FDR up to two loops

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Outline

1. Four Dimensional Regularization / Renormalization

R.P., JHEP 1211 (2012) 151
Alice M. Donati and R.P., JHEP 1304 (2013) 167
R.P., Fortsch.Phys. 63 (2015) 601-608

2. QCD up to two loops in FDR

Ben Page and R.P., JHEP 1511 (2015) 183

3. IR infinities in FDR

R.P., Eur. Phys. J. C (2014) 74:2686
Alice M. Donati and R.P., Eur. Phys. J. C (2014) 74:2864

FDR

- UV problem solved by *introducing a new kind of loop integration*
- Subtraction of UV divergences encoded in the *definition* of loop integrals

- **Scalar one-loop integral**

1. $J(q) = \dfrac{1}{(q^2 - M^2)^2}$ (log UV divergent *integrand*)
2. $q^2 \to \bar{q}^2 \equiv q^2 - \mu^2$
3. $J(q) \to J(q, \mu^2) \equiv \dfrac{1}{(\bar{q}^2 - M^2)^2}$

$$\int [d^4q] \frac{1}{(\bar{q}^2 - M^2)^2} \equiv \lim_{\mu \to 0} \int_{\mathrm{R}} d^4q \left(\frac{1}{(\bar{q}^2 - M^2)^2} - \left[\frac{1}{\bar{q}^4} \right] \right) \Bigg|_{\mu \to \mu_R}$$

$\Uparrow$ UV regulator

$\Uparrow$ in the logs

$$= \boxed{\lim_{\mu \to 0} \int d^4q \left(\frac{M^2}{\bar{q}^4(\bar{q}^2 - M^2)} + \frac{M^2}{\bar{q}^4(\bar{q}^2 - M^2)^2} \right) \Bigg|_{\mu \to \mu_R}}$$

1. Dependence on R canceled in the 2^{nd} line by *partial fractioning* $\frac{1}{\bar{q}^2 - M^2} = \frac{1}{\bar{q}^2} \left(1 + \frac{M^2}{\bar{q}^2 - M^2} \right)$
2. μ^2 serves as a provisional IR regulator in $\left[\frac{1}{\bar{q}^4} \right]$
3. FDR integration and normal integration coincide in the case of UV convergent integrands (there is nothing to subtract!)
4. FDR integration is *shift invariant* (easy to see if $\mathrm{R} =$ DReg)

- **Tensors** defined likewise ($\bar{D} \equiv \bar{q}^2 - M^2$ and $|_{\mu \to \mu_R}$ understood)

$$\int [d^4q] \frac{q_\mu q_\nu}{\bar{D}^3} \equiv \lim_{\mu \to 0} \int_{\mathrm{R}} d^4q \left(\frac{q_\mu q_\nu}{\bar{D}^3} - \left[\frac{q_\mu q_\nu}{\bar{q}^6} \right] \right)$$

$$\int [d^4q] \frac{q_\mu q_\nu}{\bar{D}^2} \equiv \lim_{\mu \to 0} \int_{\mathrm{R}} d^4q \left(\frac{q_\mu q_\nu}{\bar{D}^2} - \left[\frac{q_\mu q_\nu}{\bar{q}^4} \right] - 2M^2 \left[\frac{q_\mu q_\nu}{\bar{q}^6} \right] \right)$$

1. *In practice* subtraction terms determined by direct partial fractioning of $1/\bar{D}$ until convergent integrands are reached

$$\frac{q_\mu q_\nu}{\bar{D}^2} = \left[\frac{q_\mu q_\nu}{\bar{q}^4} \right] + 2M^2 \left[\frac{q_\mu q_\nu}{\bar{q}^6} \right] + M^4 \left(\frac{2}{\bar{D}\bar{q}^6} + \frac{1}{\bar{D}^2 \bar{q}^4} \right) q_\mu q_\nu$$

$\Uparrow$ *FDR defining expansion*

2. Divergent integrands depending solely on μ^2 are dubbed *FDR vacua* and are neglected (contain no physical scales!)

$$\boxed{\int [d^4q] \frac{q_\mu q_\nu}{\bar{D}^2} = M^4 \lim_{\mu \to 0} \int d^4q \left(\frac{2}{\bar{D}\bar{q}^6} + \frac{1}{\bar{D}^2 \bar{q}^4} \right) q_\mu q_\nu}$$

- To ensure *gauge invariance* cancellations between numerators and denominators must be preserved (that is the mechanism to prove graphical Slavnov-Taylor identities)

$$\int [d^4q] \frac{\overbrace{q^2 - \mu^2}^{\bar{q}^2} - M^2}{(\bar{q}^2 - M^2)^3} \stackrel{?}{=} \int [d^4q] \frac{1}{(\bar{q}^2 - M^2)^2}$$

- The equality works only if $\mu \to 0$ *after FDR subtracting as if* $\mu^2 = q^2$, which gives rise to *extra-integrals*

$$\int [d^4q] \frac{\mu^2}{(\bar{q}^2 - M^2)^3} = \lim_{\mu \to 0} \int d^4q \; \mu^2 \left(\frac{1}{(\bar{q}^2 - M^2)^3} - \frac{1}{\bar{q}^6} \right) = \frac{i\pi^2}{2}$$

- When q^2 appears in the numerator due to Feynman rules *it should be promoted to* $\bar{q}^2$

$\Uparrow$ *Global Prescription* (GP)

- In practice one always reduces the problem to FDR *Master Integrals* (MIs) to be computed at the end of the calculation

Tensor decomposition is *legal*

$$\int [d^4q] \frac{q_\mu q_\nu}{\bar{D}^3} = g_{\mu\nu} I$$

$$4I = \int [d^4q] \frac{\overbrace{\bar{D} + M^2 + \mu^2}^{q^2}}{\bar{D}^3} = \int [d^4q] \frac{1}{\bar{D}^2} + \int [d^4q] \frac{M^2}{\bar{D}^3} + \overbrace{\int [d^4q] \frac{\mu^2}{\bar{D}^3}}^{\frac{i\pi^2}{2}}$$

IBP is *legal*

$$\int [d^4q] \frac{\partial}{\partial q^\alpha} \frac{q^\alpha}{\bar{D}_0 \bar{D}_1} = 0$$

$$\bar{D}_0 = \bar{q}^2 - m_0^2$$
$$\bar{D}_1 = (q+p)^2 - m_1^2 - \mu^2$$

$$\int [d^4q] \left(\frac{4}{\bar{D}_0 \bar{D}_1} - 2\frac{q^2}{\bar{D}_0^2 \bar{D}_1} - 2\frac{q^2 + (q \cdot p)}{\bar{D}_0 \bar{D}_1^2} \right) = 0$$

- **Two-loop FDR** defining expansion

$$J^{\alpha\beta}(q_1,q_2,\mu^2) = \frac{q_1^\alpha q_1^\beta}{\bar{D}_1^3 \bar{D}_2 \bar{D}_{12}} \quad \begin{cases} \bar{D}_1 = \bar{q}_1^2 - m_1^2 \\ \bar{D}_2 = \bar{q}_2^2 - m_2^2 \\ \bar{D}_{12} = \bar{q}_{12}^2 - m_{12}^2 \quad (q_{12} \equiv q_1 + q_2) \end{cases}$$

$$\begin{aligned} J^{\alpha\beta}(q_1,q_2,\mu^2) &= \Bigg\{ \overbrace{\left[\frac{q_1^\alpha q_1^\beta}{\bar{q}_1^6 \bar{q}_2^2 \bar{q}_{12}^2}\right]}^{\text{Global Vacuum}} + \overbrace{\left(\frac{q_1^\alpha q_1^\beta}{\bar{D}_1^3} - \frac{q_1^\alpha q_1^\beta}{\bar{q}_1^6}\right)\left[\frac{1}{\bar{q}_2^4}\right]}^{\text{Sub-Vacuum}} \\ &\quad - q_1^\alpha q_1^\beta \left(\frac{1}{\bar{D}_1^3} - \frac{1}{\bar{q}_1^6}\right)\frac{q_1^2 + 2(q_1\cdot q_2)}{\bar{q}_2^4 \bar{q}_{12}^2} + \frac{q_1^\alpha q_1^\beta}{\bar{D}_1^3 \bar{q}_2^2 \bar{D}_{12}}\left(\frac{m_2^2}{\bar{D}_2} + \frac{m_{12}^2}{\bar{q}_{12}^2}\right)\Bigg\} \\ &= \left[J_{\rm GV}^{\alpha\beta}(q_1,q_2,\mu^2)\right] + \left[J_{\rm SV}^{\alpha\beta}(q_1,q_2,\mu^2)\right] + J_{\rm F}^{\alpha\beta}(q_1,q_2,\mu^2) \end{aligned}$$

- which leads to

$$\boxed{\int [d^4q_1][d^4q_2] J^{\alpha\beta}(q_1,q_2,\mu^2) = \lim_{\mu\to 0}\int d^4q_1 d^4q_2 J_{\rm F}^{\alpha\beta}(q_1,q_2,\mu^2)}$$

- **Two-loop IBP** is *legal*

$$
\begin{aligned}
0 &= \int [d^4q_1][d^4q_2] \frac{\partial}{\partial q_1^\alpha} \frac{q_1^\alpha q_1^\beta q_1^\gamma}{\bar{D}_1^3 \bar{D}_2 \bar{D}_{12}} \\
&= \int [d^4q_1][d^4q_2]\, q_1^\beta q_1^\gamma \left\{ \frac{6}{\bar{D}_1^3 \bar{D}_2 \bar{D}_{12}} - \frac{6q_1^2}{\bar{D}_1^4 \bar{D}_2 \bar{D}_{12}} - 2\frac{(q_1 \cdot q_{12})}{\bar{D}_1^3 \bar{D}_2 \bar{D}_{12}^2} \right\}
\end{aligned}
$$

- **Two-loop extra-integrals** appear when reducing to MIs

$$
\begin{aligned}
\int [d^4q_1][d^4q_2] \frac{\mu^2|_1}{\bar{D}_1^3 \bar{D}_2 \bar{D}_{12}} &\neq \int [d^4q_1][d^4q_2] \frac{\mu^2|_2}{\bar{D}_1^3 \bar{D}_2 \bar{D}_{12}} \\
&\neq \int [d^4q_1][d^4q_2] \frac{\mu^2|_{12}}{\bar{D}_1^3 \bar{D}_2 \bar{D}_{12}}
\end{aligned}
$$

- The index $_i$ in $\mu^2|_i$ *only denotes the FDR defining expansion to be used* (only one kind of μ^2 exists!)

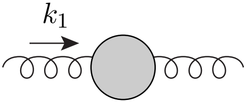
QCD up to two loops in FDR (off-shell)

- Our aim is **deriving coupling constant and quark mass shifts relating FDR to $\overline{\mathrm{MS}}$**
- FDR implies *no UV counterterms* (CTs) in $\mathcal{L}$, but a *"Canonical"* renormalization scheme based on CTs must exist that reproduces FDR correlators $G^{(\ell)}_{\mathrm{FDR}}$ at any loop order ℓ

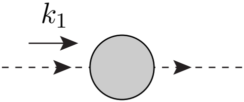
$$
\begin{aligned}
G^{(\ell)}_{\mathrm{renormalized}} &= \overbrace{G^{(\ell)}_{\mathrm{FDR}}}^{\text{computed in FDR}} \\
G^{(\ell)}_{\mathrm{renormalized}} &= \underbrace{G^{(\ell)}_{\mathrm{bare}} + (\ell\text{-loop-CTs}) + \ldots + (1\text{-loop-CTs})}_{\text{computed in DReg}}
\end{aligned}
$$

- We dub such a scheme $\mathrm{D}^{\mathrm{FDR}}_{\mathrm{Reg}}$ and look for its renormalization constants $Z^{\ell,\mathrm{FDR}}$
- The $Z^{\ell,\mathrm{FDR}}$ *relate FDR to* $\overline{\mathrm{MS}}$ (note that in a FDR calculation *there are no* $Z^{\ell,\mathrm{FDR}}$!)

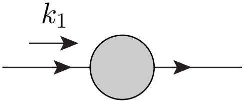
- Off-shell one-particle irreducible QCD Green's functions used in the calculation (computed in $k_1^2 = k_2^2 = k_3^2 = M^2$)



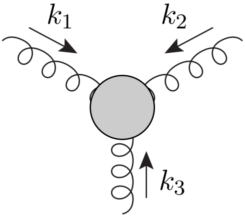
$G_1^{(\ell)} = G_{GG}^{(\ell)}$



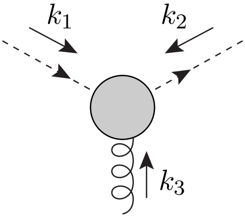
$G_2^{(\ell)} = G_{cc}^{(\ell)}$



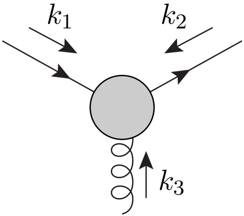
$G_3^{(\ell)} = G_{\Psi\Psi}^{(\ell)}$



$G_4^{(\ell)} = G_{GGG}^{(\ell)}$



$G_5^{(\ell)} = G_{Gcc}^{(\ell)}$



$G_6^{(\ell)} = G_{G\Psi\Psi}^{(\ell)}$

- **Strategy at 1 loop:** $\begin{cases} n = 4-2\epsilon \\ n_s = \gamma^\alpha\gamma_\alpha = g_{\alpha\beta}g^{\alpha\beta} = 4-2\lambda\epsilon \end{cases}$

$$\underbrace{\int d^nq\, J(q)}_{G^{(1)}_{\rm bare}} = \int d^nq \lim_{\mu\to 0} J(q,\mu^2) = \underbrace{\lim_{\mu\to 0}\int d^nq\, J(q,\mu^2)}_{\text{due to IR finiteness}}$$

$$= \underbrace{\lim_{\mu\to 0}\int d^4q\, J_{\rm F}(q,\mu^2)}_{G^{(1)}_{\rm FDR} = G^{(1)}_{\rm bare} + \text{(1-loop-CTs)}} + \lim_{\mu\to 0}\int d^nq\, [J_{\rm V}(q,\mu^2)]$$

$$\boxed{\text{(1-loop-CTs)} = -\lim_{\mu\to 0}\int d^nq\,[J_{\rm V}(q,\mu^2)]} = G^{(0)}\, C_1(n_s)\left(\frac{1}{\epsilon} - \gamma_E - \ln\pi\right)$$

- $C_1(n_s)$ is *independent of kinematics* so that, due to $\lambda = 1$, *universal constants* appear in $\overline{\rm MS}$ that contribute to the finite part but are fully subtracted in ${\rm D}^{\rm FDR}_{\rm Reg}$ $\Rightarrow$ $\boxed{Z^{1,\rm FDR} = Z^{1,\rm FDH}}$

- **Two loops:** $$\underbrace{\int d^nq_1 d^nq_2\, J(q_1,q_2)}_{G^{(2)}_{\rm bare}} = \lim_{\mu\to 0}\int d^nq_1 d^nq_2\, J(q_1,q_2,\mu^2)$$

$$= \underbrace{\lim_{\mu\to 0}\int d^4q_1 d^4q_2\, J_{\rm F}(q_1,q_2,\mu^2)}_{G^{(2)}_{\rm FDR} = G^{(2)}_{\rm bare} + \text{(1-loop-CTs)} + \text{(2-loop-CTs)}}$$

$$+ \lim_{\mu\to 0}\int d^nq_1 d^nq_2\, \left([J_{\rm GV}(q_1,q_2,\mu^2)] + [J_{\rm SV}(q_1,q_2,\mu^2)]\right)$$

$$\boxed{\text{(2-loop-CTs)} = -\text{(1-loop-CTs)} - \lim_{\mu\to 0}\int d^nq_1 d^nq_2\, \left([J_{\rm GV}](n_s) + [J_{\rm SV}](n_s)\right)}$$

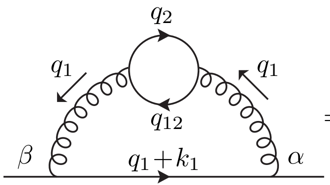
- $$\underbrace{\boxed{\text{(2-loop-CTs)} \stackrel{?}{=} G^{(0)}\, C_2}}_{\textit{Renormalizability Condition}\ (\mathrm{RC})}$$ with *no kinematics* in C_2
- Kinematics dependent part of $[J_{\rm SV}]$ *should cancel* (1-loop-CTs)!

- An explicit calculation shows that this cancellation takes place and RC fulfilled *for all correlators without external quarks*, but, *with external quarks*, $n_s \neq 4$ in $[J_{\rm SV}](n_s)$ generates extra kinematics dependent (non local!) terms *incompatible with* $\rm RC$

- **The cure:** making compatible in any diagram two-loop GP with GP at the level of each sub-diagram (*sub-prescription*)

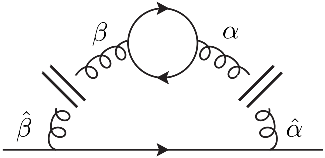
in an ℓ-loop diagram, one should be able to calculate a sub-diagram, insert the integrated form into the full diagram and get the same answer

Sub-integration consistency!



$$= \int [d^4q_1][d^4q_2] \frac{N(q_1,q_2)}{\bar{q}_1^4 \bar{q}_2^2 \bar{q}_{12}^2 (\bar{q}_1^2 + k_1^2 + 2(q_1\cdot k_1))}$$

No GP in the numerator $N(q_1,q_2)$ yet



Lorentz indices *external* to the the sub-diagram are given a hat

1.
two-loop GP: $N(q_1, q_2) \to N(q_1, q_2)$ *$i)$*
sub-prescription: $N(q_1, q_2) \to N(q_1, q_2) - 8(\not{q}_1 + \not{k}_1)\mu^2|_2$ *$ii)$*

2. To correct this mismatch one adds to the diagram *$ii) - i)$*:

$$EEI = -8 \int [d^4q_1][d^4q_2] \frac{\hat{\mu}^2|_2(\not{q}_1 + \not{k}_1)}{\bar{q}_1^4 \bar{q}_2^2 \bar{q}_{12}^2 (\bar{q}_1^2 + k_1^2 + 2(q_1 \cdot k_1))}$$

with $\hat{\mu}^2|_2$ *acting only on the q_2 sub-integral*

$$\begin{aligned} EEI &= \frac{2}{3} i\pi^2 \not{k}_1 \int [d^4q] \frac{1}{\bar{q}^2(\bar{q}^2 + k_1^2 + 2(q \cdot k_1))} = \overbrace{EEI_b}^{\text{depends on kinematics}} + EEI_a \\ &= \frac{2}{3} i\pi^2 \not{k}_1 \left(\int d^nq \frac{1}{q^2(q^2 + k_1^2 + 2(q \cdot k_1))} - \lim_{\mu \to 0} \int d^nq \frac{1}{\bar{q}^4} \right) \end{aligned}$$

- Including $EEIs$ in the calculation *restores RC at two-loops*
- $\sum_{\text{Diag}} EEI = 0$ in correlators without external quarks

- **Results:** α_S and m_q shifts up to two loops

$$\frac{Z_{\alpha_S}^{\overline{\mathsf{MS}}}}{Z_{\alpha_S}^{\mathrm{FDR}}} = \frac{\alpha_S^{\mathrm{FDR}}}{\alpha_S^{\overline{\mathsf{MS}}}} = 1 + \left(\frac{\alpha_S^{\overline{\mathsf{MS}}}}{4\pi}\right)\frac{N_c}{3} + \left(\frac{\alpha_S^{\overline{\mathsf{MS}}}}{4\pi}\right)^2 \left\{\frac{89}{18}N_c^2 + 8N_c^2 f + N_f\left[N_c - \frac{3}{2}C_F - f\left(\frac{2}{3}N_c + \frac{4}{3}C_F\right)\right]\right\}$$

$$\frac{Z_{m}^{\overline{\mathsf{MS}}}}{Z_{m}^{\mathrm{FDR}}} = \frac{m_q^{\mathrm{FDR}}}{m_q^{\overline{\mathsf{MS}}}} = 1 - C_F\left(\frac{\alpha_S^{\overline{\mathsf{MS}}}}{4\pi}\right) + C_F\left(\frac{\alpha_S^{\overline{\mathsf{MS}}}}{4\pi}\right)^2 \left\{\frac{77}{24}N_c - \frac{5}{8}C_F + f\left(9N_c + \frac{11}{3}C_F\right) + N_f\left(\frac{1}{4} - \frac{2}{3}f\right)\right\}$$

$$f = \frac{i}{\sqrt{3}}\left(\mathrm{Li}_2(e^{i\frac{\pi}{3}}) - \mathrm{Li}_2(e^{-i\frac{\pi}{3}})\right) = -1.17195361\ldots$$

- With $\alpha_S^{\mathrm{FDR}} = \alpha_S^{\mathrm{FDR}}|_{GGG} = \alpha_S^{\mathrm{FDR}}|_{Gcc} = \alpha_S^{\mathrm{FDR}}|_{G\Psi\Psi}$ (universality!)

- **Fixing two-loop FDH without evanescent quantities**

  By changing the bare two-loop FDH correlators as follows

$$\boxed{G^{(2)}_{\text{bare}}|_{n_s=4} \to G^{(2)}_{\text{bare}}|_{n_s=4} + \sum_{\text{Diag}} EEI_b|_{n_s=4}}$$

  the RC is *fulfilled*. We dub this scheme FDH′

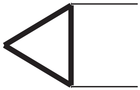
- $\alpha_S^{\text{FDH}'}$ is *universal* and $\boxed{\alpha_S^{\text{FDH}'} = \alpha_S^{\text{FDH}}\Big|_{GGG} = \alpha_S^{\text{FDH}}\Big|_{Gcc}}$
- Furthermore a quark mass shift is computable up to two loops

$$\boxed{\frac{m_q^{\text{FDH}'}}{m_q^{\overline{\text{MS}}}} = 1 - C_F\left(\frac{\alpha_S^{\overline{\text{MS}}}}{4\pi}\right) + C_F\left(\frac{\alpha_S^{\overline{\text{MS}}}}{4\pi}\right)^2\left\{\frac{29}{12}N_c - \frac{13}{2}C_F + \frac{1}{4}N_f\right\}}$$

- In DRed evanescent couplings are introduced in $\mathcal{L}$, while in FDH′ the EEI_b RC restoring terms are *directly read off from two-loop diagrams* (keeping $n_s = 4$ spin degrees of freedom!)

FDR treatment of IR infinities

- Adding μ^2 to propagators regulate **virtual** IR divergences



$$= \int [d^4q] \frac{1}{\bar{q}^2 \bar{D}_1 \bar{D}_2} \equiv \lim_{\mu \to 0} \int d^4q \frac{1}{\bar{q}^2 \bar{D}_1 \bar{D}_2}$$

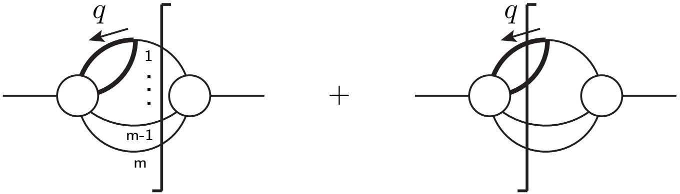
- **Real** matched via *cutting rules* $\boxed{\frac{i}{q^2 + i\varepsilon} \to (2\pi)\,\delta_+(q^2)}$

$$\left(\frac{i}{q^2 + i\varepsilon} = (2\pi)\,\delta_+(q^2) + \frac{i}{q^2 - i\varepsilon q_0} \right)$$

- When $q^2 \to \bar{q}^2 = q^2 - \mu^2$ $\boxed{\frac{i}{\bar{q}^2 + i\varepsilon} \to (2\pi)\,\delta_+(\bar{q}^2)}$

$$\left(\frac{i}{\bar{q}^2 + i\varepsilon} = (2\pi)\,\delta_+(\bar{q}^2) + \frac{i}{\bar{q}^2 - i\varepsilon q_0} \right)$$

- σ_{NLO} m-body virtual ($\sigma^{\mathrm{V}}_{\mathrm{NLO}}$) and $(m+1)$-body real ($\sigma^{\mathrm{R}}_{\mathrm{NLO}}$) contributions in which IR divergences compensate each other



- Splitting regulated by *μ-massive unobserved particles*

- The problem is changing $s_{ij}=(p_i+p_j)^2, \quad p^2_{i,j}=0$ to
$$\bar{s}_{ij}=(\bar{p}_i+\bar{p}_j)^2, \quad \bar{p}^2_{i,j}=\mu^2\to 0$$
in $\sigma^{\mathrm{R}}_{\mathrm{NLO}}$ *in a gauge invariant way*

- Easiest way $\boxed{\bar{\Phi}_{m+1} \overset{\text{mapping}}{\rightarrow} \Phi_{m+1}}$

$$\sigma^{\mathsf{R}}_{\mathsf{NLO}} = \lim_{\mu \to 0} \int_{\bar{\Phi}_{m+1}} \underbrace{d\sigma^{\mathsf{R}}_{\mathsf{NLO}}(\Phi_{m+1})}_{\text{gauge invariant!}} \prod_{i<j} \frac{s_{ij}}{\bar{s}_{ij}}$$

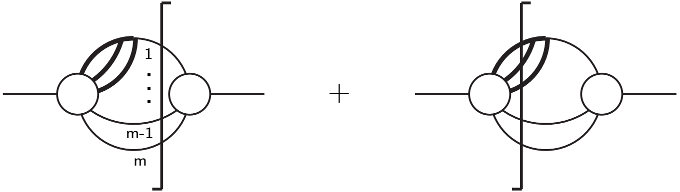
(explicitly checked with $H \to gg(g)$ at NLO)

- Based on

$$d\sigma^{\mathsf{R}}_{\mathsf{NLO}}(\Phi_{m+1}) \sim \frac{1}{s_{ij}} \quad \text{if} \quad s_{ij} \to 0$$

$\frac{s_{ij}}{\bar{s}_{ij}}$ changes IR pole $\frac{1}{s_{ij}}$ to $\frac{1}{\bar{s}_{ij}}$ when $s_{ij} \to 0$ and, due to $\mu \to 0$, *is harmless in all the other kinematical configurations*

- **NNLO ansatz:** cancellation of double unresolved singularities



- $$\boxed{\sigma = \sigma_{\text{LO}} + \sigma_{\text{NLO}} + \sigma_{\text{NNLO}}}$$

$$\sigma_{\text{LO}} = \int_{\Phi_m} d\sigma^{\text{B}}_{\text{LO}}(\Phi_m)$$

$$\sigma_{\text{NLO}} = \int_{\Phi_m} d\sigma^{\text{V}}_{\text{NLO}}(\Phi_m) + \lim_{\mu\to 0} \int_{\bar{\Phi}_{m+1}} d\sigma^{\text{R}}_{\text{NLO}}(\Phi_{m+1}) \prod_{i<j} \frac{s_{ij}}{\bar{s}_{ij}}$$

$$\sigma_{\text{NNLO}} = \int_{\Phi_m} d\sigma^{\text{VV}}_{\text{NNLO}}(\Phi_m) + \lim_{\mu\to 0} \int_{\bar{\Phi}_{m+1}} d\sigma^{\text{VR}}_{\text{NNLO}}(\Phi_{m+1}) \prod_{i<j} \frac{s_{ij}}{\bar{s}_{ij}}$$

$$+ \boxed{\lim_{\mu\to 0} \int_{\bar{\Phi}_{m+2}} d\sigma^{\text{RR}}_{\text{NNLO}}(\Phi_{m+2}) \prod_{i<j} \frac{s_{ij}}{\bar{s}_{ij}} \prod_{i<j<k} \left(\frac{s_{ijk}}{\bar{s}_{ijk}}\right)^2}$$

- Based on

$$d\sigma^{\rm RR}_{\rm NNLO}(\Phi_{m+2}) \sim \left\{ \overbrace{\begin{array}{ll} 1/s_{ij} & \text{if } s_{ij} \to 0 \\ & \\ 1/s^2_{ijk} & \text{if } s_{ijk} \to 0 \end{array}}^{\text{non integrable singularities}} \right.$$

- **To do IR list**

1. ISR
2. To prove NNLO ansatz in a simple case
3. Local cancellation of IR divergences

   Proved at NLO in $H \to gg(g)$ by using

$$\int_{\Phi_2} \Re \left(\int [d^4q] \frac{1}{\bar{q}^2 \bar{D}_1 \bar{D}_2} \right) = \int_{\bar{\Phi}_3} \frac{1}{\bar{s}_{13}\bar{s}_{23}}$$

   One drops $\frac{1}{\bar{q}^2 \bar{D}_1 \bar{D}_2}$ from $d\sigma^{\rm V}$ and corrects $d\sigma^{\rm R}$ by adding $\frac{1}{\bar{s}_{13}\bar{s}_{23}}$, which acts as a *local counterterm*

4. *Final goal:* efficient local NNLO subtraction algorithm

Conclusions

1. I have linked the FDR treatment of UV divergences to dimensional regularization up to two loops in QCD
2. This has allowed me to derive the one-loop and two-loop coupling constant and quark mass shifts necessary to translate infrared finite quantities computed in FDR to the MS renormalization scheme
3. As a by-product of this analysis I have presented a fix to FDH beyond one loop that preserves the renormalizability properties of QCD without introducing evanescent quantities
4. Finally, I have commented on the treatment of IR infinities within the FDR framework

Thanks!

Backup slides

When necessary, $\mu \to 0$ and $\big|_{\mu \to \mu_R}$ possible *at the integrand level:*

$$\int [d^4q] \frac{1}{(\bar{q}^2 - M^2)^2} = \int_{\mathrm{R}} d^4q \left(\frac{1}{(q^2 - M^2)^2} - \left[\frac{1}{(q^2 - \mu_R^2)^2} \right] \right)$$