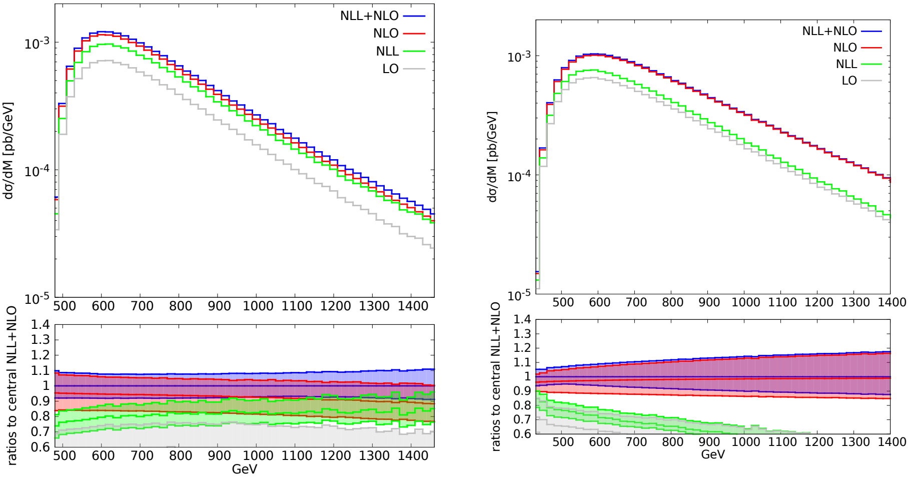
- Stop pair production @ LHC14.
- M = Born system invariant mass.
- Lines = analytic codes, dots = reweighting interface.

Preliminary results: $t\bar{t}$



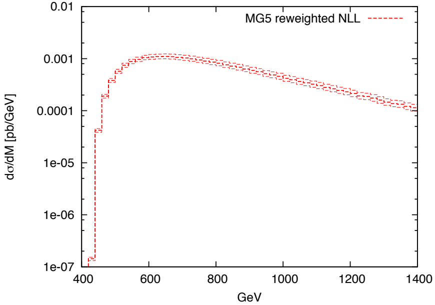
- $t\bar{t}$ production @ LHC7 (LO with NLO PDF), additive matching.

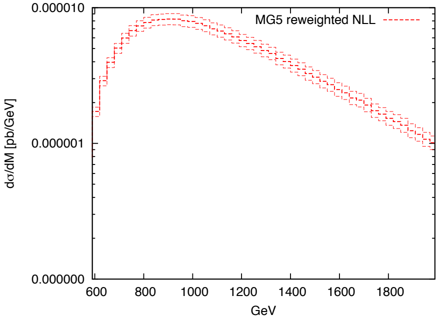
Preliminary results: $t\bar{t}H$ and $t\bar{t}W$



- $t\bar{t}H$ (left) and $t\bar{t}W^{\pm}$ (right) @ LHC13 (LO with NLO PDF), additive matching.

Preliminary results: $t\bar{t}Z$ and $t\bar{t}WW$





- $t\bar{t}Z$ (left) and $t\bar{t}W^+W^-$ (right) @ LHC13.
- Still to be matched to NLO.

Status and future directions

General interface for threshold resummation in MG5_aMC.

Current limitations

- Only processes with no final-state massless QCD.
- $\mu_r = \mu_f$.
- Only NLL for the moment.
- Only 2 coloured $\rightarrow$ 0,1,2 coloured + N colourless.

Future directions

- Real goal is NNLL, through reweighting of NLO configurations.
- Overcome the above constraints.
- Consolidate and improve the interface.

Thank you for your attention

Backup

Integration of resummation kernel

- $\mathcal{P}_{\rm NLL}(\mathbf{X}) \propto \int_\tau^1 dz\, \mathcal{L}_{ij}(\frac{\tau}{z}, \mu_f) z^{-1-\eta}(1-z)^{-1+2\eta} = \int_{-1}^1 du \int_\tau^1 dz\, \mathcal{I}(u,z)(1-z)^{-1+2\eta}$
- Bulk coming from $z \sim 1$ for small η.
- Split z-integration in $[\tau, z_0]$, and $[z_0, 1]$: $\int_{z_0}^1 = g(u,z_0) + h(u,z_0)$

$$g(u,z_0) = \int_{z_0}^1 dz \Big[\mathcal{I}(u,z) - \sum_{n=0}^m \frac{1}{n!}(1-z)^n \partial_z^n \mathcal{I}(u,1)\Big](1-z)^{-1+2\eta}$$

$$h(u,z_0) = \sum_{n=0}^m \frac{1}{n!} \partial_z^n \mathcal{I}(u,1) \frac{(1-z_0)^{n+2\eta}}{(n+2\eta)}$$

- $\partial_z^n \mathcal{I}$ analytic: PDFs on a basis of Chebyshev polynomials with fitted coefficients.
- Subtractions: make g (numerical) small and h (analytic) large, to speed up.
- Subtraction implements analytic continuation for negative η.
- Integrations by means of quadrature methods (need to be fast since one 2D integration per reweighted event).