Chris White, Nikhef

Path Integral Methods for Soft Gluon Resummation

In collaboration with E. Laenen & G. Stavenga; arXiv:0811.2067

DIS2009

Overview

What is the structure of soft gluon corrections at next-to-eikonal order?

- Review of soft gluon resummation.
- Exponentiation in (non-)abelian gauge theories - webs.
- New approach using path integral methods.
- Classification of next-to-eikonal contributions.
- Outlook.

Soft resummation

- Multiple soft gauge boson emission can lead to large corrections to cross-sections.
- If ξ is the energy carried by soft bosons, typically get contributions:

$$\frac{d\sigma}{d\xi} = \sum_{n,m} \alpha^n \left[c^0_{nm} \frac{\log^m(\xi)}{\xi} + c^1_{nm} \log^m(\xi) + \ldots \right]$$

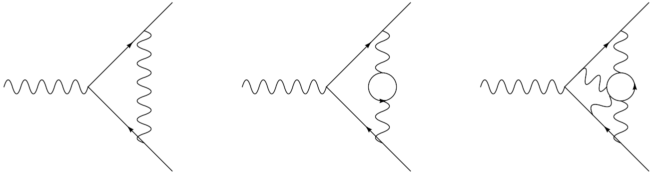
- First set of terms corresponds to *eikonal approximation*, in which momenta $k_i \to 0$ for all (soft) emissions.
- Second set of terms is *next-to-eikonal* (NE) limit i.e. first order in k_i.
- Happens in abelian and non-abelian theories.

Soft resummation - abelian case

- When ξ is small, perturbation theory breaks down - must resum problem logarithms.
- At eikonal order, have a simple result for the amplitude in abelian theories

$$\mathcal{A} = \mathcal{A}_0 \exp\left[\sum G_c\right],$$

where $\mathcal{A}_0$ is the Born amplitude, and G_c are connected subdiagrams.



- Gives eikonal logarithms at all orders in α.

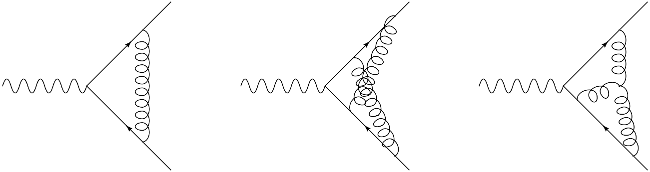
Soft resummation - nonabelian case

- Exponentation generalisable to non-abelian theories, but structure is more complicated:

$$\mathcal{A} = \mathcal{A}_0 \exp\left[\sum \bar{C}_W W\right],$$

where W are *webs* (two-eikonal line irreducible subdiagrams).

- Webs have modified colour weights $\bar{C}_W$.



- More effort than abelian case, but still predicts eikonal logs to all orders.

Generalisation to NE order

Question: Can this be extended to NE order?

- Will now introduce new framework for soft resummation.
- Old results are recovered, and can be easily generalised to sub-eikonal approximation.
- Based on key observation:

Exponentiation of connected subdiagrams looks like exponentiation of connected diagrams in QFT (a textbook result!).

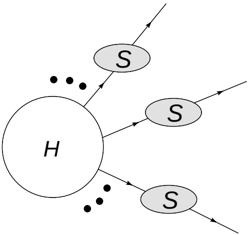
- Are they by any chance related?
- Answer: yes, after rewriting of the problem.
- Let's first look at abelian case (with scalar emittors) in detail...

Path integral method

- Consider a Green's function with a number of hard external lines, each of which may emit soft radiation.
- Can write this as:

$$G(p_1, \dots p_n) = \int \mathcal{D}A_s^\mu H(x_1, \dots, x_n) S(p_1, x_1) \dots S(p_n, x_n),$$

where H is hard interaction, and S are propagators for the emitting particles in the presence of a soft gauge field A_s^μ, sandwiched between states $|p_i\rangle$, $|x_i\rangle$.



- Propagator factors $S(p_i, x_i)$ can now be re-expressed as first-quantised path integrals...

Propagators as path integrals

- Can write the scalar free particle propagator factor as

$$S(x,p) = \int \mathcal{D}x\mathcal{D}p \exp\left[-ip(T)x(T) + i\int_0^T dt(p\dot{x} - H(p,x))\right].$$

- This is a first-quantised path integral, where $x(t)$ is the trajectory of the particle.
- For an emitting particle in a background soft gauge field, this becomes

$$S(p,x,A_s^\mu) = \int_{x(0)=0}^{p(T)=0} \mathcal{D}p\mathcal{D}x \exp\left[i\int_0^T dt(p\dot{x} - \frac{1}{2}p^2 + (p_f + p)\cdot A_s(x_i + p_f t + x) + \frac{i}{2}\partial\cdot A_s(x_i + p_f t + x) - A_s^2(x_i + p_f t + x))\right].$$

Soft photon exponentiation

- One now substitutes the propagator factors into the expression for the Green's function.
- Can carry out the path integrals over p_i (for each hard external line).
- Result has the form

$$G(p_1, \dots p_n) = \int \mathcal{D}A_s^{\mu} H(x_1, \dots x_n) \prod_i \mathcal{D}x_i e^{-ip_i \cdot x_i}$$

$$\exp\left[i \int_0^{\infty} dt \left(\frac{1}{2}\dot{x}^2 + (p_f + \dot{x}) \cdot A(x_i + p_f t + x(t)) \right.\right.$$

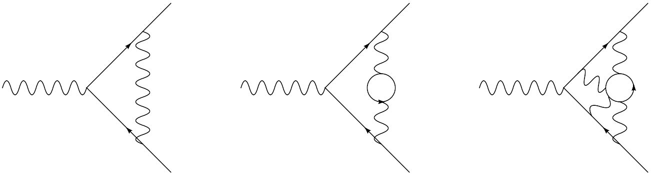
$$\left.\left. + \frac{i}{2} \partial \cdot A(x_i + p_f t + x) \right) \right].$$

- This is a quantum field theory for the soft gauge field! Terms in exponent act as *sources* for A_s^{μ}.

Soft photon exponentiation

- These sources are localised on the hard external lines.
- All possible soft photon diagrams are generated, which span the external lines.
- Field theory, so disconnected diagrams exponentiate.

$\Rightarrow$ Soft photon corrections exponentiate.



Path integral picture - summary

- Factorise Green's functions into hard interactions with outgoing (hard) legs emitting soft radiation.
- Rewrite propagators for these legs in terms of first quantised path integrals involving worldines x_i^μ.
- Get a field theory with source terms localised on the external lines,
- Exponentiation of disconnected diagrams in this field theory $\equiv$ exponentiation of soft photon subdiagrams.
- Have considered scalar external lines, and abelian gauge fields, but framework generalises...

Generalisation

- Extension to fermion emitting particles is straightforward.
- Non-abelian case is complicated, due to the fact that source vertices are matrix valued in colour space.
- Do not commute $\Rightarrow$ standard combinatorics of path integral do not work.
- Can make progress using the *replica trick* of statistical physics...

The replica trick

- Consider a theory with N copies of the soft gauge bosons.
- Now consider the Green's function raised to the power N:

$$G^N = 1 + N \log G + \mathcal{O}(N^2)$$

- It turns out that a subset of connected subdiagrams W are $\propto N$, so that

$$G = G_0 \exp\left[\sum \bar{C}_W W\right],$$

  with some colour factors $\bar{C}_W$.
- These are exactly the *webs* obtained in the old Feynman diagram approach!
- A new closed form solution for the modified colour factors is obtained, which is equivalent to previous results.

Next-to-eikonal exponentation

- The new framework does more than reproduce old results.
- Extension to NE order is easy in the new approach.
- NE corrections correspond to expanding around the classical straight-line trajectory of the emitting particles in the path integral exponent.
- Get a set of effective Feynman rules for NE emissions.
- Also get corrections due to soft emissions from within the hard interaction. General structure of matrix element:

$$\mathcal{M} = \mathcal{M}_0 \exp\left[\mathcal{M}^{E} + \mathcal{M}^{NE}\right] \times \left[1 + \mathcal{M}_{rem.}\right] + \mathcal{O}(NNE).$$

- Remainder doesn't formally exponentiate, but has an iterative structure.

Conclusions

- Have developed a new framework for examining soft gluon resummation.
- Uses path integral methods to relate exponentiation to known exponentation of field theory diagrams.
- Works for all spins of emitting particles, and for (non)-abelian gauge theories.
- Old results are recovered (i.e. webs), with more elegant solution for $\bar{C}_W$.
- Extension to next-to-eikonal corrections straightforward.
- Structure of NE corrections in matrix elements classified.

Outlook

- Have so far looked at a simple non-abelian case (two external lines only). Can extend method to more complex systems.
- In cross-sections, need corrections to phase space as well as matrix elements. Under investigation.
- Phenomenological applications: What are the $\ln(1-x)$ terms in various circumstances?