# Precision measurement and combined global analysis of PDF including $P_T$ resummation theory and data

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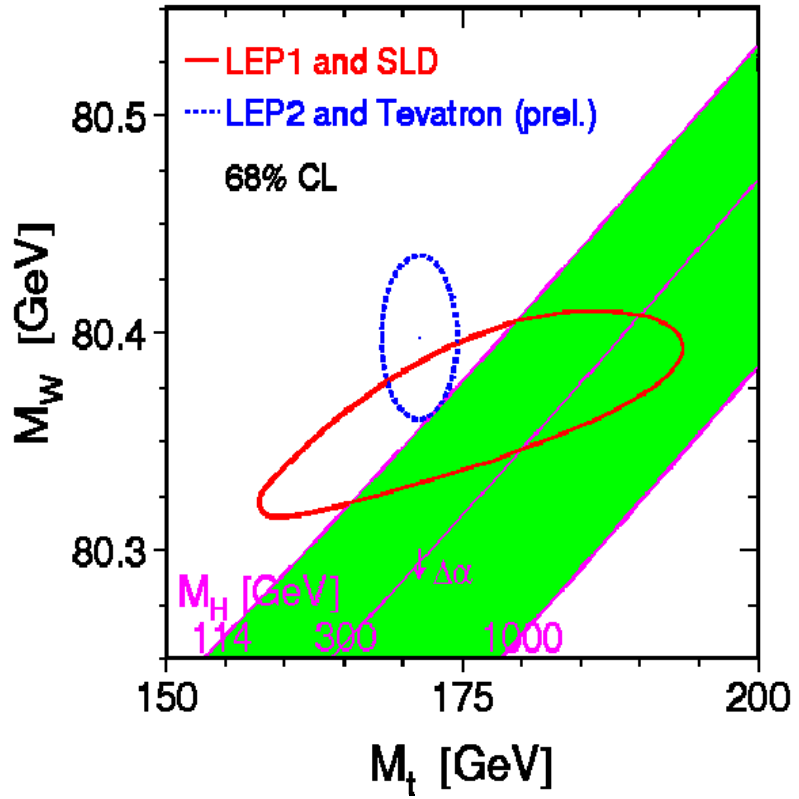
# Precision Measurement of W boson mass

- Tevatron Run II: $\Delta M_W < 30$ MeV
  LHC: $\Delta M_W = 5\sim 15$ MeV

- Precision measurement of $M_W$, along with $M_t$, constrains $M_H$ logarithmically in the SM

- $M_W$ is better determined by measuring the transverse mass of the W boson and the $P_T$'s of the decay leptons

- $Q_T$ of W boson must be predicted with high preicision



# Precision Measurement of W boson mass

The largest theory uncertainties on $M_W$ arise from

- ❑ Parton distributions (Global QCD analysis)

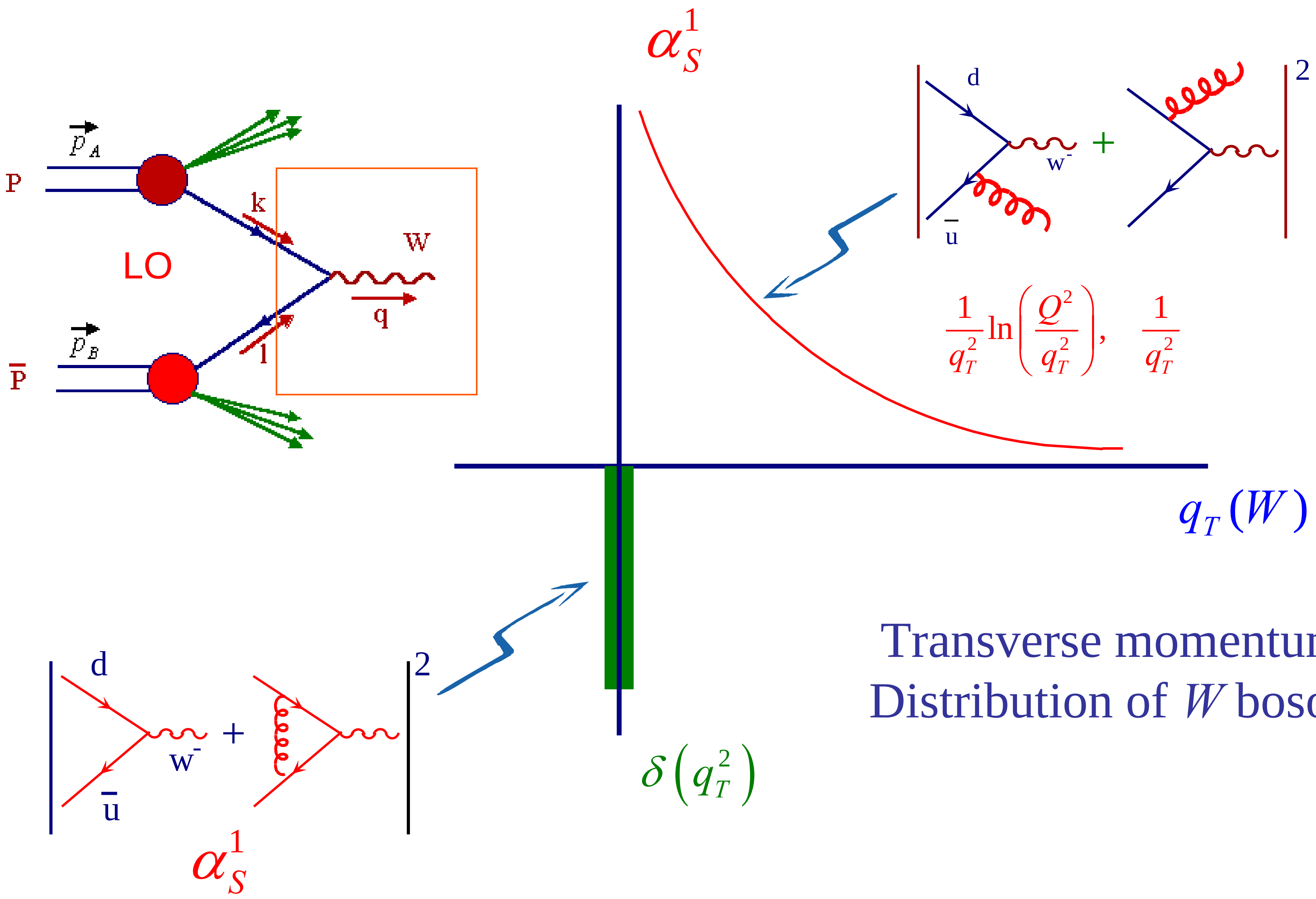
- ❑ The model for W boson's recoil in the transverse plane

  W (as well as Z, $\gamma^*$) boson acquires $Q_T$ by recoiling against perturbative and non-perturbative radiation

  (Pt resummation)

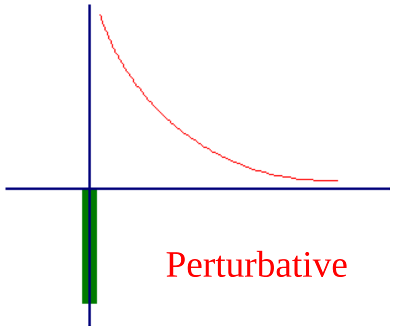
# Predicting $P_T$ distributions at fixed orders

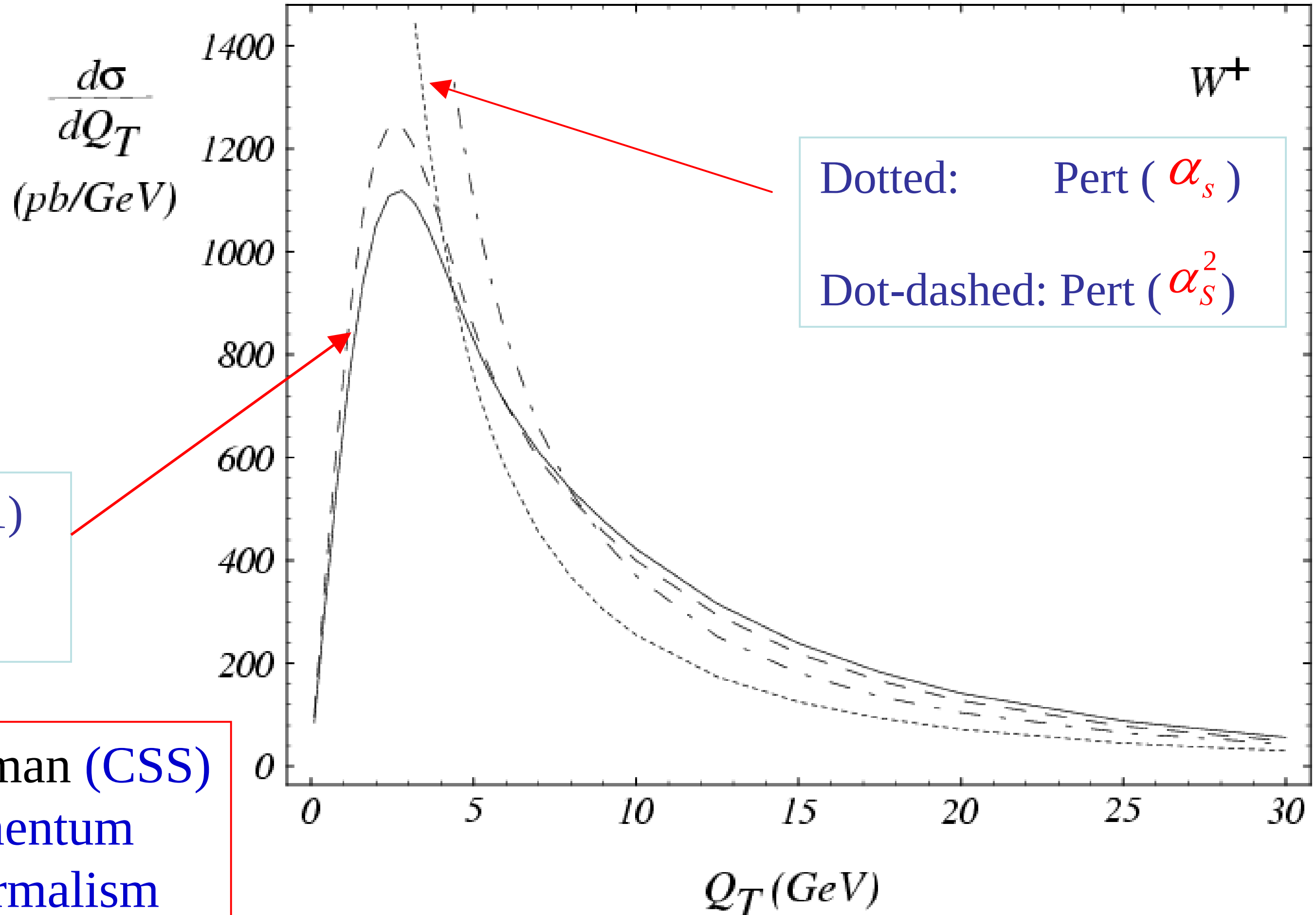
LO and Next to Leading order QCD corrections




Transverse momentum Distribution of $W$ boson

To describe data ⟹ All order QCD Resummation is needed.




Dotted: Pert ( $\alpha_s$ )

Dot-dashed: Pert ( $\alpha_S^2$ )

Dashed: CSS (1,1,1)

Solid: CSS (2,2,1)

Collins-Soper-Sterman (CSS)
Transverse momentum
Resummation formalism

# What's QCD Resummation?

- Perturbative expansion

$$\frac{d\hat{\sigma}}{dq_T^2} \sim \alpha_S \left\{ 1 + \alpha_S + \alpha_S^2 + \cdots \right\}$$

- The singular pieces, as $\frac{1}{q_T^2}$ (1 or log's)

$$\frac{d\hat{\sigma}}{dq_T^2} \sim \frac{1}{q_T^2} \sum_{n=1}^{\infty} \sum_{m=0}^{2n-1} \alpha_S^{(n)} \ln^{(m)}\left(\frac{Q^2}{q_T^2}\right)$$

$$\sim \frac{1}{q_T^2}\{ \quad \alpha_S \left(\underline{L+1}\right)$$

$$+\alpha_S^2 \left(\underline{L^3 + L^2} + \underline{L+1}\right)$$

$$+\alpha_S^3 \left(\underline{L^5 + L^4} + \underline{L^3 + L^2} + \underline{L+1}\right)$$

$$+ \cdots \quad \}$$

$$L \equiv \ln\left(\frac{Q^2}{q_T^2}\right)$$

Resummation is to reorganize the results in terms of the large Log's.

# Resummed results:

Determined by $A^{(1)}$ and $B^{(1)}$

$$\frac{d\sigma}{dq_T^2} \sim \frac{1}{q_T^2}\Big\{ \left[\alpha_S\left(L+1\right) \quad + \alpha_S^2\left(L^3+L^2\right) + \alpha_S^3\left(L^5+L^4\right) \quad + \cdots \right]$$

$$+\left[ \qquad + \alpha_S^2\left(L+1\right) \quad + \alpha_S^3\left(L^3+L^2\right) \quad + \cdots \right]$$

Determined by $A^{(2)}$ and $B^{(2)}$

$$+\left[ \qquad\qquad + \alpha_S^3\left(L+1\right) \quad + \cdots \right]$$

$$+ \cdots \Big\}$$

Determined by $A^{(3)}$ and $B^{(3)}$

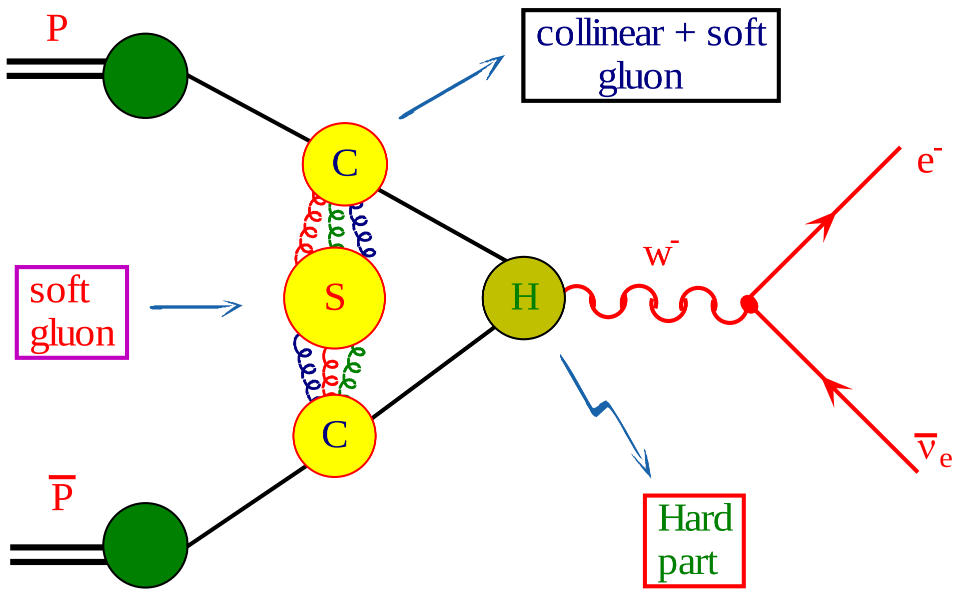
⟹ QCD Resummation

In the formalism by Collins-Soper-Sterman, in addition to these perturbative results, the effects from physics beyond the leading twist is also implemented as

[non-perturbative function].

# CSS Resummation Formalism

Resum all order effects in large logs




NPB 250 ('85)

Collins-Soper-Sterman (CSS)
Transverse momentum
Resummation formalism

# CSS Resummation Formalism

$$\frac{d\sigma}{dq_T^2\, dy\, dQ^2} = \frac{\pi}{S}\sigma_0\delta\left(Q^2 - M_W^2\right)\cdot$$

$$\left\{\frac{1}{(2\pi)^2}\int d^2b \;\; e^{i\vec{q}_T\cdot\vec{b}}\tilde{W}\left(b,Q,x_A,x_B\right)\cdot\text{[Non-perturbative function]}\right.$$

$$\left. + Y\left(q_T, y, Q\right)\right\}$$

$$\tilde{W} = e^{-S(b)}\cdot C\otimes f\left(x_A\right)\cdot C\otimes f\left(x_B\right)$$

$$C\otimes f\left(x_A\right) \rightarrow \sum_j\int_{x_A}^1 \frac{d\xi_A}{\xi_A} C_{qj}\left(\frac{x_A}{\xi_A}, b, \mu\right)\cdot f_{j/A}\left(\xi_A,\mu\right)$$

$$C\otimes f\left(x_B\right) \rightarrow \sum_k\int_{x_B}^1 \frac{d\xi_B}{\xi_B} C_{qk}\left(\frac{x_A}{\xi_A}, b, \mu\right)\cdot f_{k/B}\left(\xi_B,\mu\right)$$

$e^{-S(b)} \rightarrow$ Sudakov form factor

$$S\left(b\right) = \int_{\left(\frac{b_0}{b}\right)^2}^{Q^2}\frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right)A\left(\bar{\mu}\right) + B\left(\bar{\mu}\right)\right]$$

[Non-perturbative functions] are functions of $(b,Q,x_A,x_B)$ which include QCD effects beyond Leading Twist.

[non-perturbative function] is a function of $(b,Q,x_A,x_B)$, implemented to include effects beyond Leading Twist.

Until we know how to calculate QCD non-perturbatively, (Lattice Gauge Theory?), these functions can only be parameterized. However, the same functions should describe Drell-Yan, $W^{\pm}$, $Z^0$ data.

→

- Test QCD in problems involving multiple scales.
- Measuring these non-perturbative functions may help in understanding the non-perturbative part of QCD.

[non-perturbative function] , dependent of Q, b, $x_A$, $x_B$, is necessary to describe $q_T$ – distribution of Drell-Yan, $W^{\pm}$,$Z^0$ events.

# Conventional $P_T$ Resummation Global Analysis

- Global analysis of Drell-Yan pair and $Z$ boson $P_T$ distributions
- Use a fixed PDF set and allow a few parameters in non-pert. function

**Typical non-pert. Function** : Brock-Landry-Nadolsky-Yuan (BLNY)

hep-ph/0212159

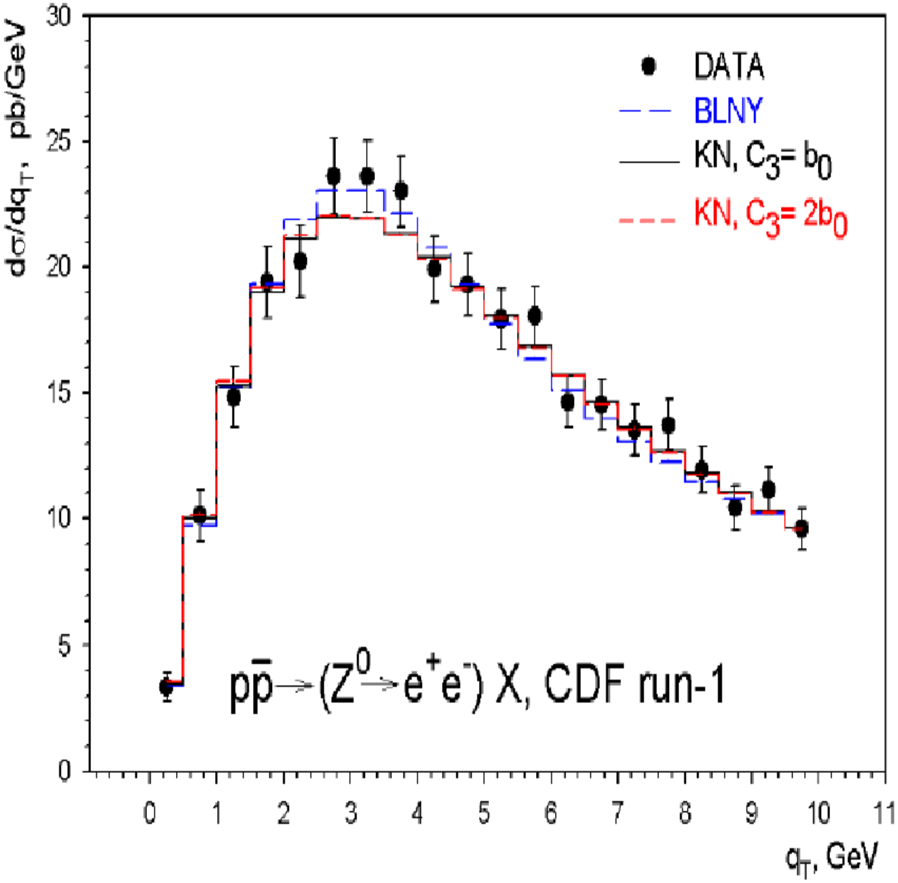
$$\exp(-S_{NP}) = \exp\left[-g_1 b^2 - g_2 b^2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 b^2 \ln\left(100 x_A x_B\right)\right]$$

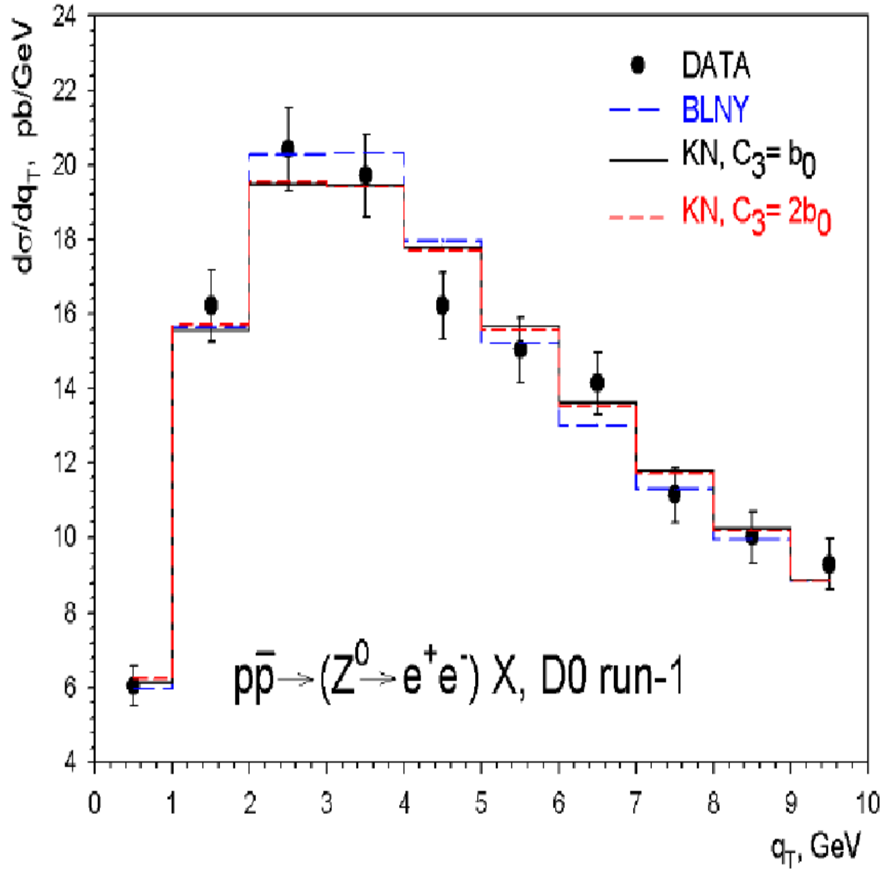
# Experiments included in $P_T$ global fits

| Experiment | CM Energy (GeV) | Boson Mass (GeV) | Process |
|---|---|---|---|
| R209 | 62.0 | 5-11 | $p + p \to \mu^+ \mu^- + X$ |
| CDF I:Z | 1800.0 | 91.19 | $p + \overline{p} \to Z$ |
| DØ I:Z | 1800.0 | 91.19 | $p + \overline{p} \to Z$ |
| E288 | 27.4 | 5-9 | $p + Cu \to \mu^+ \mu^- + X$ |
| E605 | 38.8 | 7-9<br>10.5-18 | $p + Cu \to \mu^+ \mu^- + X$ |

# Universality of the non-perturbative function in CSS resummation formalism

hep-ph/0506225
Konychev, Nadolsky



DØ Note 5755-CONF

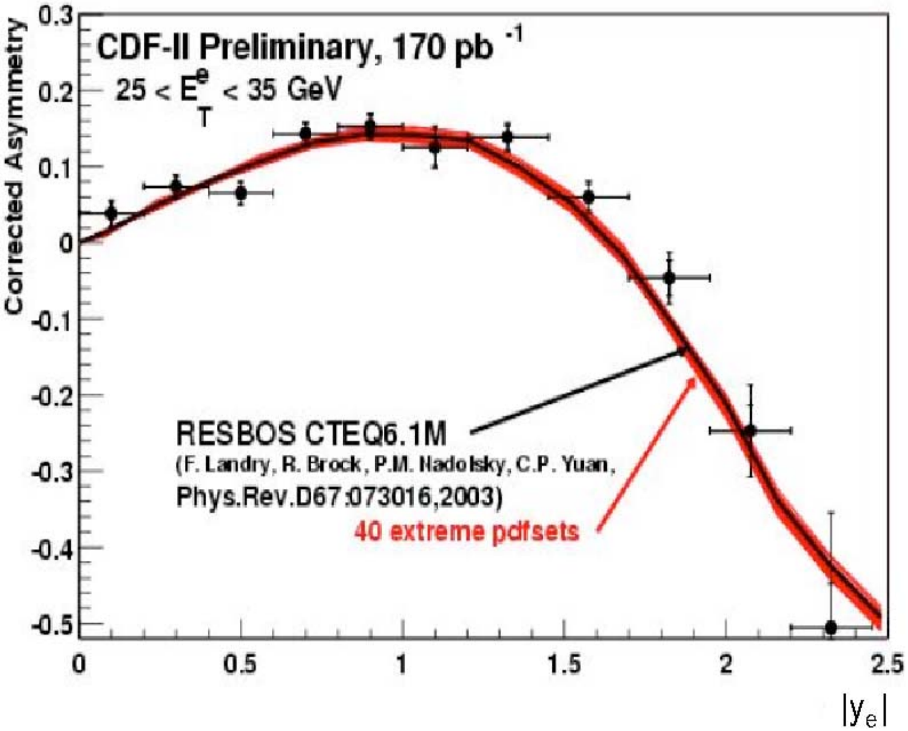
Version: 1.5

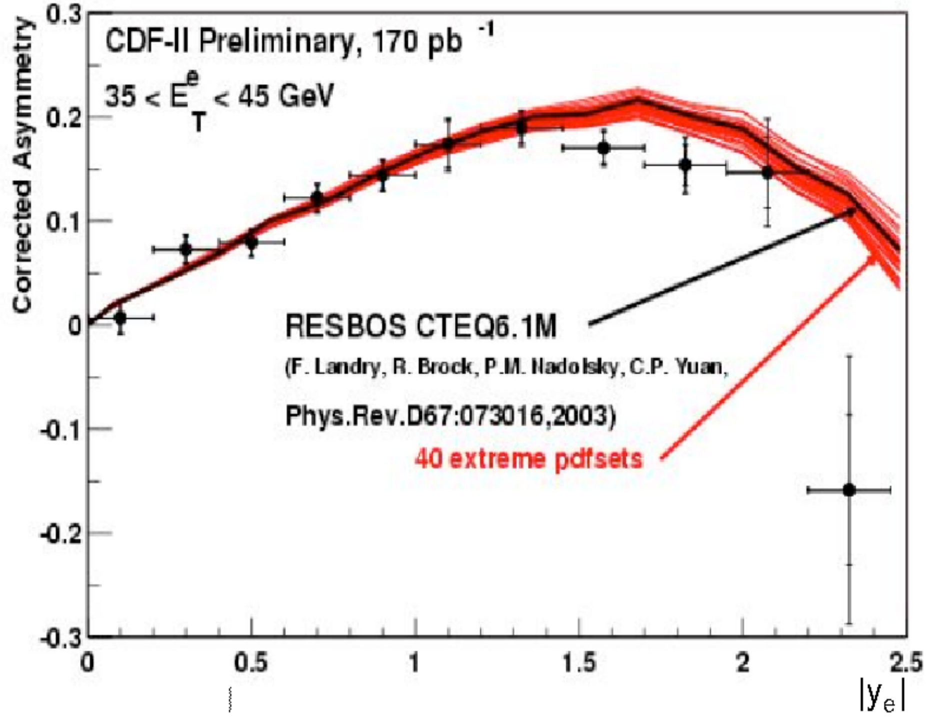
# A Study of $Z \rightarrow e^+e^-$ and $Z \rightarrow \mu^+\mu^-$ Events Produced at Low Transverse Momentum Using a Novel Technique

The $Z$ boson transverse momentum, $p_T^Z$, can be decomposed into two components, $a_T$ and $a_L$, that are transverse and parallel, respectively, to the di-lepton thrust axis. Using the $a_T$ distribution of $Z$ decays observed with the DØ detector, we measure $g_2$, a phenomenological parameter in the BLNY non-perturbative form factor. In a combined measurement with di-muon and di-electron decay channels, using approximately 2 fb$^{-1}$ of data, we measure $g_2 = 0.63 \pm 0.02 \pm 0.04$ GeV$^2$. The first uncertainty is experimental and the second uncertainty is due to the PDF dependence of the theoretical prediction.

# Conventional PDF Global Analysis

- Data: DIS, Drell-Yan, inclusive jet, and charged lepton asymmetry (from *W*-decay)




(In order to describe Lasy data, resummed calculation had been applied)

# New Task of Global Analysis

## Including Transverse Momentum $p_T$ distributions

- Nonpert. part of collinear PDF $f_a(x,\mu)$ and nonpert. function $S_{NP}(b,x,Q)$ from $k_T$ dependent PDF's share the common origin, indicating the need for simultaneous analysis
- Best-fit $S_{NP}$ is found to be correlated with PDF set => combined analysis will have effects on EW precision measurements
- Adding $p_T$ data will provide the constraints on previously less-known degrees of freedom in $f_a(x,\mu)$

# Combined PDF+ $p_T$ Global Analysis

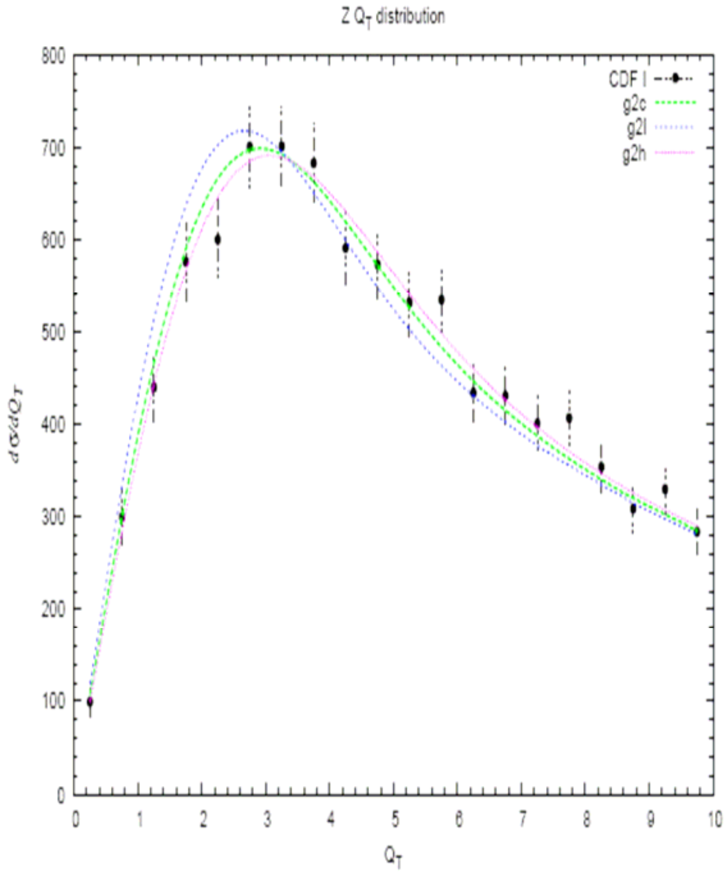
## New Inputs:

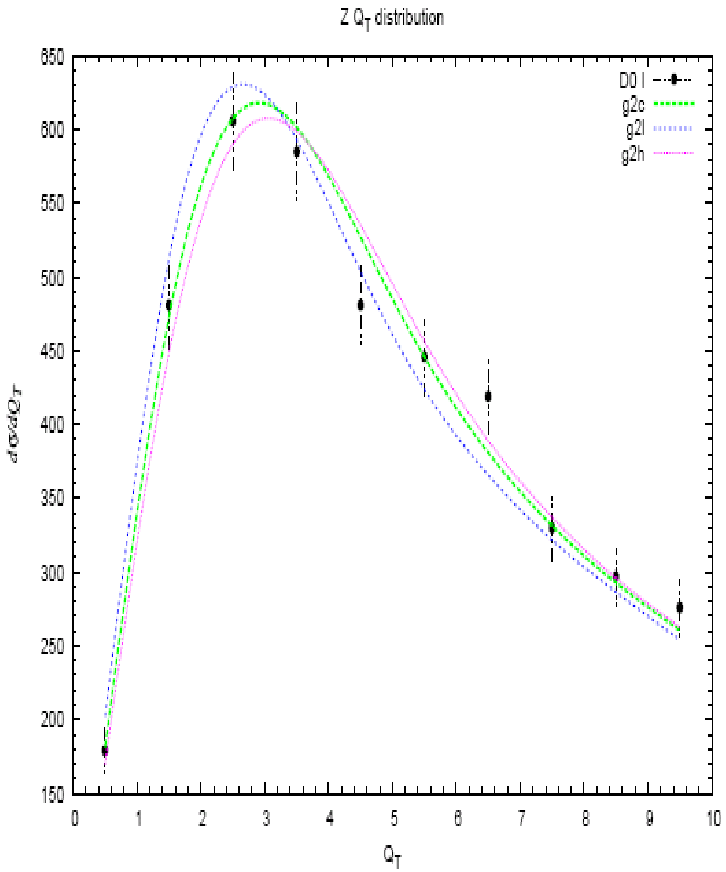
- Experimentally: include not only rapidity ($y$) but also $p_T$ of Drell-Yan pairs and $Z$ bosons
- Theoretically: include $p_T$ Resummation formalism to account for the soft physics that entangle with multi-scale measurements

## New Outputs:

- $S_{NP}$ of non-perturbative function is simultaneously determined, In addition to the PDF $f_a(x,\mu)$.

# Preliminary results



# Preliminary results

| | CTEQ66 (BLNY) | CTEQ66 (refit g's) | g2c | g2l | g2h |
|---|---|---|---|---|---|
| $g_1$ | .210 | .234 | .294 | .409 | .219 |
| $g_2$ | .680 | .600 | .566 | .415 | .660 |
| $g_1 g_3$ | -.126 | -.174 | -.194 | -.194 | -.194 |
| $S_{NP}(M_Z)/b^2$ | 2.68 | 2.51 | 2.49 | 2.10 | 2.73 |
| $\chi^2_{pt}$ (111 Pts) | 403 | 165 | 135 | 155 | 155 |
| $\Delta\chi^2_{pt}$ | 267 | 30 | 0 | 20 | 20 |

DØ Note 5755-CONF

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Translated to $S_{NP}$ : $S_{NP}(M_Z)/b^2 = 2.51 \pm 0.15$

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# Conclusion

➢ Precision measurements related Pt at the Tevatron and LHC cannot be possibly further improved without concurrent understandings of PDF and Pt resummation effects

➢ A combined PDF+Pt global analysis is able to determine both PDF $f_a(x,\mu)$ and $S_{NP}(b,x,Q)$ simultaneously, and thus gives a better estimate on quantities that rely on both.

➢ The range of $S_{NP}(M_Z)$ determined by the combined analysis is slightly larger than that reported by D0 collaboration.

# Backup Slides

# $P_T$ Experiments included in combined global analysis

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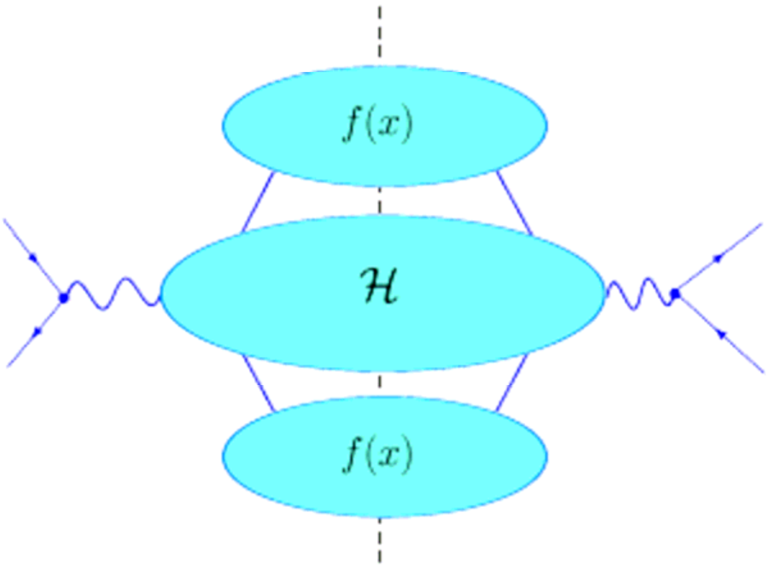
# QCD factorization in hard and soft regions

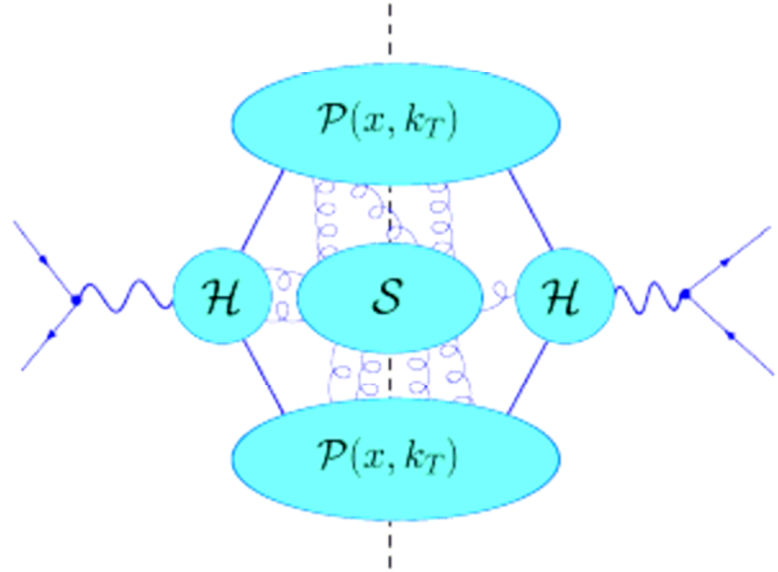
Finite-order (FO) factorization

$\Lambda_{QCD}^2 \ll q_T^2 \sim Q^2$

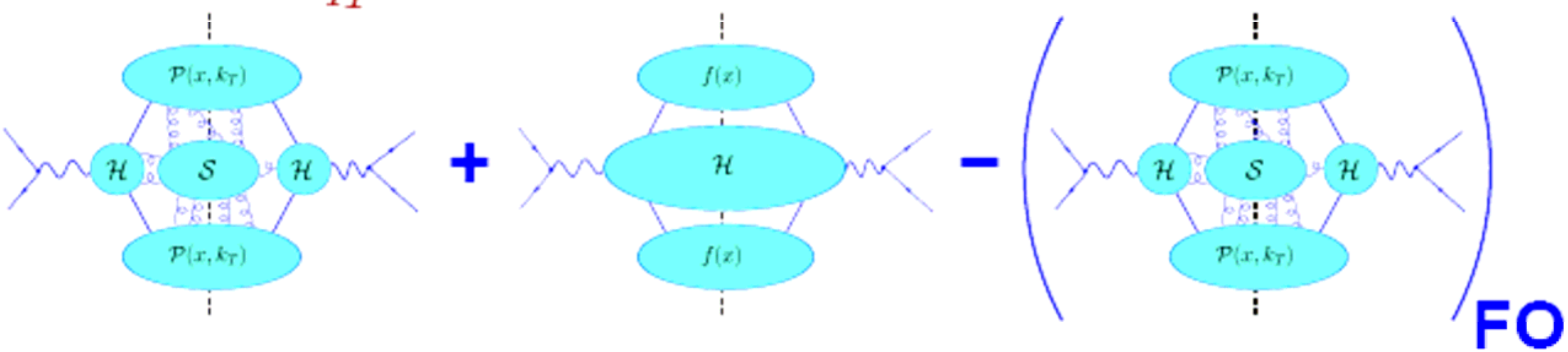


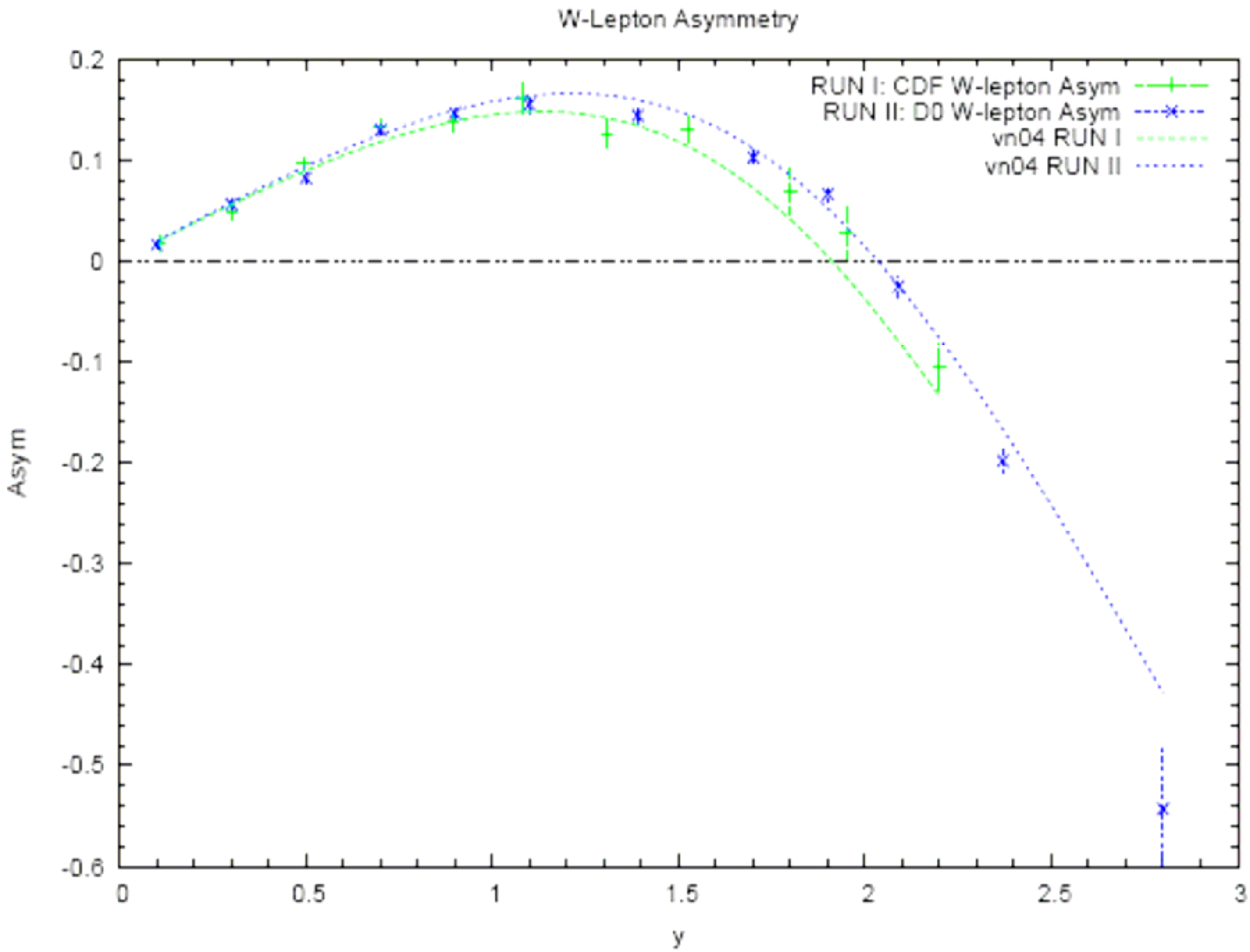

Small-$q_T$ factorization

$\Lambda_{QCD}^2 \ll q_T^2 \ll Q^2$

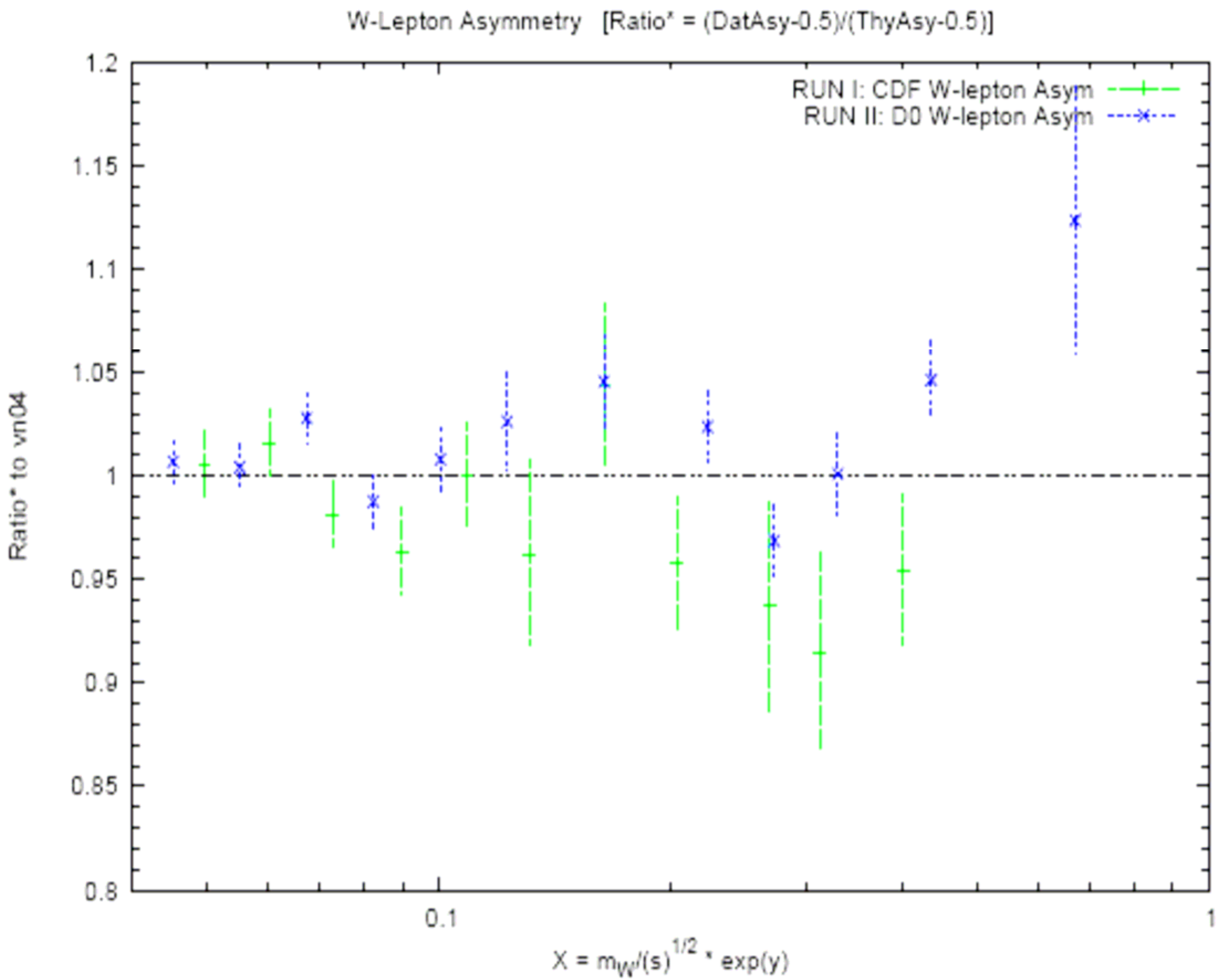



Solution for all $q_T$:





W-Lepton Asymmetry
RUN I: CDF W-lepton Asym
RUN II: D0 W-lepton Asym
vn04 RUN I
vn04 RUN II
Asym
y
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
0
0.5
1
1.5
2
2.5
3


W-Lepton Asymmetry [Ratio* = (DatAsy-0.5)/(ThyAsy-0.5)]
RUN I: CDF W-lepton Asym
RUN II: D0 W-lepton Asym
Ratio* to vn04
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0.1
1
X = m_W/(s)^1/2 * exp(y)