

Nuclear enhancement and suppression of diffractive structure functions at high energies

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Outline

- ▶ Computing F_2^D in the dipole model (IPsat) ,
- ▶ Results vs. β , Q^2 , $x_{\mathbb{P}}$, A .

Talk based on: H. Kowalski, T. L., C. Marquet and R. Venugopalan,
Phys. Rev. C **78** (2008) 045201, [arXiv:0805.4071 [hep-ph]].

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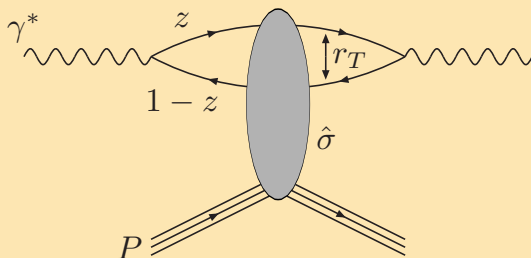
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Context: measure diffraction in eA at EIC

See <http://www.eic.bnl.gov/>

Also first evaluation of F_2^D in ep with IPsat parametrization

DIS in dipole frame



Use:

- ▶ S-matrix real
- ▶ optical theorem

- ▶ $\Psi_{L,T}^\gamma \sim K_{0,1} \left(\sqrt{z(1-z)} Q |r_T| \right)$
- ▶ momentum scale $Q^2 \sim 1/r_T^2$
- ▶ Diffractive: t is FT of \mathbf{b}_T .

See talk by A. Luszczak

Observables from dipole cross section

$$\sigma_{\text{dip}}(x, \mathbf{r}_T, \mathbf{\Delta}) = \int d^2\mathbf{b}_T \frac{d^2\sigma_{\text{dip}}(x, \mathbf{r}_T, \mathbf{b}_T)}{d^2\mathbf{b}_T} e^{i\mathbf{b}_T \cdot \mathbf{\Delta}}, \quad \mathbf{\Delta}^2 = -t$$



Inclusive

$$\sigma_{L,T}^{\gamma^*p} = \int d^2\mathbf{r}_T \int dz \left| \Psi_{L,T}^{\gamma}(Q^2, \mathbf{r}_T, z) \right|^2 \sigma_{\text{dip}}(x, \mathbf{r}_T)$$

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Inclusive **diffractive** (elastic dipole–target)

$$\sigma_{L,T}^{\gamma^*p} = \int d^2\mathbf{r}_T \int dz \left| \Psi_{L,T}^{\gamma}(Q^2, \mathbf{r}_T, z) \right|^2 \sigma_{\text{dip}}(x, \mathbf{r}_T)$$

$$\frac{\sigma_{L,T}^{D,tot}}{dt} = \frac{1}{16\pi} \int d^2\mathbf{r}_T \int dz \left| \Psi_{L,T}^{\gamma}(Q^2, \mathbf{r}_T, z) \right|^2 \sigma_{\text{dip}}^2(x, \mathbf{r}_T, \mathbf{\Delta})$$

Observables from dipole cross section

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Inclusive diffractive (elastic dipole–target) **exclusive diff.**

$$\sigma_{L,T}^{\gamma^* p} = \int d^2\mathbf{r}_T \int dz \left| \Psi_{L,T}^{\gamma}(Q^2, \mathbf{r}_T, z) \right|^2 \sigma_{\text{dip}}(x, \mathbf{r}_T)$$

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$$\frac{\sigma_{L,T}^{D,V}}{dt} = \frac{1}{16\pi} \left| \int d^2\mathbf{r}_T \int dz \left(\Psi^{\gamma} \Psi^{*V} \right)_{L,T} \sigma_{\text{dip}}(x, \mathbf{r}_T, \mathbf{\Delta}) \right|^2$$

IPsat model: protons

Use here the IPSat model [Kowalski, Teaney](#)

- ▶ DGLAP evolution: improves description of large Q^2 .
- ▶ Consistent \mathbf{b}_T dependence for total and diffractive
(no separate B_D and σ_0)

$$\sigma_{\text{dip}} = 2 \int d^2 \mathbf{b}_T \left(1 - \exp \left\{ \overbrace{-\frac{\pi^2}{2N_c} \alpha_s(\mu^2) x g(x, \mu^2) T_p(\mathbf{b}_T) r_T^2}^{-\mathbf{r}_T^2 Q_s^2 / 4}} \right\} \right)$$

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$$\sigma_{\text{dip}} = 2 \int d^2\mathbf{b}_T \left(1 - \exp \left\{ \overbrace{-\frac{\pi^2}{2N_c} \alpha_s(\mu^2) xg(x, \mu^2) T_p(\mathbf{b}_T) r_T^2}^{-r_T^2 Q_s^2/4}} \right\} \right)$$

4 parameters fit to HERA data

- ▶ $xg(x, \mu^2)$ is evolved with DGLAP, 2 parameters
- ▶ $\mu^2 = \frac{4}{r_T^2} + \mu_0^2$
- ▶ Gaussian $T_p(\mathbf{b}_T)$ ▶ B_D

IPsat: nuclei

Straightforward generalization to nuclei:

$$\frac{d\sigma_{\text{dip}}^A}{d^2\mathbf{b}_T} = 2 \left(1 - \exp \left\{ -\frac{\pi^2}{2N_c} \alpha_s(\mu^2) x g(x, \mu^2) \sum_{i=1}^A T_p(\mathbf{b}_T - \mathbf{b}_{T_i}) r_T^2 \right\} \right)$$

\mathbf{b}_{Tj} : nucleon positions \blacktriangleright average $\langle \cdot \rangle_N$ from Woods-Saxon;
no additional nuclear parameters

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$$R_p \sim 0.6\text{fm} \ll d_{NN}$$

► **lumpy** nucleus seen in **incoherent** diffraction $\left\langle \left(\sigma_{\text{dip}}^A \right)^2 \right\rangle_N$

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Coherent: Glauber

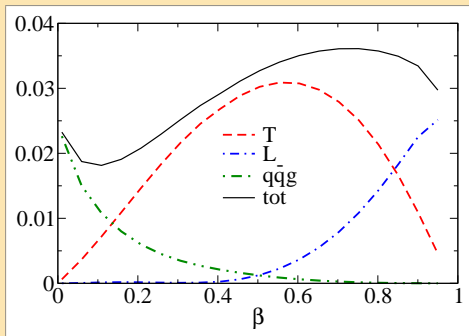
$$\left\langle \frac{d\sigma_{\text{dip}}^A}{d^2\mathbf{b}_T} \right\rangle_N \approx_{A \rightarrow \infty} 2 \left[1 - e^{-\frac{AT_A(\mathbf{b}_T)}{2} \sigma_{\text{dip}}^p} \right]$$

Diffractive structure function $F_2^D(x_{\mathbb{P}}, \beta, Q^2)$

$$\beta = \frac{Q^2}{M_X^2 + Q^2} \quad x_{\mathbb{P}} = \frac{M_X^2 + Q^2}{W^2 + Q^2} \quad x = \beta x_{\mathbb{P}}$$

Essential regimes:

- ▶ $\beta \ll 1$: dominated by higher Fock ($q\bar{q}g$ etc.)
- ▶ $\beta \sim 0.5$: dominated by transverse $q\bar{q}$
- ▶ $\beta \rightarrow 1$: longitudinal $q\bar{q}$.



(Proton, $Q^2 = 5\text{GeV}^2$, $x_{\mathbb{P}} = 10^{-3}$)

Computing F_2^D : $q\bar{q}$

$$x_{\mathbb{P}} F_2^D = \underbrace{x_{\mathbb{P}} F_{T,q\bar{q}}^D + x_{\mathbb{P}} F_{L,q\bar{q}}^D}_{\alpha_s^0} + \underbrace{x_{\mathbb{P}} F_{T,q\bar{q}g}^D}_{\alpha_s} + \underbrace{\text{higher Fock states}}_{\alpha_s^2}$$

Need Bessel transform of dipole cross section

$$\Phi_n = \int d^2\mathbf{b}_T \left[\int_0^\infty dr r K_n(\varepsilon r) J_n(kr) \frac{d\sigma_{\text{dip}}}{d^2\mathbf{b}_T}(\mathbf{b}_T, \mathbf{r}_T, x_{\mathbb{P}}) \right]^2,$$

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Transverse part of $q\bar{q}$

$$x_{\mathbb{P}} F_{T,q\bar{q}}^D(x_{\mathbb{P}}, \beta, Q^2) = \frac{N_c Q^4}{16\pi^3 \beta} \sum_f e_f^2 \int_{z_0}^{1/2} dz z (1-z) \\ \times \left[\varepsilon^2 (z^2 + (1-z)^2) \Phi_1 + m_f^2 \Phi_0 \right]$$

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Longitudinal part of $q\bar{q}$

$$x_{\mathbb{P}} F_{L,q\bar{q}}^D(x_{\mathbb{P}}, \beta, Q^2) = \frac{N_c Q^6}{4\pi^3 \beta} \sum_f e_f^2 \int_{z_0}^{1/2} dz z^3 (1-z)^3 \Phi_0$$

Computing F_2^D : $q\bar{q}$

$$x_{\mathbb{P}} F_2^D = \underbrace{x_{\mathbb{P}} F_{T,q\bar{q}}^D + x_{\mathbb{P}} F_{L,q\bar{q}}^D}_{\alpha_s^0} + \underbrace{x_{\mathbb{P}} F_{T,q\bar{q}g}^D}_{\alpha_s} + \underbrace{\text{higher Fock states}}_{\alpha_s^2}$$

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$\int dt$ from $-\infty$ to 0 ► “easy” evaluation integrating over \mathbf{b}_T .

Incoherent: $\int d^2\mathbf{b}_T$ really 2-dimensional.

Computing F_2^D II: $q\bar{q}g$ – component

Combine two known limits

Large Q^2 , finite β (“GBW” Golec-Biernat, Wusthoff)

$$\begin{aligned} x_{\mathbb{P}} F_{T,q\bar{q}g}^D(\text{GBW}) &= \frac{\alpha_s \beta}{8\pi^4} \sum_f e_f^2 \int d^2 \mathbf{b}_T \int_{\beta}^1 dz \left[\left(1 - \frac{\beta}{z}\right)^2 + \left(\frac{\beta}{z}\right)^2 \right] \\ &\times \int_0^{Q^2} dk^2 k^4 \ln \frac{Q^2}{k^2} \left[\int_0^{\infty} dr r K_2(\sqrt{z}kr) J_2(\sqrt{1-z}kr) \frac{d\tilde{\sigma}_{\text{dip}}}{d^2 \mathbf{b}_T}(\mathbf{b}_T, \mathbf{r}_T, x_{\mathbb{P}}) \right]^2 \end{aligned}$$

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Small $\beta \rightarrow 0$, large N_c , (“MS” Munier, Shoshi)

$$x_{\mathbb{P}} F_{T,q\bar{q}g}^D(\text{MS}) = \frac{C_F \alpha_s Q^2}{4\pi^4 \alpha_{\text{em}}} \int d^2\mathbf{b}_T d^2\mathbf{r}_T d^2\mathbf{r}_{T'} \int_0^1 dz \left| \Psi_T^{\gamma^*}(r, Q, z) \right|^2 \\ \frac{r_T^2}{(\mathbf{r}_{T'})^2 (\mathbf{r}_T - \mathbf{r}_{T'})^2} \left[\mathcal{N}(\mathbf{r}_{T'}) + \mathcal{N}(\mathbf{r}_T - \mathbf{r}_{T'}) - \mathcal{N}(\mathbf{r}_T) - \mathcal{N}(\mathbf{r}_{T'}) \mathcal{N}(\mathbf{r}_T - \mathbf{r}_{T'}) \right]^2$$

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Interpolate between these Marquet 2007

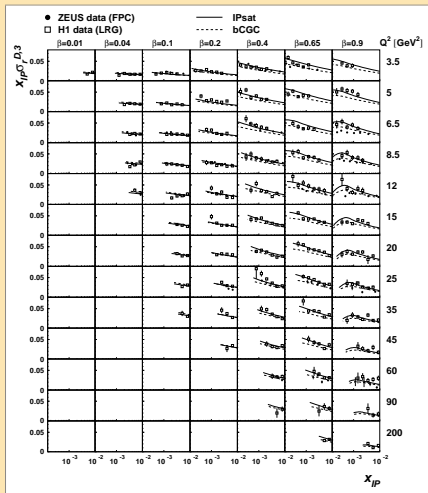
$$x_{\mathbb{P}} F_2^D(x_{\mathbb{P}}, \beta, Q^2) = \frac{x_{\mathbb{P}} F_{T,q\bar{q}g}^D(\text{GBW})(x_{\mathbb{P}}, \beta, Q^2) \times x_{\mathbb{P}} F_{T,q\bar{q}g}^D(\text{MS})(x_{\mathbb{P}}, Q^2)}{x_{\mathbb{P}} F_{T,q\bar{q}g}^D(\text{GBW})(x_{\mathbb{P}}, \beta = 0, Q^2)}$$

First: HERA data

Agree with HERA ep data
($x_{\mathbb{P}} < 0.01$) with
 $\chi^2/\text{d.o.f.} \sim 1$.

Need small $\alpha_s = 0.14$
(coefficient of the $q\bar{q}g$ -term)
use same value for nuclei.

- ▶ Mostly cancels in ratios
- ▶ Larger $q\bar{q}g$: natural from b -dependence (more on this next slide)



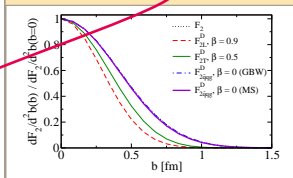
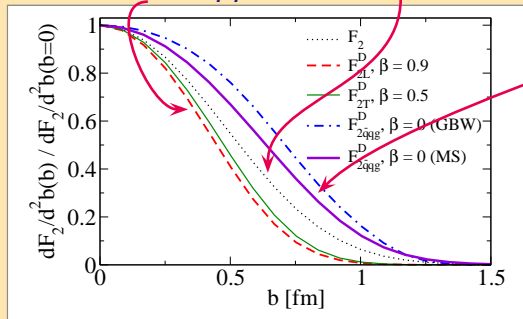
b_T -dependence of different components

Dominant impact parameters different

$$b^{\text{diff}}(q\bar{q}) < b^{\text{incl}} < b^{\text{diff}}(q\bar{q}g)$$

Integrand vs. b_T for

diff. $q\bar{q}$ — inclusive — diff $q\bar{q}g$



$$Q^2 = 100 \text{ GeV}^2$$

$$(x_{\mathbb{P}} = 10^{-3})$$

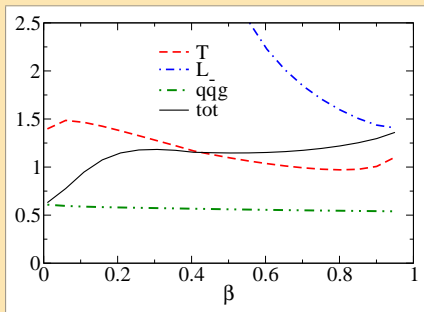
$$Q^2 = 1 \text{ GeV}^2$$

Results: β -dependence

Ratios of nucleus/proton

$$\frac{F_{2A(x)}^D}{AF_{2p(x)}^D} \text{ with } x = L, T(q\bar{q}), q\bar{q}g, \text{ tot.}$$

- ▶ $\beta \ll 1$: $q\bar{q}g$ strongly suppressed (black disk limit)
- ▶ $\beta \sim 0.5$: transverse $q\bar{q}$ enhanced.
- ▶ $\beta \rightarrow 1$: longitudinal $q\bar{q}$ very much enhanced.



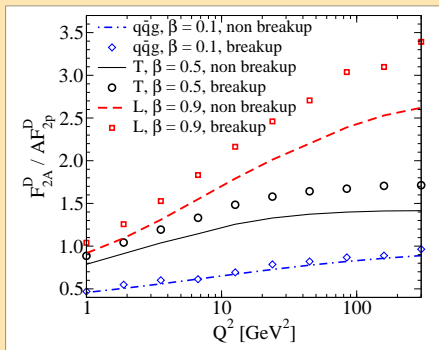
(Au at $Q^2 = 5\text{GeV}^2$ and $x_p = 10^{-3}$)

Results: Q^2 -dependence

Nuclei have smaller Q^2/Q_s^2 at same $x_{\mathbb{P}}$, Q^2

► Nuclear enhancement of F_2^D 's grows with Q^2

- Non breakup = coherent
- Breakup = coherent + incoherent



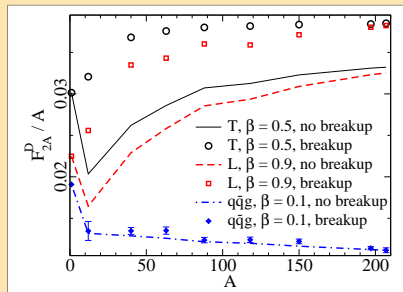
(Au at $x_{\mathbb{P}} = 10^{-3}$)

Results: A-dependence

Small A more dilute than p :
coherent diff. suppressed

(Different components at

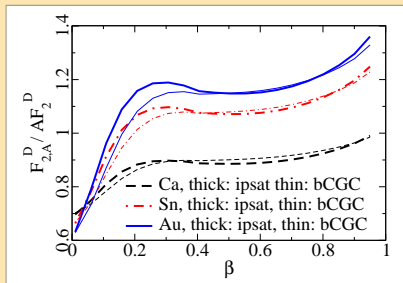
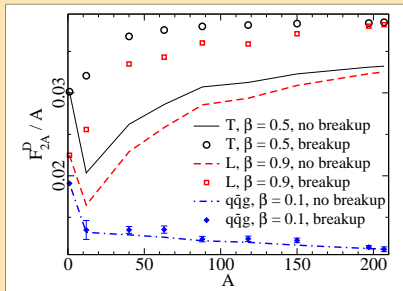
$x_{\mathbb{P}} = 10^{-3}$ and $Q^2 = 5\text{GeV}^2$) \blacktriangleright



Results: A-dependence

Small A more dilute than p:
coherent diff. suppressed

(Different components at
 $x_{\text{p}} = 10^{-3}$ and $Q^2 = 5\text{GeV}^2$) ▶



(Different nuclei vs β)

Conclusions

Computed diffractive structure functions F_2^D for nuclei

- ▶ Realistic b -dependence essential
- ▶ Nuclear suppression at small β , enhancement at large β
- ▶ Large A close to black disk: diffraction enhanced
- ▶ Coherent diffraction at small A suppressed by diluteness

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Further questions:

- ▶ t -dependence in nuclei?
- ▶ Beyond Glauber-like (independent scattering) treatment of nucleons?

Thank you.