Chiral-odd and Chiral-even Generalized Parton Distributions in Transverse and Longitudinal Position Space

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- Chiral Odd GPDs: Why they are interesting
- Overlap representation of Chiral-odd GPDs
- Simple example: electron at one loop
- GPDs in position space
- Phenomenological study

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In collaboration with D. Chakrabarti (IIT Kanpur), R. Manohar (IIT Bombay)
Deeply virtual Compton scattering (DVCS) Amplitude

Deeply virtual Compton scattering:

\[
M^{IJ}(\vec{q}_\perp, \vec{q}_\perp', \zeta) = \epsilon^I_\mu \epsilon^{*J}_\nu M^{\mu\nu}(\vec{q}_\perp, \vec{q}_\perp', \zeta) = -e_q^2 \frac{1}{2P^+} \int_{\zeta-1}^{1} dx \\
\times \left\{ t^{IJ}(x, \zeta) \bar{U}(P') \left[ H(x, \zeta, t) \gamma^+ + E(x, \zeta, t) \frac{i}{2M} \sigma^{+\alpha}(-\Delta \alpha) \right] U(P) \right\},
\]

where \( \bar{P} = \frac{1}{2}(P' + P) \),

\[
t^{\uparrow\uparrow}(x, \zeta) = t^{\downarrow\downarrow}(x, \zeta) = \frac{1}{x - i\epsilon} + \frac{1}{x - \zeta + i\epsilon}
\]
DVCS contd.

DVCS amplitude contains

\[
\int \frac{dy^-}{8\pi} e^{ixP^+ y^- / 2} \left. \langle P', \lambda' | \bar{\psi}(0) \gamma^+ \psi(y) | P, \lambda \rangle \right|_{y^+ = 0, y_\perp = 0} = \frac{1}{2P^+} \bar{U}(P', \lambda') \left[ H(x, \zeta, t) \gamma^+ + E(x, \zeta, t) \frac{i}{2M} \sigma^{+\alpha}(-\Delta_\alpha) \right] U(P, \lambda),
\]

\(H(x, \zeta, t)\) and \(E(x, \zeta, t)\) are chiral even GPDs; as well as \(\tilde{H}(x, \zeta, t)\) and \(\tilde{E}(x, \zeta, t)\)

\[
\int \frac{dy^-}{8\pi} e^{ixP^+ y^- / 2} \left. \langle P', \lambda' | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(y) | P, \lambda \rangle \right|_{y^+ = 0, y_\perp = 0} = \frac{1}{2P^+} \bar{U}(P', \lambda') \left[ \tilde{H}(x, \zeta, t) \gamma^+ \gamma_5 + \tilde{E}(x, \zeta, t) \frac{\gamma_5 \Delta^+}{2M} \right] U(P, \lambda),
\]

Momentum transfer \(\Delta = P - P', t = \Delta^2, \zeta = \frac{\Delta^+}{P^+}\)

\(x\) is the fraction of the proton momentum carried by the active quark
**Chiral-odd GPDs**

Defined as the non-forward matrix elements of light-like correlations of tensor charge

We use the parametrization

\[
P^+ \int \frac{dz^-}{2\pi} e^{iP^+z^-} \langle P', \lambda' | \bar{\psi}(\frac{-z^-}{2})\sigma^+ j \gamma_5 \psi(\frac{z^-}{2}) | P, \lambda \rangle_{z^+=0, z_\perp=0} = H_T(x, \zeta, t)\bar{u}(P')\sigma^+ j \gamma_5 u(P) - \tilde{H}_T(x, \zeta, t)\varepsilon^+ j \alpha \beta \bar{u}(P')\frac{\Delta \alpha P_\beta}{M^2} u(P)
\]

\[
- E_T(x, \xi, t)\varepsilon^+ j \alpha \beta \bar{u}(P')\frac{\Delta \alpha \gamma \beta}{2M} u(P) + \tilde{E}_T(x, \zeta, t)\varepsilon^+ j \alpha \beta \bar{u}(P')\frac{P_\alpha \gamma \beta}{M} u(P).
\]


Momenta of initial and final protons:

\[
P = \left( P^+, \vec{0}_\perp, \frac{M^2}{P^+} \right), P' = \left( (1 - \zeta)P^+, -\vec{\Delta}_\perp, \frac{M^2 + \vec{\Delta}_\perp^2}{(1 - \zeta)P^+} \right)
\]

Involves quark helicity flip

In the forward limit $H_T(x, 0, 0)$ reduces to the transversity distribution (generalized transversity)
Chiral Odd GPDs : Why ? How to measure ?

- \( \int dxdy[H_T(x, 0, 0) + 2\bar{H}_T(x, 0, 0) + E_T(x, 0, 0)] \) related to the transverse angular momentum carried by transversely polarized quarks in an unpolarized target: similar to Ji’s relation

- \( \int dx[2\bar{H}_T(x, 0, 0) + E_T(x, 0, 0)] \) tells us in which direction the average position of the quarks with spin in the \( x \) direction is shifted in the \( y \) direction for an unpolarized target w.r.t the transverse center of momentum: can determine the sign of Boer-Mulders function

M. Burkardt (2005)

- Certain combinations of chiral-odd GPDs also give the correlation between the transverse quark spin and target spin
- Several proposals to measure \( H_T \): for example in photo or electroproduction of two vector mesons on a nucleon target


- Exclusive process \( \gamma^* P \rightarrow \pi^0 P \): to measure the tensor charge

  S. Ahmad, G. Goldstein, S. Liuti (2008)
Overlap Representation

- The target state is expanded in terms of multiparticle light-front wave functions in Fock space; choose light-front gauge $A^+ = 0$

GPDs are given in terms of overlaps of the light-front wave functions

- Both chiral even and chiral odd GPDs; diagonal parton number conserving $n \rightarrow n$ overlap in the kinematical regime $\zeta < x < 1$ and $\zeta - 1 < x < 0$

- Off-diagonal $n + 1 \rightarrow n - 1$ overlap for $0 < x < \zeta$ where the parton number is decreased by two: higher Fock sector LFWF

  Diehl, Feldman, Jacob, Kroll (2001);
  Brodsky, Diehl, Huang (2001)
  Chakrabarti, Manohar, Mukherjee (2008)

- Both contributions needed to get the complete kinematical region as well as to calculate the moments at non-zero $\zeta$

- Only diagonal overlap when $\zeta$ is zero

- In chiral odd GPDs, overlap of different quark helicities whereas in chiral even GPDs no helicity flip
Overlap Representation for Chiral-odd GPDs

\[ F_{T\lambda',\lambda}^{n\rightarrow n} = (1 - \zeta)^{1 - \frac{n}{2}} \sum_{n,\lambda_i} \int \Pi_{i=1}^{n} \frac{dx_id^2k_i^\perp}{16\pi^3} 16\pi^3 \delta(1 - \sum_j x_j)\delta^2(\sum_{j=1}^{n} k_j^\perp)\delta(x - x_1) \]

\[ \psi_n^\lambda (x_i', k_i^\perp, \lambda_i) \psi_n^\lambda (x_i, k_i^\perp, \lambda_i) \delta_{\lambda_1', -\lambda_1} \delta_{\lambda_i', \lambda_i} (i \neq 1); \]

where \( x_i' = \frac{x_i}{1 - \zeta}; \ k_i'^\perp = k_i^\perp + \frac{x_i}{1 - \zeta} \Delta_\perp \) for \( i = 2, \ldots, n \) and

\( x_1' = \frac{x_1 - \zeta}{1 - \zeta}; \ k_1'^\perp = k_1^\perp - \frac{1 - x_1}{1 - \zeta} \Delta_\perp. \)

\[ F_{T\lambda',\lambda}^{n+1\rightarrow n-1} = (1 - \zeta)^{3/2 - n/2} \sum_{n,\lambda_i} \int \Pi_{i=1}^{n+1} \frac{dx_id^2k_i^\perp}{16\pi^3} (16\pi^3)^2 \delta(1 - \sum_{j=1}^{n+1} x_j)\delta^2(\sum_{j=1}^{n+1} k_j^\perp) \]

\[ \delta(x_{n+1} + x_1 - \zeta)\delta^2(k_{\perp n+1} + k_{1\perp} - \Delta_\perp)\delta(x - x_1)\psi_{n-1}^\lambda (x_i', k_i'^\perp, \lambda_i) \psi_{n+1}^\lambda (x_i, k_i^\perp, \lambda_i) \delta_{\lambda_1', -\lambda_{n+1}} \delta_{\lambda_i', \lambda_i} (i = 2, \ldots, n). \]

where \( x_i' = \frac{x_i}{1 - \zeta}; \ k_i'^\perp = k_i^\perp + \frac{x_i}{1 - \zeta} \Delta_\perp, \) for \( i = 2, \ldots, n \) label \( n - 1 \) spectators.
Consider a dressed electron state instead of a proton

State is expanded in Fock space: $|e^-\gamma\rangle$ and $|e^-e^-e^+\rangle$ contribute to $O(\alpha)$

- Generalized form of QED: mass $M$ to the external electrons, $m$ to the internal electron lines, $\lambda$ to the internal photon lines → composite fermion state with mass $M$: a fermion and a vector ‘diquark’ constituents

  Brodsky, Drell (1980); Brodsky, Chakrabarti, Harindranath, Mukherjee, Vary (2006)

  $M < m + \lambda$ to prevent decay

- Two and three particle LFWFs are systematically evaluated in perturbation theory

  - $2 \rightarrow 2$ overlap in the region $\zeta < x < 1$ and $3 \rightarrow 1$ overlap in the region $0 < x < \zeta$

- There is also a contribution from the single particle sector: wave function renormalization; contributes at $x = 1$, not included in this analysis

- A field theory inspired model satisfying the properties like polynomiality and positivity of GPDs: gives an intuitive picture of spin and orbital angular momentum of a composite relativistic system
Chiral Odd GPDs: Analytic forms for $\zeta < x < 1$

\[ E_T(x, \zeta, t) = \frac{e^2}{8\pi^3} \frac{2M\pi}{1-\zeta} (M - \frac{m}{x})x(1-x)I_3, \]

\[ \tilde{E}_T(x, \zeta, t) = \frac{e^2}{8\pi^3} \frac{M\pi}{1-\zeta} \left[ - (1-x) \left\{ (M - \frac{m}{x})x + (M - \frac{m}{x'})x' \right\} I_1 
+ (M - \frac{m}{x})x(1-x)I_2 \right], \]

\[ H_T(x, \zeta, t) = \frac{e^2}{8\pi^3} \frac{\pi}{2} \left[ \frac{x + x'}{2(1-x)} \ln(\frac{\Lambda^4}{DD'}) + \left\{ \frac{x + x'}{2(1-x)} B(x, \zeta) 
+ \frac{\zeta M}{1-\zeta} (M - \frac{m}{x})x(1-x) \right\} I_2 
- \frac{\zeta M}{1-\zeta} \left\{ (M - \frac{m}{x})x(1-\zeta) + (M - \frac{m}{x'})x' \right\} (1-x')I_1 \right]. \]
Chiral Odd GPDs : Analytic forms (contd.)

\[ B(x, \zeta) = M^2 x'(1 - x') - m^2(1 - x') - \lambda^2 x' + M^2 x(1 - x) - m^2(1 - x) - \lambda^2 x \]

\[ I_1 = \int_0^1 dy \frac{1 - y}{Q(y)} \]

\[ I_2 = \int_0^1 dy \frac{1}{Q(y)} \]

\[ I_3 = \int_0^1 dy \frac{y}{Q(y)} \]

\[ Q(y) = y(1 - y)(1 - x')^2 \Delta_{\perp}^2 - y(M^2 x(1 - x) - m^2(1 - x) - \lambda^2 x) \]
\[ - (1 - y)(M^2 x'(1 - x') - m^2(1 - x') - \lambda^2 x') \]

\[ D = M^2 x(1 - x) - m^2(1 - x) - \lambda^2 x \text{ and } D' = M^2 x'(1 - x') - m^2(1 - x') - \lambda^2 x' \]
For all plots, $M = 0.51$ MeV, $m = 0.5$ Mev, $\lambda = 0.02$ Mev; normalization $\frac{e^2}{(2\pi)^3} = 1$

$E_T$ is independent of $x$ for small and medium $x$, for $x \to 1$ it is independent of $t$ and goes to 0

Increases in magnitude with increase of $\zeta$ for fixed $x$; finite at $\zeta = 0$

Decreases in magnitude with increase of $t$
One has to include also the higher Fock space contribution for $\zeta > x$ in order to calculate $x$ moments.

Behaviour for $x > \zeta$ from these plots.

$\tilde{E}_T$ is zero at $\zeta = 0$. 

$\tilde{E}_T(x, \zeta, t)$
- $H_T$ decreases with increase of $\zeta$ for fixed $x$
- Finite at $\zeta = 0$
- Reduces to transversity in the forward limit: coefficient of Log term gives the correct splitting function for leading order evolution
- Single particle Fock space sector contribution important to get the correct behaviour at $x \to 1$: no divergence
Generalized Parton Distributions in Impact Parameter Space

Fourier transform with respect to the transverse momentum transfer \( \Delta_\perp \) gives GPDs in impact parameter space

\[
\mathcal{H}(x, \zeta, b_\perp) = \frac{1}{(2\pi)^2} \int d^2 \Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H(x, \zeta, t)
\]

\[
= \frac{1}{2\pi} \int \Delta d\Delta J_0(\Delta b) H(x, \zeta, t),
\]

where \( \Delta = |\Delta_\perp| \) and \( b = |b_\perp| \)

Other chiral even and chiral odd GPDS in impact parameter space are defined in the same way

Probability interpretation when \( \zeta = 0 \): impact parameter dependent parton distributions
GPDs in Longitudinal Position Space

Connection with Wigner distribution: Belitsky, Ji, Yuan (2003)

DVCS amplitude in longitudinal position space: analogy with diffraction pattern in optics
Brodsky, Chakrabarti, Harindranath, Mukherjee, Vary (2006)

We define a boost invariant impact parameter conjugate to the longitudinal momentum transfer as
\[ \sigma = \frac{1}{2} b^- P^+ \]

\[ \mathcal{H}(x, \sigma, t) = \frac{1}{2\pi} \int_0^{\zeta_f} d\zeta e^{i \frac{1}{2} P^+ \zeta b^-} H(x, \zeta, t) \]
\[ = \frac{1}{2\pi} \int_0^{\zeta_f} d\zeta e^{i \sigma \zeta} H(x, \zeta, t). \]

Upper limit is the maximum \( \zeta \) value allowed for fixed \( -t \)

To get the complete picture both \( x > \zeta \) and \( x < \zeta \) contributions will have to be considered: chiral even calculated in the reference above

Similarly for other chiral even and chiral odd GPDS
Two Particle Light-front Wave Functions
Chiral Odd GPDs in Position Space (1 > x > ζ)

- As \( \bar{H}_T = 0 \) in this model, \( E_T \) in impact parameter space gives the distortion in the distribution of transversely polarized quarks in an unpolarized proton.
- Related to the spin-orbit correlation in the two-particle LFWF.
- In longitudinal position space one can observe diffraction pattern.
Chiral Odd GPDs in Position Space ($1 > x > \zeta$)

- Zero at $\zeta = 0$: odd function of $\zeta$
- First study in position space
- No diffraction pattern in longitudinal position space
Chiral Odd GPDs in Position Space \((1 > x > \zeta)\)

- Delta function peaked at \(b_\perp = 0\) for a free Dirac particle; smearing in \(b_\perp\) due to two-particle LFWF

- Reduces to transversity in the forward limit: gives the correlation of the transverse spin of the quark and the transverse spin of the nucleon in a polarized target in impact parameter space

- Diffraction pattern in impact parameter space
- Used parametrization of Ahmad, Honkanen, Liuti, Taneja (2007)

- At zero $\zeta$, parametrization obtained by simultaneously fitting the experimental data on nucleon form factor and DIS structure functions

- Spectator model with Regge-type term at input scale

- We have used parametrizations set I (u quark) at scale $0.09 \, GeV^2$
GPD model in position space

- Used parametrization of Ahmad, Honkanen, Liuti, Taneja (2007)

- We have used parametrizations set I (u quark) at scale 0.09 GeV²
Summary

• Studied the chiral-odd GPDs in transverse and longitudinal position space for non-zero skewness $\zeta$

• Presented complete overlap formulas both for $1 > x > \zeta$ and $0 < x < \zeta$ in terms of light-front wave functions in light-front gauge

• Used a self consistent field theory inspired relativistic two-body model, namely for the quantum fluctuation of an electron at one loop in QED: most general form of this model may act as a template for the quark spin-one diquark structure of the proton

• Only the diagonal $2 \rightarrow 2$ overlap contributes for $x > \zeta$: spin-orbit correlations in the two-particle LFWFs as well as the correlations between the quark transverse spin and the target spin

• Also presented recent parametrization of GPDs in position space