Inclusive Single Hadron Production in Neutral Current Deep-Inelastic Scattering at Next-to-Leading Order

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Outline

1. Introduction
   - Motivation
   - Parton Model

2. First Order in $\alpha_s$
   - Partonic Subprocesses
   - HERA data

3. Second order in $\alpha_s$
   - Subtraction Method
   - HERA data

4. Conclusions
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Motivation

- Test of perturbative QCD and factorization.
- Universality of Fragmentation Functions.
- Direct comparison with experimental data.
- Precise data from H1 and ZEUS Collaborations.
Single Production of Hadrons

\[ \begin{align*}
  k & \quad k' \\
  q & \quad p_b \\
  p_q & \quad p_h \\
  P & 
\end{align*} \]
Single Production of Hadrons

Process

\[ e(k) + p(P) \rightarrow e(k') + h(p) + X \]

Cross Section

\[ \frac{d^4 \sigma^h}{dx dy dz d\phi} = \int_{\bar{x}}^{1} \frac{dx}{x} \int_{\bar{z}}^{1} \frac{dz}{z} \sum_{ab} f_a \left( \frac{\bar{x}}{x}, \mu^2 \right) \frac{d^3 \sigma^{ab}}{dx dy dz} D_b \left( \frac{\bar{z}}{z}, \mu^2 \right) \]

Variables

\[ x_B = \bar{x} = \frac{Q^2}{2P \cdot q}, \quad y = \frac{p_a \cdot q}{p_a \cdot k}, \quad z = \frac{P \cdot p_h}{P \cdot q} \]
\[ x = \frac{Q^2}{2p_a \cdot q}, \quad z = \frac{p_a \cdot p_b}{p_a \cdot q} \]
Partonic Cross Section

\[
\frac{d^3 \sigma^{ab}}{dx dy dz} = \frac{\alpha^2}{16\pi^2} \frac{y}{Q^4} \lambda_{ab} L^{\mu \nu} H^{ab}_{\mu \nu}
\]

- Lepton Tensor:

\[
L^{\mu \nu} = \frac{Q^2}{2y} \left( 2 - 2y + y^2 \right) \left( -g^{\mu \nu} \right) + \frac{2Q^4}{sh^2} \left( \frac{y^2 - 6y + 6}{y^4} \right) p_\mu p_\nu
\]

\[
\pm i \frac{Q^2}{sh} \left( \frac{y - 2}{y^2} \right) \varepsilon^{\mu \nu \alpha \beta} p_\alpha q_\beta
\]

with \(sh = \frac{Q^2}{xy}\).

- Cross section in transverse, longitudinal and axial components:

\[
H_T^{ab} = -g^{\mu \nu} H^{ab}_{\mu \nu}, \quad H_L^{ab} = p_\mu p_\nu H^{ab}_{\mu \nu}, \quad H_A^{ab} = \pm i\varepsilon^{\mu \nu \alpha \beta} p_\alpha q_\beta H^{ab}_{\mu \nu}
\]

\[
H^{ab}_{\mu \nu} = \sum_{\text{spins}} M^\dagger_{\mu} M_\nu
\]
Partonic Cross Section

\[
\lambda_{ab}^{T,L} = e_f^2 - 2e_f v_f v_e \chi_Z(Q^2) + (a_f^2 + v_f^2) (a_e^2 + v_e^2) \chi_Z^2(Q^2)
\]

\[
\lambda_{ab}^A = -2e_f a_f a_e \chi_Z(Q^2) + 4a_f a_e v_f v_e \chi_Z^2(Q^2)
\]

\[
a_{e,f} = T_{e,f}^3
\]

\[
v_{e,f} = T_{e,f}^3 - 2e_{e,f} \sin^2 \theta_W
\]

\[
\chi_Z(Q^2) = \frac{1}{4 \sin^2 \theta_W \cos^2 \theta_W \frac{Q^2}{Q^2 + M_Z^2}}
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Partonic Subprocesses

LO

\[ \gamma^*, Z^0 + q \rightarrow q \]

Real Corrections:

\[ \gamma^*, Z^0 + q \rightarrow q + g \]
\[ \gamma^*, Z^0 + q \rightarrow g + q \]
\[ \gamma^*, Z^0 + g \rightarrow q + \bar{q} \]
Charged Particle Production


\[ 0.05 < y < 0.6 \quad 100 < Q^2 < 20000 \text{ GeV}^2 \quad 10^\circ < \theta_e < 150^\circ \]

\[ E_{e^+} = 27.6 \text{ GeV} \quad E_p = 920 \text{ GeV} \]

\[ x_p = \frac{2p_h}{\sqrt{s}} \]

Fragmentation and Scale


\[ \mu^2 = Q^2 \]
\[ \frac{1}{\sigma} \frac{d\sigma(h^\pm)}{dx_p} \]

Graphs showing data for different ranges of x_p, Q(GeV) for H1 data, AKK (γ), and AKK (γ+Z) for 0.1<x_p<0.2, 0.2<x_p<0.3, 0.3<x_p<0.4, 0.4<x_p<0.5, 0.5<x_p<0.7, and 0.7<x_p<1.
- \( Z^0 \) boson contribution not noticeable in multiplicities.
- Ratios of cross sections show contributions of up to 15\% for \( Q^2 > 10000 \) GeV\(^2\).
- Similar behaviour for all \( x_p \) bins.
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Partonic Subprocesses

**Born**

\[ \gamma^*, Z^0 + q \rightarrow q + g \]
\[ \gamma^*, Z^0 + q \rightarrow g + q \]
\[ \gamma^*, Z^0 + g \rightarrow q + \bar{q} \]

**Virtual Corrections**

- Dimensional Regularization.
- UV and IR singularities.
- UV singularities removed through renormalization of the wave functions and the strong coupling constant.
- The remaining soft and collinear singularities should cancel partly against counterparts originating from the phase space integration of the real correction.
Subtraction Method

\[ \sigma^{NLO} = \int_{m+1} \left[ (d\sigma^R)_{\epsilon=0} - (d\sigma^A)_{\epsilon=0} \right] + \int_m \left[ d\sigma^V + \int_1 d\sigma^A \right]_{\epsilon=0} \]

- \( d\sigma^A \) acts as a local counterterm for \( d\sigma^R \)
- After phase space integration \( \epsilon \to 0 \) in the integrand on the first term on the right hand side.
- Singularities on the other two terms cancel each other.
Subtraction Method

$d\sigma^A$

- It has to be obtained independently from the process considered.
- It has to match the singular behaviour of $d\sigma^R$ in $d$ dimensions.
- Its form has to be particularly convenient for Monte Carlo integration techniques.
- It has to be exactly integrable analytically in $d$ dimensions over the single parton subspaces leading to soft and collinear divergences.

\[ d\sigma^A = \sum_{dipoles} d\sigma^B \times dV_{dipole} \]

\[ \int_{m+1} \int_{m} d\sigma^A = \int_{m} d\sigma^B \times I \]
Production of hadrons with finite $p_T^*$


$0.1 < y < 0.6 \quad 2 < Q^2 < 70 \text{ GeV}^2 \quad 5^\circ < \theta < 25^\circ$

$E_{e^+} = 27.6 \text{ GeV} \quad E_p = 820 \text{ GeV}$

$p_T^* > 2.5 \text{ GeV}$

Fragmentation and Scale

- AKK Fragmentation functions

$$\mu = \sqrt{Q^2 + (p_T^*)^2}$$
$\pi^0$

**Introduction**

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**Second Order in $\alpha_s$**

**Conclusions**
Production of hadrons with finite $p_T^{*}$


$0.05 < y < 0.7 \quad 2 < Q^2 < 100 \text{ GeV}^2$

$E_{e^+} = 27.6 \text{ GeV} \quad E_p = 920 \text{ GeV}$

$p_T^{*} > 2.0 \text{ GeV}$

Fragmentation and Scale


$$\mu = \sqrt{Q^2 + (p_T^{*})^2}$$
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$D^{*\pm}$

H1 data

KKK

$\mu = 1/2 \mu, 2 \mu$

$\frac{d\sigma(D^*)}{dQ^2}$ (nb/GeV$^2$)

$Q^2$ (GeV$^2$)

$\frac{d\sigma(D^*)}{dx_B}$ (nb)

$x_B$

$\frac{d\sigma(D^*)}{dp_T}$ (nb/GeV)

$p_T$ (GeV)
Ratios of cross sections show contributions of up to 5% for $Q^2 > 10000 \text{ GeV}^2$.

The behaviour is similar for both $\pi^0$ and $D^{*\pm}$ production.
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- At NLO, virtual and real corrections have been calculated for single hadron production in the case of neutral currents.
- Results agree with the data for $\pi^0$ and $D^{*\pm}$ production using AKK and KKKS fragmentation functions.
- Results not sensitive to choice of PDF set.
- Effect of $Z^0$ boson in the cross sections found to be up to 15% in the first order calculation and up to 5% in the second order calculation.