2- and 3-Loop Heavy Flavor Corrections to Transversity

B.Tödtli, DESY

in Collaboration with J. Blümlein and S. Klein

- Introduction
- Heavy Flavor Wilson Coefficients
- Massive Operator Matrix Elements at $O\left(a_s^2\right)$ and $O(a_s^3)$
- Conclusions

1. Introduction

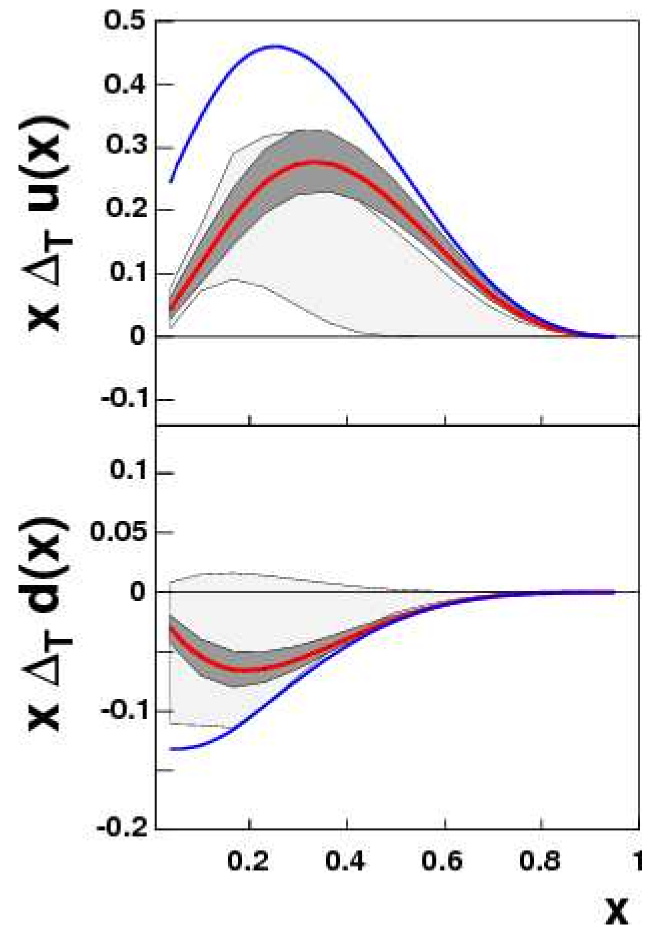
- Twist-2 distribution functions:

$$\begin{array}{ll} \text{unpol.} & f(x,Q^2) \equiv f^{\rightarrow}(x,Q^2) + f^{\leftarrow}(x,Q^2) \\ \text{long. pol.} & \Delta f(x,Q^2) \equiv f^{\rightarrow}(x,Q^2) - f^{\leftarrow}(x,Q^2) \\ \text{trans. pol.} & \Delta_T f(x,Q^2) \equiv f^{\uparrow}(x,Q^2) - f^{\downarrow}(x,Q^2) \end{array}$$

- Transversity is a leading twist quantity: twist-2, non-singlet

- Some results from lattice simulations:

Ref.	$\Delta_T u - \Delta_T d$		$\Delta_T u$		$\Delta_T d$	
	$N=1$	$N=2$	$N=1$	$N=2$	$N=1$	$N=2$
QCDSF (2005)	1.165(67)		+0.963(59)		-0.202(36)	
Detmold et al. 2002	1.187	0.490				
QCDSF	1.068(16)	0.322(6)	+0.857(13)	+0.268(6)	-0.212(5)	-0.052(2)
Orginos et al. (2006)	1.192(30)					
Aoki (1996)	1.124(59)		+0.893(22)		-0.231(55)	



[Anselmino et al., 0807.0173 (hep-ph)]

Observables

- Transversity is not observable in fully inclusive DIS
- We consider twist-2 level only
  - + higher order calculations in the QCD improved parton model are possible
  - − set of possible observables is reduced: No $k_\perp$-dependence
- Semi-inclusive DIS: A hadron h is detected in the final state:

$$\ell + N \to \ell' + h + X$$

$$\frac{d^3\Delta_T\sigma}{dxdydz} = -\frac{4\pi\alpha_{\rm em}^2 s}{Q^4}\sum_{a=q,\bar{q}} e_a^2 x\left\{(1-y)|\mathbf{S}_\perp||\mathbf{S}_{h\perp}|\cos\left(\phi_S+\phi_{S_h}\right)\mathsf{C}_a^{\mathsf{NS,TR}}\left(x,\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right)\right.$$
$$\left.\otimes\,\Delta_T f_a(x,Q^2)\Delta_T D_a(z,Q^2)\right\}$$

- Heavy flavor can be tagged in the final state

Observables

- Transversity can also be measured in the transversely polarized Drell-Yan process

$$h^{\uparrow}\, h^{\uparrow\downarrow} \to \ell^{+}\, \ell^{-} + X$$

$$\frac{\tau d\Delta_T \sigma}{d\tau d\phi} = \frac{1}{2}\left[\frac{\tau d\sigma^{\uparrow\uparrow}}{d\tau d\phi} - \frac{\tau d\sigma^{\uparrow\downarrow}}{d\tau d\phi}\right]$$

$$\frac{d\Delta_T \sigma^N}{d\phi} = \frac{\alpha_{\rm em}^2}{9s}\cos(2\phi)\Delta_T H(N, M^2)\cdot \Delta_T \mathsf{C}_{\mathsf{q}}^{\rm DY}(\mathsf{N}, \mathsf{M}^2)$$

$$\Delta_T H(N, Q^2) = \sum_q e_q^2 \left[\Delta_T q_1(N, Q^2)\Delta_T \overline{q}_2(N, Q^2) + \Delta_T \overline{q}_1(N, Q^2)\Delta_T q_2(N, Q^2)\right]$$

- Drell-Yan: The heavy quarks contributions affect the initial state only

2. Heavy Flavor Wilson Coefficients

- Heavy Flavor contribution to the Wilson coefficient:

$$\mathsf{C}_q^{\rm TR}\left(x, \frac{Q^2}{\mu^2}\right) = C_q^{\rm TR, light}\left(x, \frac{Q^2}{\mu^2}\right) + H_q^{\rm TR}\left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right)$$

- For $Q^2/m^2 \gtrsim 10$: Factorization of the heavy flavor Wilson coefficient into light flavor Wilson coefficient and heavy flavor operator matrix element:[Van Neerven et al. 1995]

$$H_q^{\rm TR}\left(x, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}\right) = C_q^{\rm TR}\left(x, \frac{Q^2}{\mu^2}\right) \otimes \underbrace{A_{qq,Q}^{\rm TR}\left(x, \frac{m^2}{\mu^2}\right)}_{\text{process independent}}$$

- In detail:

$$\begin{aligned} H_q^{\mathsf{TR}}(n_f) \quad = \quad & a_s^2 \left[A_{qq,Q}^{\mathsf{TR},(2)}(n_f) + \hat{C}_q^{\mathsf{TR},(2)}(n_f+1)\right] \\ & + a_s^3 \left[A_{qq,Q}^{\mathsf{TR},(3)}(n_f) + A_{qq,Q}^{\mathsf{TR},(2)}(n_f) C_q^{\mathsf{TR},(1)} + \hat{C}_q^{\mathsf{TR},(3)}(n_f+1)\right] \end{aligned}$$

3. Massive Operator Matrix Elements at $O\left(a_s^2\right)$ and $O\left(a_s^3\right)$

- Transversity Operator and Operator matrix element

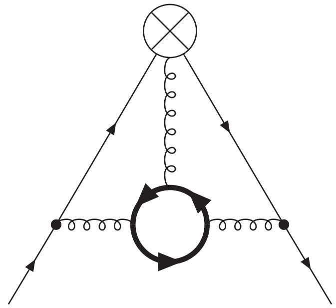
$$\begin{aligned} O^{\mathsf{NS,T}}_{F,a;\mu\mu_1\ldots\mu_n} &= i^n \mathbf{S}\left[\overline{\psi}\gamma_5\sigma_{\mu\mu_1}D_{\mu_2}\ldots D_{\mu_n}\frac{\lambda_a}{2}\psi\right] - \text{trace terms} \\ A_{qq,Q} &= \langle q|O^{\rm NS,T}|q\rangle \end{aligned}$$

- Un-renormalized operator matrix elements: $\hat{\hat{A}} = a_s^2 \hat{\hat{A}}^{(2)} + a_s^3 \hat{\hat{A}}^{(3)} + O\left(a_s^4\right)$

[Bierenbaum, Blümlein, Klein,DESY 09-057]

$$\begin{aligned} \hat{\hat{A}}^{(2),\mathsf{NS}}_{qq,Q} &= \left(\frac{\hat{m}^2}{\mu^2}\right)^{\varepsilon}\left(\frac{\beta_{0,Q}\gamma_{qq}^{(0)}}{\varepsilon^2} + \frac{\hat{\gamma}_{qq}^{(1),\mathsf{NS}}}{2\varepsilon} + a_{qq,Q}^{(2),\mathsf{NS}} + \overline{a}_{qq,Q}^{(2),\mathsf{NS}}\varepsilon\right) \\ \hat{\hat{A}}^{(3),\mathsf{NS}}_{qq,Q} &= \left(\frac{\hat{m}^2}{\mu^2}\right)^{3\varepsilon/2}\Biggl\{-\frac{4\gamma_{qq}^{(0)}\beta_{0,Q}}{3\varepsilon^3}\left(\beta_0 + 2\beta_{0,Q}\right) + \frac{1}{\varepsilon^2}\Biggl(\frac{2\gamma_{qq}^{(1),\mathsf{NS}}\beta_{0,Q}}{3} - \frac{4\hat{\gamma}_{qq}^{(1),\mathsf{NS}}}{3}\left[\beta_0 + \beta_{0,Q}\right] \\ &\quad + \frac{2\beta_{1,Q}\gamma_{qq}^{(0)}}{3} - 2\delta m_1^{(-1)}\beta_{0,Q}\gamma_{qq}^{(0)}\Biggr) + \frac{1}{\varepsilon}\Biggl(\frac{\hat{\gamma}_{qq}^{(2),\mathsf{NS}}}{3} - 4a_{qq,Q}^{(2),\mathsf{NS}}\left[\beta_0 + \beta_{0,Q}\right] + \beta_{1,Q}^{(1)}\gamma_{qq}^{(0)} \\ &\quad + \frac{\gamma_{qq}^{(0)}\beta_0\beta_{0,Q}\zeta_2}{2} - 2\delta m_1^{(0)}\beta_{0,Q}\gamma_{qq}^{(0)} - \delta m_1^{(-1)}\hat{\gamma}_{qq}^{(1),\mathsf{NS}}\Biggr) + a_{qq,Q}^{(3),\mathsf{NS}}\Biggr\} \end{aligned}$$

- 129 3-loop Feynman graphs with an operator insertion
  We used MATAD for the 3-loop calculation [Steinhauser 2001]



Renormalized Operator matrix elements:

- We use $\overline{\text{MS}}$-scheme except for mass renormalization (on-mass-shell scheme)
  $\rightarrow$ all ultra-violet and collinear singularities removed

$$A_{qq,Q}^{(2),\mathsf{NS},\overline{\mathsf{MS}}} = \frac{\beta_{0,Q}\gamma_{qq}^{(0)}}{4}\ln^2\left(\frac{m^2}{\mu^2}\right) + \frac{\hat{\gamma}_{qq}^{(1),\mathsf{NS}}}{2}\ln\left(\frac{m^2}{\mu^2}\right) + a_{qq,Q}^{(2),\mathsf{NS}} - \frac{\beta_{0,Q}\gamma_{qq}^{(0)}}{4}\zeta_2$$

$$\begin{aligned} A_{qq,Q}^{(3),\mathsf{NS},\overline{\mathsf{MS}}} = & -\frac{\gamma_{qq}^{(0)}\beta_{0,Q}}{6}\Big(\beta_0 + 2\beta_{0,Q}\Big)\ln^3\left(\frac{m^2}{\mu^2}\right) + \frac{1}{4}\Big\{2\gamma_{qq}^{(1),\mathsf{NS}}\beta_{0,Q} - 2\hat{\gamma}_{qq}^{(1),\mathsf{NS}}\Big(\beta_0 + \beta_{0,Q}\Big) \\ & +\beta_{1,Q}\gamma_{qq}^{(0)}\Big\}\ln^2\left(\frac{m^2}{\mu^2}\right) + \frac{1}{2}\Big\{\hat{\gamma}_{qq}^{(2),\mathsf{NS}} - \Big(4a_{qq,Q}^{(2),\mathsf{NS}} - \zeta_2\beta_{0,Q}\gamma_{qq}^{(0)}\Big)(\beta_0 + \beta_{0,Q}) \\ & +\gamma_{qq}^{(0)}\beta_{1,Q}^{(1)}\Big\}\ln\left(\frac{m^2}{\mu^2}\right) + 4\overline{a}_{qq,Q}^{(2),\mathsf{NS}}(\beta_0 + \beta_{0,Q}) - \gamma_{qq}^{(0)}\beta_{1,Q}^{(2)} - \frac{\gamma_{qq}^{(0)}\beta_0\beta_{0,Q}\zeta_3}{6} \\ & -\frac{\gamma_{qq}^{(1),\mathsf{NS}}\beta_{0,Q}\zeta_2}{4} + 2\delta m_1^{(1)}\beta_{0,Q}\gamma_{qq}^{(0)} + \delta m_1^{(0)}\hat{\gamma}_{qq}^{(1),\mathsf{NS}} + 2\delta m_1^{(-1)}a_{qq,Q}^{(2),\mathsf{NS}} \\ & + a_{qq,Q}^{(3),\mathsf{NS}} \end{aligned}$$

Transversity Anomalous Dimension

- One-loop anomalous dimension [Baldracchini et al. (1981)]:

$$\gamma_{qq}^{\mathrm{TR},(0)}(N) = 2C_F\left[-3+4S_1(N)\right]$$

- Two-loop anomalous dimension [Hayashigaki et al., 1997, hep-ph/9707208, Kumano et al., 1997, hep-ph/9706420, Vogelsang 1998, hep-ph/9706511]:

$$\begin{aligned}\gamma_{qq}^{\mathrm{TR},(1)}(N) &= 4C_F^2\left[S_2(N)-2S_1(N)-\frac{1}{4}\right]+C_FC_G\left[-16S_1(N)S_2(N)-\frac{58}{3}S_2(N)+\frac{572}{9}S_1(N)-\frac{20}{3}\right]\\ &-8\left(C_F^2-\frac{1}{2}C_FC_G\right)\left[4S_1(N)\left\{S_2'\left(\frac{N}{2}\right)-S_2(N)-\frac{1}{4}\right\}-8\widetilde{S}(N)+S_3'\left(\frac{N}{2}\right)-\frac{5}{2}S_2(N)+\frac{(1-(-1)^N)}{(N)(N+1)}+\frac{1}{4}\right]\\ &+\frac{32}{9}C_FT_R\left[3S_2(N)-5S_1(N)+\frac{3}{8}\right]\end{aligned}$$

- Three-loop moments $N = 1\ldots8$ [Gracey, 2003, hep-ph/0304113, hep-ph/0306163; Gracey 2006, hep-ph/0609231, hep-ph/0611071]: e.g.

$$\begin{aligned}\gamma_{qq}^{\mathrm{TR},(2)}(N=8) = &-\left(\frac{2324068794763}{68068350000}+\frac{43153}{1225}\zeta_3\right)C_F^3\\ &-\left(\frac{350888781989C_A}{5186160000}+\frac{129459\,(C_A-C_F)}{1225}\zeta_3\right)C_F^2+\left(\frac{177184521133}{444528000}C_F+\frac{43153}{1225}\zeta_3C_F\right)C_A^2\\ &-C_FT_F\left\{\frac{711801943}{41674500}n_f^2T_F+n_f\left[\left(\frac{6056338297}{66679200}C_A+\frac{8816}{35}(C_A-C_F)\zeta_3\right)+\frac{849420853541}{3889620000}C_F\right]\right\}\end{aligned}$$

- Harmonic sums [Blümlein et al. 1998; Vermaseren 1998]:

$$S_{b,\vec{a}}(N) = \sum_{k=1}^{N}\frac{(\mathrm{sign}\,(b))^k}{k^{|b|}}S_{\vec{a}}(k)$$

Massive OMEs at $O\left(a_s^2\right)$ and $O\left(a_s^3\right)$

- At $O\left(a_s^2\right)$, the calculation yields

$$\text{1-loop anom. dim.} \quad \gamma_{qq}^{(0),\mathrm{TR}}(N) = 2C_F\left[-3+4S_1(N)\right]$$

$$\text{2-loop anom. dim.} \quad \hat{\gamma}_{qq}^{(1),\mathrm{TR}}(N) = \tfrac{32}{9}C_FT_F\left[3S_2(N)-5S_1(N)+\tfrac{3}{8}\right]$$

are obtained from the Pole parts of the massive operator matrix element

- We also obtain the finite parts to all N:

Two-loop finite part:

$$a_{qq,Q}^{\mathrm{TR},(2)}(N) \quad = \quad C_FT_F\left\{-\frac{8}{3}S_3(N)+\frac{40}{9}S_2(N)-\left[\frac{224}{27}+\frac{8}{3}\zeta_2\right]S_1(N)+2\zeta_2+\frac{\left(24+73\,N+73\,N^2\right)}{18N\left(1+N\right)}\right\}$$

Two-loop $O\left(\varepsilon\right)$ part:

$$\overline{a}_{qq,Q}^{\mathrm{TR},(2)}(N) = C_FT_F\left\{-\left[\frac{656}{81}+\frac{20}{9}\zeta_2+\frac{8}{9}\zeta_3\right]S_1(N)+\left[\frac{112}{27}+\frac{4}{3}\zeta_2\right]S_2(N)-\frac{20}{9}S_3(N)\right.$$
$$\left.+\frac{4}{3}S_4(N)+\frac{1}{6}\zeta_2+\frac{2}{3}\zeta_3+\frac{1}{216}\frac{\left(-144-48\,N+757\,N^2+1034\,N^3+517\,N^4\right)}{N^2\left(1+N\right)^2}\right\}$$

- At $O\left(a_s^3\right)$, we obtain for the moments $N = 1, \ldots, 13$ the anomalous dimension

$$
\begin{aligned}
\hat{\gamma}_{qq}^{(3),\mathrm{TR}}(13) &= C_F T_F \Biggl[-\frac{383379490933459}{19469534584500} T_F(1+2n_f) - \frac{38283693844132279}{389390691690000} C_A \\
&\quad -\frac{1237841854306528417}{4612782040020000} C_F + \frac{1043696}{3465} (C_F - C_A)\zeta_3\Biggr]
\end{aligned}
$$

and the finite part

$$
\begin{aligned}
a_{qq,Q}^{(3),\mathrm{TR}}(13) =& C_F T_F \Biggl\{ -\Biggl[\frac{1245167831299024242467303}{7114266063630605880000} + \frac{93611152819}{7304587290} \zeta_2 - \frac{7005784}{173745} \zeta_3\Biggr] n_f T_F \\
& - \Biggl[\frac{19689788786597173029530 3}{3557133031815302940000} + \frac{93611152819}{3652293645} \zeta_2 + \frac{112092544}{1216215} \zeta_3\Biggr] T_F \\
& + \Biggl[-\frac{4306332196155232788830 51}{64675146033005508000000} + \frac{15314434459241}{1460917458000} \zeta_2 \\
& \qquad + \frac{327241423}{935550} \zeta_3 - \frac{3502892}{15015} \zeta_4 + \frac{3502892}{135135} B_4\Biggr] C_A \\
& + \Biggl[\frac{7068044558560857730886158289 3}{12208080565190119690080000 0} + \frac{449066258795623169}{4387135126374000} \zeta_2 \\
& \qquad - \frac{81735983092}{243486243} \zeta_3 + \frac{3502892}{15015} \zeta_4 - \frac{7005784}{135135} B_4\Biggr] C_F \Biggr\}
\end{aligned}
$$

- Note the constant ζ_2
  $B_4 = -4\zeta_2 \ln^2(2) + \frac{2}{3} \ln^4(2) - \frac{13}{2}\zeta_4 + 16\,\mathrm{Li}_4\left(\frac{1}{2}\right)$

- We obtain the renormalized operator matrix elements as e.g.

$$
\begin{aligned}
A_{qq,Q}^{(3),\mathrm{T_R}}(N=13) = C_F T_F \Bigg\{ & \left[\frac{130462}{8505} C_A + \left(-\frac{521848}{93555} n_f + \frac{521848}{93555}\right) T_F\right] \ln^3\left(\frac{m^2}{\mu^2}\right) \\
& + \left[-\frac{72872472641}{4322241000} C_A + \frac{535118971}{21611205} T_F - \frac{8747715742963}{998437671000} C_F\right] \ln^2\left(\frac{m^2}{\mu^2}\right) \\
& + \left[\left(\frac{9502263527285869}{194695345845000} - \frac{521848}{3465}\zeta_3\right) C_A + \left(\frac{521848}{3465}\zeta_3 + \frac{64739900033491583}{9225564080040000}\right) C_F \right. \\
& \quad \left. - \left(\frac{2627793779850259}{38939069169000} n_f + \frac{383379490933459}{38939069169000}\right) T_F\right] \ln\left(\frac{m^2}{\mu^2}\right) \\
& + \left[\frac{260924}{10395} B_4 - \frac{1893582238878051016363}{765386343585864000000} - \frac{260924}{1155}\zeta_4 + \frac{3784910159}{12162150}\zeta_3\right] C_A \\
& + \left[\left(-\frac{160323682499449516 6513}{21048124448611260000} + \frac{4174784}{93555}\zeta_3\right) n_f - \frac{1043696}{13365}\zeta_3 + \frac{1671087052774352737487}{420962488972225200000}\right] T_F \\
& + \left[-\frac{468587596}{1440747}\zeta_3 - \frac{521848}{10395} B_4 + \frac{18774218738469536610 63773}{42743883495641328000000} + \frac{260924}{1155}\zeta_4\right] C_F \Bigg\}
\end{aligned}
$$

- ζ_2 is not present.

- Comparison with Gracey:

$$
\begin{aligned}
\gamma_{qq}^{\mathrm{TR},(2)}(N=8) = \\
& -\left(\frac{2324068794763}{68068350000} + \frac{43153}{1225}\zeta_3\right) C_F^3 \\
& -\left(\frac{350888781989 C_A}{5186160000} + \frac{129459\,(C_A - C_F)}{1225}\zeta_3\right) C_F^2 \\
& +\left(\frac{177184521133}{444528000} C_F + \frac{43153}{1225}\zeta_3 C_F\right) C_A^2 \\
& - C_F T_F \Bigg\{ \frac{711801943}{41674500} n_f^2 T_F \\
& \qquad + n_f \Bigg[\left(\frac{6056338297}{66679200} C_A + \frac{8816}{35}(C_A - C_F)\zeta_3\right) \\
& \qquad\qquad + \frac{849420853541}{3889620000} C_F \Bigg] \Bigg\}
\end{aligned}
$$

- From the Pole terms of $\hat{\hat{A}}_{qq,Q}^{(3),\mathrm{TR}}$ extract $\hat{\gamma}_{qq}^{\mathrm{TR},(2)}(n_f) = \gamma_{qq}^{\mathrm{TR},(2)}(n_f+1) - \gamma_{qq}^{\mathrm{TR},(2)}(n_f)$

$$
\begin{aligned}
\hat{\gamma}_{qq}^{\mathrm{TR},(2)}(N=8) = -C_F T_F \Bigg\{ & \frac{711801943}{41674500}(2n_f+1)\, T_F \\
& + \frac{6056338297}{66679200} C_A + \frac{8816}{35}(C_A - C_F)\zeta_3 \\
& + \frac{849420853541}{3889620000} C_F \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
H_q^{\mathsf{TR}}(n_f) \quad = \quad & a_s^2 \left[A_{qq,Q}^{\mathsf{TR},(2)}(n_f) + \hat{C}_q^{\mathsf{TR},(2)}(n_f+1) \right] \\
& + a_s^3 \left[A_{qq,Q}^{\mathsf{TR},(3)}(n_f) + A_{qq,Q}^{\mathsf{TR},(2)}(n_f) C_q^{\mathsf{TR},(1)} + \hat{C}_q^{\mathsf{TR},(3)}(n_f+1) \right]
\end{aligned}
$$

- Expansion valid for all processes
- Light flavor Wilson coefficients:
  - Drell-Yan: known to $O(a_s)$ + soft resummation
  - SIDIS: not yet available
- Check if the Soffer bound for Transversity:

$$
\left| \Delta_T F(x, Q^2) \right| \leq \frac{1}{2} \left[F(x, Q^2) + \Delta F(x, Q^2) \right]
$$

is valid at higher order

4. Conclusions

- In the region $Q^2 \gg m^2$ the heavy flavor Wilson coefficients factorize into massive operator matrix elements and the massless Wilson coefficients.
- We calculated the 2-loop massive operator matrix elements for transversity analytically for general values of N.
- In the 3-loop case, we calculated the massive OMEs for fixed values of $N = 1 \ldots 13$
- These are process independent quantities, bearing all the mass dependence in the region where power corrections $(m^2/Q^2)^k$ can be disregarded.
- Together with the corresponding massless Wilson coefficients for the different processes the operator matrix elements assemble to the heavy flavor Wilson coefficients.
- We also obtained the moments $N = 1, \ldots, 13$ for the complete 2-loop anomalous dimension and the terms $\propto T_F$ in the 3-loop anomalous dimension, which confirm and extend results in the literature.