Chiral-odd and Chiral-even Generalized Parton Distributions in Transverse and Longitudinal Position Space

Asmita Mukherjee
Indian Institute of Technology, Mumbai, India

- Chiral Odd GPDs : Why they are interesting
- Overlap representation of Chiral-odd GPDs
- Simple example : electron at one loop
- GPDs in position space
- Phenomenological study

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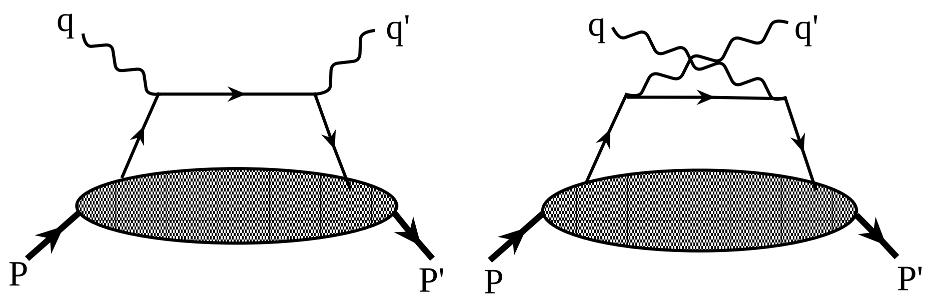
In collaboration with D. Chakrabarti (IIT Kanpur), R. Manohar (IIT Bombay)

Deeply Virtual Compton Scattering (DVCS) Amplitude

Deeply virtual Compton scattering :



$$M^{IJ}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) = \epsilon^I_\mu \, \epsilon^{*J}_\nu \, M^{\mu\nu}(\vec{q}_\perp, \vec{\Delta}_\perp, \zeta) = -e_q^2 \, \frac{1}{2\bar{P}^+} \int_{\zeta-1}^{1} \mathrm{d}x$$

$$\times \quad \Big\{ t^{IJ}(x,\zeta)\, \bar{U}(P') \Big[H(x,\zeta,t)\, \gamma^+ + E(x,\zeta,t)\, \frac{i}{2M}\, \sigma^{+\alpha}(-\Delta_\alpha) \Big] U(P) \Big\},$$

where $\bar{P} = \frac{1}{2}(P' + P)$,

$$t^{\uparrow\uparrow}(x,\zeta) = \quad t^{\downarrow\downarrow}(x,\zeta) = \frac{1}{x - i\epsilon} + \frac{1}{x - \zeta + i\epsilon}$$

DVCS contd.

DVCS amplitude contains

$$\int \frac{dy^-}{8\pi}\, e^{ixP^+y^-/2}\, \langle P',\lambda'|\bar{\psi}(0)\,\gamma^+\,\psi(y)\,|P,\lambda\rangle\Big|_{y^+=0,y_\perp=0}$$

$$= \frac{1}{2\bar{P}^+}\,\bar{U}(P',\lambda')\left[H(x,\zeta,t)\,\gamma^+ + E(x,\zeta,t)\,\frac{i}{2M}\,\sigma^{+\alpha}(-\Delta_\alpha)\right]U(P,\lambda),$$

$H(x,\zeta,t)$ and $E(x,\zeta,t)$ are chiral even GPDs; as well as $\tilde{H}(x,\zeta,t)$ and $\tilde{E}(x,\zeta,t)$

$$\int \frac{dy^-}{8\pi}\, e^{ixP^+y^-/2}\, \langle P',\lambda'|\bar{\psi}(0)\,\gamma^+\gamma_5\,\psi(y)\,|P,\lambda\rangle\Big|_{y^+=0,y_\perp=0}$$

$$= \frac{1}{2\bar{P}^+}\,\bar{U}(P',\lambda')\left[\tilde{H}(x,\zeta,t)\,\gamma^+\gamma_5 + \tilde{E}(x,\zeta,t)\,\frac{\gamma_5\Delta^+}{2M}\right]U(P,\lambda),$$

Momentum transfer $\Delta = P - P'$, $t = \Delta^2$, $\zeta = \frac{\Delta^+}{P^+}$

x is the fraction of the proton momentum carried by the active quark

Chiral-odd GPDs

Defined as the non-forward matrix elements of light-like correlations of tensor charge
We use the parametrization

$$P^+ \int \frac{dz^-}{2\pi} \qquad e^{\frac{iP^+z^-}{2}} \langle P', \lambda' \mid \bar{\psi}(\frac{-z^-}{2})\sigma^{+j}\gamma_5\psi(\frac{z^-}{2}) \mid P, \lambda\rangle_{z^+=0, z_\perp=0}$$

$$= H_T(x,\zeta,t)\bar{u}(P')\sigma^{+j}\gamma_5 u(P) - \tilde{H}_T(x,\zeta,t)\varepsilon^{+j\alpha\beta}\bar{u}(P')\frac{\Delta_\alpha P_\beta}{M^2}u(P)$$

$$-E_T(x,\xi,t)\varepsilon^{+j\alpha\beta}\bar{u}(P')\frac{\Delta_\alpha\gamma_\beta}{2M}u(P) + \tilde{E}_T(x,\zeta,t)\varepsilon^{+j\alpha\beta}\bar{u}(P')\frac{P_\alpha\gamma_\beta}{M}u(P).$$

M. Burkardt (2005), M. Diehl (2003), M. Diehl, P. Haegler (2005)

Momenta of initial and final protons :

$$P = \left(P^+ \; , \; \vec{0}_\perp \; , \; \frac{M^2}{P^+} \right) \; , P' = \left((1-\zeta)P^+ \; , \; -\vec{\Delta}_\perp \; , \; \frac{M^2 + \vec{\Delta}_\perp^2}{(1-\zeta)P^+} \right)$$

Involves quark helicity flip
In the forward limit $H_T(x,0,0)$ reduces to the transversity distribution (generalized transversity)

Chiral Odd GPDs : Why ? How to measure ?

- $\int dx x \left[H_T(x,0,0) + 2\tilde{H}_T(x,0,0) + E_T(x,0,0) \right]$ related to the transverse angular momentum carried by transversely polarized quarks in an unpolarized target : similar to Ji's relation
- $\int dx \left[2\tilde{H}_T(x,0,0) + E_T(x,0,0) \right]$ tells us in which direction the average position of the quarks with spin in the x direction is shifted in the y direction for an unpolarized target w.r.t the transverse center of momentum : can determine the sign of Boer-Mulders function

M. Burkardt (2005)

- Certain combinations of chiral-odd GPDs also give the correlation between the transverse quark spin and target spin
- Several proposals to measure H_T : for example in photo or electroproduction of two vector mesons on a nucleon target

Ivanov, Pire, Szymanowski, Teryaev (2002), Enberg, Pire, Szymanowski (2006)

- Exclusive process $\gamma^* P \to \pi^0 P$: to measure the tensor charge

S. Ahmad, G. Goldstein, S. Liuti (2008)

Overlap Representation

• The target state is expanded in terms of multiparticle light-front wave functions in Fock space; choose light-front gauge $A^{+} = 0$

GPDs are given in terms of overlaps of the light-front wave functions

• Both chiral even and chiral odd GPDs ; diagonal parton number conserving $n \rightarrow n$ overlap in the kinematical regime $\zeta < x < 1$ and $\zeta - 1 < x < 0$

• Off-diagonal $n + 1 \rightarrow n - 1$ overlap for $0 < x < \zeta$ where the parton number is decreased by two : higher Fock sector LFWF

Diehl, Feldman, Jacob, Kroll (2001);
Brodsky, Diehl, Huang (2001)
Chakrabarti, Manohar, Mukherjee (2008)

• Both contributions needed to get the complete kinematical region as well as to calculate the moments at non-zero ζ

• Only diagonal overlap when ζ is zero

• In chiral odd GPDs, overlap of different quark helicities whereas in chiral even GPDs no helicity flip

Overlap Representation for Chiral-odd GPDs

$$F^{n\to n}_{T\lambda',\lambda} = (1-\zeta)^{1-\frac{n}{2}} \sum_{n,\lambda_i} \int \Pi^n_{i=1} \frac{dx_i d^2 k^i_\perp}{16\pi^3} 16\pi^3 \delta(1-\sum_j x_j)\delta^2(\sum_{j=1}^{n} k^j_\perp)\delta(x-x_1)$$
$$\psi^{\lambda' *}_n(x'_i, {k'}^i_\perp, \lambda'_i)\psi^\lambda_n(x_i, k^i_\perp, \lambda_i)\delta_{\lambda'_1,-\lambda_1}\delta_{\lambda'_i,\lambda_i}(i \neq 1);$$

where ${x'}_i = \frac{x_i}{1-\zeta}$; ${k'}^i_\perp = k^i_\perp + \frac{x_i}{1-\zeta}\Delta_\perp$ for $i = 2, ...n$ and
${x'}_1 = \frac{x_1-\zeta}{1-\zeta}$; ${k'}^1_\perp = k^1_\perp$ - $\frac{1-x_1}{1-\zeta}\Delta_\perp$.

$$F^{n+1\to n-1}_{T\lambda',\lambda} = (1-\zeta)^{3/2-n/2} \sum_{n,\lambda_i} \int \Pi^{n+1}_{i=1} \frac{dx_i d^2 k^i_\perp}{16\pi^3} (16\pi^3)^2 \delta(1-\sum_{j=1}^{n+1} x_j)\delta^2(\sum_{j=1}^{n+1} k^i_\perp)$$
$$\delta(x_{n+1}+x_1-\zeta)\delta^2(k_{\perp n+1}+k_{1\perp}-\Delta_\perp)\delta(x-x_1)\psi^{\lambda' *}_{n-1}(x'_i, {k'}^i_\perp, \lambda'_i)$$
$$\psi^\lambda_{n+1}(x_i, k^i_\perp, \lambda_i)\delta_{\lambda'_1,-\lambda_{n+1}}\delta_{\lambda'_i,\lambda_i}(i = 2, ..n).$$

where ${x'}_i = \frac{x_i}{1-\zeta}$, ${k'}^i_\perp = k^i_\perp + \frac{x_i}{1-\zeta}\Delta_\perp$, for $i = 2, ...n$ label $n-1$ spectators

DVCS in QED at one loop

• Consider a dressed electron state instead of a proton

State is expanded in Fock space : $| e^- \gamma\rangle$ and $| e^- e^- e^+\rangle$ contribute to $O(\alpha)$

• Generalized form of QED : mass M to the external electrons, m to the internal electron lines λ to the internal photon lines $\longrightarrow$ composite fermion state with mass M : a fermion and a vector 'diquark' constituents

Brodsky, Drell (1980); Brodsky, Chakrabarti, Harindranath, Mukherjee, Vary (2006)

$M < m + \lambda$ to prevent decay

• Two and three particle LFWFs are systematically evaluated in perturbation theory

• $2 \longrightarrow 2$ overlap in the region $\zeta < x < 1$ and $3 \longrightarrow 1$ overlap in the region $0 < x < \zeta$

• There is also a contribution from the single particle sector : wave function renormalization; contributes at $x = 1$, not included in this analysis

• A field theory inspired model satisfying the properties like polynomiality and positivity of GPDs : gives an intuitive picture of spin and orbital angular momentum of a composite relativistic system

Chiral Odd GPDs : Analytic forms for $\zeta < x < 1$

$$E_T(x,\zeta,t) = \frac{e^2}{8\pi^3}\frac{2M\pi}{1-\zeta}(M-\frac{m}{x})x(1-x)I_3,$$

$$\begin{aligned}\tilde{E}_T(x,\zeta,t) \quad &= \quad \frac{e^2}{8\pi^3}\frac{M\pi}{1-\zeta}\Big[-(1-x)\Big\{(M-\frac{m}{x})x+(M-\frac{m}{x'})x'\Big\}I_1 \\ &+(M-\frac{m}{x})x(1-x)I_2\Big],\end{aligned}$$

$$\begin{aligned}H_T(x,\zeta,t) \quad &= \quad \frac{e^2}{8\pi^3}\frac{\pi}{2}\Big[\frac{x+x'}{2(1-x)}\ln(\frac{\Lambda^4}{DD'})+\Big\{\frac{x+x'}{2(1-x)}B(x,\zeta) \\ &+\frac{\zeta M}{1-\zeta}(M-\frac{m}{x})x(1-x)\Big\}I_2 \\ &-\frac{\zeta M}{1-\zeta}\Big\{(M-\frac{m}{x})x(1-\zeta)+(M-\frac{m}{x'})x'\Big\}(1-x')I_1\Big],\end{aligned}$$

Chiral Odd GPDs : Analytic forms (contd.)

$$B(x,\zeta) = M^2 x'(1-x') - m^2(1-x') - \lambda^2 x' + M^2 x(1-x) - m^2(1-x) - \lambda^2 x$$

$$I_1 = \int_0^1 dy \frac{1-y}{Q(y)}$$

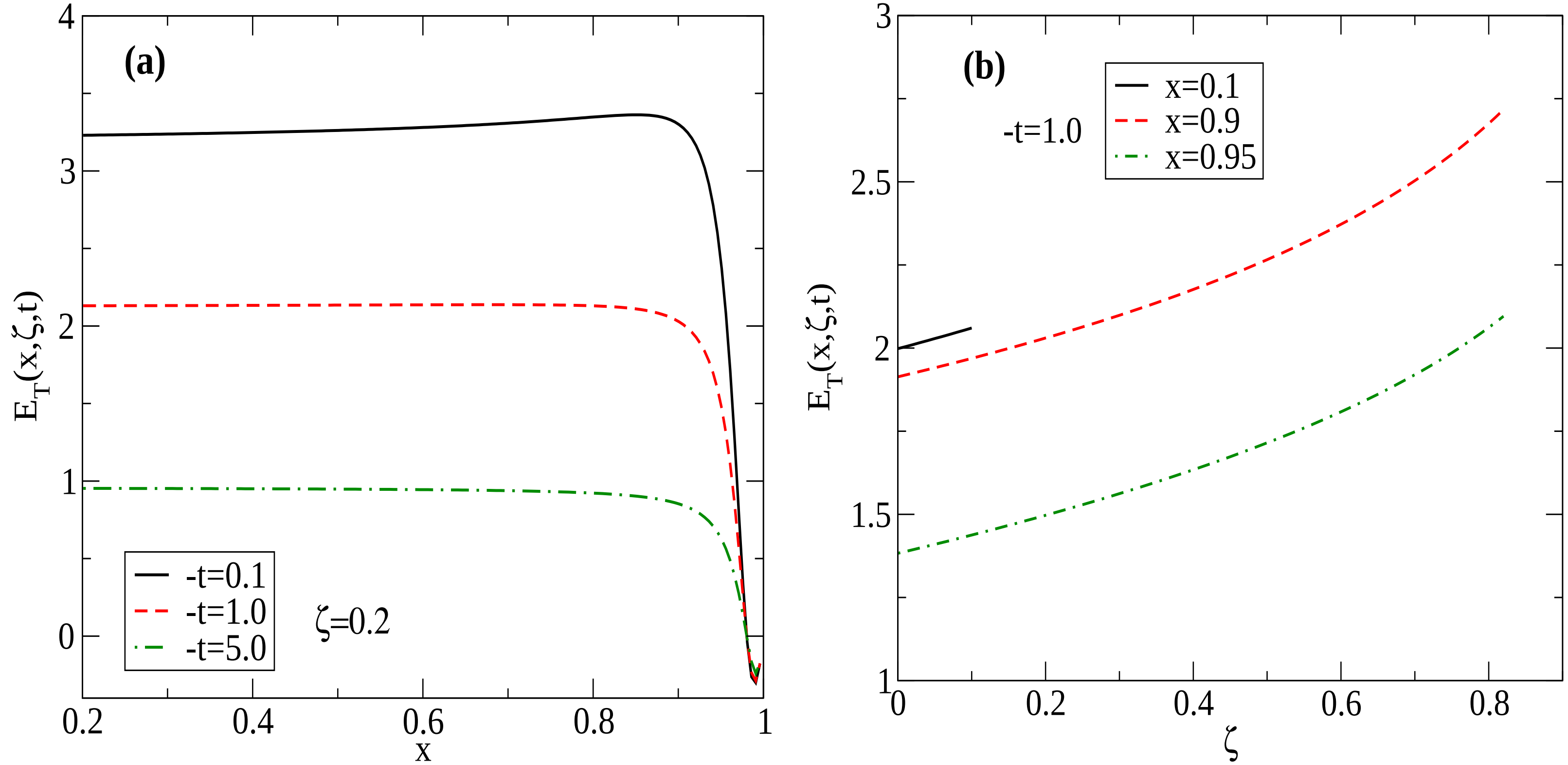
$$I_2 = \int_0^1 dy \frac{1}{Q(y)}$$

$$I_3 = \int_0^1 dy \frac{y}{Q(y)}$$

$$\begin{aligned} Q(y) \quad = \quad & y(1-y)(1-x')^2\Delta_\perp^2 - y(M^2 x(1-x) - m^2(1-x) - \lambda^2 x) \\ & -(1-y)(M^2 x'(1-x') - m^2(1-x') - \lambda^2 x') \end{aligned}$$

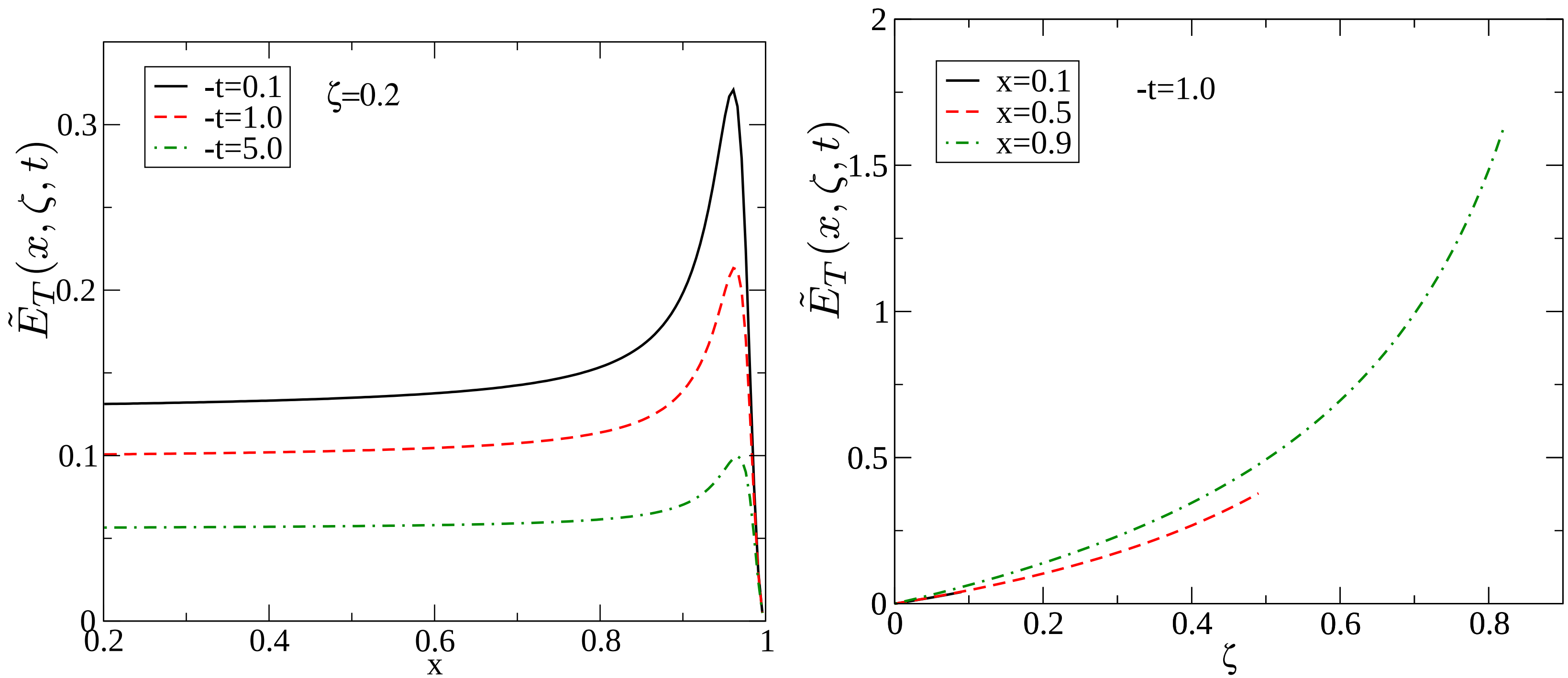
$D = M^2 x(1-x) - m^2(1-x) - \lambda^2 x$ and $D' = M^2 x'(1-x') - m^2(1-x') - \lambda^2 x'$

Chiral Odd GPDs : Numerical Results



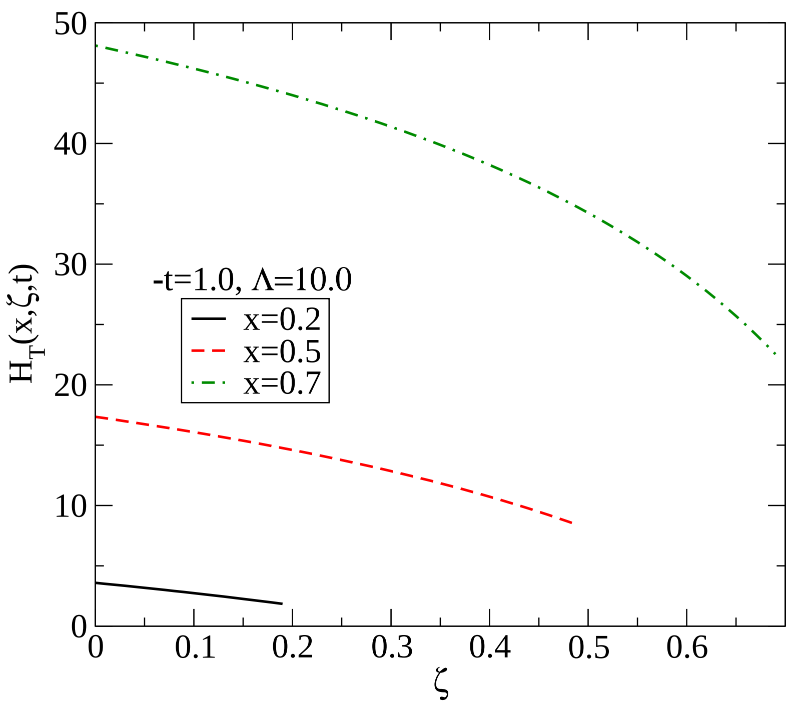
- For all plots, $M = 0.51$ MeV, $m = 0.5$ Mev, $\lambda = 0.02$ Mev; normalization $\frac{e^2}{(2\pi)^3} = 1$
- E_T is independent of x for small and medium x, for $x \to 1$ it is independent of t and goes to 0
- Increases in magnitude with increase of ζ for fixed x; finite at $\zeta = 0$
- Decreases in magnitude with increase of t

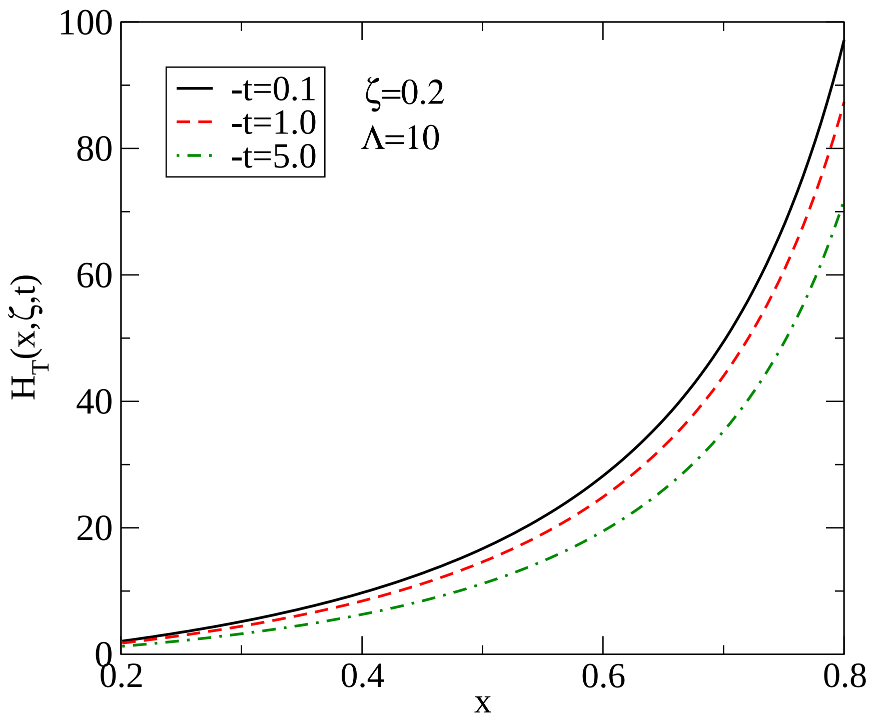
Chiral Odd GPDs : Numerical Results



- One has to include also the higher Fock space contribution for $\zeta > x$ in order to calculate x moments
- Behaviour for $x > \zeta$ from these plots
- $\tilde{E}_T$ is zero at $\zeta = 0$

Chiral Odd GPDs : Numerical Results





- H_T decreases with increase of ζ for fixed x
- Finite at $\zeta = 0$
- Reduces to transversity in the forward limit : coefficient of Log term gives the correct splitting function for leading order evolution
- Single particle Fock space sector contribution important to get the correct behaviour at $x \to 1$: no divergence

Generalized Parton Distributions in Impact Parameter Space

Fourier transform with respect to the transverse momentum transfer $\Delta_\perp$ gives GPDs in impact parameter space

$$\mathcal{H}(x,\zeta,b_\perp) = \frac{1}{(2\pi)^2}\int d^2\Delta_\perp e^{-i\Delta_\perp\cdot b_\perp} H(x,\zeta,t)$$
$$= \frac{1}{2\pi}\int \Delta d\Delta J_0(\Delta b) H(x,\zeta,t),$$

where $\Delta = |\Delta_\perp|$ and $b = |b_\perp|$

Other chiral even and chiral odd GPDS in impact parameter space are defined in the same way

Probability interpretation when $\zeta = 0$: impact parameter dependent parton distributions

GPDs in Longitudinal Position Space

Connection with Wigner distribution : Belitsky, Ji, Yuan (2003)

DVCS amplitude in longitudinal position space : analogy with diffraction pattern in optics

Brodsky, Chakrabarti, Harindranath, Mukherjee, Vary (2006)

We define a boost invariant impact parameter conjugate to the longitudinal momentum transfer as $\sigma = \frac{1}{2} b^- P^+$

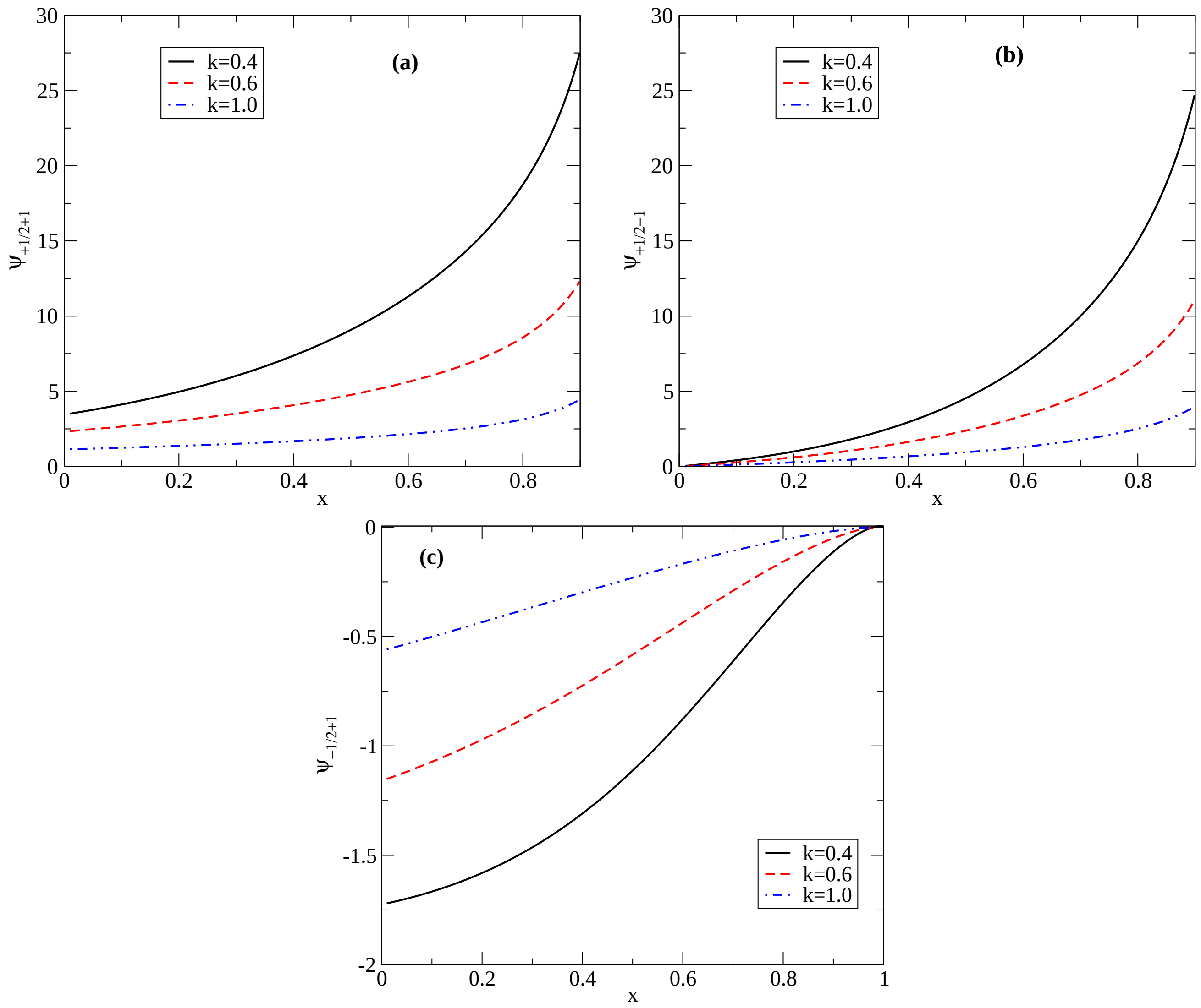
$$\begin{aligned} \mathcal{H}(x,\sigma,t) &= \frac{1}{2\pi}\int_0^{\zeta_f} d\zeta e^{i\frac{1}{2}P^+\zeta b^-} H(x,\zeta,t) \\ &= \frac{1}{2\pi}\int_0^{\zeta_f} d\zeta e^{i\sigma\zeta} H(x,\zeta,t). \end{aligned}$$

Upper limit is the maximum ζ value allowed for fixed $-t$

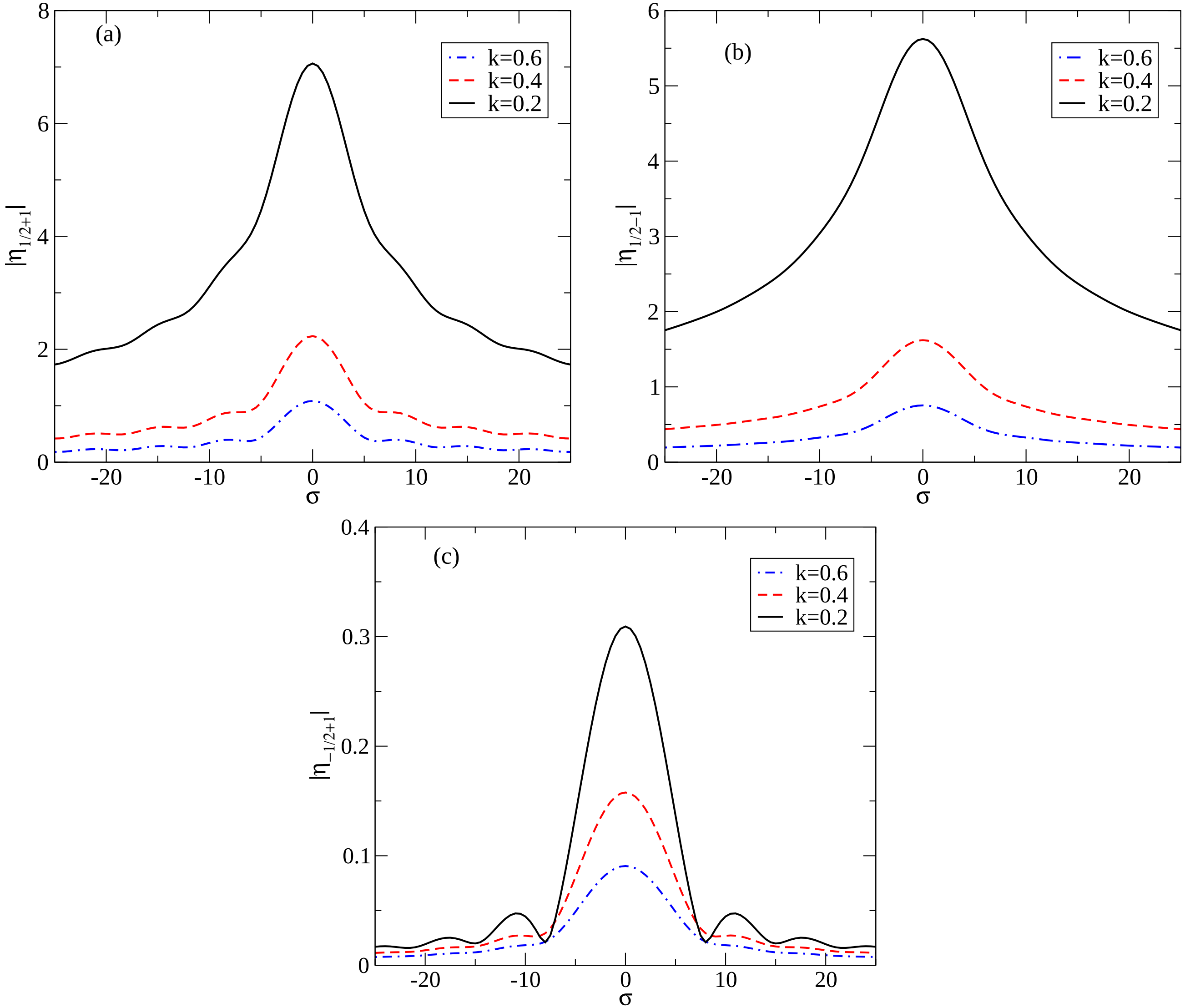
To get the complete picture both $x > \zeta$ and $x < \zeta$ contributions will have to be considered : chiral even calculated in the reference above

Similarly for other chiral even and chiral odd GPDS

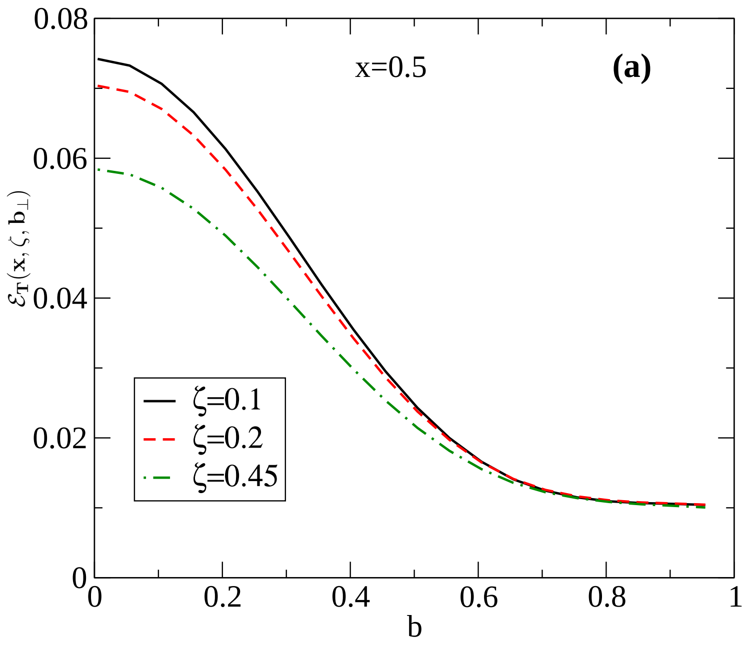
Two Particle Light-front Wave Functions

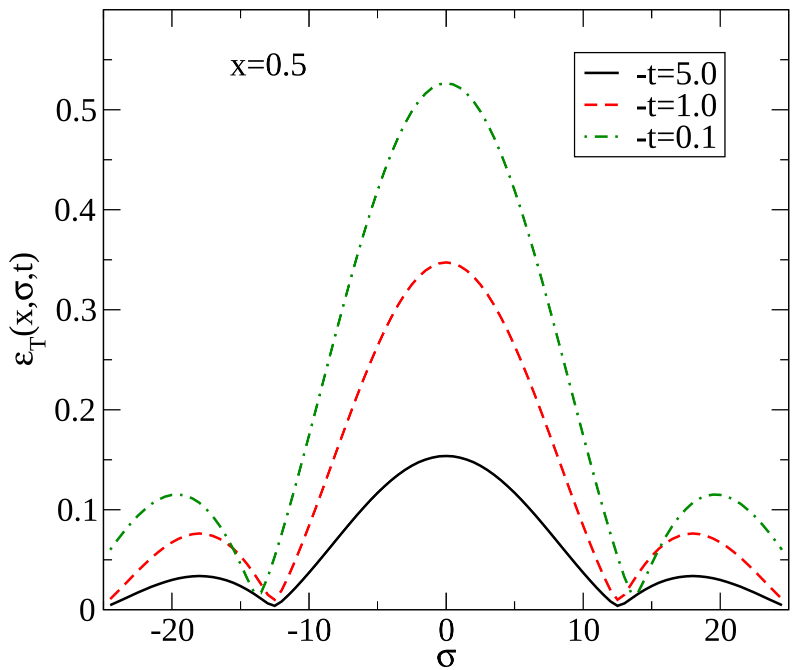


Two Particle Light-front Wave Functions in Longitudinal Position Space



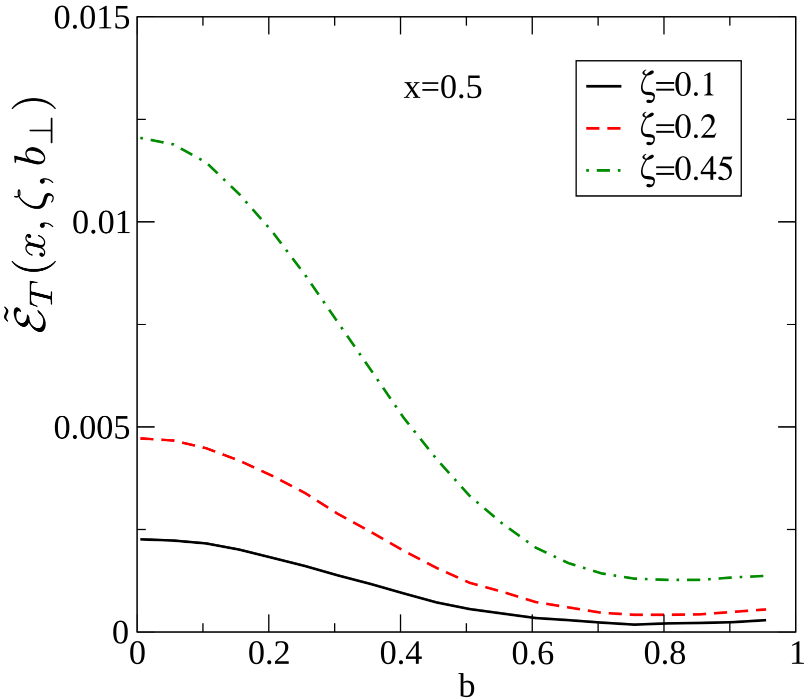
Chiral Odd GPDs in Position Space ($1 > x > \zeta$)

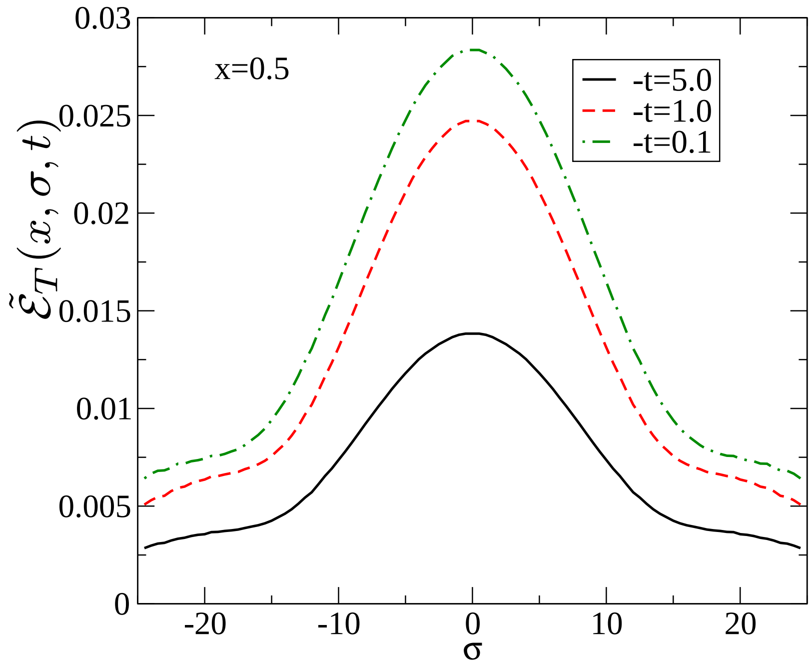




- As $\tilde{H}_T = 0$ in this model, E_T in impact parameter space gives the distortion in the distribution of transversely polarized quarks in an unpolarized proton
- Related to the spin-orbit correlation in the two-particle LFWF
- In longitudinal position space one can observe diffraction pattern

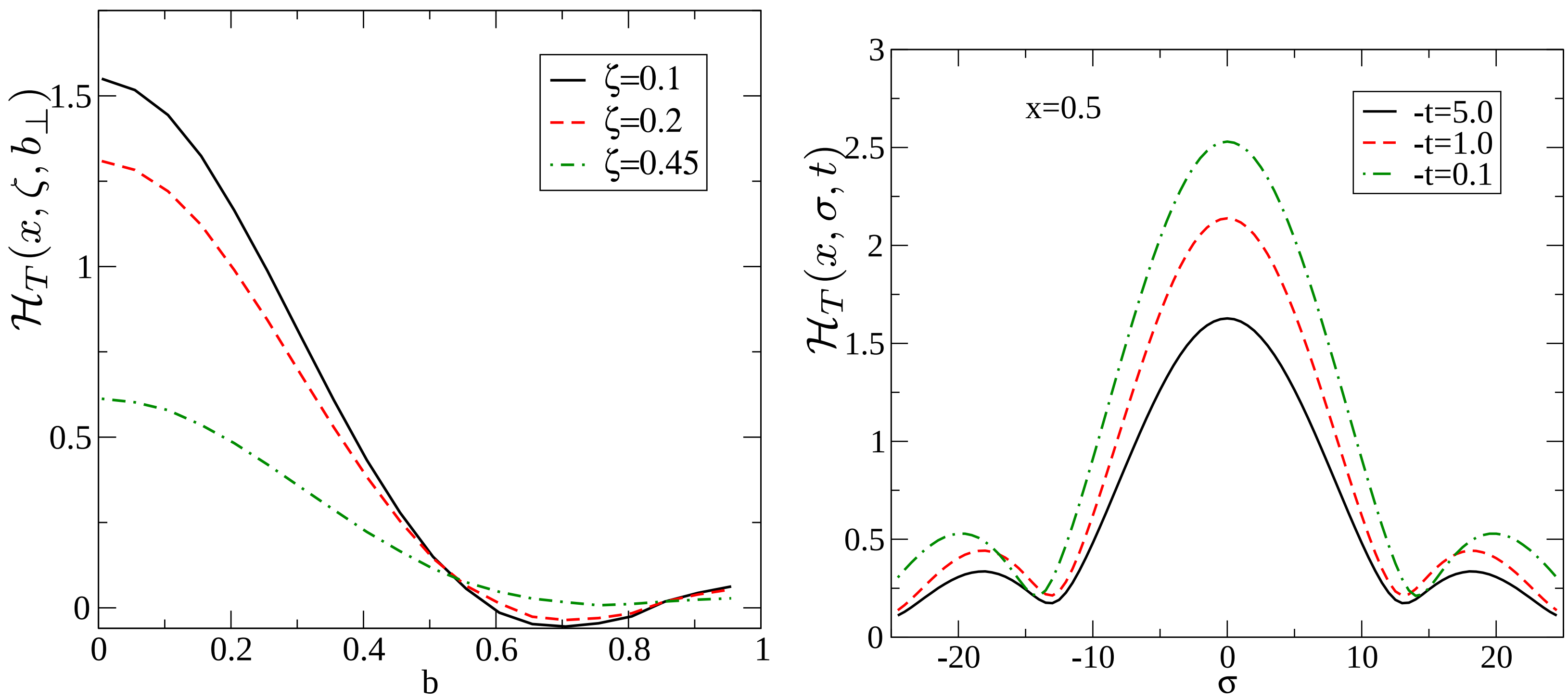
Chiral Odd GPDs in Position Space ($1 > x > \zeta$)





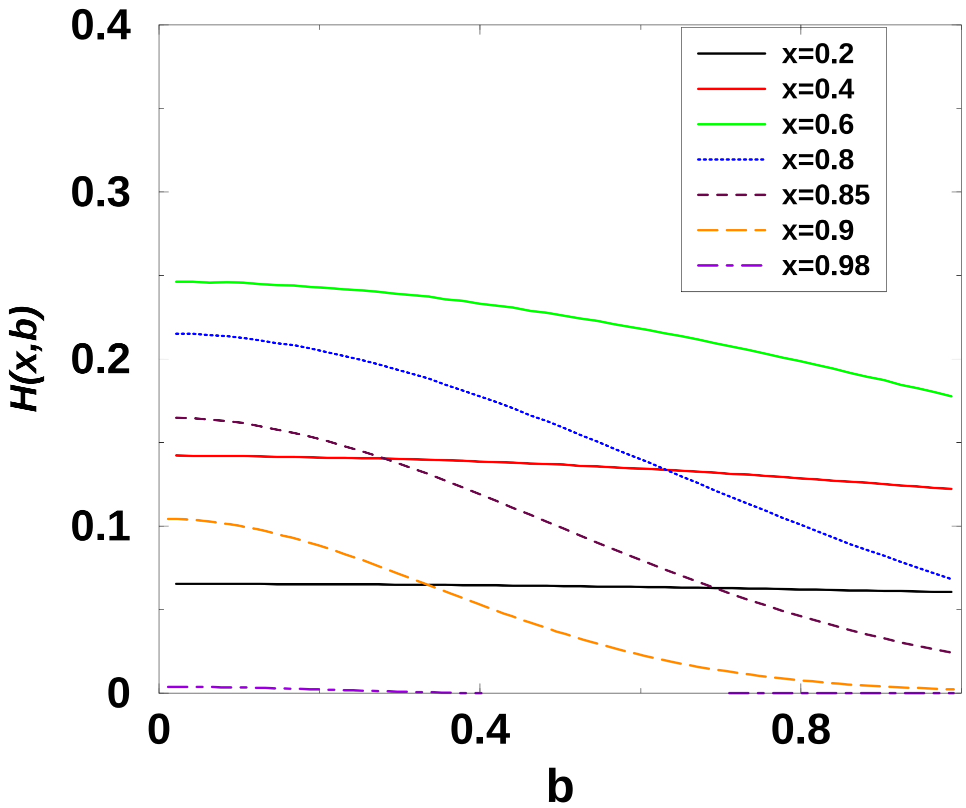
- Zero at $\zeta = 0$: odd function of ζ
- First study in position space
- No diffraction pattern in longitudinal position space

Chiral Odd GPDs in Position Space ($1 > x > \zeta$)



- Delta function peaked at $b_\perp = 0$ for a free Dirac particle ; smearing in $b_\perp$ due to two-particle LFWF
- Reduces to transversity in the forward limit : gives the correlation of the transverse spin of the quark and the transverse spin of the nucleon in a polarized target in impact parameter space
- Diffraction pattern in impact parameter space

GPD model in position space



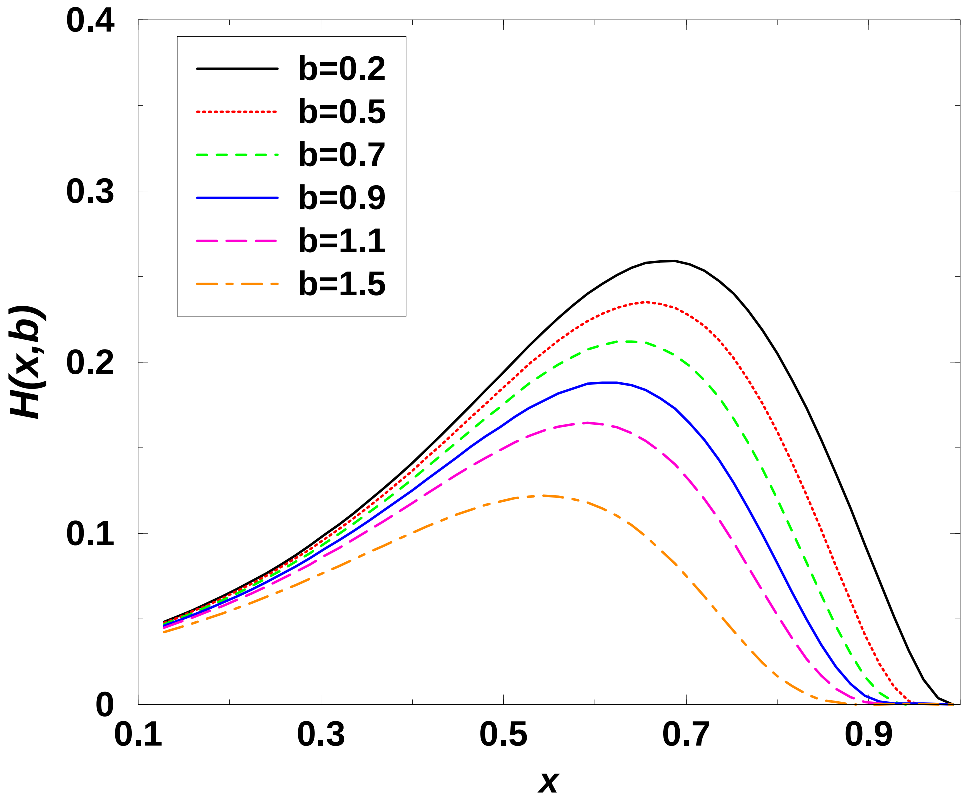
- Used parametriztion of

Ahmad, Honkanen, Liuti, Taneja (2007)

- At zero ζ, parametrization obtained by simultaneously fitting the experimental data on nucleon form factor and DIS structure functions

- Spectator model with Regge-type term at input scale

GPD model in position space



- Used parametriztion of

Ahmad, Honkanen, Liuti, Taneja (2007)

- We have used parametrizations set I (u quark) at scale 0.09 GeV^2

Summary

- Studied the chiral-odd GPDs in transverse and longitudinal position space for non-zero skewness ζ
- Presented complete overlap formulas both for $1 > x > \zeta$ and $0 < x < \zeta$ in terms of light-front wave functions in light-front gauge
- Used a a self consistent field theory inspired relativistic two-body model, namely for the quantum fluctuation of an electron at one loop in QED : most general form of this model may act as a template for the quark spin-one diquark structure of the proton
- Only the diagonal $2 \to 2$ overlap contributes for $x > \zeta$: spin-orbit correlations in the two-particle LFWFs as well as the correlations between the quark transverse spin and the target spin
- Also presented recent parametrization of GPDs in position space