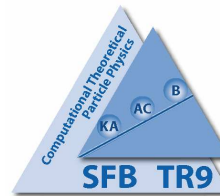


Moments of the 3-loop corrections to the heavy flavor contribution to $F_2(x, Q^2)$ for $Q^2 \gg m^2$

Sebastian Klein, DESY

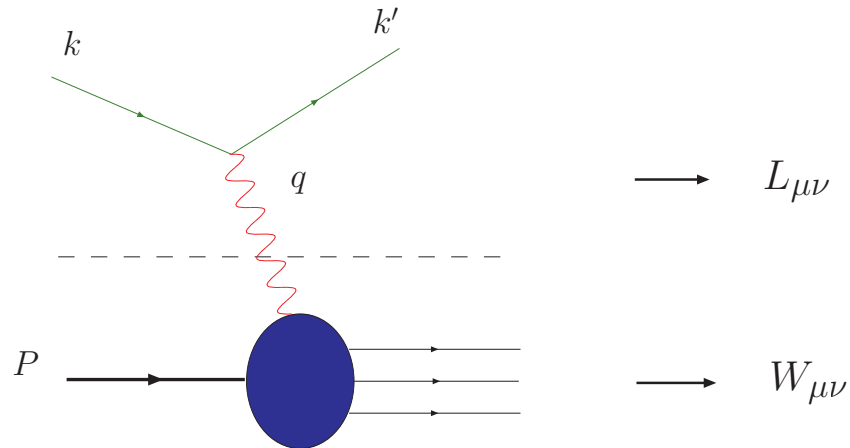
in collaboration with I. Bierenbaum and J. Blümlein [0904.3536 [hep-ph]]



- Introduction and Theory Status
- The Method
- Renormalization
- Asymptotic 3-Loop Results (Fixed Moments) & Anomalous Dimensions
- Conclusions

1. Introduction

Deep-Inelastic Scattering (DIS):



$$Q^2 := -q^2, \quad x := \frac{Q^2}{2pq} \quad \text{Bjorken-}x$$

$$\nu := \frac{Pq}{M},$$

$$\frac{d\sigma}{dQ^2 dx} \sim W_{\mu\nu} L^{\mu\nu}$$

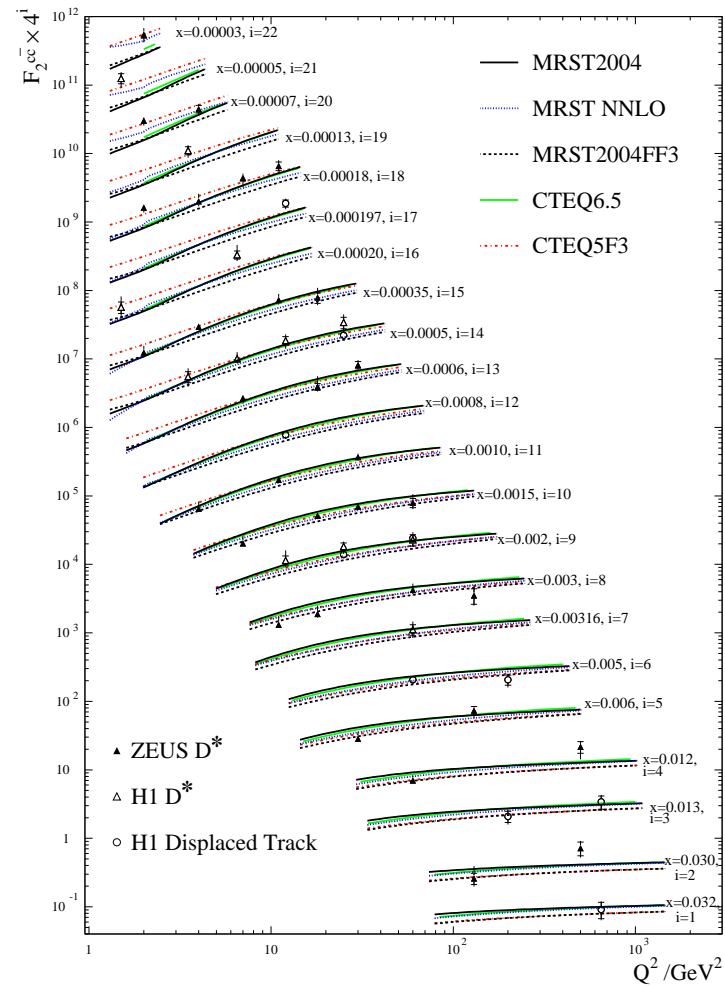
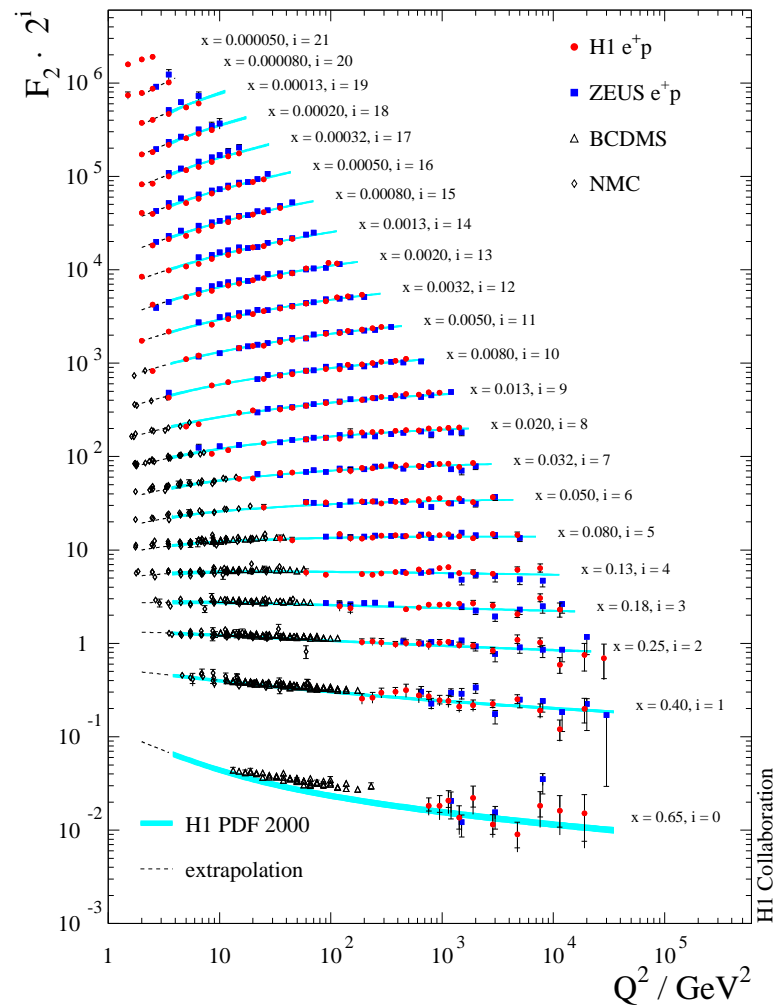
$$W_{\mu\nu}(q, P, s) = \frac{1}{4\pi} \int d^4\xi \exp(iq\xi) \langle P, s | [J_\mu^{em}(\xi), J_\nu^{em}(0)] | P, s \rangle$$

$$\text{unpol.} \left\{ = \frac{1}{2x} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) F_L(x, Q^2) + \frac{2x}{Q^2} \left(P_\mu P_\nu + \frac{q_\mu P_\nu + q_\nu P_\mu}{2x} - \frac{Q^2}{4x^2} g_{\mu\nu} \right) F_2(x, Q^2) \right.$$

$$\left. \text{pol.} \left\{ -\frac{M}{2Pq} \varepsilon_{\mu\nu\alpha\beta} q^\alpha \left[s^\beta g_1(x, Q^2) + \left(s^\beta - \frac{sq}{Pq} p^\beta \right) g_2(x, Q^2) \right] \right. \right.$$

Structure Functions: $F_{2,L}$, $g_{1,2}$

contain light and heavy quark contributions.



[Thompson, 2007]

- High statistics for F_2 and F_2^{cc} \implies Accuracy will increase in the future.
- Different scaling violations.

Leading Order : $F_{2,L}(x, Q^2)$ [Witten, 1976; Babcock, Sivers, 1978; Shifman, Vainshtein, Zakharov, 1978; Leveille, Weiler, 1979; Glück, Reya, 1979; Glück, Hoffmann, Reya, 1982.]

Leading Order : $g_1(x, Q^2)$ [Watson, 1982; Glück, Reya, Vogelsang, 1991; Vogelsang, 1991]

Leading Order : $g_2(x, Q^2)$ [Blümlein, Ravindran, van Neerven, 2003]

Soft resummation: $F_{2,L}(x, Q^2)$ [Laenen & Moch, 1998; Alekhin & Moch, 2008]

Next-to-Leading Order : $F_{2,L}(x, Q^2)$ [Laenen, Riemersma, Smith, van Neerven, 1993, 1995]

asymptotic: [Buza, Matiounine, Smith, Migneron, van Neerven, 1996; Bierenbaum, Blümlein, S.K., 2007]

Mellin space expressions: [Alekhin, Blümlein, 2003].

Next-to-Leading Order : $g_1(x, Q^2)$ asymptotic: [Buza, Matiounine, Smith, Migneron, van Neerven, 1997; Bierenbaum, Blümlein, S.K., 2009]

Next-to-Next-to-Leading Order : $F_L(x, Q^2)$ asymptotic:

[Blümlein, De Freitas, S.K., van Neerven, 2006.]

\implies 3-Loop corrections needed to obtain the same accuracy as reached for the light flavor contributions.

2. The Method

- **massless RGE** and **light-cone expansion** in Bjorken-limit $\{Q^2, \nu\} \rightarrow \infty$, x fixed:

$$\lim_{\xi^2 \rightarrow 0} [J(\xi), J(0)] \propto \sum_{i,N,\tau} c_{i,\tau}^N(\xi^2, \mu^2) \xi_{\mu_1} \dots \xi_{\mu_N} O_{i,\tau}^{\mu_1 \dots \mu_N}(0, \mu^2) .$$

- **mass factorization** of the structure functions into **Wilson coefficients** and **parton densities**:

$$F_i(x, Q^2) = \sum_j C_i^j \left(x, \frac{Q^2}{\mu^2} \right) \otimes f_j(x, \mu^2) ; \quad \text{Twist } \tau = 2$$

- Light flavor **Wilson coefficients**: **process dependent** ($O(a_s^3)$): [Moch, Vermaseren, Vogt, 2005.]

$$C_{(2,L);i}^{\text{fl}} \left(\frac{Q^2}{\mu^2} \right) = \delta_{i,q} + \sum_{l=1}^{\infty} a_s^l C_{(2,L),i}^{\text{fl},(l)}, \quad i = q, g$$

- Heavy quark contributions given by heavy quark Wilson coefficients

$$H_{(2,L),i}^{\text{S}} \left(\frac{Q^2}{\mu^2}, \frac{m^2}{Q^2} \right) = \underbrace{H_{(2,L),i}^{\text{S}} \left(\frac{Q^2}{\mu^2}, \frac{m^2}{Q^2} \right)}_{\gamma + q_{\text{heavy}} \rightarrow X} + \underbrace{L_{(2,L),i}^{\text{S,NS}} \left(\frac{Q^2}{\mu^2}, \frac{m^2}{Q^2} \right)}_{\gamma + q_{\text{light}} \rightarrow X}$$

- Consider only one species of heavy quarks

- Factorization for $F_2^{Q\bar{Q}}(x, Q^2)$ at the level of twist $\tau = 2$:

$$\begin{aligned}
F_2^{Q\bar{Q}}(n_f, x, Q^2, m^2) = & \sum_{k=1}^{n_f} e_k^2 \left\{ \begin{aligned} & L_{2,q}^{\text{NS}} \left(n_f, x, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes \left[f_k(n_f, x, \mu^2) + f_{\bar{k}}(n_f, x, \mu^2) \right] \\ & + \tilde{L}_{2,q}^{\text{PS}} \left(n_f, x, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes \Sigma(n_f, x, \mu^2) \\ & + \tilde{L}_{2,g}^{\text{S}} \left(n_f, x, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes G(n_f, x, \mu^2) \end{aligned} \right\} \\
& + e_Q^2 \left\{ \begin{aligned} & H_{2,q}^{\text{PS}} \left(n_f, x, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes \Sigma(n_f, x, \mu^2) \\ & + H_{2,g}^{\text{S}} \left(n_f, x, \frac{Q^2}{m^2}, \frac{m^2}{\mu^2} \right) \otimes G(n_f, x, \mu^2) \end{aligned} \right\} .
\end{aligned}$$

- In the limit $Q^2 \gg m^2$ [$Q^2 \approx 10 m^2$ for F_2, g_1]: **massive RGE**, derivative $m^2 \partial / \partial m^2$ acts on Wilson coefficients only: all terms but power corrections calculable through **partonic operator matrix elements**, $\langle i | A_l | j \rangle$, which are **process independent objects!**

$$H_{(2,L),i}^S \left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2} \right) = \underbrace{A_{k,i}^S \left(\frac{m^2}{\mu^2} \right)}_{\text{massive OMEs}} \otimes \underbrace{C_{(2,L),k}^S \left(\frac{Q^2}{\mu^2} \right)}_{\text{light-parton-Wilson coefficients}}.$$

- Similar formula for $L_{(2,L),i}^{S,NS}$. Holds for **polarized** and **unpolarized** case.
- OMEs obey expansion

$$A_{k,i}^{S,NS} \left(\frac{m^2}{\mu^2} \right) = \langle i | O_k^{S,NS} | i \rangle = \delta_{k,i} + \sum_{l=1}^{\infty} a_s^l A_{k,i}^{S,NS,(l)} \left(\frac{m^2}{\mu^2} \right), \quad i = q, g$$

[Buza, Matiounine, Migneron, Smith, van Neerven, 1996; Buza, Matiounine, Smith, van Neerven, 1997.]

- **OMEs** occur as well in the definition of a **variable flavor number scheme** starting from a **fixed flavor number scheme**.

[Buza, Matiounine, Smith, van Neerven, 1998; Chuvakin, Smith, van Neerven, 1998.]

- Expansion up to $O(a_s^3)$ for $F_2^{Q\bar{Q}}(x, Q^2)$ reads

$$L_{2,q}^{\text{NS}}(n_f) = a_s^2 \left[A_{qq,Q}^{\text{NS},(2)}(n_f) + \hat{C}_{2,q}^{\text{NS},(2)}(n_f) \right] + a_s^3 \left[A_{qq,Q}^{\text{NS},(3)}(n_f) + A_{qq,Q}^{\text{NS},(2)}(n_f) C_{2,q}^{\text{NS},(1)}(n_f) + \hat{C}_{2,q}^{\text{NS},(3)}(n_f) \right]$$

$$\tilde{L}_{2,q}^{\text{PS}}(n_f) = a_s^3 \left[\tilde{A}_{qq,Q}^{\text{PS},(3)}(n_f) + A_{gq,Q}^{(2)}(n_f) \tilde{C}_{2,g}^{(1)}(n_f + 1) + \hat{C}_{2,q}^{\text{PS},(3)}(n_f) \right]$$

$$\begin{aligned} \tilde{L}_{2,g}^{\text{S}}(n_f) = & a_s^2 \left[A_{gg,Q}^{(1)}(n_f) \tilde{C}_{2,g}^{(1)}(n_f + 1) \right] + a_s^3 \left[\tilde{A}_{gq,Q}^{(3)}(n_f) + A_{gg,Q}^{(1)}(n_f) \tilde{C}_{2,g}^{(2)}(n_f + 1) \right. \\ & \left. + A_{gg,Q}^{(2)}(n_f) \tilde{C}_{2,g}^{(1)}(n_f + 1) + A_{Qg}^{(1)}(n_f) \tilde{C}_{2,q}^{\text{PS},(2)}(n_f + 1) + \hat{C}_{2,g}^{(3)}(n_f) \right] \end{aligned}$$

$$H_{2,q}^{\text{PS}}(n_f) = a_s^2 \left[A_{Qq}^{\text{PS},(2)} + \tilde{C}_{2,q}^{\text{PS},(2)}(n_f + 1) \right] + a_s^3 \left[A_{Qq}^{\text{PS},(3)} + \tilde{C}_{2,q}^{\text{PS},(3)}(n_f + 1) + A_{gq,Q}^{(2)} \tilde{C}_{2,g}^{(1)}(n_f + 1) + A_{Qq}^{\text{PS},(2)} C_{2,q}^{\text{NS},(1)}(n_f + 1) \right]$$

$$\begin{aligned} H_{2,g}^{\text{S}}(n_f) = & a_s \left[A_{Qg}^{(1)} + \tilde{C}_{2,g}^{(1)}(n_f + 1) \right] + a_s^2 \left[A_{Qg}^{(2)} + A_{Qg}^{(1)} C_{2,q}^{\text{NS},(1)}(n_f + 1) + A_{gg,Q}^{(1)} \tilde{C}_{2,g}^{(1)}(n_f + 1) + \tilde{C}_{2,g}^{(2)}(n_f + 1) \right] \\ & + a_s^3 \left[A_{Qg}^{(3)} + A_{Qg}^{(2)} C_{2,q}^{\text{NS},(1)}(n_f + 1) + A_{gg,Q}^{(2)} \tilde{C}_{2,g}^{(1)}(n_f + 1) + A_{gg,Q}^{(1)} \tilde{C}_{2,g}^{(2)}(n_f + 1) \right. \\ & \left. + A_{Qg}^{(1)} \left[C_{2,q}^{\text{NS},(2)}(n_f + 1) + \tilde{C}_{2,q}^{\text{PS},(2)}(n_f + 1) \right] + \tilde{C}_{2,g}^{(3)}(n_f + 1) \right]. \end{aligned}$$

- n_f -dependence non-trivial: $\hat{f}(n_f) \equiv f(n_f + 1) - f(n_f)$, $\tilde{f}(n_f) \equiv f(n_f)/n_f$.
- Highlighted terms are (partially) due to **heavy quark insertions on external legs** and have to be included in the $\overline{\text{MS}}$ -scheme \implies not considered in previous **NLO** analyses.
- Comparison to exact order $O(a_s^2)$ result: asymptotic formulae valid for $Q^2 \gtrsim 20$ (GeV/c)² in case of $F_2^{c\bar{c}}(x, Q^2)$ and $Q^2 \gtrsim 800$ (GeV/c)² for $F_L^{c\bar{c}}(x, Q^2)$

3. Renormalization

- **Mass** renormalization (on mass shell scheme)
- **Charge** renormalization: **MOM scheme** for the gluon propagator.
MOM scheme \rightarrow $\overline{\text{MS}}$ scheme:

$$a_s^{\text{MOM}} = a_s^{\overline{\text{MS}}} - \beta_{0,Q} \ln\left(\frac{m^2}{\mu^2}\right) a_s^{\overline{\text{MS}}} + \left[\beta_{0,Q}^2 \ln^2\left(\frac{m^2}{\mu^2}\right) - \beta_{1,Q} \ln\left(\frac{m^2}{\mu^2}\right) - \beta_{1,Q}^{(1)} \right] a_s^{\overline{\text{MS}}}.$$

\implies Accounts for difference due to **heavy quark insertions on external legs**.

- Renormalization of **ultraviolet** singularities
 \implies are absorbed into Z -factors given in terms of **anomalous dimensions** γ_{ij} .
- Factorization of **collinear** singularities
 \implies are factored into Γ -factors Γ_{NS} , $\Gamma_{ij,S}$ and $\Gamma_{qq,PS}$.

Generic formula for operator renormalization and mass factorization:

$$A_{ij} = Z_{il}^{-1} \hat{A}_{lm} \Gamma_{mj}^{-1}$$

- \implies $O(\varepsilon)$ -terms of the **2-loop OMEs** are needed for renormalization at 3 loops.
- \implies All 2-loop $O(\varepsilon)$ -terms in the unpolarized case are known.

[Bierenbaum, Blümlein, S.K., Schneider, 2008; Bierenbaum, Blümlein, S.K., 2009.]

4. Fixed moments at 3–Loop

Contributing OMEs:

$$\begin{array}{lcl}
 \text{Singlet} & A_{Qg} & A_{gg,Q} & A_{gq,Q} \\
 \text{Pure–Singlet} & & A_{Qq}^{\text{PS}} & \\
 \text{Non–Singlet} & A_{qq,Q}^{\text{NS,+}} & A_{qq,Q}^{\text{NS,-}} & A_{qq,Q}^{\text{NS,v}}
 \end{array}
 \left. \vphantom{\begin{array}{l} \\ \\ \\ \end{array}} \right\} \text{mixing}$$

- **Unpolarized anomalous dimensions** are known up to $O(a_s^3)$ [Moch, Vermaseren, Vogt, 2004.]
 \implies All terms needed for the renormalization of **unpolarized 3–Loop heavy OMEs** are present.
 \implies The calculation provides first independent checks on $\gamma_{qg}^{(2)}$, $\gamma_{qq}^{(2),\text{PS}}$ and on respective color projections of $\gamma_{qq}^{(2),\text{NS}\pm}$, $\gamma_{gg}^{(2)}$ and $\gamma_{gq}^{(2)}$.
- The calculation proceeds in the same way in the **polarized** case.
- Calculation in **Mellin space**.
 For fixed N : three–loop “self-energy” type diagrams with an operator insertion
 \implies Calculation using **MATAD** [Steinhauser, 2001] and **FORM** [Vermaseren, 2000].

General structure of the result: the PS –case

$$\begin{aligned}
A_{Qq}^{(3),\text{PS},\overline{\text{MS}}} &= \frac{\hat{\gamma}_{qg}^{(0)} \gamma_{gq}^{(0)}}{48} \left\{ \gamma_{gg}^{(0)} - \gamma_{qq}^{(0)} + 6\beta_0 + 16\beta_{0,Q} \right\} \ln^3 \left(\frac{m^2}{\mu^2} \right) \\
&+ \frac{1}{8} \left\{ -4\hat{\gamma}_{qq}^{(1),\text{PS}} (\beta_0 + \beta_{0,Q}) + \hat{\gamma}_{qg}^{(0)} (\hat{\gamma}_{gq}^{(1)} - \gamma_{gq}^{(1)}) - \gamma_{gq}^{(0)} \hat{\gamma}_{qg}^{(1)} \right\} \ln^2 \left(\frac{m^2}{\mu^2} \right) \\
&+ \frac{1}{16} \left\{ 8 \hat{\gamma}_{qq}^{(2),\text{PS}} - 8n_f \hat{\gamma}_{qq}^{(2),\text{PS}} - 32a_{Qq}^{(2),\text{PS}} (\beta_0 + \beta_{0,Q}) + 8\hat{\gamma}_{qg}^{(0)} a_{gq,Q}^{(2)} - 8\gamma_{gq}^{(0)} a_{Qg}^{(2)} \right. \\
&\quad \left. - \zeta_2 \hat{\gamma}_{qg}^{(0)} \gamma_{gq}^{(0)} (\gamma_{gg}^{(0)} - \gamma_{qq}^{(0)} + 6\beta_0 + 8\beta_{0,Q}) \right\} \ln \left(\frac{m^2}{\mu^2} \right) \\
&+ 4(\beta_0 + \beta_{0,Q}) \bar{a}_{Qq}^{(2),\text{PS}} + \gamma_{gq}^{(0)} \bar{a}_{Qg}^{(2)} - \hat{\gamma}_{qg}^{(0)} \bar{a}_{gq,Q}^{(2)} + \zeta_3 \frac{\gamma_{gq}^{(0)} \hat{\gamma}_{qg}^{(0)}}{48} (\gamma_{gg}^{(0)} - \gamma_{qq}^{(0)} + 6\beta_0) \\
&+ \frac{\hat{\gamma}_{qg}^{(0)} \gamma_{gq}^{(1)} \zeta_2}{16} + C_F \left(-(4 + \frac{3}{4}\zeta_2) \hat{\gamma}_{qg}^{(0)} \gamma_{gq}^{(0)} - 4\hat{\gamma}_{qq}^{(1),\text{PS}} + 12a_{Qq}^{(2),\text{PS}} \right) + \bar{a}_{Qq}^{(3),\text{PS}} .
\end{aligned}$$

All terms but $a_{Qq}^{(3),\text{PS}}$ known for all N.

- There are similar formulas for the remaining OMEs.

5. Results

Using **MATAD**, we calculated the terms (≈ 250 days of computer time)

$$\begin{aligned}
 A_{Qq}^{(3),PS} &: (2, 4, \dots, 12); & A_{qq,Q}^{(3),PS}, A_{gq,Q}^{(3)} &: (2, 4, \dots, 14); \\
 A_{qq,Q}^{(3),NS\pm} &: (2, 3, \dots, 14); & A_{Q(q)g}^{(3)}, A_{gg,Q}^{(3)} &: (2, 4, \dots, 10);
 \end{aligned}$$

and find **agreement** with the predictions obtained from renormalization.

Example: non-logarithmic term of $A_{Qg}^{(3)}$ for $N = 2$

$$\begin{aligned}
 A_{Qg}^{(3),\overline{MS}}(\mu^2 = m^2, N = 2) &= T_F C_A^2 \left(\frac{174055}{4374} - \frac{88}{9} B_4 + 72 \zeta_4 - \frac{29431}{324} \zeta_3 \right) \\
 &+ T_F C_F C_A \left(-\frac{18002}{729} + \frac{208}{9} B_4 - 104 \zeta_4 + \frac{2186}{9} \zeta_3 - \frac{64}{3} \zeta_2 + 64 \zeta_2 \ln(2) \right) \\
 &+ T_F C_F^2 \left(-\frac{8879}{729} - \frac{64}{9} B_4 + 32 \zeta_4 - \frac{701}{81} \zeta_3 + 80 \zeta_2 - 128 \zeta_2 \ln(2) \right) + T_F^2 C_A \left(-\frac{21586}{2187} + \frac{3605}{162} \zeta_3 \right) \\
 &+ T_F^2 C_F \left(-\frac{55672}{729} + \frac{889}{81} \zeta_3 + \frac{128}{3} \zeta_2 \right) + n_f T_F^2 C_A \left(-\frac{7054}{2187} - \frac{704}{81} \zeta_3 \right) + n_f T_F^2 C_F \left(-\frac{22526}{729} + \frac{1024}{81} \zeta_3 - \frac{64}{3} \zeta_2 \right).
 \end{aligned}$$

The constant terms: $N = 10$ $a_{Qg}^{(3)}$ & $a_{gg,Q}^{(3)}$:

$$\begin{aligned}
a_{Qg}^{(3)} \Big|_{N=10} &= T_F \left(\frac{m^2}{\mu^2} \right)^{3\epsilon/2} \left\{ n_f T_F \left(C_A \left[-\frac{1505896}{245025} \zeta_3 + \frac{189965849}{188669250} \zeta_2 + \frac{297277185134077151}{15532837481700000} \right] \right. \right. \\
&+ C_F \left[\frac{62292104}{13476375} \zeta_3 - \frac{49652772817}{93391278750} \zeta_2 - \frac{1178560772273339822317}{107642563748181000000} \right] \left. \right) + C_A^2 \left[-\frac{563692}{81675} B_4 \right. \\
&+ \frac{483988}{9075} \zeta_4 - \frac{103652031822049723}{415451499724800} \zeta_3 - \frac{20114890664357}{581101290000} \zeta_2 \\
&+ \left. \frac{6830363463566924692253659}{685850575063965696000000} \right] + C_A C_F \left[\frac{1286792}{81675} B_4 - \frac{643396}{9075} \zeta_4 \right. \\
&- \frac{761897167477437907}{33236119977984000} \zeta_3 + \frac{15455008277}{660342375} \zeta_2 + \left. \frac{872201479486471797889957487}{2992802509370032128000000} \right] \\
&+ C_F^2 \left[-\frac{11808}{3025} B_4 + \frac{53136}{3025} \zeta_4 + \frac{9636017147214304991}{7122025709568000} \zeta_3 + \frac{14699237127551}{15689734830000} \zeta_2 \right. \\
&- \left. \frac{247930147349635960148869654541}{148143724213816590336000000} \right] + T_F C_A \left[\frac{4206955789}{377338500} \zeta_2 + \frac{123553074914173}{5755172290560} \zeta_3 \right. \\
&+ \left. \frac{23231189758106199645229}{633397356480430080000} \right] + T_F C_F \left[-\frac{502987059528463}{113048027136000} \zeta_3 + \frac{24683221051}{46695639375} \zeta_2 \right. \\
&- \left. \frac{18319931182630444611912149}{1410892611560158003200000} \right] - \frac{896}{1485} T_F^2 \zeta_3 \left. \right\} . \\
a_{gg,Q}^{(3)} \Big|_{N=10} &= n_f T_F^2 \left(\frac{m^2}{\mu^2} \right)^{3\epsilon/2} \left\{ C_A \left[-\frac{1505896}{245025} \zeta_3 + \frac{1109186999}{377338500} \zeta_2 + \frac{6542127929072987}{191763425700000} \right] \right. \\
&+ \left. C_F \left[\frac{62292104}{13476375} \zeta_3 - \frac{83961181063}{93391278750} \zeta_2 - \frac{353813854966442889041}{21528512749636200000} \right] \right\}
\end{aligned}$$

- We obtain e.g. for the **moments** of the $\hat{\gamma}_{qq}^{(2)}$ **anomalous dimension**

N	$\hat{\gamma}_{qq}^{(2)}/T_F$
2	$(1 + 2n_f)T_F \left(\frac{8464}{243}C_A - \frac{1384}{243}C_F \right) + \frac{\zeta_3}{3} \left(-416C_A C_F + 288C_A^2 + 128C_F^2 \right) - \frac{7178}{81}C_A^2 + \frac{556}{9}C_A C_F - \frac{8620}{243}C_F^2$
4	$(1 + 2n_f)T_F \left(\frac{4481539}{303750}C_A + \frac{9613841}{3037500}C_F \right) + \frac{\zeta_3}{25} \left(2832C_A^2 - 3876C_A C_F + 1044C_F^2 \right) - \frac{295110931}{303750}C_A^2 + \frac{278546497}{2025000}C_A C_F - \frac{757117001}{12150000}C_F^2$
6	$(1 + 2n_f)T_F \left(\frac{86617163}{11668860}C_A + \frac{1539874183}{340341750}C_F \right) + \frac{\zeta_3}{735} \left(69864C_A^2 - 94664C_A C_F + 24800C_F^2 \right) - \frac{58595443051}{653456160}C_A^2 + \frac{1199181909343}{8168202000}C_A C_F - \frac{2933980223981}{40841010000}C_F^2$
8	$(1 + 2n_f)T_F \left(\frac{10379424541}{2755620000}C_A + \frac{7903297846481}{1620304560000}C_F \right) + \zeta_3 \left(\frac{128042}{1575}C_A^2 - \frac{515201}{4725}C_A C_F + \frac{749}{27}C_F^2 \right) - \frac{24648658224523}{289340100000}C_A^2 + \frac{4896295442015177}{32406091200000}C_A C_F - \frac{4374484944665803}{56710659600000}C_F^2$
10	$(1 + 2n_f)T_F \left(\frac{1669885489}{988267500}C_A + \frac{1584713325754369}{323600780868750}C_F \right) + \zeta_3 \left(\frac{1935952}{27225}C_A^2 - \frac{2573584}{27225}C_A C_F + \frac{70848}{3025}C_F^2 \right) - \frac{21025430857658971}{255684567600000}C_A^2 + \frac{926990216580622991}{6040547909550000}C_A C_F - \frac{1091980048536213833}{13591232796487500}C_F^2$

- **Agreement** for the terms $\propto T_F$ of the **anomalous dimensions** $\gamma_{ij}^{(2),NS^\pm, S, PS}$ with [Larin, Nogueira, Ritbergen, Vermaseren, 1997; Moch, Vermaseren, Vogt, 2004.]
- How far can we go ? $N = 14$ in some cases; generally: $N = 10 \implies$ Phenomenology
- Unfortunately not enough to perform the automatic **fixed moments** \rightarrow all moments turn. [Blümlein, Kauers, S.K., Schneider, 0902.4091 [hep-ph]].
- **Recently** with B. Tödtli: Calculation of moments $N = 1, \dots, 13$ of the **transversity heavy OMEs** $A_{qq}^{h,(2,3)}$ (cf. Spin Physics Session, Wednesday) \implies Agreement with **anomalous dimensions** $\gamma_{qq}^{h,(1,2)}$ from [Kumano, 1997; 2-Loop: Hayashigaki, Kanazawa, Koike, 1997; Vogelsang, 1998; 3 Loop, $N \leq 8$: Gracey, 2006]

6. Conclusions

- The heavy flavor contributions to F_2 are rather large in the region of lower values of x .
- QCD precision analyses require the description of the heavy quark contributions to 3 loops.
- Complete analytic results are known in the region $Q^2 \gg m^2$ at NLO for $F_{2,L}^{Q\bar{Q}}(x, Q^2), g_{1,2}^{Q\bar{Q}}(x, Q^2)$. They are expressed in terms of massive operator matrix elements and the corresponding massless Wilson coefficients.
- $F_L^{Q\bar{Q}}(x, Q^2)$ is known to NNLO for $Q^2 \gg m^2$.
- The calculation of fixed moments of the massive operator matrix elements at $O(a_s^3)$ has been finished for $N = 10, 12, 14$
 $\implies F_2^{Q\bar{Q}}(x, Q^2)$ to NNLO for $Q^2 \gg m^2$.
 \implies Logarithmic terms are known for all N.
- We also calculate the matrix elements necessary to transform from the **FFNS** to the **VFNS**.
- We corrected left outs even at NLO in the literature.
- First phenomenological parameterization to come up soon.
- Moments of the fermionic contributions to the 3-loop anomalous dimensions have been confirmed for the first time by an independent calculation.