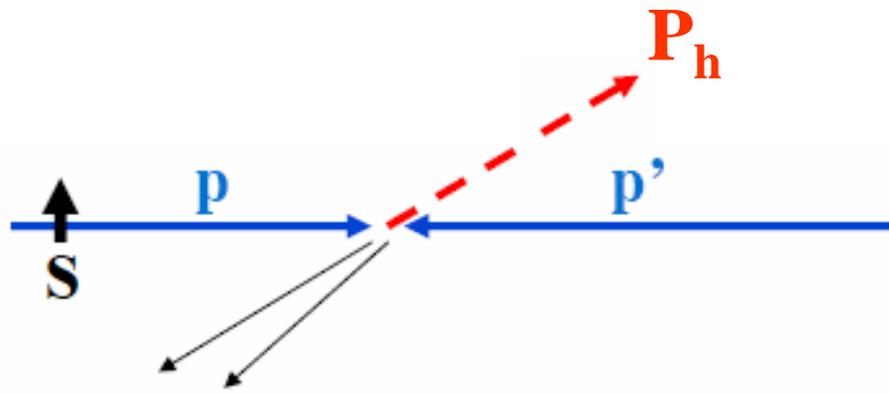


# Theoretical update of twist-3 single-spin asymmetry in semi-inclusive DIS

**Kazuhiro Tanaka (Juntendo U)**

with Y. Koike (Niigata U)

# Single (Transverse) Spin Asymmetry **SSA**



$$d\sigma \sim \vec{S}_\perp \cdot \left( \vec{p} \times \vec{P}_h \right)$$

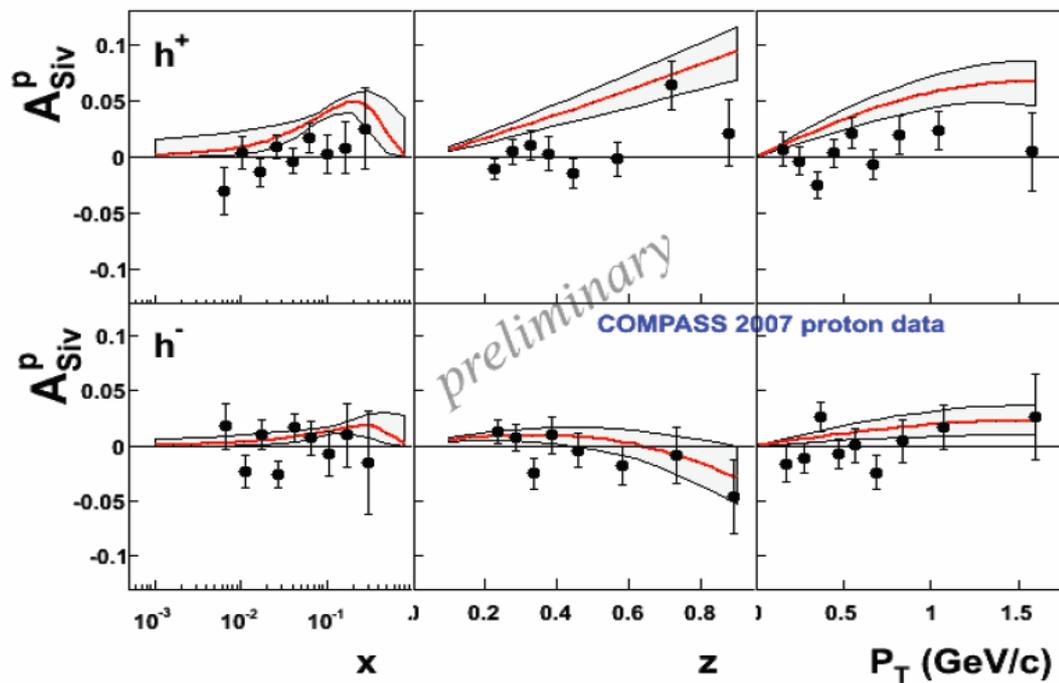
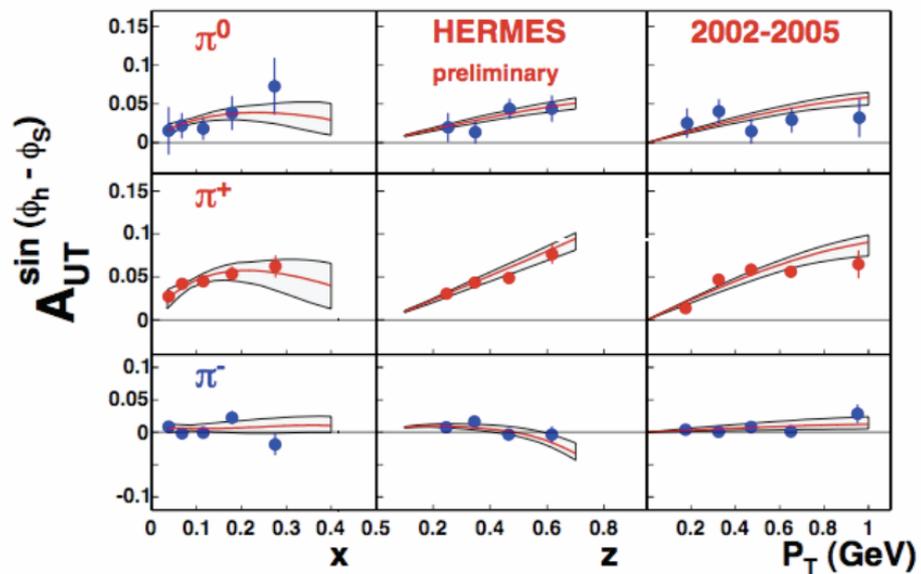
$$A = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

$p^\uparrow p \rightarrow \pi X$  **FNAL-E704, RHIC-STAR**  
 $A_N \sim 0.3$  at large  $x_F$

**Drell-Yan**  $p^\uparrow p \rightarrow l^+ l^- X$   
**Direct  $\gamma$**   $p^\uparrow p \rightarrow \gamma X$  } **RHIC, JPARC, ...**

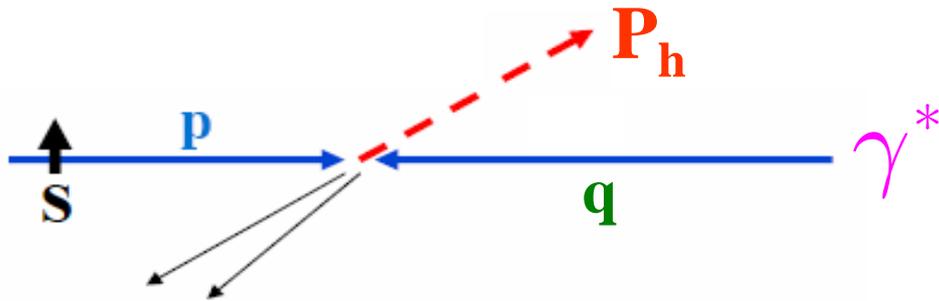
**SIDIS**  $ep^\uparrow \rightarrow e\pi X$  **HERMES, COMPASS**

# ★ Siverson asymmetry

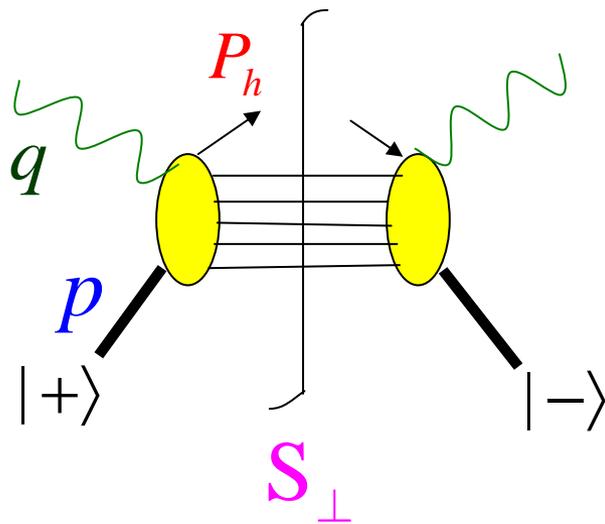


SSA in  $e + p^\uparrow \rightarrow e + \pi + X$

$$d\sigma \sim \vec{S}_\perp \cdot \left( \vec{p} \times \vec{P}_h \right)$$



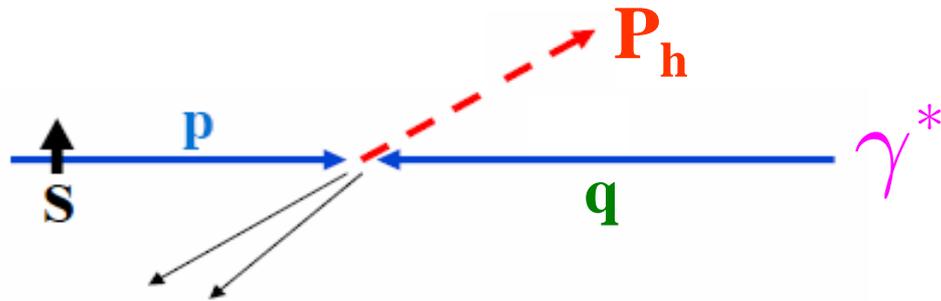
$$A = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$



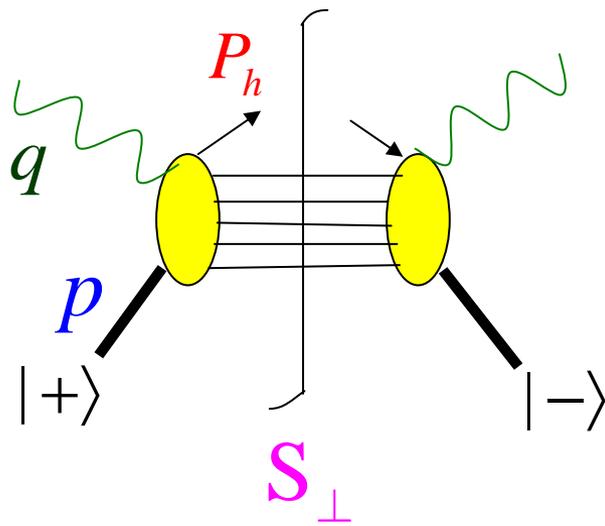
1.  $P_{h\perp} \neq 0$ :  $k_\perp$  of quark or gluon
2. proton helicity flip
3. interaction phase: beyond Born

SSA in  $e + p^\uparrow \rightarrow e + \pi + X$

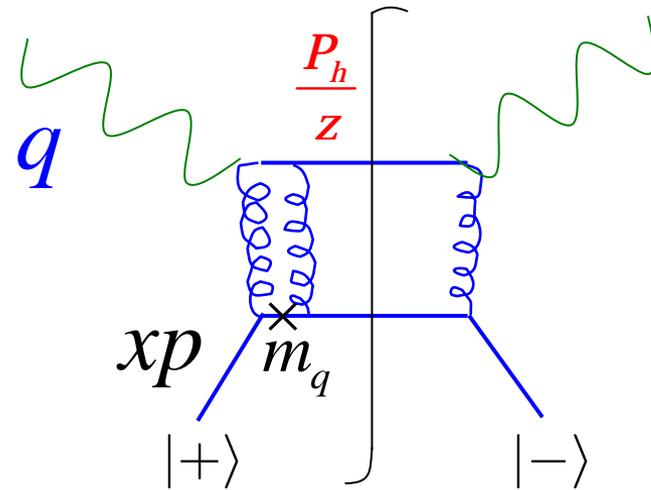
$$d\sigma \sim \vec{S}_\perp \cdot \left( \vec{p} \times \vec{P}_h \right)$$



$$A = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$



1.  $P_{h\perp} \neq 0$ :  $k_\perp$  of quark or gluon P
2. proton helicity flip P
3. interaction phase: beyond Born P



$$A \sim \frac{\alpha_s m_q}{P_{h\perp}}$$

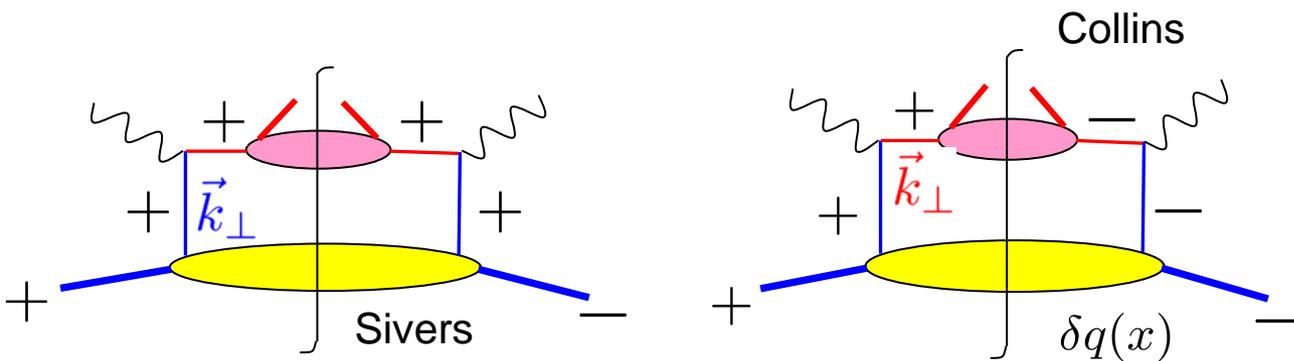
**tiny!**

Conventional twist-2 mechanism:

1.  $P_{h\perp} \neq 0$ :  $k_{\perp}$  of quark or gluon
2. proton helicity flip
3. interaction phase: beyond Born

NP  
NP  
NP

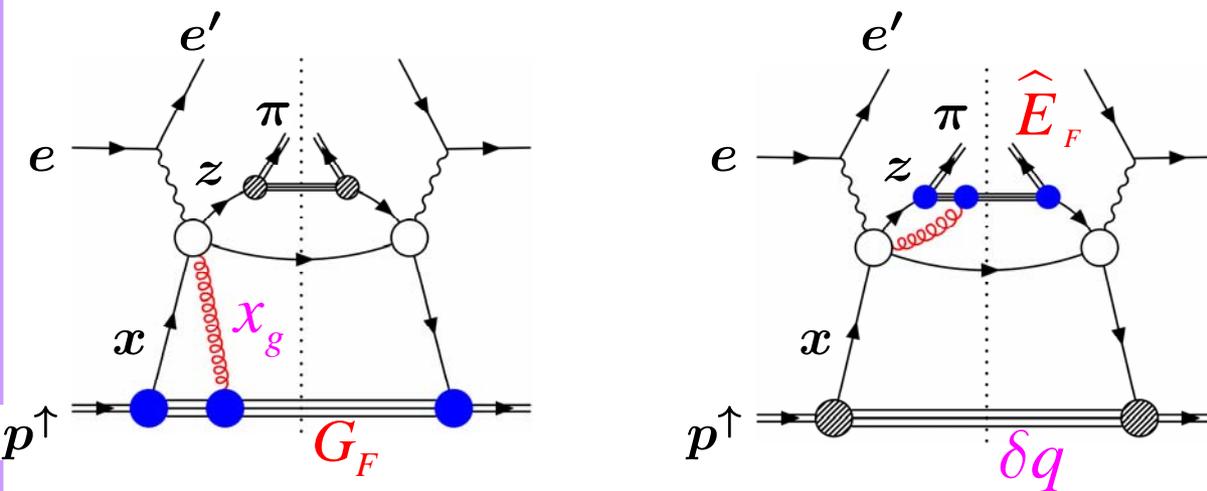
P  
NP  
P&NP



$$P_{h\perp} \sim \Lambda_{\text{QCD}}$$

**TMD-factorization**  
**Sivers & Collins functions**

Sivers ('90); Collins ('93)  
Boer, Mulders ('98)  
Ji, Qiu, Vogelsang, Yuan ('06)



$$P_{h\perp} \gg \Lambda_{\text{QCD}}$$

**collinear-factorization**  
**twist-3 functions**

Efremov, Teryaev ('82)  
Qiu, Serman ('91)  
Eguchi, Koike, Tanaka ('06, '07)

# Recent developments in theoretical basis for twist-3 mechanism and its underlying universal structure

**SISIS**, Drell-Yan, Direct  $\gamma$ ,  $p^\uparrow p \rightarrow \pi X$ , ...

Proof of Factorization and Gauge Invariance of the twist-3 single-spin-dep. cross section in the leading order QCD

Eguchi, Koike, Tanaka, NPB752 ('06) 1; NPB763 ('07) 198

Master formula allowing to derive twist-3 "SGP" cross section directly from twist-2 cross section

Koike, Tanaka, PLB646 ('07) 232; PRD76 ('07) 11502

Connection between the twist-3 mechanism and the TMD Sivers mechanism for intermediate region of  $P_{h\perp}$

Ji, Qiu, Vogelsang, Yuan, PRL97 ('06) 08200; PRD73 ('06) 094017; PLB638 ('06) 178

Koike, Vogelsang, Yuan PLB659 ('07) 878

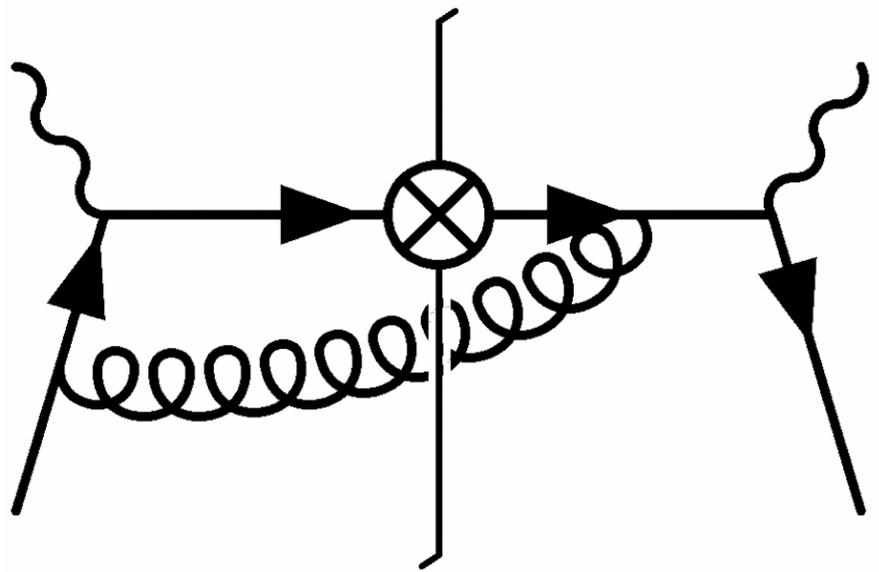
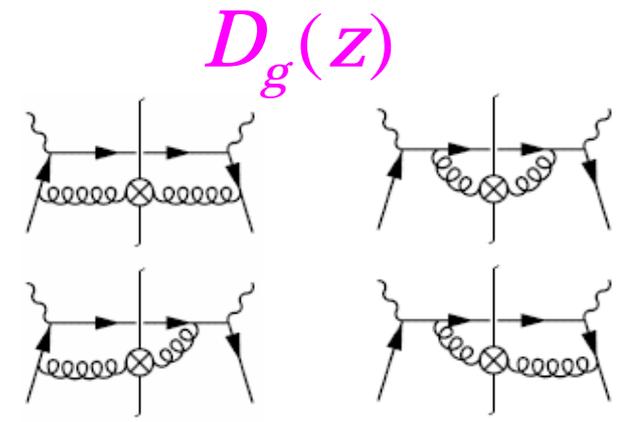
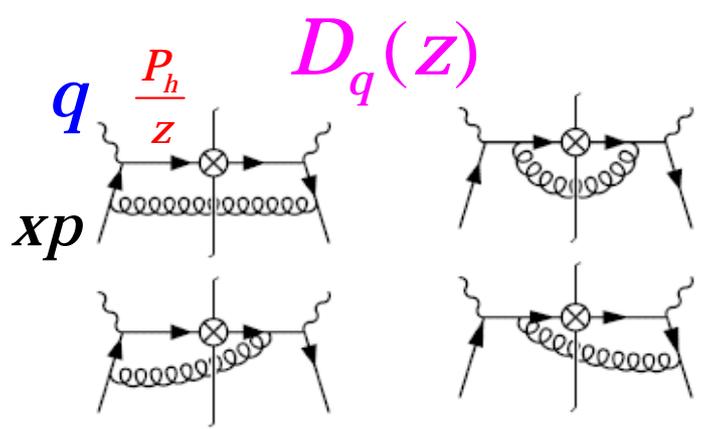
## update & extension are needed!

Koike, Tanaka, PLB668 ('08) 458(E) & in preparation

1.  $P_{h\perp} \neq 0$ :  $k_{\perp}$  of quark or gluon **P**
2. proton helicity flip **NP**
3. interaction phase: beyond Born **P&NP**

$P_{h\perp} \gg \Lambda_{\text{QCD}}$

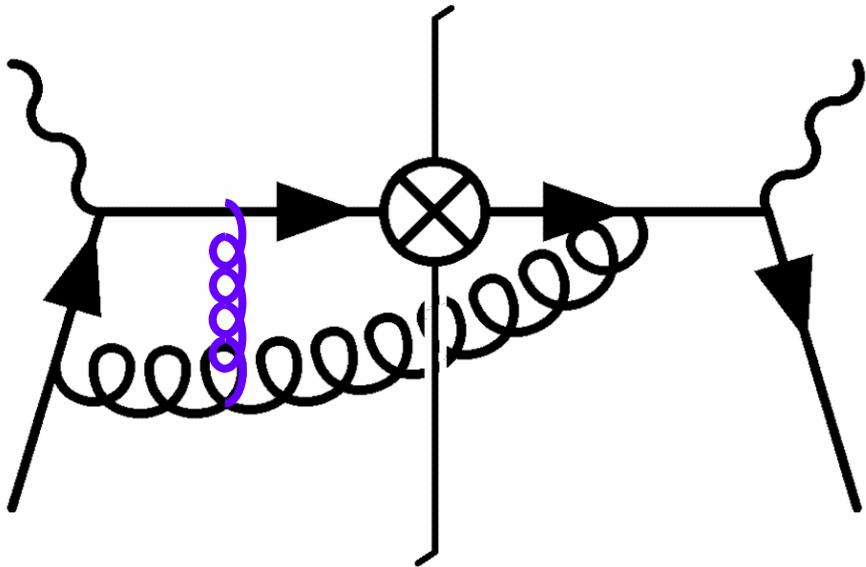
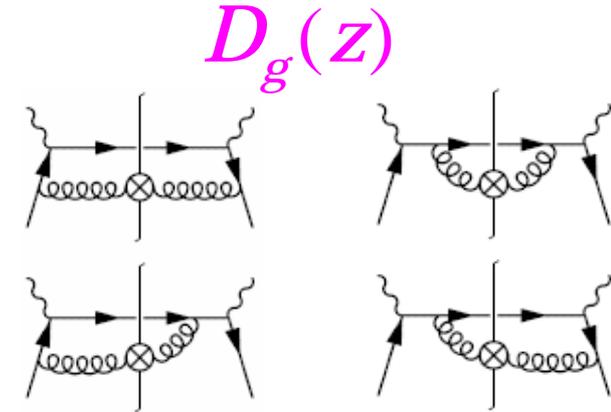
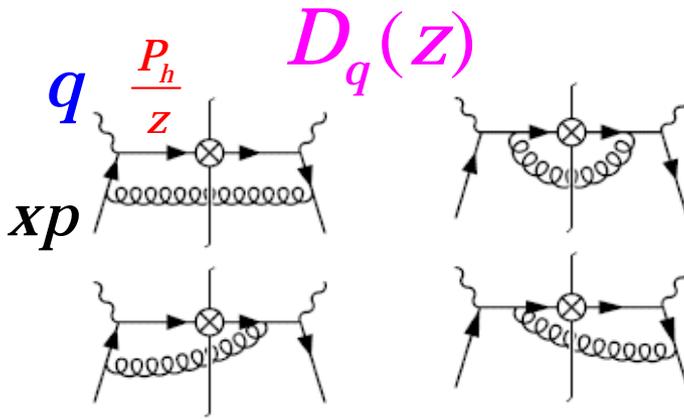
twist-2



1.  $P_{h\perp} \neq 0$ :  $k_\perp$  of quark or gluon **P**
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3. interaction phase: beyond Born **P**

$$P_{h\perp} \gg \Lambda_{\text{QCD}}$$

twist-2



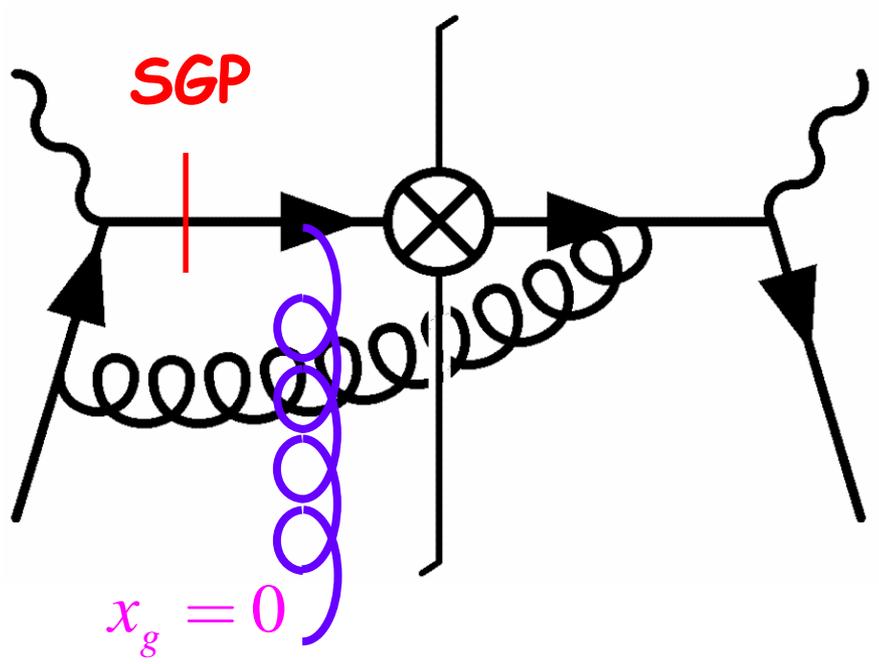
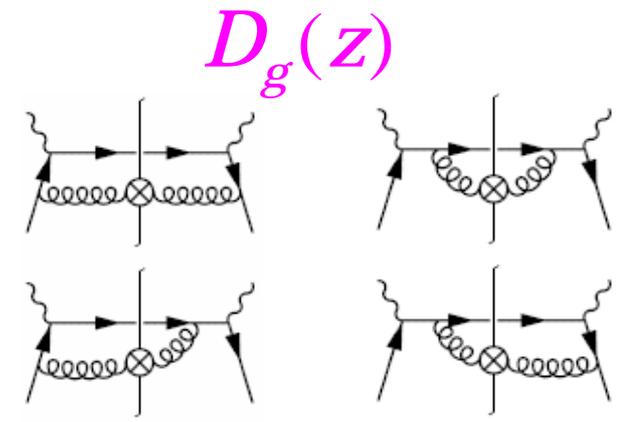
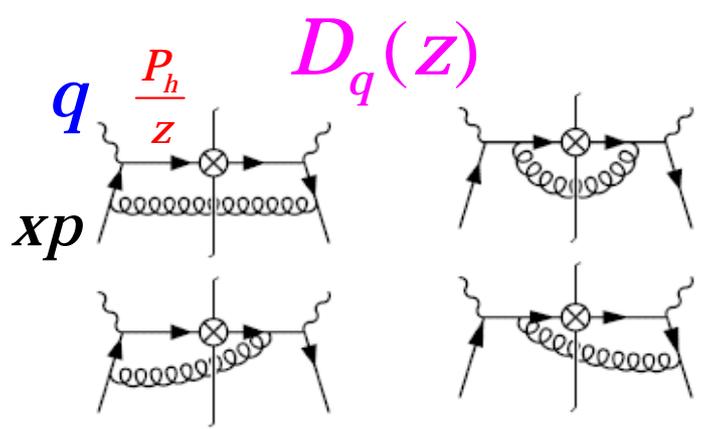
$$\sim \frac{\alpha_s m_q}{P_{h\perp}}$$

tiny!

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twist-2

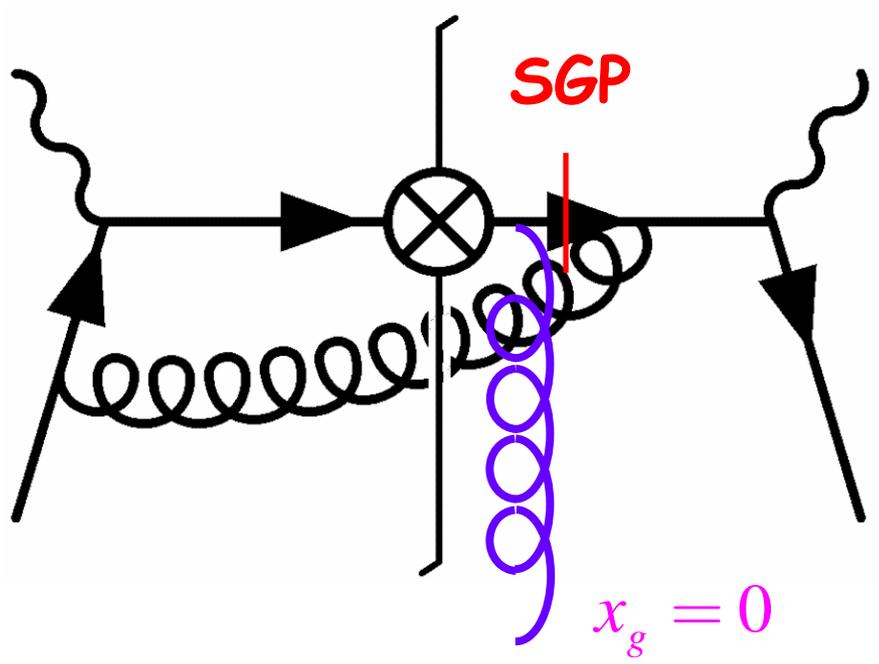
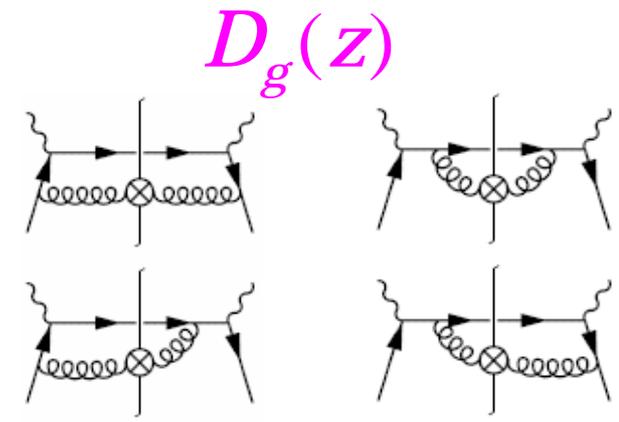
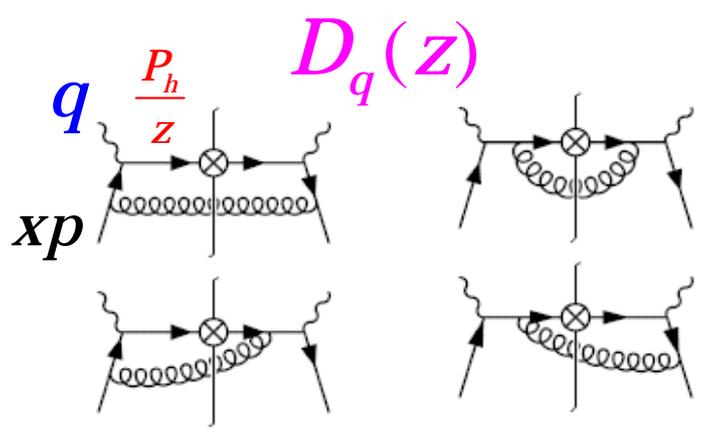


$$\frac{1}{k^2 + i\epsilon} = \text{P} \frac{1}{k^2} - i\pi\delta(k^2)$$

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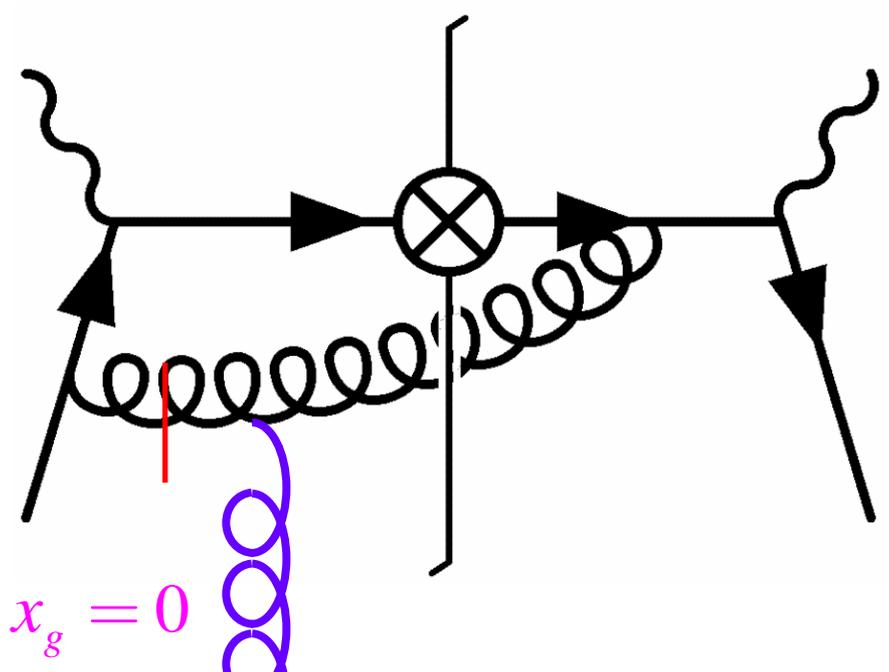
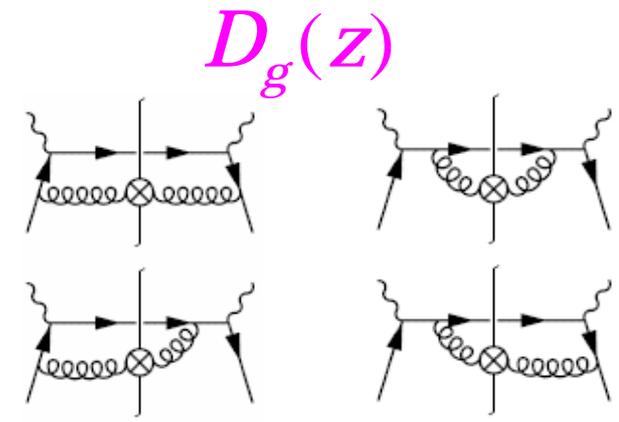
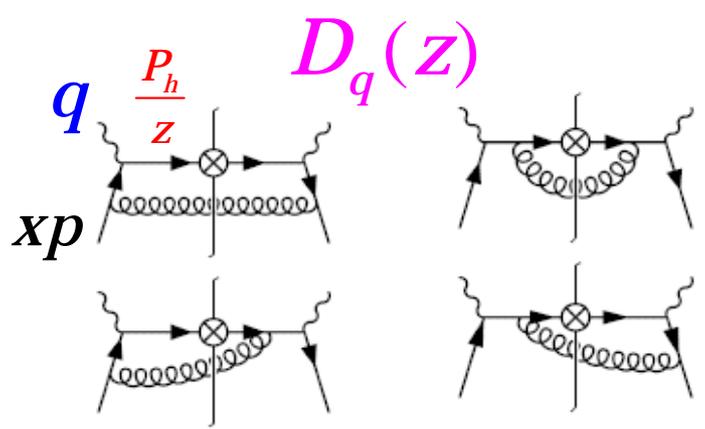


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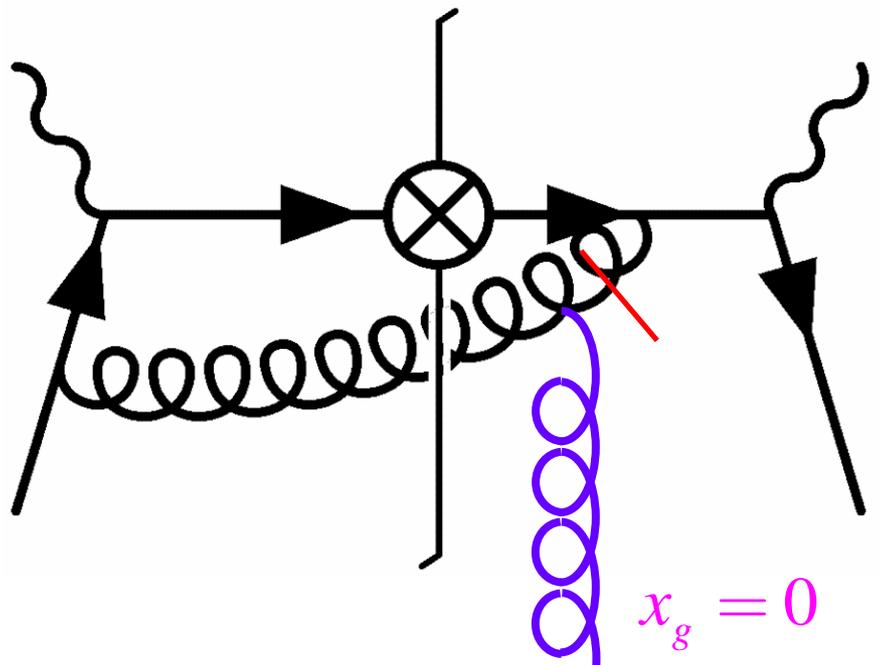
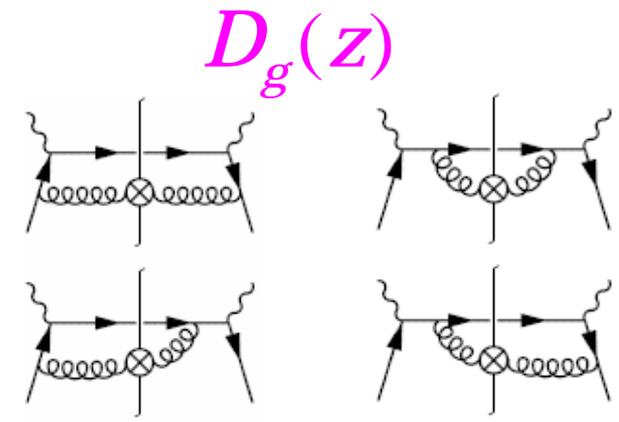
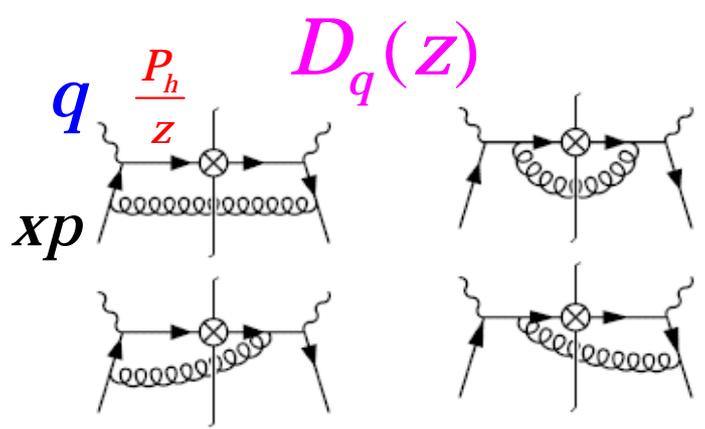


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twist-2

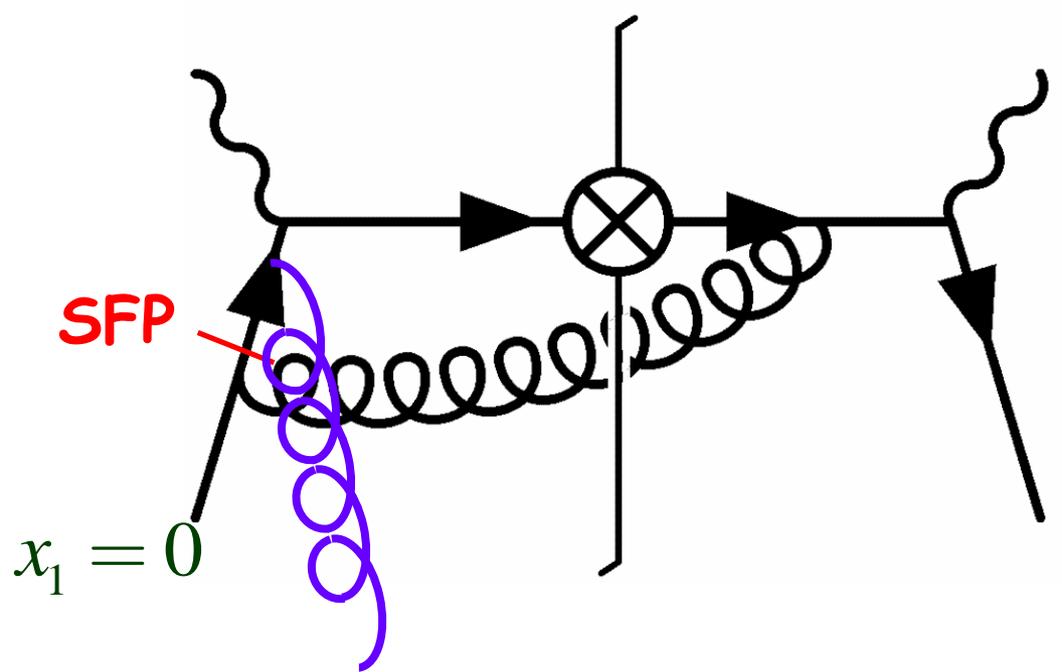
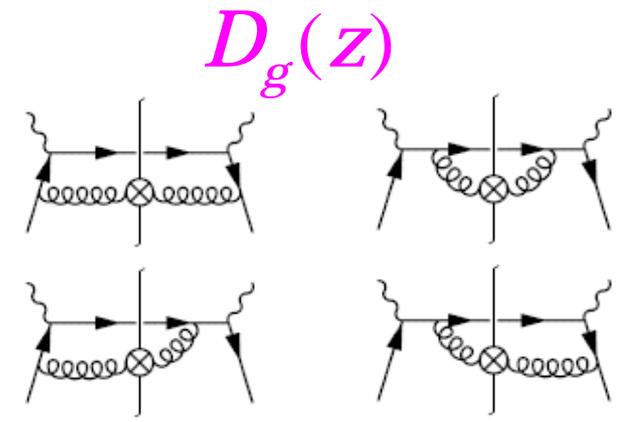
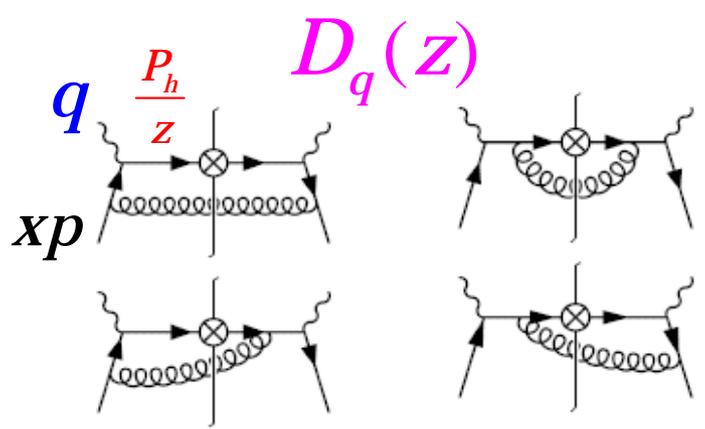


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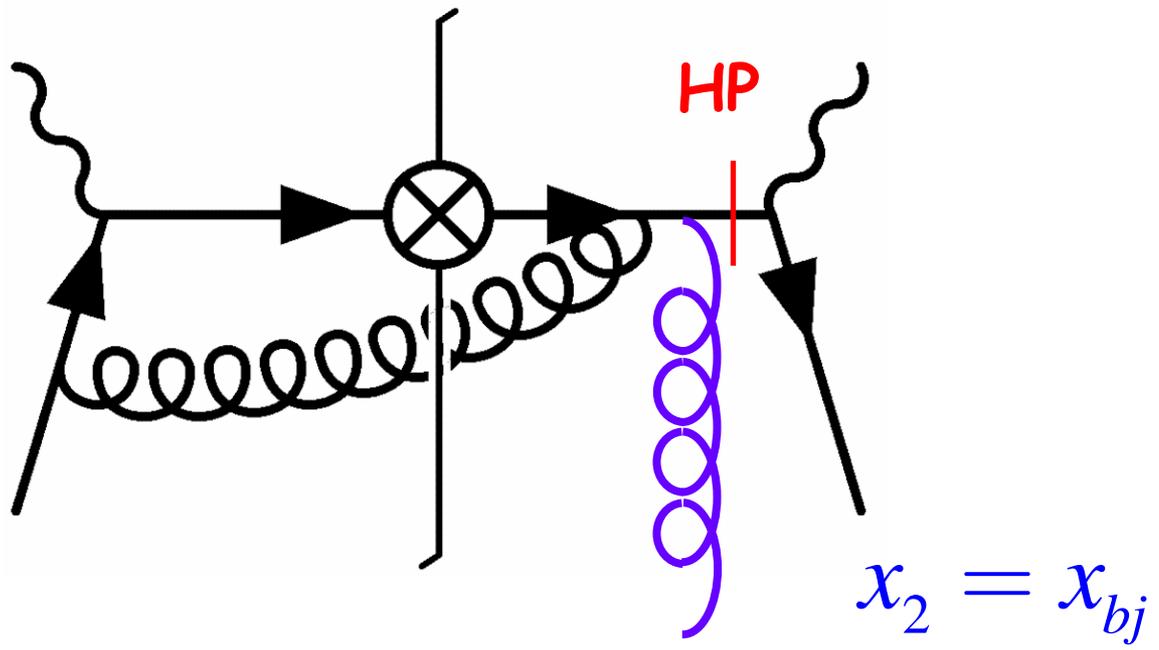
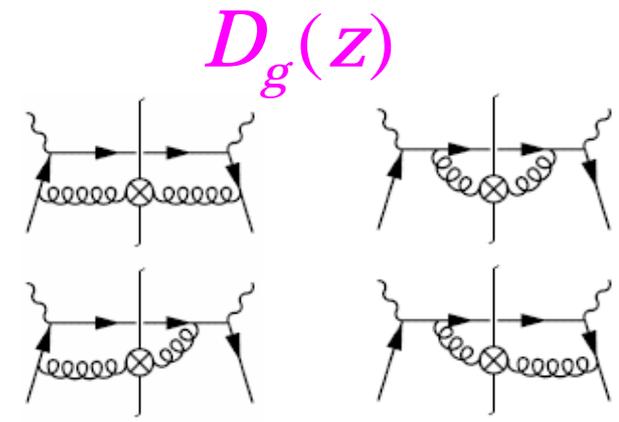
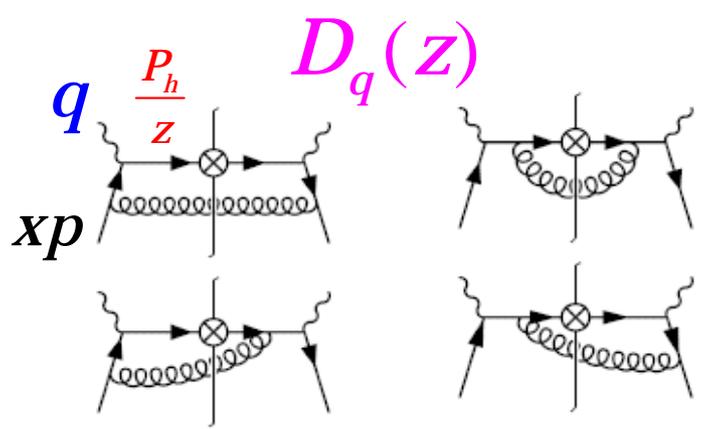


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twist-2

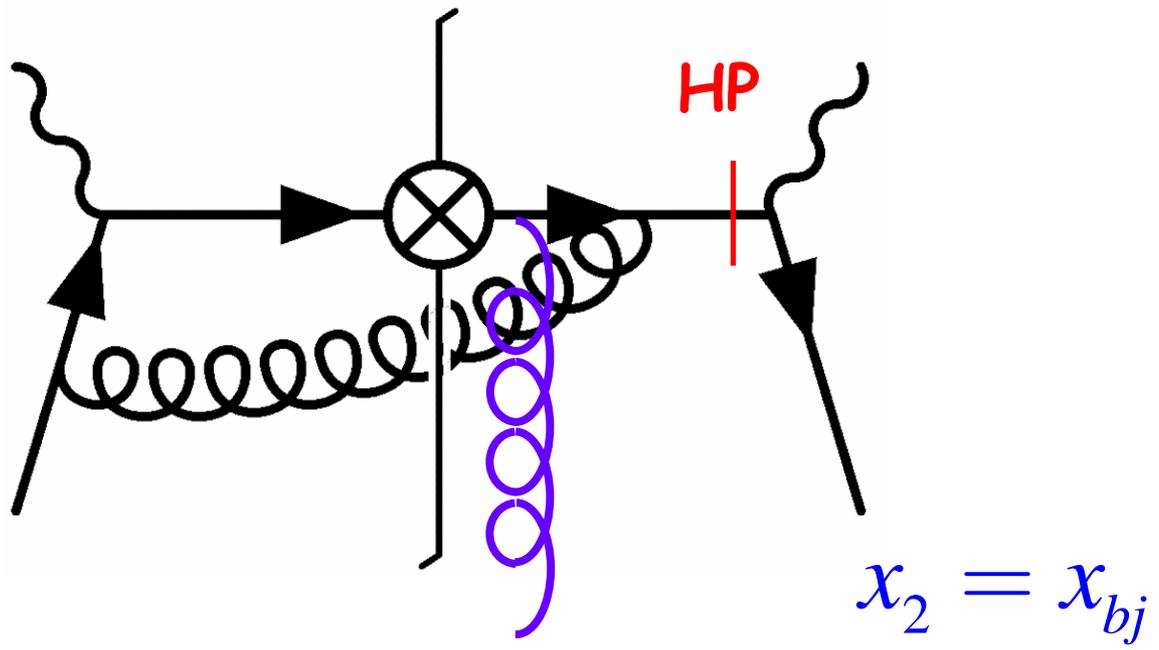
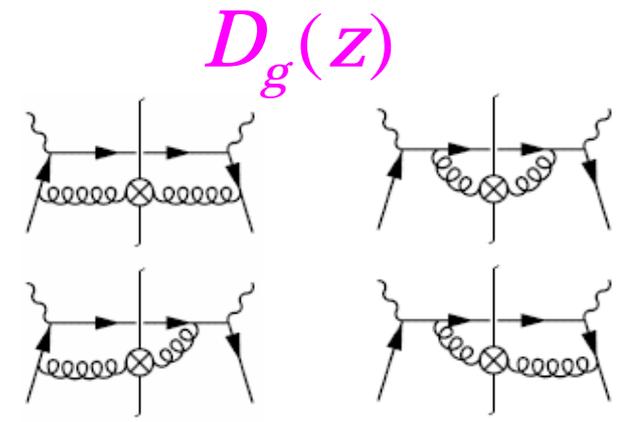
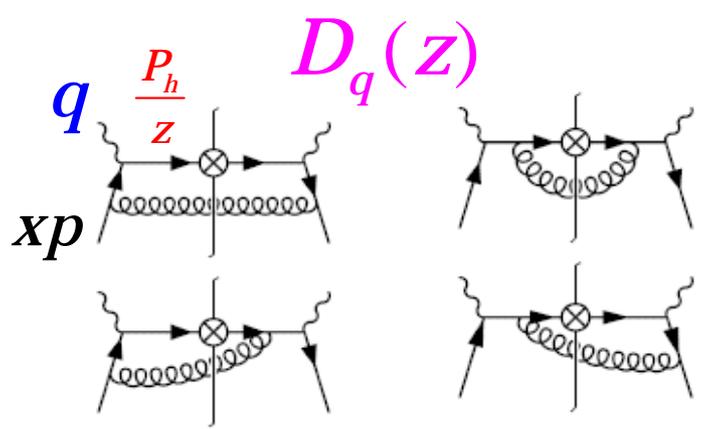


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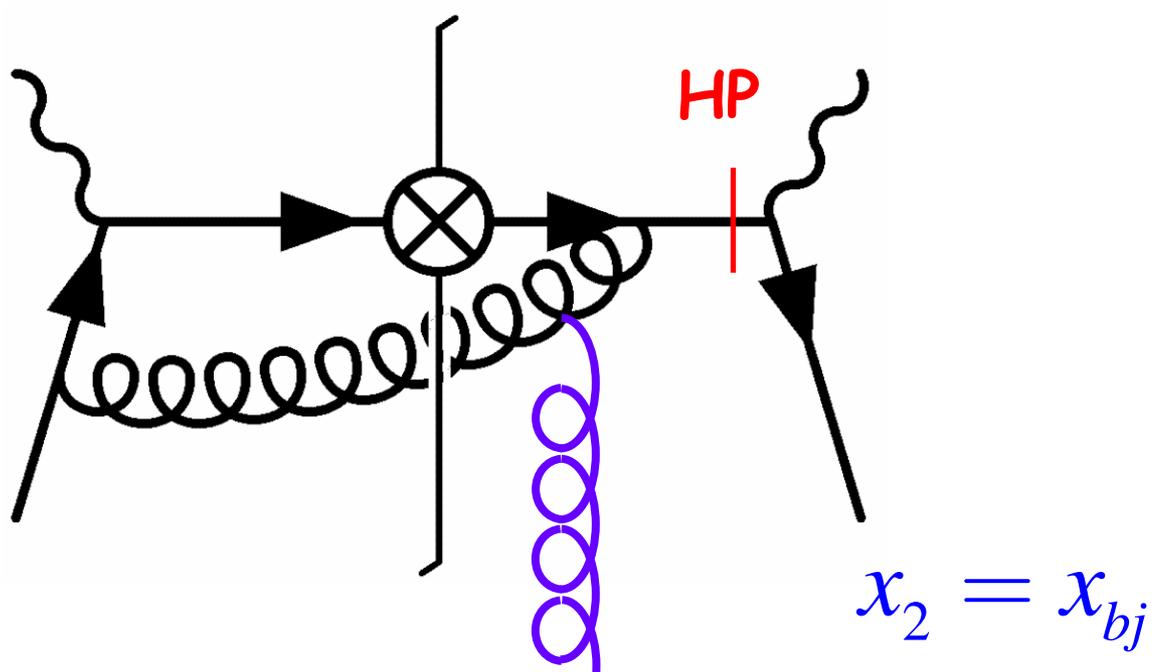
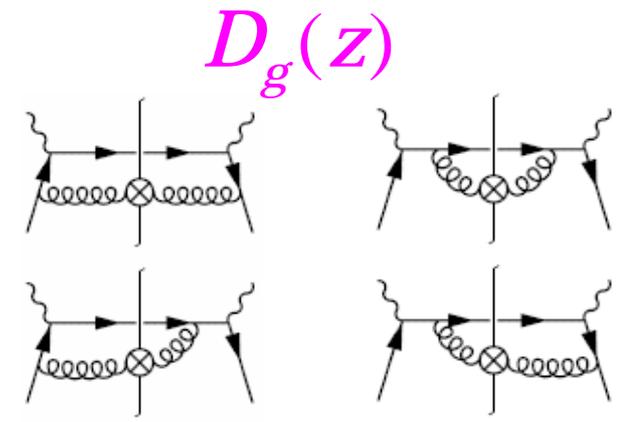
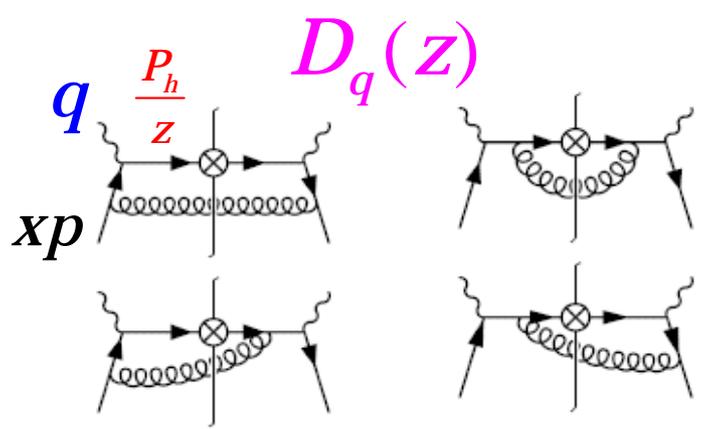


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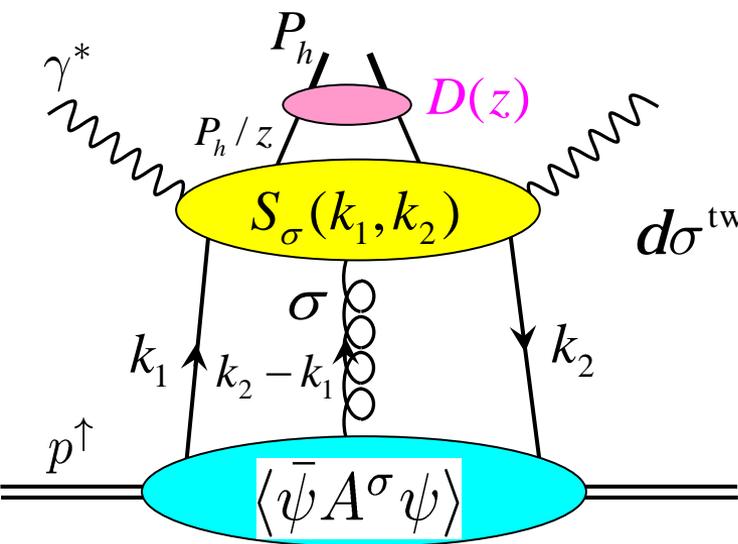
twist-2



$$\frac{1}{k^2 + i\epsilon} = \text{P} \frac{1}{k^2} - i\pi\delta(k^2)$$

Total twist-3 contribution can be written in a gauge-invariant way:

Eguchi, Koike, Tanaka, NPB763 ('07) 198



$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \Big|_{k_i=x_i p} \right] \not{p}$$

$$\otimes (\sim) G_F(x_1, x_2) \otimes D(z)$$

$$\int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\zeta(x_2-x_1)} \langle p S_{\perp} | \bar{\psi}(0) g F^{\mu+}(\zeta n) \psi(\lambda n) | p S_{\perp} \rangle$$

$$= \frac{M_N}{4} \not{p} S_{\perp \alpha} p_{\beta} \epsilon^{\alpha\beta\mu+} G_F(x_1, x_2) + i \frac{M_N}{4} \gamma_5 \not{p} S_{\perp}^{\mu} \tilde{G}_F(x_1, x_2)$$

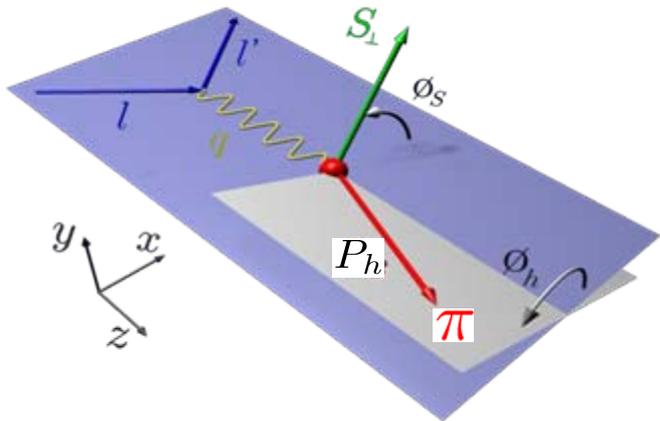
$S_{\sigma}(k_1, k_2)$  in Feynman gauge calculation

$$(1) \quad \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_1^{\alpha}} \Big|_{k_i=x_i p} = - \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_2^{\alpha}} \Big|_{k_i=x_i p}$$

Ward Identity:

$$(2) \quad (x_2 - x_1) \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_2^{\alpha}} \Big|_{k_i=x_i p} + S_{\alpha}(x_1 p, x_2 p) = 0$$

Kinematics for  $e(\ell) + p(p) \rightarrow e(\ell') + \pi(P_h) + X$



$$S_{ep} = (\ell + p)^2, \quad q = \ell - \ell', \quad x_{bj} = \frac{Q^2}{2p \cdot q}, \quad z_f = \frac{p \cdot P_h}{p \cdot q}$$

$P_{h\perp}$  :  $\perp$ -mom. of final  $\pi$

$\phi_h$ : azimuth. angle of hadron plane

$\phi_S$ : azimuth. angle of  $\vec{S}_\perp$

“Trento convention”

- twist-2 unpol. cross section

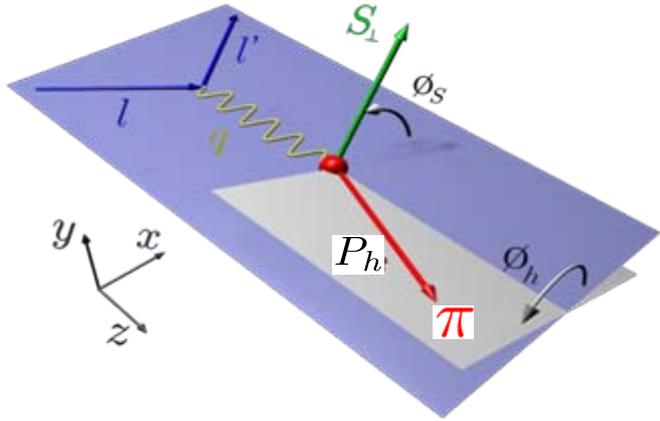
$$\frac{d^5 \sigma^{\text{tw}2}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sigma_1^{\text{tw}2} + \sigma_2^{\text{tw}2} \cos(\phi_h) + \sigma_3^{\text{tw}2} \cos(2\phi_h)$$

- twist-3 single-spin cross section

$$\frac{d^5 \sigma^{\text{tw}3}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sin(\phi_h - \phi_S) [\sigma_1^{\text{tw}3} + \sigma_2^{\text{tw}3} \cos(\phi_h) + \sigma_3^{\text{tw}3} \cos(2\phi_h)]$$

Eguchi, Koike, Tanaka, NPB752 ('06) 1; NPB763 ('07) 198

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$\phi_h$ : azimuth. angle of hadron plane

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“Trento convention”

- twist-2 unpol. cross section

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- twist-3 single-spin cross section

$$\frac{d^5 \sigma^{\text{tw}3}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sin(\phi_h - \phi_S) \left[ \sigma_1^{\text{tw}3} + \sigma_2^{\text{tw}3} \cos(\phi_h) + \sigma_3^{\text{tw}3} \cos(2\phi_h) \right]$$

update #1  $\rightarrow$   $+ \cos(\phi_h - \phi_S) \left[ \sigma_4^{\text{tw}3} \sin(\phi_h) + \sigma_5^{\text{tw}3} \sin(2\phi_h) \right]$

update #2: new partonic subprocesses

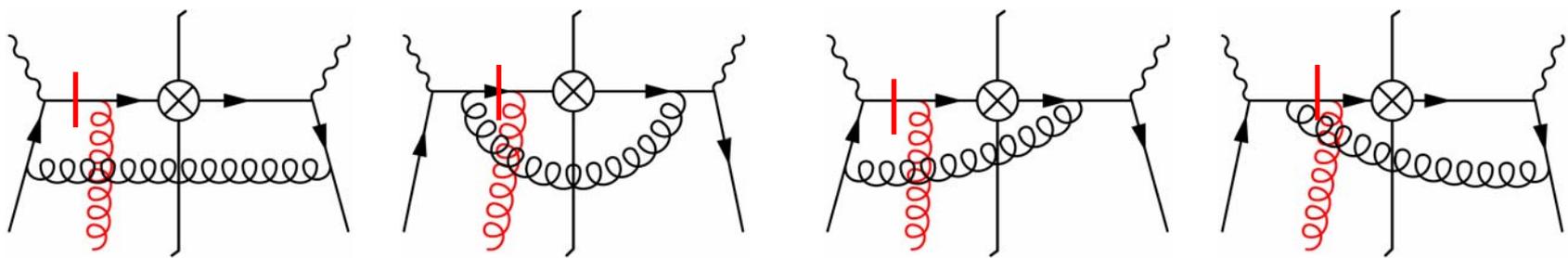
(no counterpart in TMD approach)

# Relevant diagrams for SGP

$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \right]_{k_i = x_i p} \not{p} \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

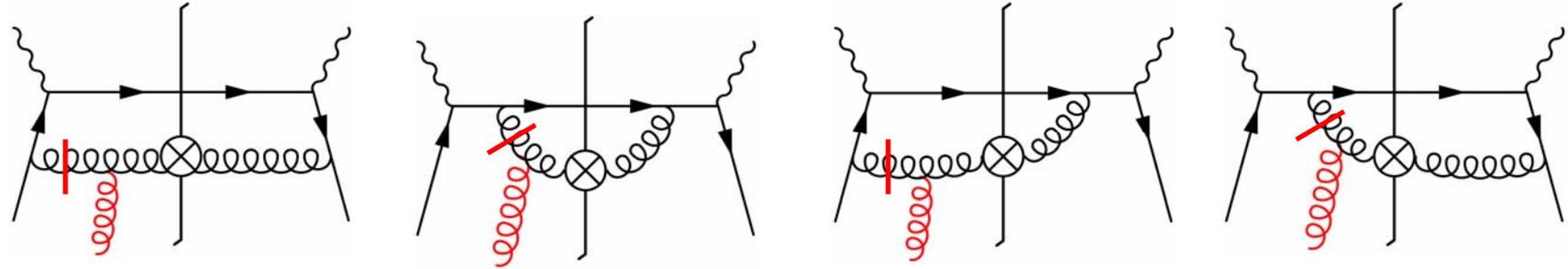
$$\frac{d}{dx} G_F(x, x), G_F(x, x) \quad (\tilde{G}_F(x, x) = 0)$$

$D_q(z)$



+mirror diagrams

$D_g(z)$



+mirror diagrams

$$\frac{d\sigma_{\text{SGP}}^{\text{tw}3}}{dx_{bj}dQ^2dz_fdq_T^2d\phi} = \frac{\alpha_{em}^2\alpha_s e_q^2}{8\pi x_{bj}^2 S_{ep}^2 Q^2} \frac{\pi M_N}{C_F} \sum_{j=q,g} \mathcal{C}_j$$

$$\times \int \frac{dz}{z} \int \frac{dx}{x} D_j(z) \left[ \frac{q_T}{Q^2} \sin(\phi_h - \phi_S) \sum_{k=1}^4 \mathcal{A}_k \left\{ \frac{\hat{x}}{1-\hat{z}} \hat{\sigma}_k^{jq} x \frac{dG_F^q(x,x)}{dx} \right. \right.$$

$$\left. \left. + \left( \frac{1}{\hat{z}} Q^2 \frac{\partial \hat{\sigma}_k^{jq}}{\partial q_T^2} - \frac{\hat{x}}{1-\hat{z}} \frac{\partial(\hat{x} \hat{\sigma}_k^{jq})}{\partial \hat{x}} \right) G_F^q(x,x) \right\} \right.$$

$$\left. - \frac{\cos(\phi_h - \phi_S)}{\hat{z} q_T} \left( \frac{1}{2} \mathcal{A}_8 \hat{\sigma}_3^{jq} + \mathcal{A}_9 \hat{\sigma}_4^{jq} \right) G_F^q(x,x) \right] \delta \left( \frac{q_T^2}{Q^2} - \left( \frac{1}{\hat{x}} - 1 \right) \left( \frac{1}{\hat{z}} - 1 \right) \right).$$

$$\hat{x} = \frac{x_{bj}}{x}, \quad \hat{z} = \frac{z_f}{z}$$

$$\mathcal{C}_q = \frac{1}{2N_c}, \quad \mathcal{C}_g = \frac{N_c}{2}$$

$$P_{h\perp} \equiv z_f q_T$$

$$\mathcal{A}_1 = 1 + \cosh^2 \psi, \quad \mathcal{A}_2 = -2, \quad \mathcal{A}_3 = -\cos \phi_h \sinh 2\psi, \quad \mathcal{A}_4 = \cos 2\phi_h \sinh^2 \psi,$$

$$\mathcal{A}_8 = -\sin \phi_h \sinh 2\psi, \quad \mathcal{A}_9 = \sin 2\phi_h \sinh^2 \psi.$$

$$\text{with } \cosh \psi = 2x_{bj} S_{ep} / Q^2 - 1.$$

$$\frac{d^5 \sigma_{\text{unpol}}^{\text{tw}2}}{dx_{bj} dQ^2 dz_f dq_T^2 d\phi_h} = \frac{\alpha_{em}^2 \alpha_s}{8\pi x_{bj}^2 S_{ep}^2 Q^2} \sum_{\text{flavors}} e_f^2 \sum_{k=1}^4 \mathcal{A}_k$$

$$\times \int \frac{dx}{x} \frac{dz}{z} \left[ \hat{\sigma}_k^{qq} D_q(z) q(x) + \hat{\sigma}_k^{gq} D_g(z) q(x) \right] \delta \left( \frac{q_T^2}{Q^2} - \left( \frac{1}{\hat{x}} - 1 \right) \left( \frac{1}{\hat{z}} - 1 \right) \right)$$

$$\hat{\sigma}_1^{qq} = 2C_F \hat{x} \hat{z} \left\{ \frac{1}{Q^2 q_T^2} \left( \frac{Q^4}{\hat{x}^2 \hat{z}^2} + (Q^2 - q_T^2)^2 \right) + 6 \right\}, \quad \hat{\sigma}_2^{qq} = 2\hat{\sigma}_4^{qq} = 8C_F \hat{x} \hat{z}, \quad \hat{\sigma}_3^{qq} = 4C_F \hat{x} \hat{z} \frac{Q^2 + q_T^2}{Q q_T}$$

$$\hat{\sigma}_1^{gq} = 2C_F \hat{x} (1 - \hat{z}) \left\{ \frac{1}{Q^2 q_T^2} \left( \frac{Q^4}{\hat{x}^2 \hat{z}^2} + \frac{(1 - \hat{z})^2}{\hat{z}^2} \left( Q^2 - \frac{\hat{z}^2 q_T^2}{(1 - \hat{z})^2} \right)^2 \right) + 6 \right\}, \quad \hat{\sigma}_2^{gq} = \dots$$

twist-2 hard cross section

$$\begin{aligned} \frac{d\sigma_{SGP}^{tw3}}{[d\omega]} &= \frac{\pi M_N}{2C_F} \epsilon^{\sigma pn S_\perp} \sum_{j=q,g} c_j \int \frac{dz}{z} \int \frac{dx}{x} D_j(z) \frac{\partial H_{jq}(x, z, q_T^2)}{\partial (P_{h\perp}^\sigma/z)} G_F^q(x, x) \\ &= \frac{\pi M_N}{C_F z_f^2} \left( \epsilon^{pn S_\perp P_{h\perp}} \frac{\partial}{\partial q_T^2} - \boxed{(P_{h\perp} \cdot S_\perp) \frac{1}{2q_T^2} \frac{\partial}{\partial \phi_h}} \right) \frac{d\sigma_{unpol}^{tw2}}{[d\omega]} \Bigg|_{f_q(x) \rightarrow G_F^q(x, x), D_j(z) \rightarrow C_j z D_j(z)} \end{aligned}$$

$$[d\omega] \equiv dx_{bj} dQ^2 dz_f dq_T^2 d\phi_h$$

$$\mathcal{A}_1 = 1 + \cosh^2 \psi, \mathcal{A}_2 = -2, \mathcal{A}_3 = -\cos \phi_h \sinh 2\psi, \mathcal{A}_4 = \cos 2\phi_h \sinh^2 \psi,$$

$$\mathcal{A}_8 = -\sin \phi_h \sinh 2\psi, \mathcal{A}_9 = \sin 2\phi_h \sinh^2 \psi.$$

$$\text{with } \cosh \psi = 2x_{bj} S_{ep}/Q^2 - 1.$$

$$\frac{d^5 \sigma_{unpol}^{tw2}}{dx_{bj} dQ^2 dz_f dq_T^2 d\phi_h} = \frac{\alpha_{em}^2 \alpha_s}{8\pi x_{bj}^2 S_{ep}^2 Q^2} \sum_{\text{flavors}} e_f^2 \sum_{k=1}^4 \mathcal{A}_k$$

$$\times \int \frac{dx}{x} \frac{dz}{z} \left[ \hat{\sigma}_k^{qq} D_q(z) q(x) + \hat{\sigma}_k^{gq} D_g(z) q(x) \right] \delta \left( \frac{q_T^2}{Q^2} - \left( \frac{1}{\hat{x}} - 1 \right) \left( \frac{1}{\hat{z}} - 1 \right) \right)$$

$$\hat{\sigma}_1^{qq} = 2C_F \hat{x} \hat{z} \left\{ \frac{1}{Q^2 q_T^2} \left( \frac{Q^4}{\hat{x}^2 \hat{z}^2} + (Q^2 - q_T^2)^2 \right) + 6 \right\}, \hat{\sigma}_2^{qq} = 2\hat{\sigma}_4^{qq} = 8C_F \hat{x} \hat{z}, \hat{\sigma}_3^{qq} = 4C_F \hat{x} \hat{z} \frac{Q^2 + q_T^2}{Q q_T}$$

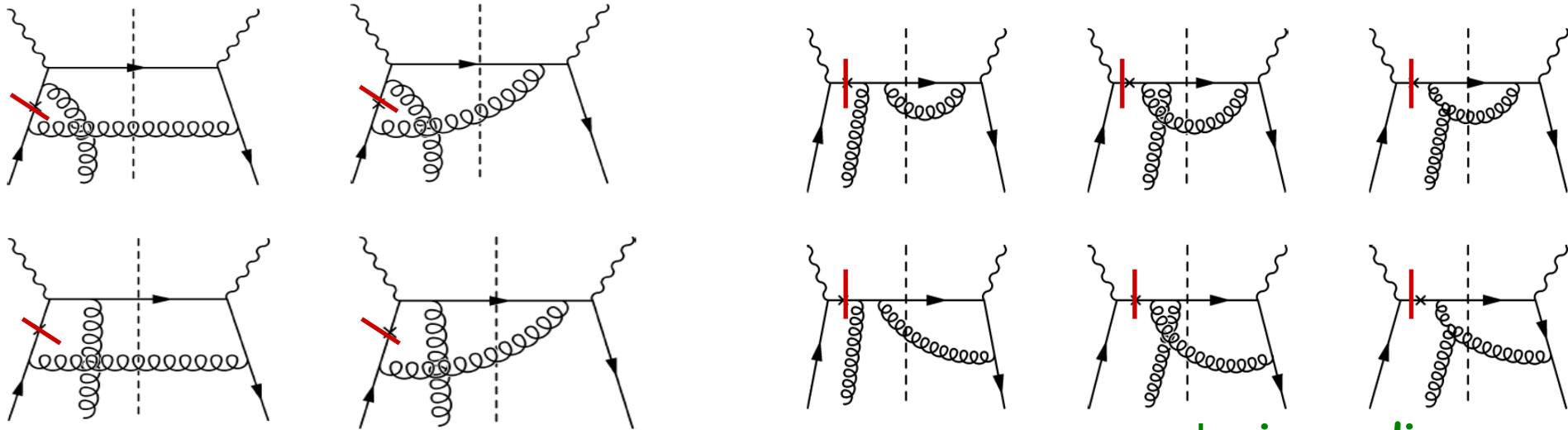
$$\hat{\sigma}_1^{gq} = 2C_F \hat{x} (1 - \hat{z}) \left\{ \frac{1}{Q^2 q_T^2} \left( \frac{Q^4}{\hat{x}^2 \hat{z}^2} + \frac{(1 - \hat{z})^2}{\hat{z}^2} \left( Q^2 - \frac{\hat{z}^2 q_T^2}{(1 - \hat{z})^2} \right)^2 \right) + 6 \right\}, \hat{\sigma}_2^{gq} = \dots$$

# Relevant diagrams for SFP & HP

$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \right]_{k_i = x_i p} \not{p} \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

$$(x_2 - x_1) \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_2^{\alpha}} \Big|_{k_i = x_i p} + S_{\alpha}(x_1 p, x_2 p) = 0$$

$$\begin{array}{lll} G_F(0, X) & \tilde{G}_F(0, X) & \text{SFP} \\ G_F(X_{bj}, X) & \tilde{G}_F(X_{bj}, X) & \text{HP} \end{array}$$



+mirror diagrams

+mirror diagrams

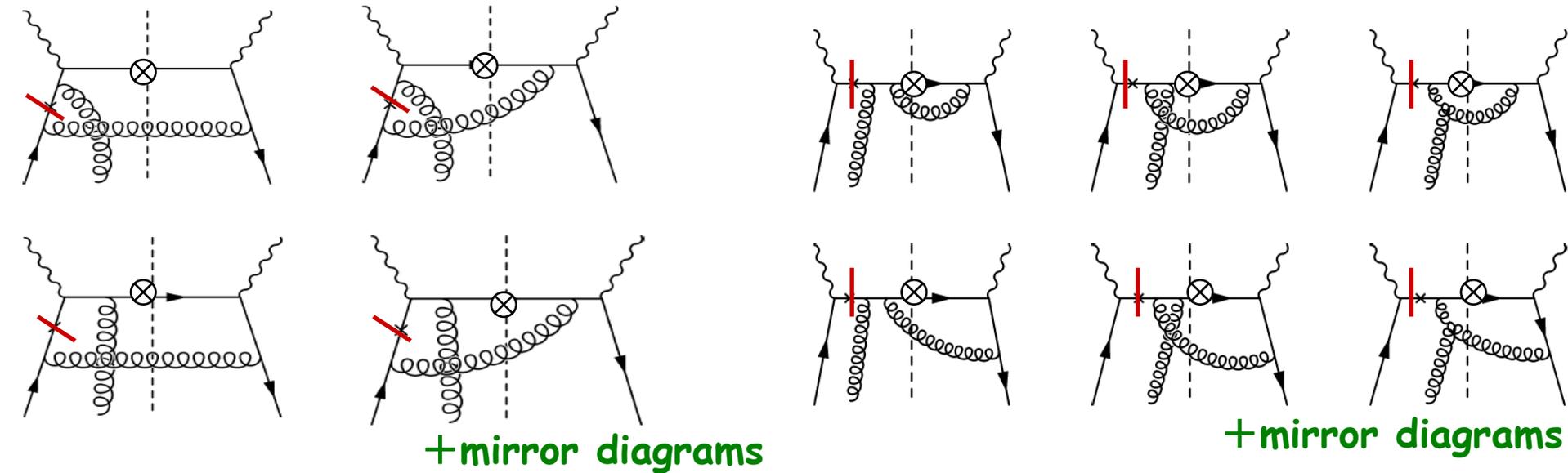
# Relevant diagrams for SFP & HP

$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \right]_{k_i = x_i p} \not{p} \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

$$(x_2 - x_1) \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_2^{\alpha}} \Big|_{k_i = x_i p} + S_{\alpha}(x_1 p, x_2 p) = 0$$

$$\begin{array}{ccc} G_F(0, X) & \tilde{G}_F(0, X) & \text{SFP} \\ G_F(X_{bj}, X) & \tilde{G}_F(X_{bj}, X) & \text{HP} \end{array}$$

$D_q(z)$ :



$$\sin(\phi_h - \phi_S) \left[ \sigma_1^{\text{tw}3} + \sigma_2^{\text{tw}3} \cos(\phi_h) + \sigma_3^{\text{tw}3} \cos(2\phi_h) \right]$$

$$+ \cos(\phi_h - \phi_S) \left[ \sigma_4^{\text{tw}3} \sin(\phi_h) + \sigma_5^{\text{tw}3} \sin(2\phi_h) \right]$$

$$\sigma_2^{\text{tw}3, \text{HP}} = \sigma_4^{\text{tw}3, \text{HP}}$$

$$\sigma_3^{\text{tw}3, \text{HP}} = \sigma_5^{\text{tw}3, \text{HP}}$$

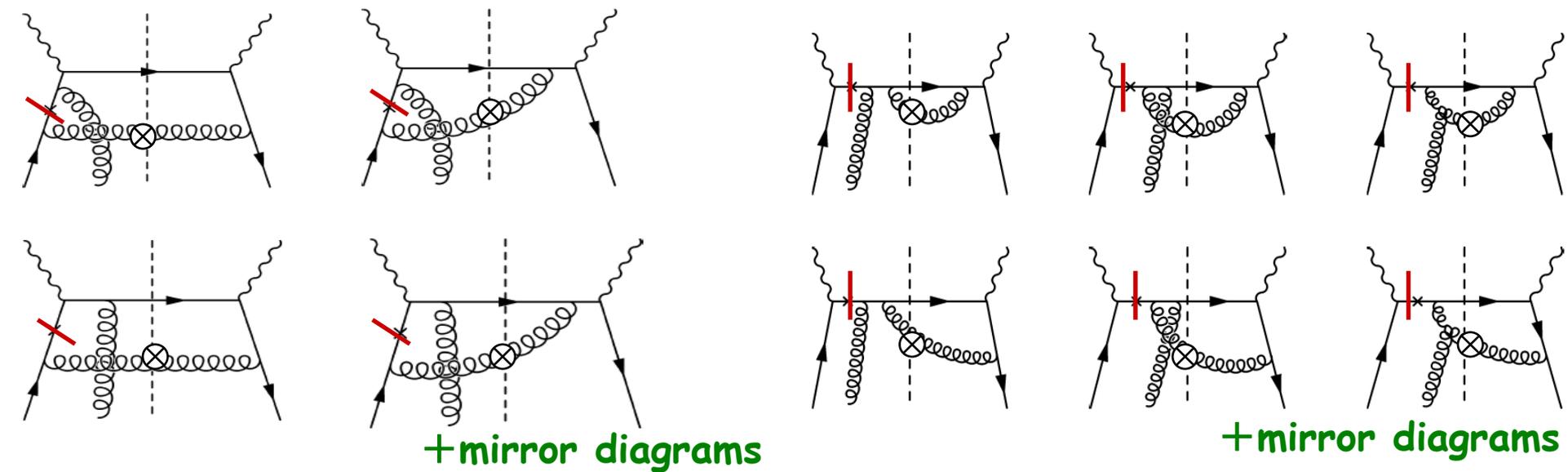
# Relevant diagrams for SFP & HP

$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \right]_{k_i = x_i p} \not{p} \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

$$(x_2 - x_1) \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_2^{\alpha}} \Big|_{k_i = x_i p} + S_{\alpha}(x_1 p, x_2 p) = 0$$

$$\begin{array}{ccc} G_F(0, X) & \tilde{G}_F(0, X) & \text{SFP} \\ G_F(X_{bj}, X) & \tilde{G}_F(X_{bj}, X) & \text{HP} \end{array}$$

$D_g(z)$ :



$$\begin{aligned} & \sin(\phi_h - \phi_S) \left[ \sigma_1^{\text{tw}3} + \sigma_2^{\text{tw}3} \cos(\phi_h) + \sigma_3^{\text{tw}3} \cos(2\phi_h) \right] \\ & + \cos(\phi_h - \phi_S) \left[ \sigma_4^{\text{tw}3} \sin(\phi_h) + \sigma_5^{\text{tw}3} \sin(2\phi_h) \right] \end{aligned}$$

$$\begin{aligned} \sigma_2^{\text{tw}3, \text{HP}} &= \sigma_4^{\text{tw}3, \text{HP}} \\ \sigma_3^{\text{tw}3, \text{HP}} &= \sigma_5^{\text{tw}3, \text{HP}} \end{aligned}$$

**update #2: new partonic subprocesses**

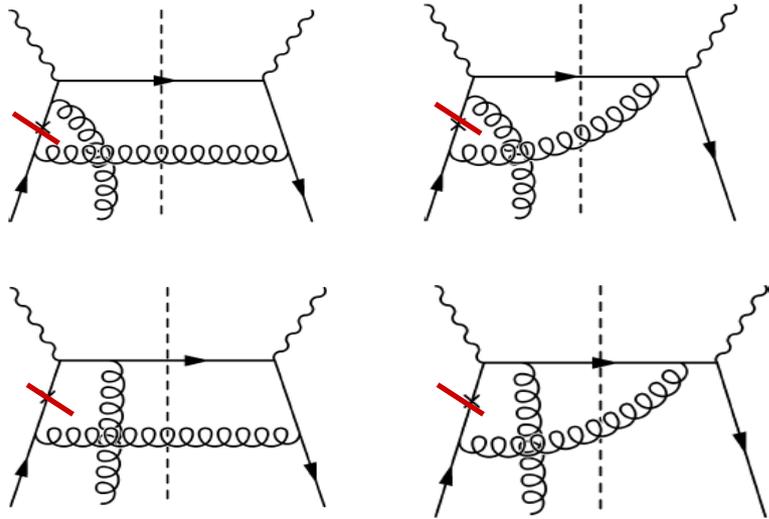
# new partonic subprocesses for SFP

$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \right]_{k_i = x_i p}$$

$$\not{p} \left[ \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z) \right]$$



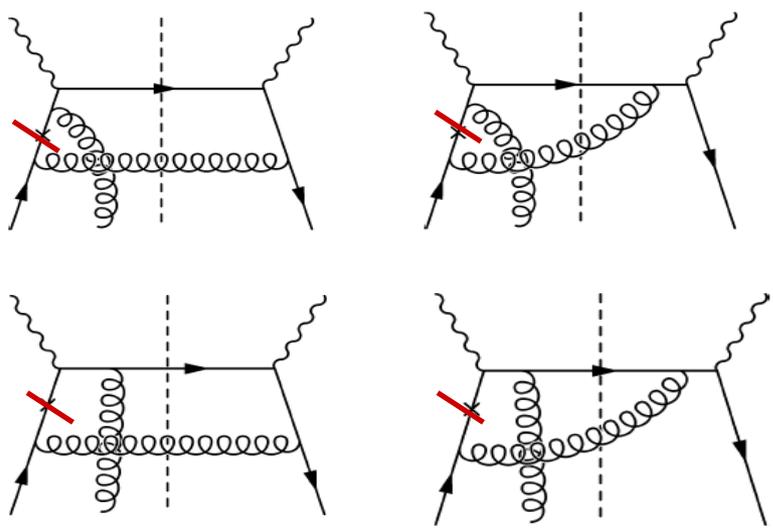
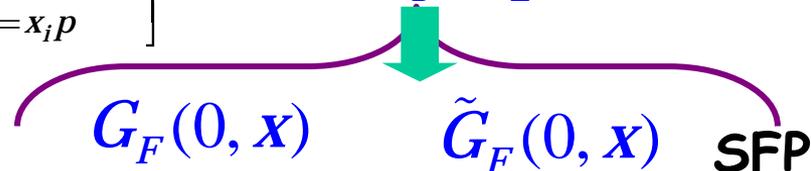
$G_F(0, x) \quad \tilde{G}_F(0, x) \quad \text{SFP}$



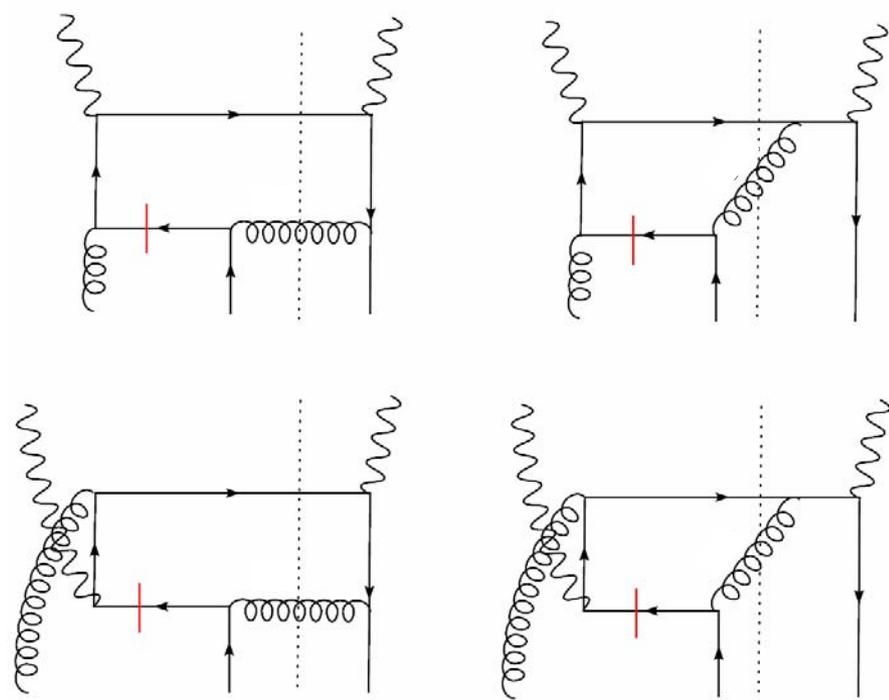
+mirror diagrams

# new partonic subprocesses for SFP

$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \right]_{k_i = x_i p} \not{p} \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$



+mirror diagrams

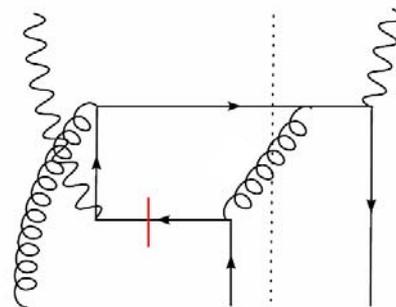
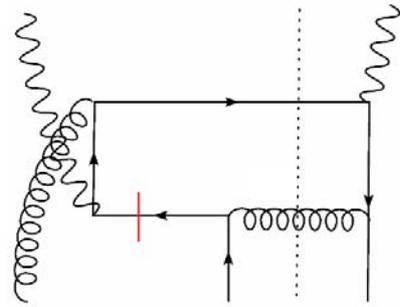
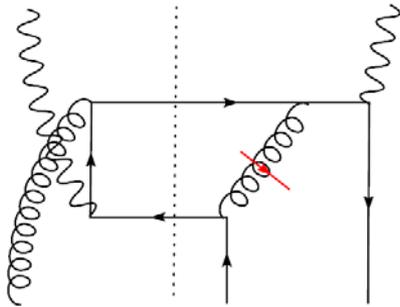
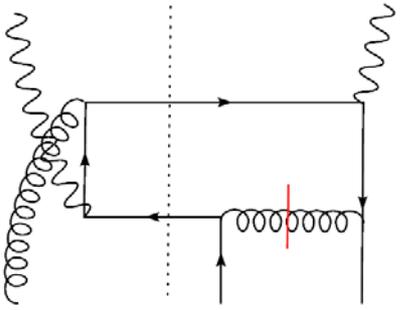
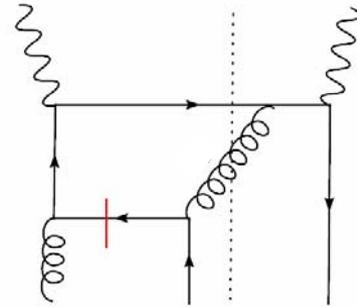
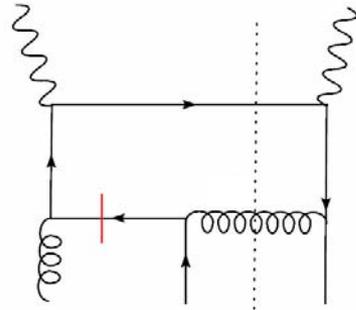
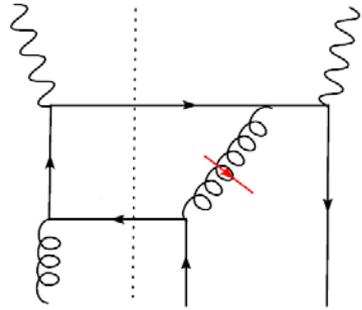
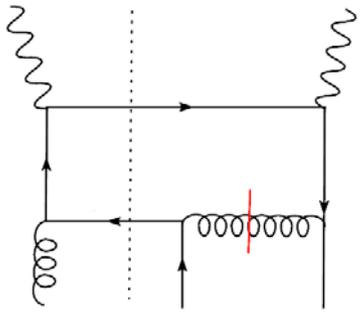


+mirror diagrams

# new partonic subprocesses for SFP

$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \Big|_{k_i = x_i p} \not{p} \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z) \right]$$

$G_F(0, x) \quad \tilde{G}_F(0, x) \quad \text{SFP}$



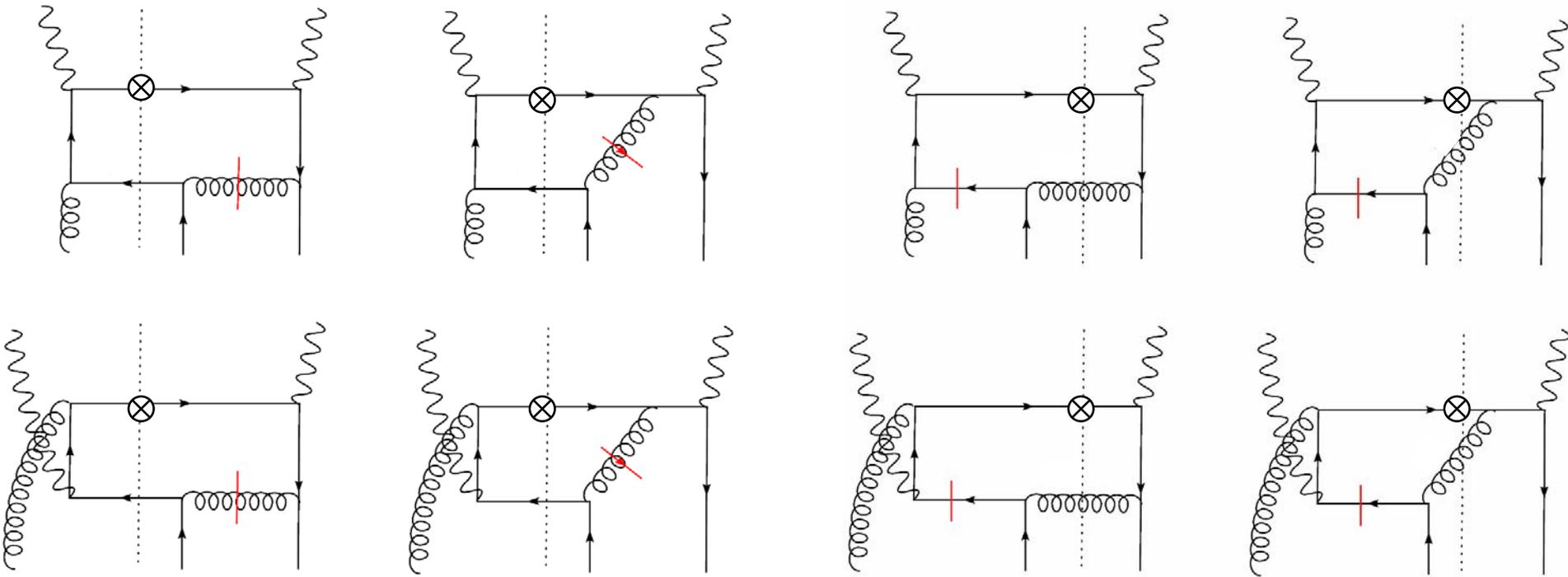
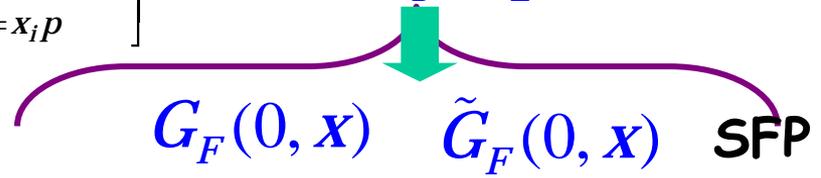
+mirror diagrams

+mirror diagrams

# new partonic subprocesses for SFP

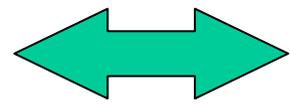
$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \right]_{k_i = x_i p} \not{p} \otimes \tilde{G}_F(x_1, x_2) \otimes D(z)$$

$D_q(z)$ :



+mirror diagrams

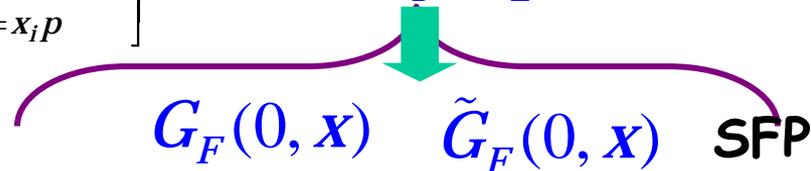
+mirror diagrams



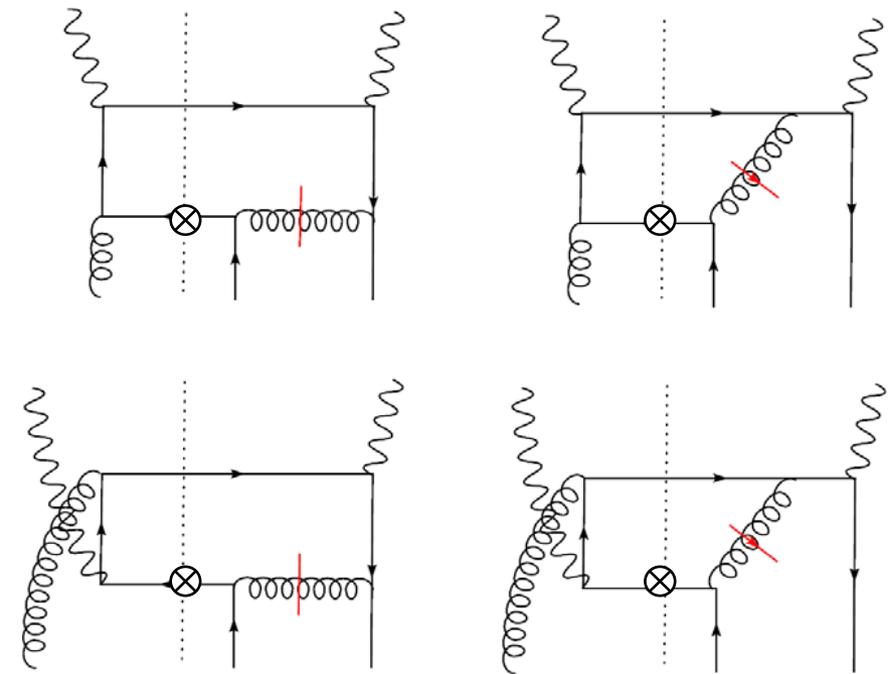
**Cancel!!**

# new partonic subprocesses for SFP

$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial \mathbf{S}_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \right]_{k_i = x_i p} \not{p} \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

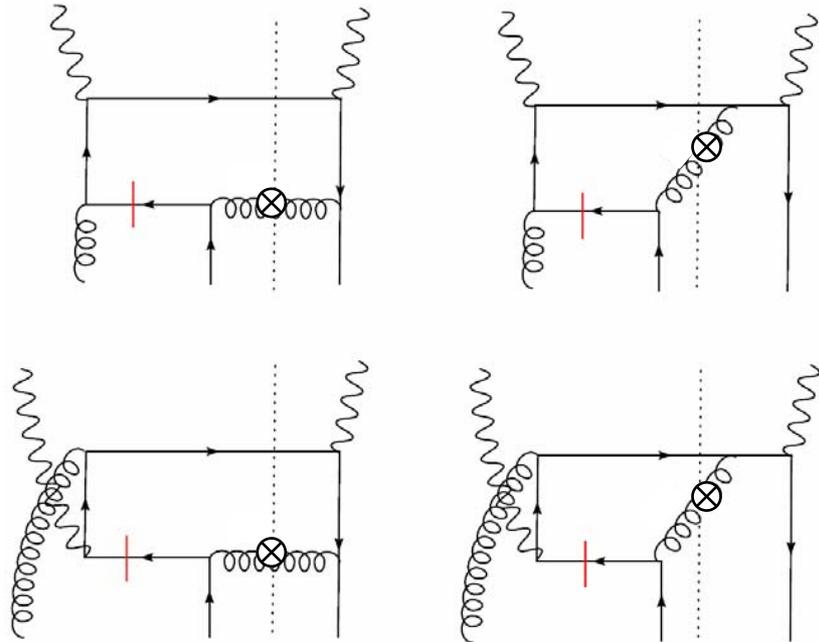


$D_{\bar{q}}(z)$ :



+mirror diagrams

$D_g(z)$ :



+mirror diagrams

$$\hat{\sigma}_{\text{SFP}}(\bar{q} \rightarrow \pi) = -\hat{\sigma}_{\text{SFP}}(g \rightarrow \pi)$$

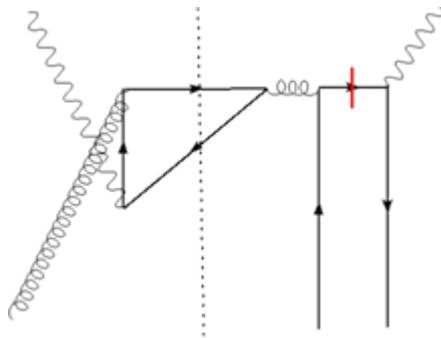
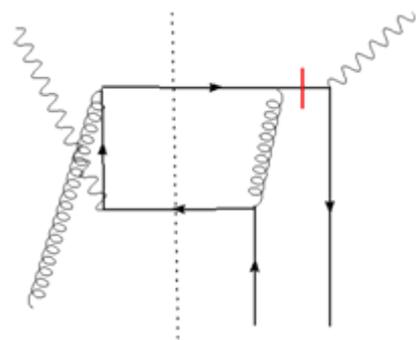
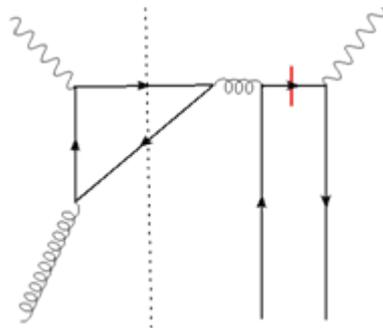
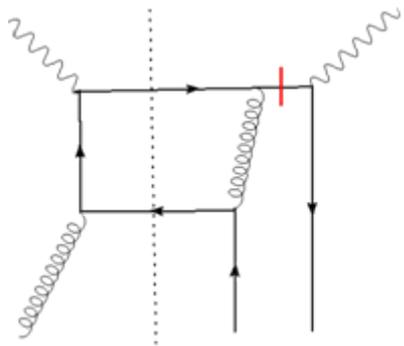
$$\begin{aligned} & \sin(\phi_h - \phi_S) \left[ \sigma_1^{\text{tw}3} + \sigma_2^{\text{tw}3} \cos(\phi_h) + \sigma_3^{\text{tw}3} \cos(2\phi_h) \right] \\ & + \cos(\phi_h - \phi_S) \left[ \sigma_4^{\text{tw}3} \sin(\phi_h) + \sigma_5^{\text{tw}3} \sin(2\phi_h) \right] \end{aligned}$$

# new partonic subprocesses for HP

$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \Big|_{k_i = x_i p} \right] \not{p} \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

$$G_F(x_{bj}, x_{bj} - x) \quad \tilde{G}_F(x_{bj}, x_{bj} - x)$$

$$x_{bj} - x < 0$$

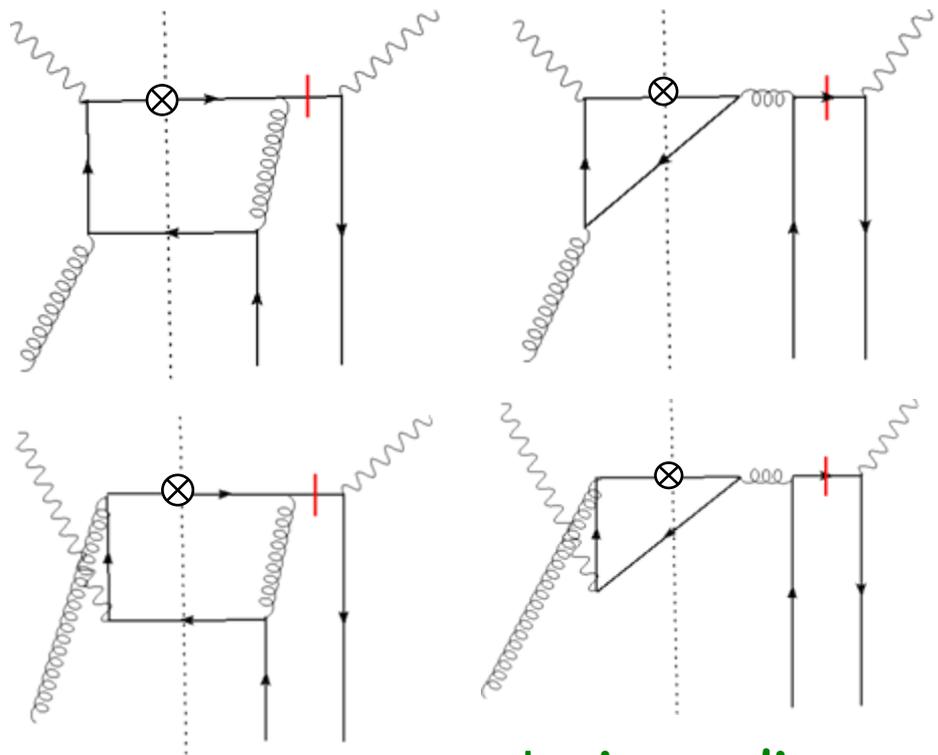


+ mirror diagrams

# new partonic subprocesses for HP

$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \Big|_{k_i = x_i p} \right] \otimes \tilde{G}_F(x_1, x_2) \otimes D(z)$$

$D_q(z)$ :



+ mirror diagrams

$$G_F(x_{bj}, x_{bj} - x) \quad \tilde{G}_F(x_{bj}, x_{bj} - x)$$

$$x_{bj} - x < 0$$

$$\begin{aligned} & \sin(\phi_h - \phi_S) \left[ \sigma_1^{\text{tw}3} + \sigma_2^{\text{tw}3} \cos(\phi_h) \right. \\ & \quad \left. + \sigma_3^{\text{tw}3} \cos(2\phi_h) \right] \\ & + \cos(\phi_h - \phi_S) \left[ \sigma_4^{\text{tw}3} \sin(\phi_h) + \sigma_5^{\text{tw}3} \sin(2\phi_h) \right] \end{aligned}$$

$$\sigma_2^{\text{tw}3, \text{HP}} \neq \sigma_4^{\text{tw}3, \text{HP}}$$

$$\sigma_3^{\text{tw}3, \text{HP}} \neq \sigma_5^{\text{tw}3, \text{HP}}$$

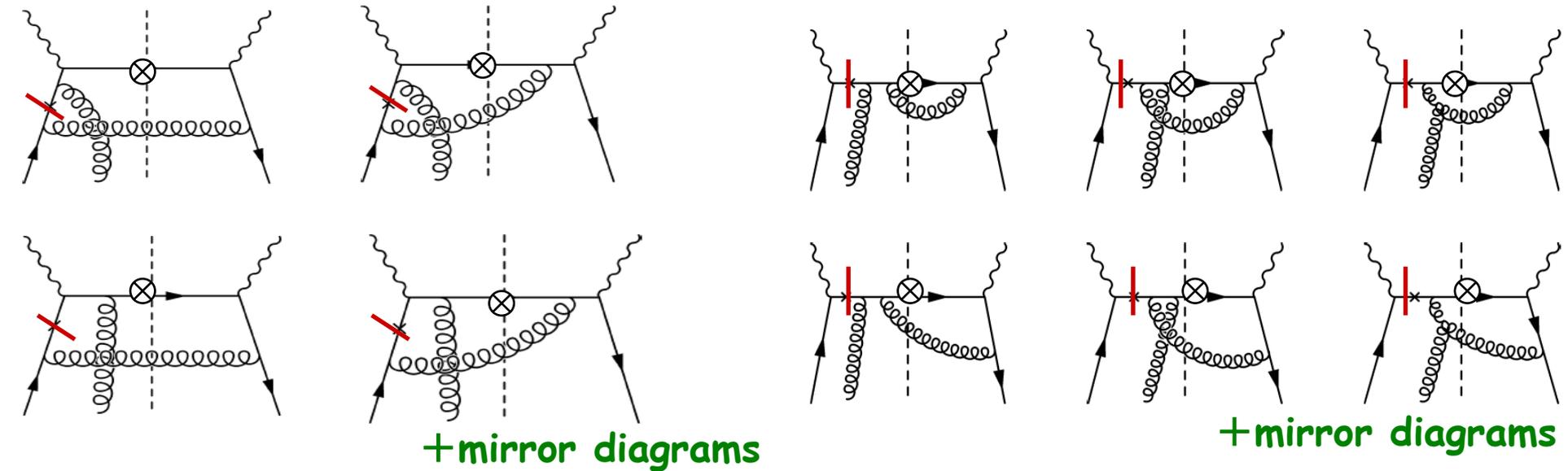
# Relevant diagrams for SFP & HP

$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \right]_{k_i = x_i p} \not{p} \otimes \overset{(\sim)}{G}_F(x_1, x_2) \otimes D(z)$$

$$(x_2 - x_1) \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_2^{\alpha}} \Big|_{k_i = x_i p} + S_{\alpha}(x_1 p, x_2 p) = 0$$

$$\begin{array}{ccc} G_F(0, X) & \tilde{G}_F(0, X) & \text{SFP} \\ G_F(X_{bj}, X) & \tilde{G}_F(X_{bj}, X) & \text{HP} \end{array}$$

$D_q(z)$ :



$$\sin(\phi_h - \phi_S) \left[ \sigma_1^{\text{tw}3} + \sigma_2^{\text{tw}3} \cos(\phi_h) + \sigma_3^{\text{tw}3} \cos(2\phi_h) \right]$$

$$+ \cos(\phi_h - \phi_S) \left[ \sigma_4^{\text{tw}3} \sin(\phi_h) + \sigma_5^{\text{tw}3} \sin(2\phi_h) \right]$$

$$\sigma_2^{\text{tw}3, \text{HP}} = \sigma_4^{\text{tw}3, \text{HP}}$$

$$\sigma_3^{\text{tw}3, \text{HP}} = \sigma_5^{\text{tw}3, \text{HP}}$$

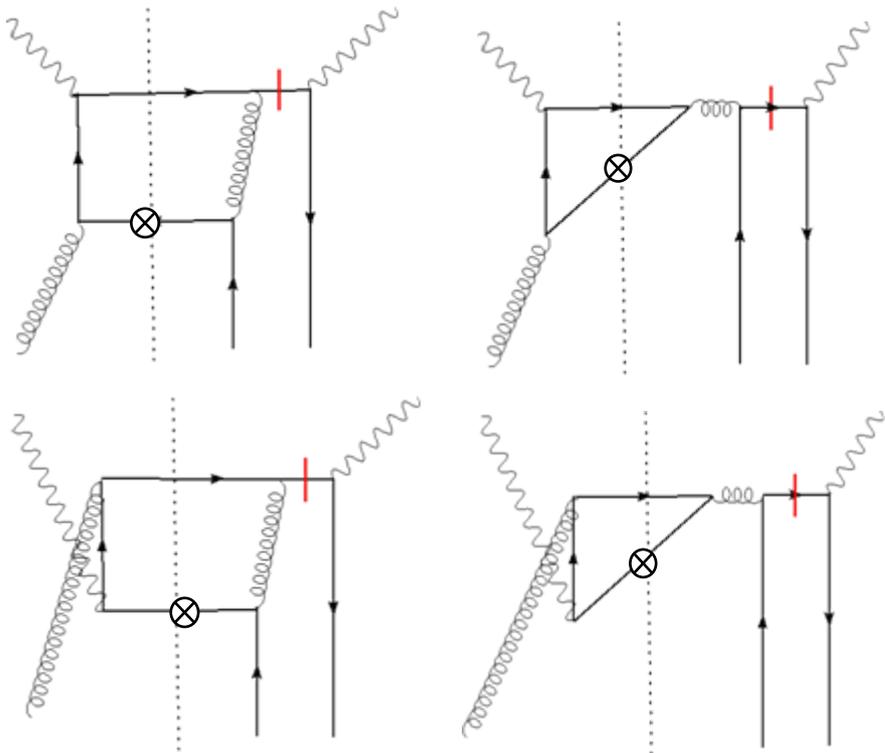
# new partonic subprocesses for HP

$$d\sigma^{\text{tw}3} \sim \epsilon^{\alpha p n S_{\perp}} \int dx_1 dx_2 dz \text{Tr} \left[ \frac{\partial S_{\sigma}(k_1, k_2) p^{\sigma}}{\partial k_{2\perp}^{\alpha}} \Big|_{k_i = x_i p} \right] \otimes \tilde{G}_F(x_1, x_2) \otimes D(z)$$

$D_{\bar{q}}(z)$ :

$$G_F(x_{bj}, x_{bj} - x) \quad \tilde{G}_F(x_{bj}, x_{bj} - x)$$

$$x_{bj} - x < 0$$



+ mirror diagrams

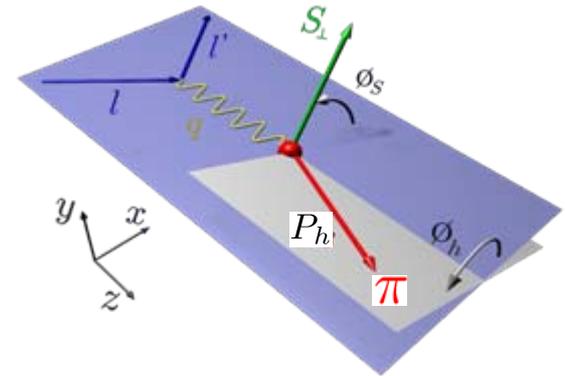
$$\begin{aligned} & \sin(\phi_h - \phi_S) \left[ \sigma_1^{\text{tw}3} + \sigma_2^{\text{tw}3} \cos(\phi_h) \right. \\ & \quad \left. + \sigma_3^{\text{tw}3} \cos(2\phi_h) \right] \\ & + \cos(\phi_h - \phi_S) \left[ \sigma_4^{\text{tw}3} \sin(\phi_h) + \sigma_5^{\text{tw}3} \sin(2\phi_h) \right] \end{aligned}$$

$$\sigma_2^{\text{tw}3, \text{HP}} \neq \sigma_4^{\text{tw}3, \text{HP}}$$

$$\sigma_3^{\text{tw}3, \text{HP}} \neq \sigma_5^{\text{tw}3, \text{HP}}$$

Complete twist-3 SSA for  $ep^\uparrow \rightarrow e\pi X$  (EKT'07 and updates # 1 & 2)

$$\begin{aligned}
 \frac{d^5\sigma^{\text{tw3}}}{dx_{bj}dQ^2dz_f dP_{h\perp}^2 d\phi} &= \frac{\alpha_{em}^2\alpha_S}{8\pi x_{bj}^2 S_{ep}^2 Q^2} \sum_k \mathcal{A}_k \mathcal{S}_k \left( \frac{-\pi M_N}{4} \right) \int \frac{dx}{x} \int \frac{dz}{z} \\
 &\times \sum_{a=q,\bar{q}} e_a^2 \sum_{j=q,\bar{q},g} D_j(z) \left[ \hat{\sigma}_{Dk}^{ja} \left( x \frac{d}{dx} G_F^a(\mathbf{x}, \mathbf{x}) \right) + \hat{\sigma}_{Gk}^{ja} G_F^a(\mathbf{x}, \mathbf{x}) + \hat{\sigma}_{Fk}^{ja} G_F^a(\mathbf{0}, \mathbf{x}) \right. \\
 &+ \left. \hat{\sigma}_{Hok}^{ja} G_F^a(\mathbf{x}_{bj}, \mathbf{x}) + \hat{\sigma}_{Hnk}^{ja} G_F^a(\mathbf{x}_{bj}, \mathbf{x}_{bj} - \mathbf{x}) \right] \delta \left( \frac{P_{h\perp}^2}{z_f^2 Q^2} - \left( 1 - \frac{1}{\hat{x}} \right) \left( 1 - \frac{1}{\hat{z}} \right) \right) + (\tilde{G}_F \text{ terms}) \\
 &= \sin(\phi_h - \phi_S) \left[ \sigma_1^{\text{tw3}} + \sigma_2^{\text{tw3}} \cos(\phi_h) + \sigma_3^{\text{tw3}} \cos(2\phi_h) \right] \\
 &\quad + \cos(\phi_h - \phi_S) \left[ \sigma_4^{\text{tw3}} \sin(\phi_h) + \sigma_5^{\text{tw3}} \sin(2\phi_h) \right] \\
 &= \sin(\phi_h - \phi_S) F^{\sin(\phi_h - \phi_S)} + \sin(2\phi_h - \phi_S) F^{\sin(2\phi_h - \phi_S)} \\
 &\quad + \sin(\phi_S) F^{\sin(\phi_S)} + \sin(3\phi_h - \phi_S) F^{\sin(3\phi_h - \phi_S)} \\
 &\quad + \sin(\phi_h + \phi_S) F^{\sin(\phi_h + \phi_S)}
 \end{aligned}$$



five independent azimuthal components similarly as in the TMD factorization approach (cf. Bachetta et. al. JHEP('07))

# Small $P_{h\perp}$ behavior: connection with TMD approach

$$\Lambda_{\text{QCD}} \ll P_{h\perp} \ll Q$$

$$\frac{d^5 \sigma^{\text{tw}3}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sin(\phi_h - \phi_S) F^{\sin(\phi_h - \phi_S)} + \sin(2\phi_h - \phi_S) F^{\sin(2\phi_h - \phi_S)} \\ + \sin(\phi_S) F^{\sin(\phi_S)} + \sin(3\phi_h - \phi_S) F^{\sin(3\phi_h - \phi_S)} + \sin(\phi_h + \phi_S) F^{\sin(\phi_h + \phi_S)}$$

$$F^{\sin(\phi_h - \phi_S)} \sim \frac{M_N \alpha_s}{P_{h\perp}^3} [\text{SGP, HPo, HPn}] \quad \frac{M_N \alpha_s}{P_{h\perp}^3} [\text{SGP, HPo}] \longleftrightarrow \text{TMD-Sivers}$$

Ji, Qiu, Vogelsang, Yuan ('06)

Koike, Vogelsang, Yuan ('07)

$$F^{\sin(2\phi_h - \phi_S)} \sim \frac{M_N \alpha_s}{QP_{h\perp}^2} [\text{SGP, HPo, HPn}]$$

$$F^{\sin(\phi_S)} \sim \frac{M_N \alpha_s}{QP_{h\perp}^2} [\text{SGP, HPn}]$$

$$F^{\sin(3\phi_h - \phi_S)} \sim \frac{M_N \alpha_s}{Q^2 P_{h\perp}} [\text{SGP, HPo}]$$

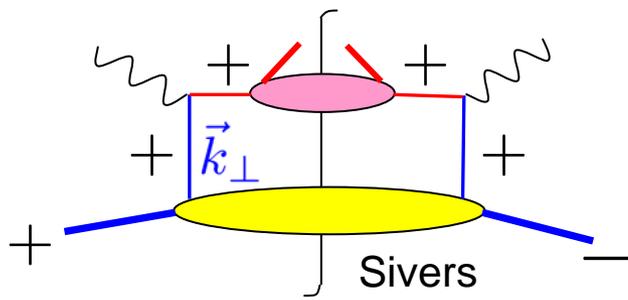
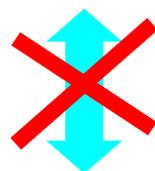
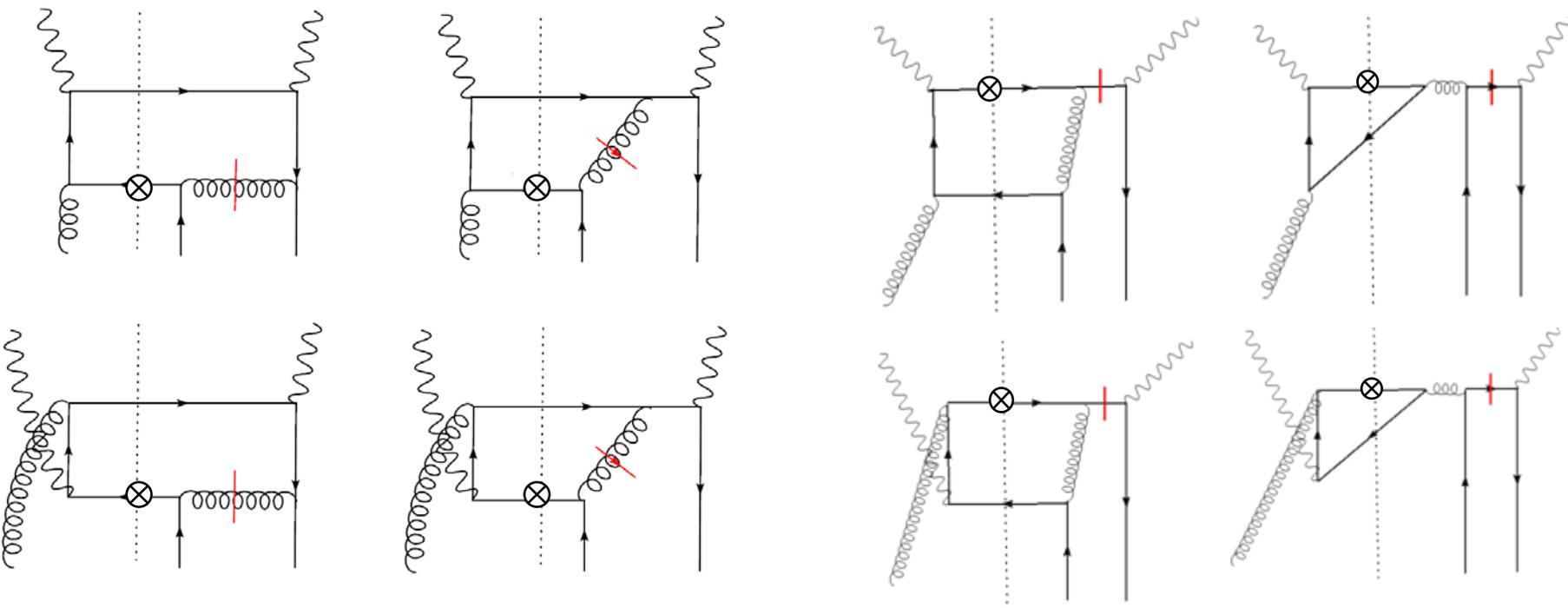
$$F^{\sin(\phi_h + \phi_S)} \sim \frac{M_N \alpha_s}{P_{h\perp}} [\text{SGP, HPn}] \quad \left[ F^{\sin(\phi_h + \phi_S)} \sim \frac{M_N \alpha_s}{P_{h\perp}^3} [\widehat{\text{SGP}}] \longleftrightarrow \text{TMD-Collins} \right]$$

With the present updates,  
no change for the power behaviors

SGP contribution is suppressed in all azimuthal structures,  
but new HP contribution (HPn) survives!

# "new" SFP

# "new" HP



# Summary

update of twist-3 SSA for  $ep^\uparrow \rightarrow e\pi X$  associated with twist-3 quark-gluon correlation functions  $G_F(x_1, x_2)$  and  $\tilde{G}_F(x_1, x_2)$

Five structure functions with different azimuthal-dependence contribute (  $\sim$  TMD factorization approach)

$$\frac{d^5 \sigma^{\text{tw3}}}{dQ^2 dx_{bj} dz_f dP_{h\perp}^2 d\phi_h} = \sin(\phi_h - \phi_S) \left[ \sigma_1^{\text{tw3}} + \sigma_2^{\text{tw3}} \cos(\phi_h) + \sigma_3^{\text{tw3}} \cos(2\phi_h) \right] \\ + \cos(\phi_h - \phi_S) \left[ \sigma_4^{\text{tw3}} \sin(\phi_h) + \sigma_5^{\text{tw3}} \sin(2\phi_h) \right]$$

Not only  $qg$  final states, but also  $q\bar{q}$  final states

new SFP & HP contributions

Small  $P_{h\perp}$  behavior  $\Lambda_{\text{QCD}} \ll P_{h\perp} \ll Q$

SFP contribution is subleading in all azimuthal structures

(new) HP contribution is leading in most azimuthal structures, e.g. in that for the Sivers asymmetry

connection with TMD approach ?  
quantitative roles ?

The twist3 single-spin dep. cross section vanishes in inclusive DIS limit  
cancellation of final-state interactions