New higher-order results on coefficient functions

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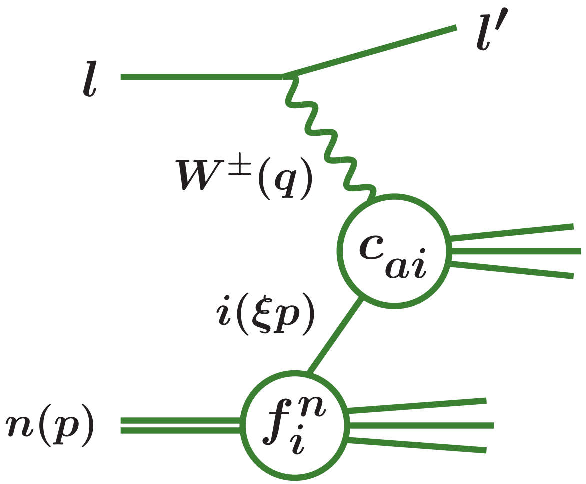
Collaborations with Sven Moch (DESY) and Jos Vermaseren (NIKHEF) *

- **Introduction**
- **Three-loop coefficient function of the structure function $F_3^{\nu+\bar{\nu}}$**
- **Physical evolution kernels and large-x coefficient functions**
- **Summary and outlook**

* **MVV, arXiv: 0812.4168 (NPB); MV, arXiv: 0902.2342 (JHEP); MV, to appear**

Hard processes in perturbative QCD (I)

Example: inclusive deep-inelastic scattering (DIS) via W-exchange



Hard scale, Bjorken variable

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Q^2 &= -q^2 \\
x &= Q^2/(2p\cdot q)
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Lowest order, quarks: $x = \xi$

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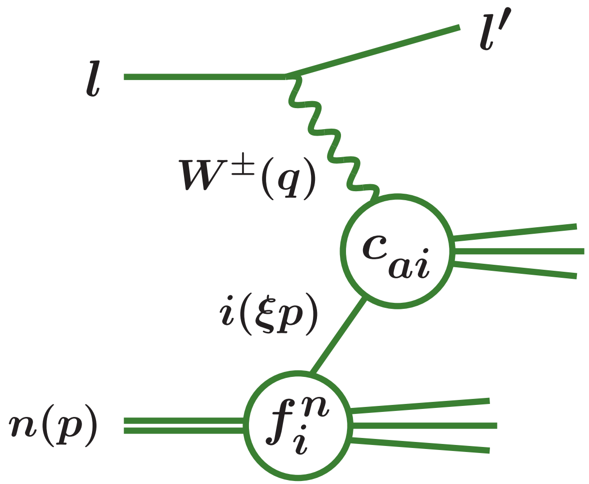

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Structure functions $F_{2,3,L}$ ($\eta_{2,L} = 1$, $\eta_3 = 0$), Mellin convolutions

$$x^{-\eta_a} F_a^n(x, Q^2) = \sum_i \int_x^1 \frac{d\xi}{\xi}\, C_{a,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) f_i^n(\xi, \mu^2)$$

Coefficient functions: scheme ($\overline{\text{MS}}$), scale $\mu = \mathcal{O}(Q)$ (calc.: $= Q$)

Not included: $1/Q^2$ corrections, extract or suppress by data cuts

Hard processes in perturbative QCD (II)

Parton distributions f_i: renormalization-group evolution equations

$$\frac{d}{d\ln\mu^2} f_i(\xi,\mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2)) \otimes f_k(\mu^2)\right](\xi)$$

$\otimes$ **= Mellin convolution. Initial conditions incalculable in pert. QCD**

$\Rightarrow$ **predictions: fits of suitable reference processes, universality**

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Expansions in α_s: splitting functions P, coefficient functions c_a

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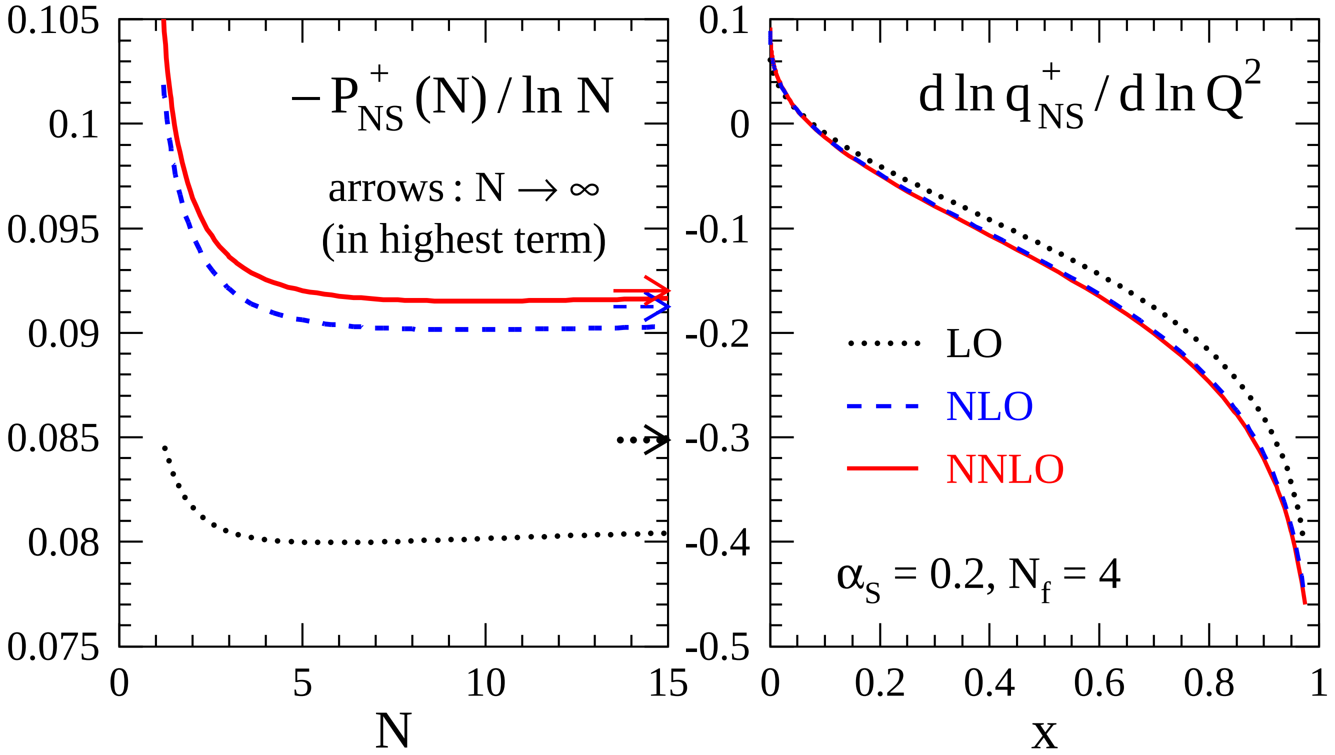
N^{3}LO, $P^{(3)}$, $c_a^{(3)}$: highest present precision (approx., 3-loop c_a)

Splitting functions at large N/ large x

Moments $A^N = \int_0^1 dx\, x^{N-1} A(x)$. **Non-singlet**$^+$: $u + \bar{u} - (d + \bar{d})$ **etc**



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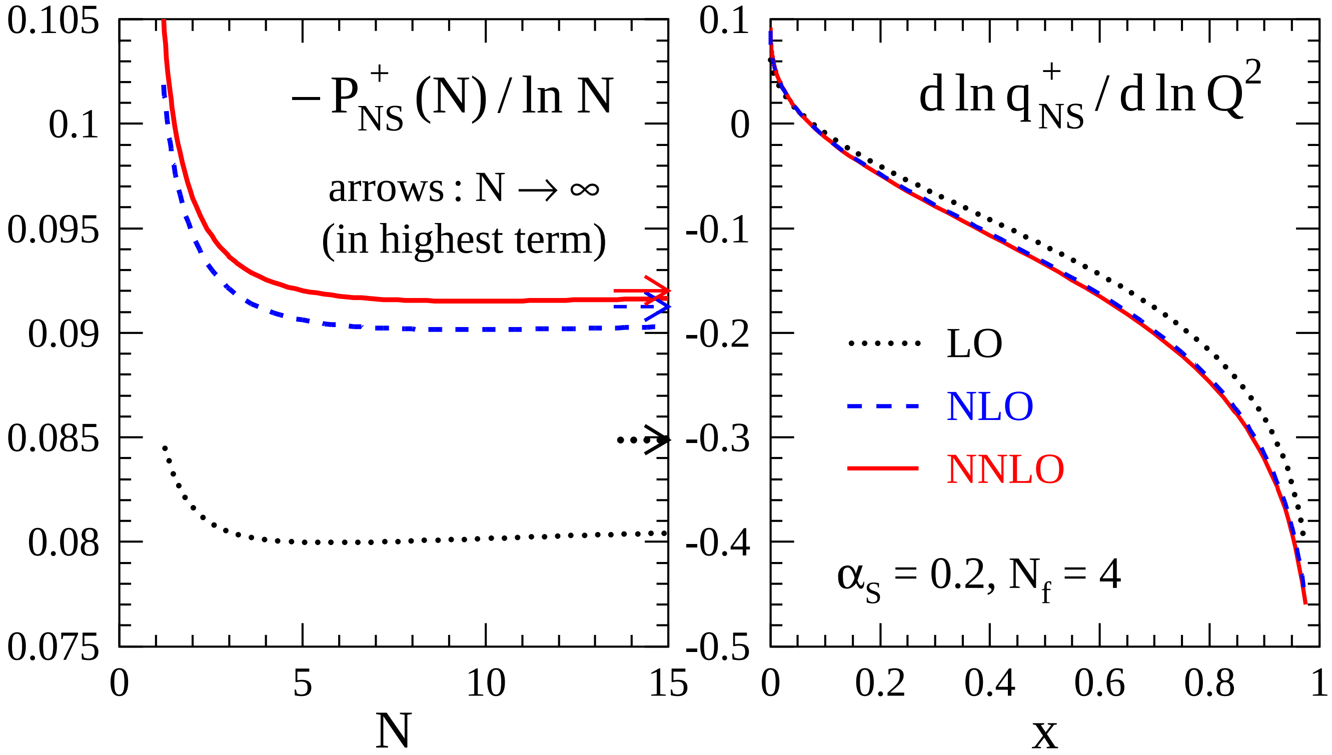


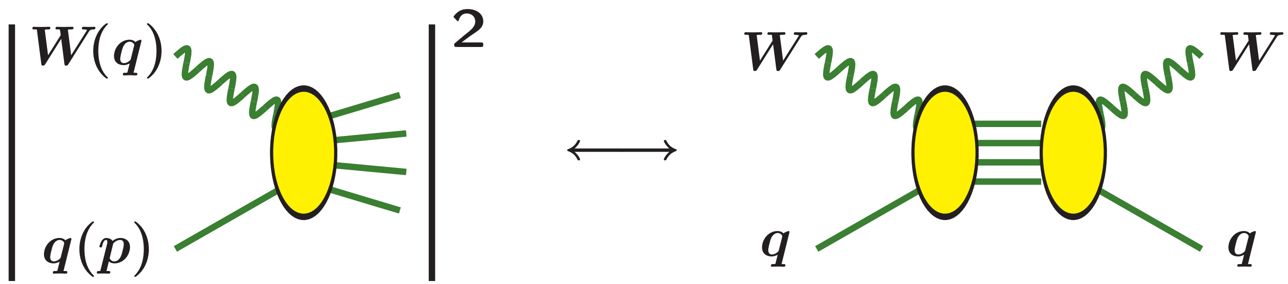
N^{3}LO: $P_{\rm ns}^{+}$ **computed for** $N=2\,,\ n_f = 3$ **Baikov, Chetyrkin (06)**

$$P_{\rm ns}^{+} \;=\; -\,0.283\,\alpha_{\rm s}\,[\,1 \,+\, 0.869\,\alpha_{\rm s} \,+\, 0.798\,\alpha_{\rm s}^{2} \,+\, 0.926\,\alpha_{\rm s}^{3} \,+\, \ldots\,]$$

$N > 2\,,\ n_f > 3$: **similar / smaller** $\ln N$ **coeff's.** $\simeq$ **1% accuracy at** $\alpha_{\rm s} \lesssim 0.25$

The three-loop calculation of $F_3^{\nu+\bar{\nu}}$

Optical theorem: W^*q **total cross sections** $\leftrightarrow$ **forward amplitudes**



Coefficient of $(2p\cdot q)^N$ $\leftrightarrow$ N**-th moment** $A^N = \int_0^1 dx\, x^{N-1}\, A(x)$

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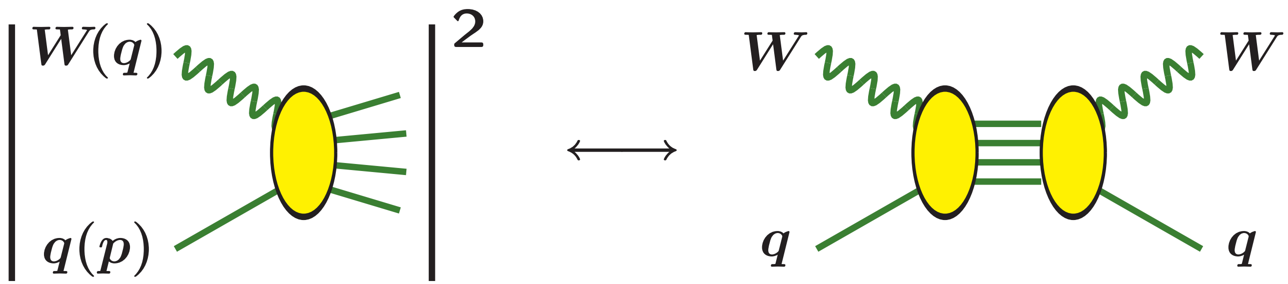

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F_3**: VA interference, projection of hadronic tensor** ($\mathrm{dim.} = 4 - 2\varepsilon$)

$$P_3^{\mu\nu} \;=\; -\mathrm{i}\,\frac{1}{(1-2\varepsilon)(1-\varepsilon)}\,\epsilon^{\mu\nu\alpha\beta}\,\frac{p_\alpha q_\beta}{2\,p\cdot q}$$

$\nu+\bar{\nu}$ **combination (large** $\leftrightarrow$ **valence quark distr.): odd moments ('$-$')**

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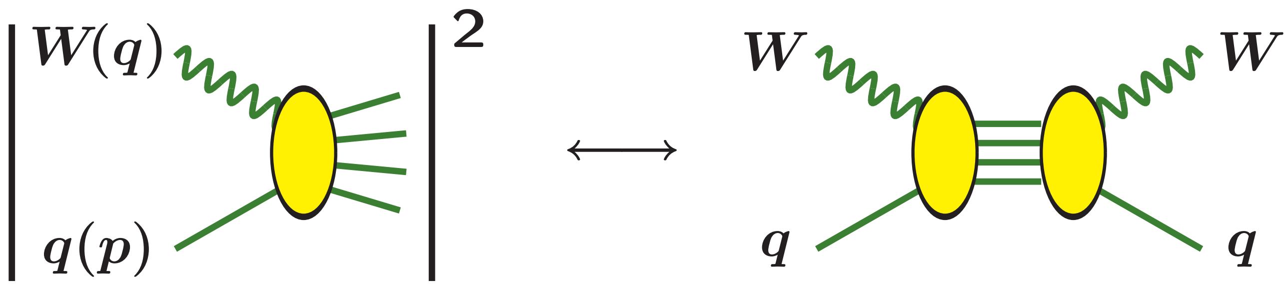

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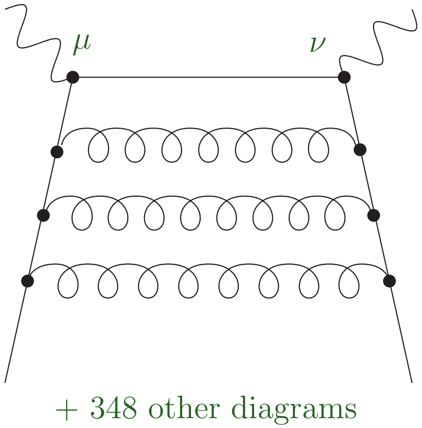
$N = 1$: **Larin, Vermaseren (91);** $N = 3, \ldots, 13$: **Retey, Vermaseren (00)**

From there: γ_5 **treatment (‘Larin scheme’), renormalizations** Z_A, Z_5

Analytic properties of the three-loop results

New feature at third order: SU(n_c) group invariant $d^{abc}d_{abc}$ (right)

fl_2:



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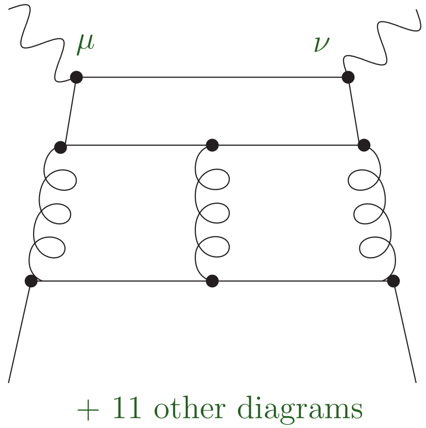

Suppressed as $x \longrightarrow 1$, but large effect at small x. Leading logarithm:

$$c_{3,-}^{(3)}\Big|_{\ln^5 x} = \frac{2}{5}\,C_A^2 C_F - \frac{29}{15}\,C_A C_F^2 + \frac{53}{30}\,C_F^3 - \frac{32}{15}\,\frac{d^{abc}d_{abc}}{n_c}$$

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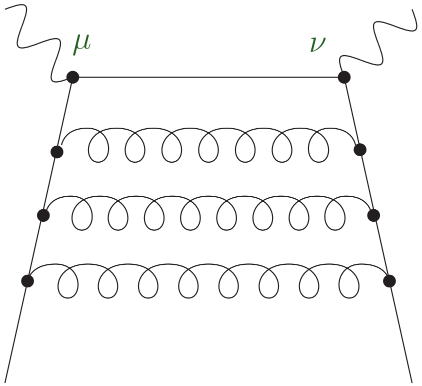

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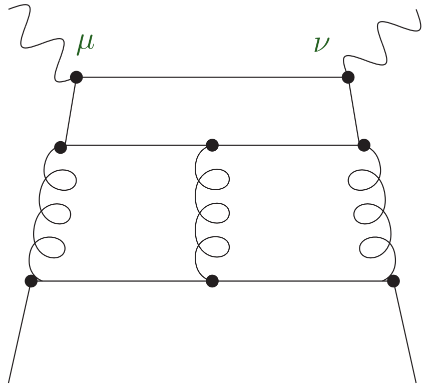

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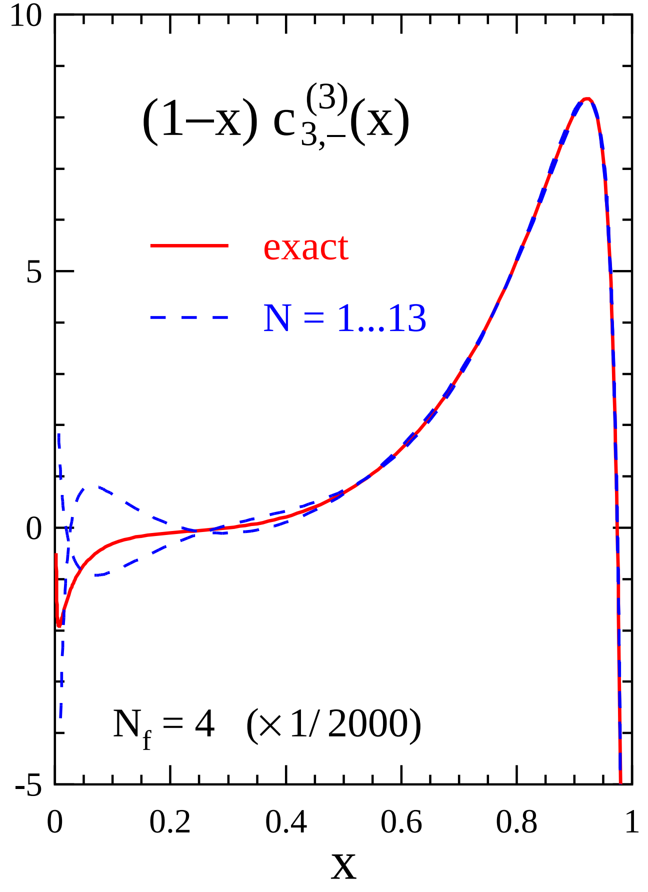
Surprise: subleading large-x terms, $\ln^k(1-x)$, same for F_3 and F_1

$$C_1(x,\alpha_s) \equiv C_2(x,\alpha_s) - C_L(x,\alpha_s) = C_3(x,\alpha_s) + \mathcal{O}(1-x)$$

Recall: $C_3 = C_{g_1,\mathrm{ns}}$ (mod. d_{abc}), helicity-flip suppression in ΔP_{ij} ...

The third-order coefficient function $c^{(3)}_{3,-}(x)$



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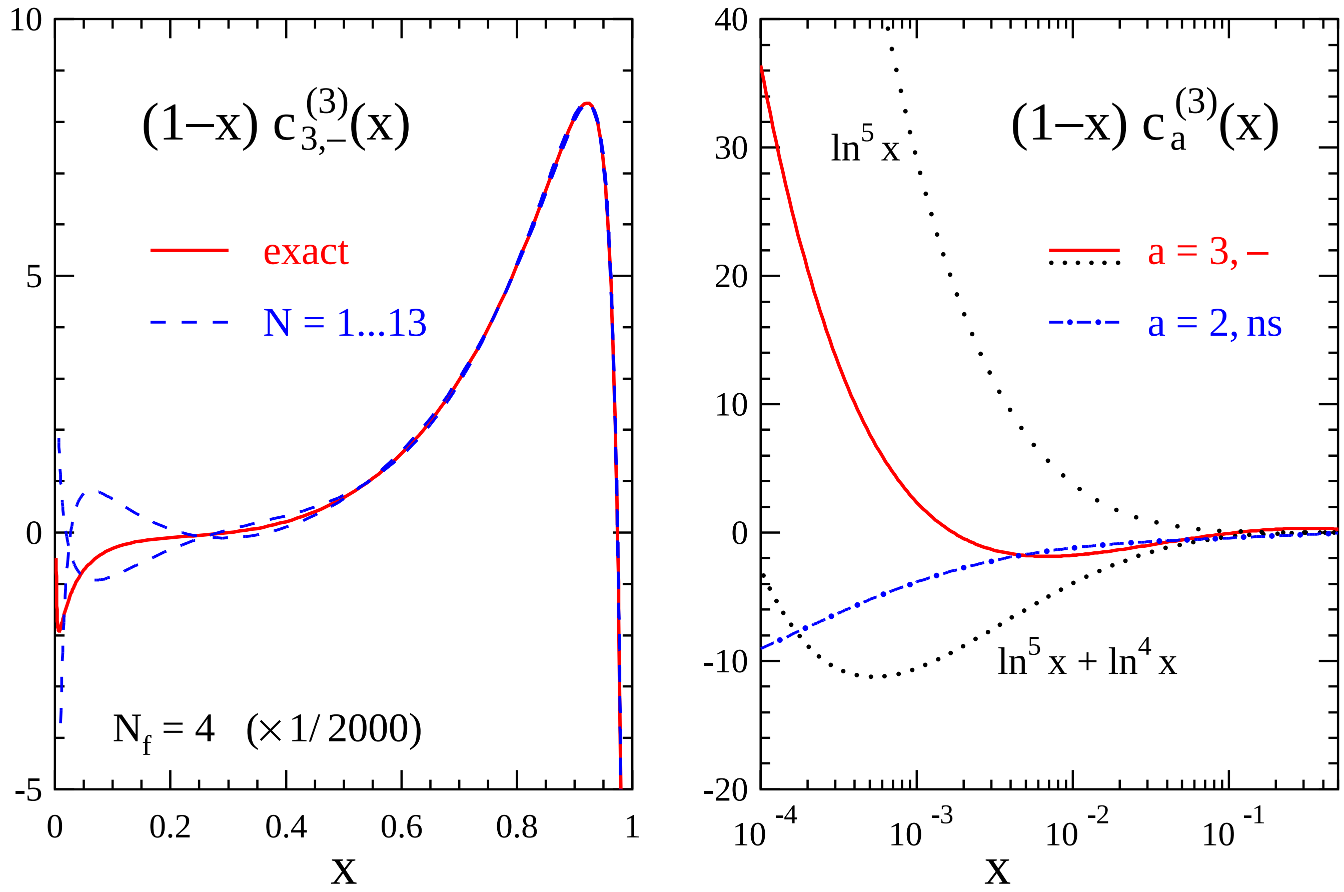

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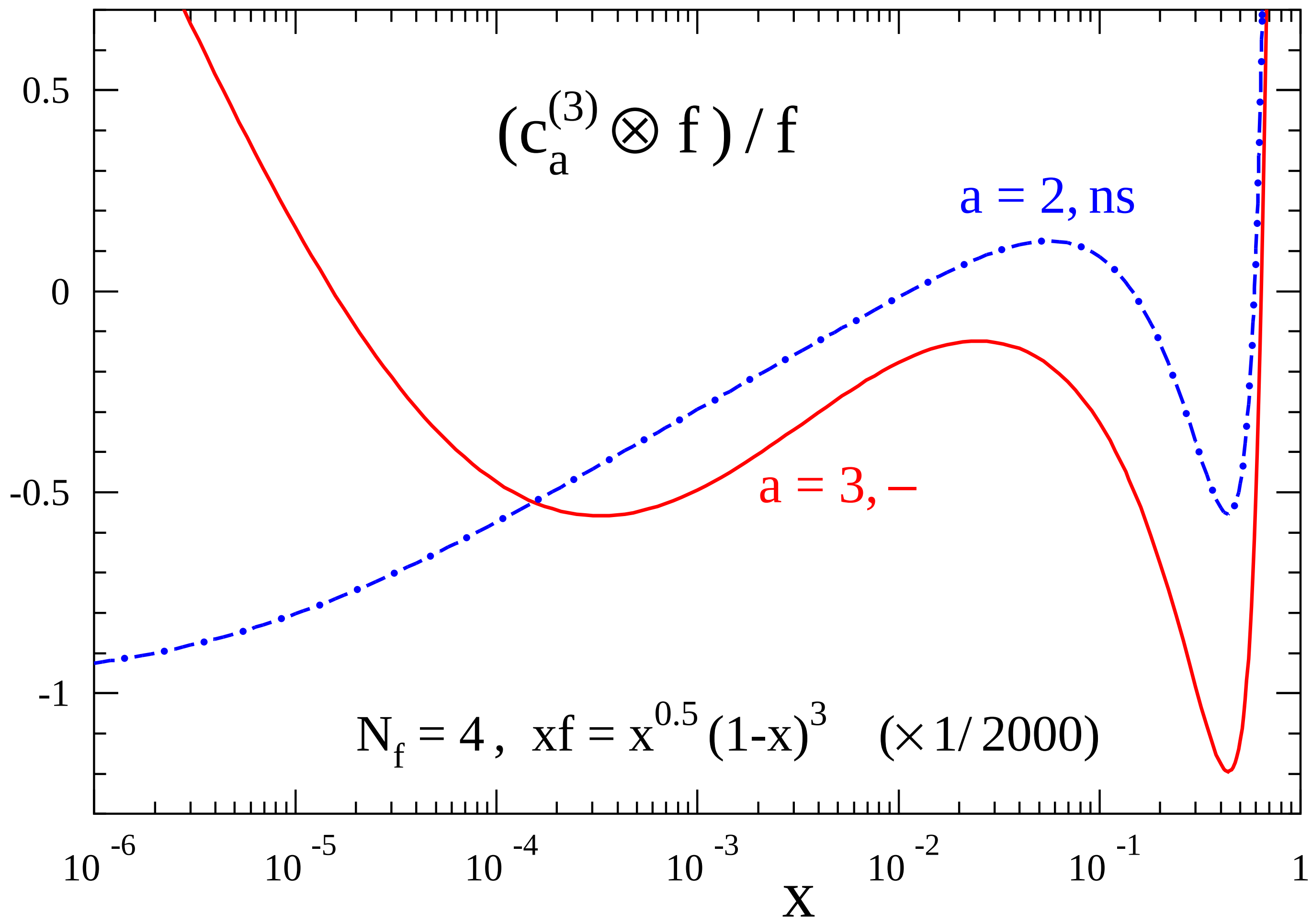
Huge low-x rise from $x \simeq 10^{-3}$ – very different from coeff. fct. for $F_{2,\mathrm{ns}}$

Convolution with a typical quark distribution



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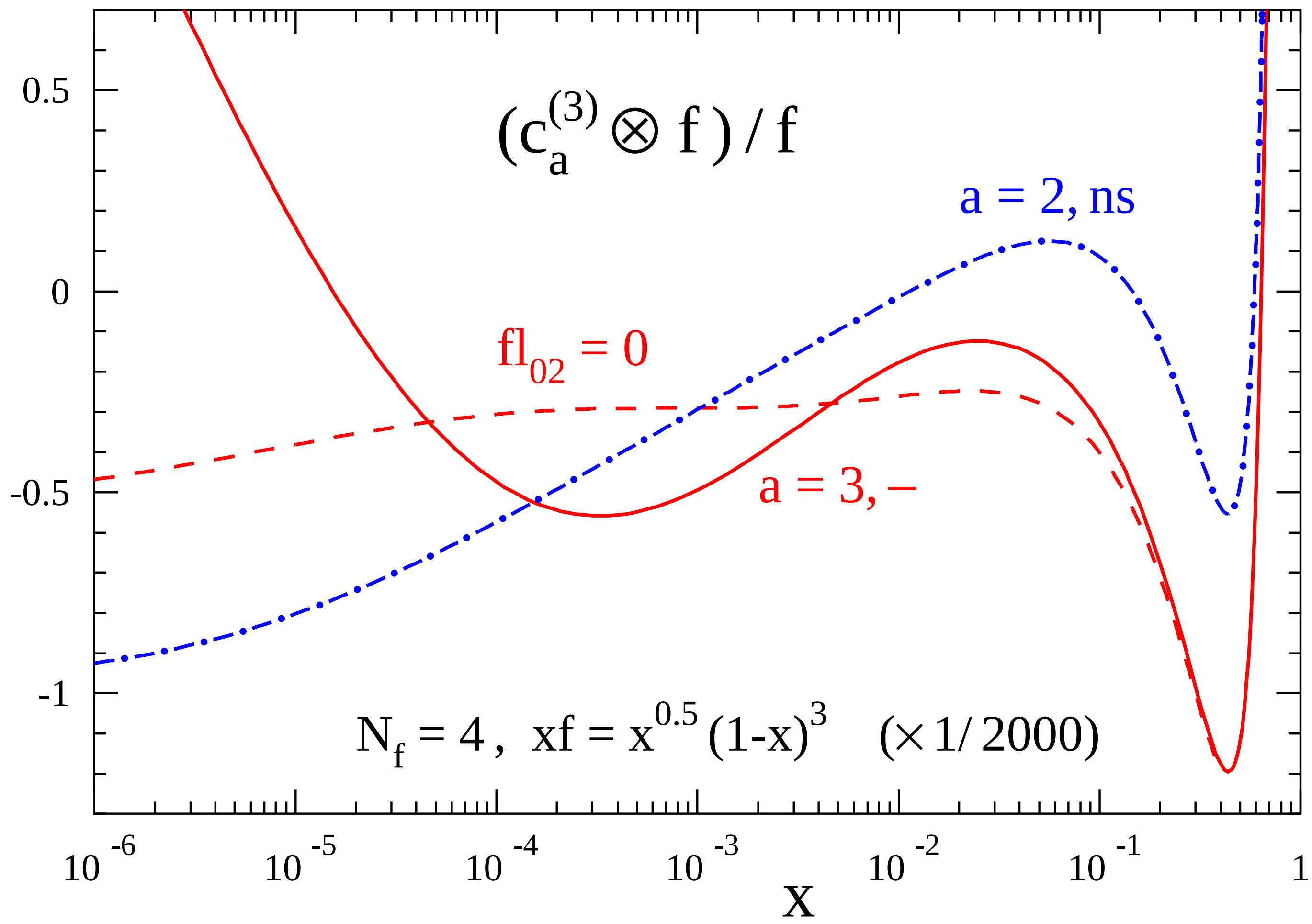

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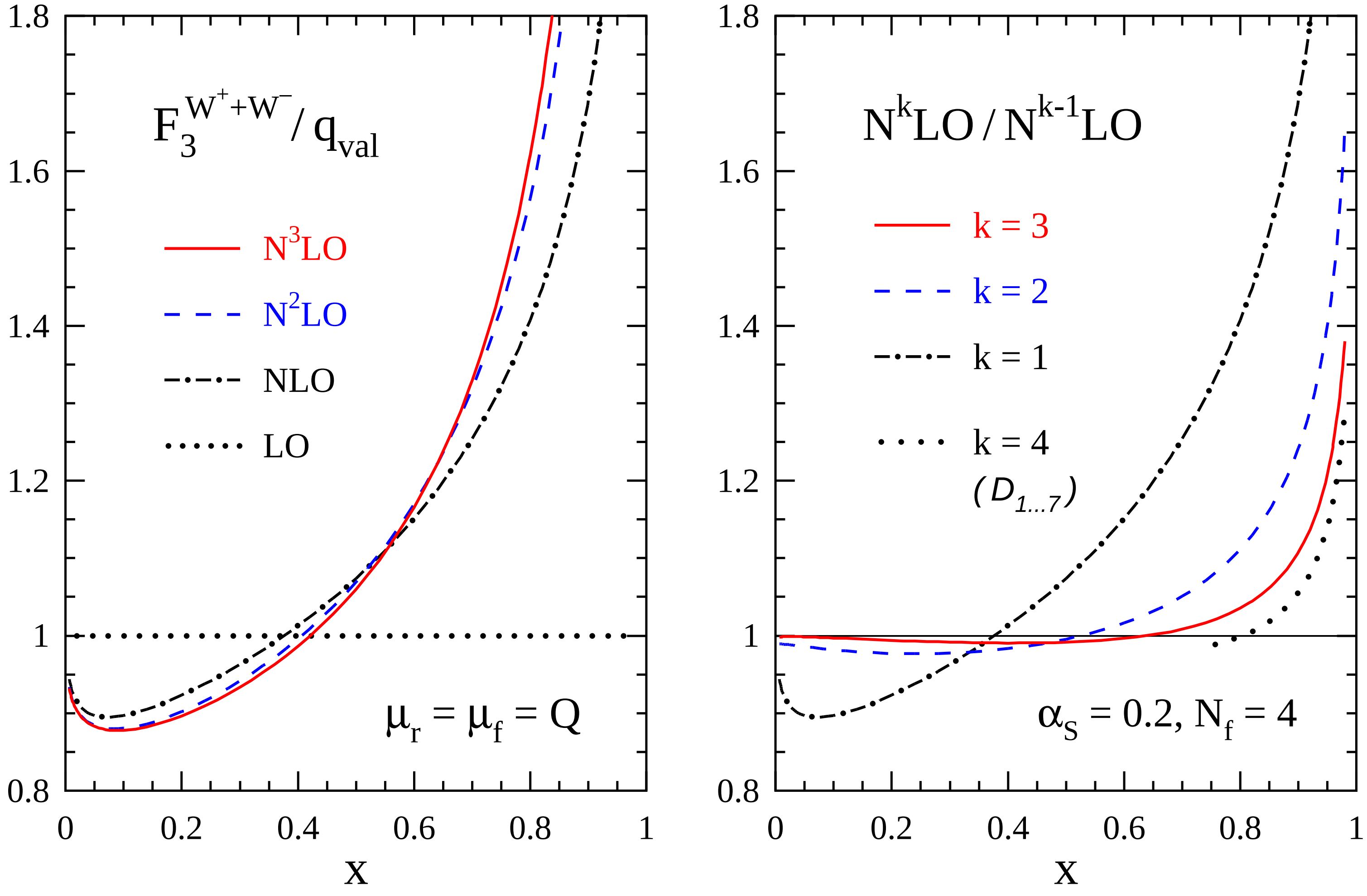
Small-x rise (delayed to $x \simeq 10^{-4}$ by convolution) entirely due to d_{abc} part

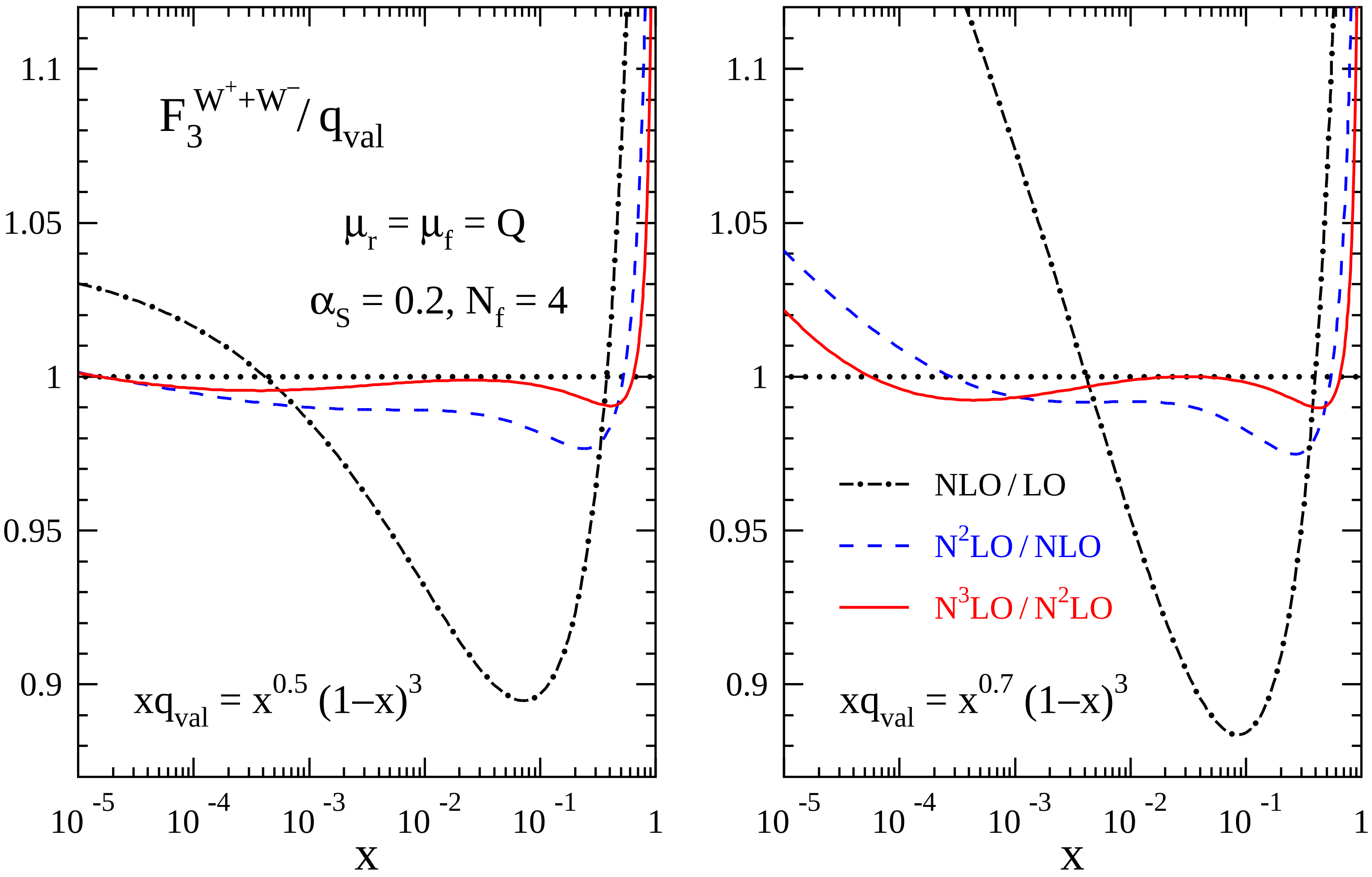
Perturbative expansion of $F_3^{\nu+\bar{\nu}}$ at large x



Higher order: soft-gluon rise towards $x = 1$ steeper, confined to larger x

Relative l-loop corr. $> 5\%$ only at $x >$ 0.46, 0.7, 0.83 for $l =$ 1, 2, 3, resp.

Perturbative expansion of $F_3^{\nu+\bar{\nu}}$ at small x

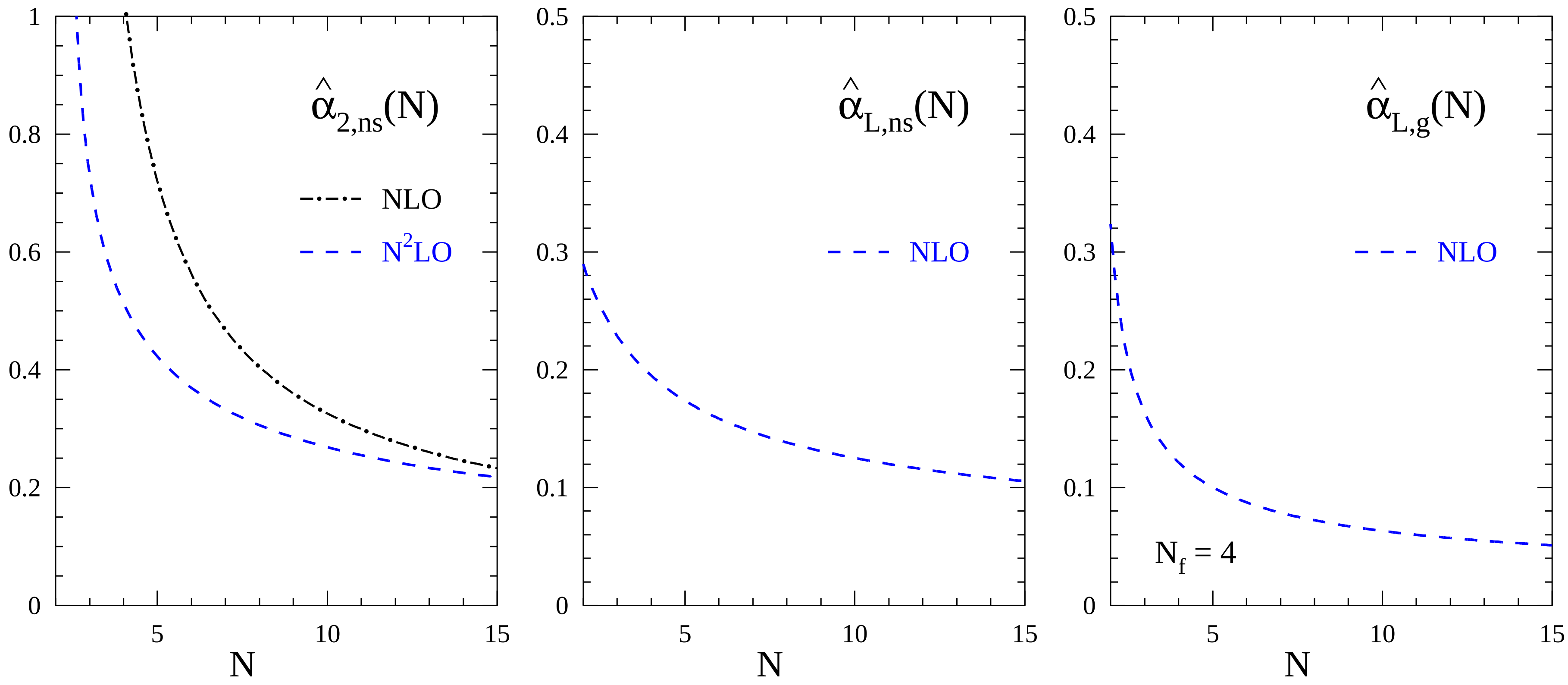


Steeper $xq_{\rm val}$: corr's larger. N^{3}LO $> 1\%$ only at $x < 2 \cdot 10^{-5}$ for $xq_{\rm val} \sim x^{0.7}$

d_{abc} 3-loop dominance: no 4$^{\rm th}$-order estimate possible (unlike large-x limit)

Coefficient functions for $F_{2,L}$ in N-space

$\widehat{\alpha}_a^{(n)}(N)$: α_s for which effect of $c_a^{(n)}(N)$ is half that of $c_a^{(n-1)}(N)$



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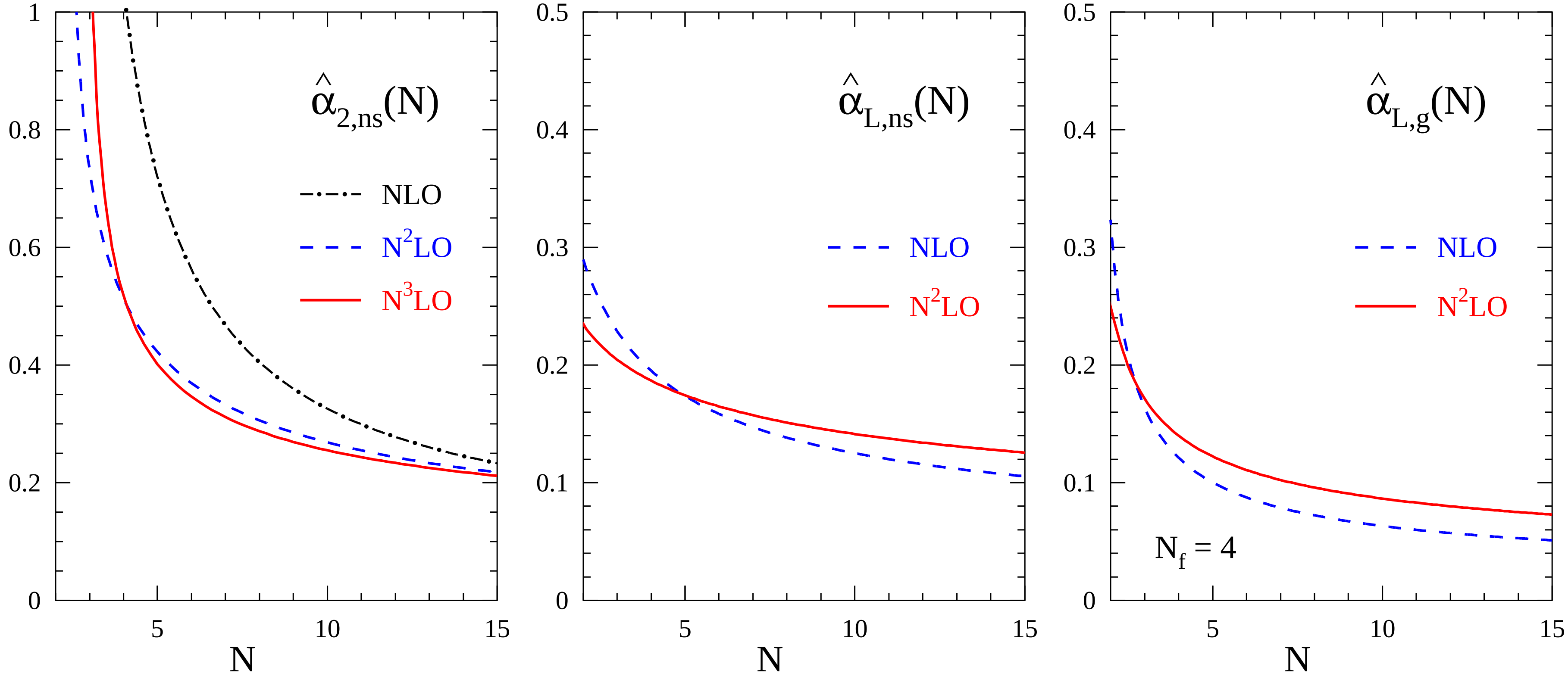

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No sign of asymptotic character (yet). Very slow convergence for F_L

Coefficient functions and physical kernels (I)

Flavour non-singlet structure functions for scale $\mu = Q$ ($a_s \equiv \alpha_s / 4\pi$)

$$\begin{aligned}
\mathcal{F}_{a=2,L}(x,Q^2) &\equiv x^{-1} F_{a,\mathrm{ns}}(x,Q^2) \;=\; C_{a,\mathrm{ns}}(x,\alpha_s) \otimes q_{\mathrm{ns}}(x,Q^2) \\
&= \Big[(1-\delta_{aL})\delta(1-x) + \sum_{n=1} a_s^n \, c_{a,q}^{(n)}(x) \Big] \otimes q_{\mathrm{ns}}(x,Q^2)
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Coefficient functions at large x: double-logarithmic enhancements

$$c_{2,q}^{(n)} \;:\; \mathcal{D}_k \equiv \left[\frac{\ln^k(1-x)}{1-x}\right]_+ , \quad L_x^k \equiv \ln^k(1-x) \;, \quad \ldots , \quad k = 0, \ldots, 2n-1$$

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Physical evolution kernel: eliminate quark density q_{ns} from $d\mathcal{F}_a/d\ln Q^2$

$$\frac{d}{d\ln Q^2}\mathcal{F}_a \;=\; \Big\{P_{\mathrm{ns}}(a_s) + \beta(a_s)\,\frac{d}{da_s}\ln C_a(a_s)\Big\} \otimes \mathcal{F}_a$$
$$\equiv\; K_a \otimes \mathcal{F}_a \;=\; \sum_{n=1} a_s^n K_a^{(n)} \otimes \mathcal{F}_a$$

Beta fct. $\beta(a_s) = -\beta_0 a_s^2 - \beta_1 a_s^3 - \ldots,$ $\;d\ln C_a/da_s$ defined via moments

Coefficient functions and physical kernels (II)

Insert coefficient functions to three loops, expand at $x = 1$ (done in FORM)

$$
\begin{aligned}
K_{2,L}^{(n)}(x) \quad &= \quad 4C_F(-\beta_0)^{n-1}\,\mathcal{D}_{n-1} + O(\mathcal{D}_{n-2}) \\
&\overset{\text{M-trf}}{=} \quad -\frac{4C_F\beta_0^{n-1}}{n}\ln^{n} N + O(\ln^{n-1} N) \quad :
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same single-log enhancement for both F_2, F_L established to $n = 4,\, 3$, resp.

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Soft-gluon resummation   Sterman (87); Catani, Trentadue (89), . . . , MVV(05)

$$C_{2,\mathrm{ns}}(N, a_\mathrm{s}) = g_2^{(0)}(a_\mathrm{s})\exp[Lg_2^{(1)}(a_\mathrm{s}L) + g_2^{(2)}(a_\mathrm{s}L) + \ldots]\,, \;\; g_2^{(i)}(\lambda) = \sum g_{2j}^{(i)}\lambda^j$$

$$\Downarrow$$

$$K_2(N, a_\mathrm{s}) \;\; = \;\; -(A_1 a_\mathrm{s} + A_2 a_\mathrm{s}^2)\ln N - (\beta_0 + \beta_1 a_\mathrm{s})\lambda^2\frac{dg_2^{(1)}}{d\lambda} - a_\mathrm{s}\beta_0\lambda\frac{dg_2^{(2)}}{d\lambda}$$

$$+\,\mathcal{O}(a_s^2(f(\lambda))) \qquad \text{with} \qquad \lambda = a_\mathrm{s}L \quad :$$

all-order single-logarithmic large-N behaviour of F_2 van Neerven, A.V. (01)

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all-order single-logarithmic large-N behaviour of F_2 **van Neerven, A.V. (01)**

Conversely: single-log $K_a \Rightarrow$ expon. $C_{a,\mathrm{ns}}$, prediction of higher-order logs

Conjecture and predictions for F_L

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$\Rightarrow$ **prediction of the three highest logarithms at all orders, e.g.,**

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c^{(4)}_{L,q}(x) \;=\;& \frac{16}{3}\, C_F^4\, L_x^6 + \Big\{ [\,72 - 64\,\zeta_2]\, C_F^4 - \Big[\frac{728}{9} - 32\,\zeta_2\Big] C_F^3 C_A + \frac{80}{9}\, C_F^3 n_f \Big\} L_x^5 \\
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&\quad + \Big[\frac{3388}{9} - \frac{1360}{9}\,\zeta_2 - 64\,\zeta_3\Big] C_F^2 C_A^2 - \Big[\frac{880}{9} - \frac{352}{9}\,\zeta_2\Big]\, C_F^2 C_A n_f + \frac{16}{3}\, C_F^2 n_f^2 \Big\}\, L_x^4 \\
&+ O(L_x^3)
\end{aligned}
$$

L_x^6 as predicted by us before (04), rest new

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&+ \left\{ [\,32\,\zeta_2 - 160\,\zeta_3]\, C_F^4 - \left[\frac{904}{3} - \frac{1856}{9}\,\zeta_2 - 208\,\zeta_3\right] C_F^3 C_A + \left[\frac{160}{3} - \frac{704}{9}\,\zeta_2\right] C_F^3 n_f \right. \\
&\left. + \left[\frac{3388}{9} - \frac{1360}{9}\,\zeta_2 - 64\,\zeta_3\right] C_F^2 C_A^2 - \left[\frac{880}{9} - \frac{352}{9}\,\zeta_2\right] C_F^2 C_A n_f + \frac{16}{3}\, C_F^2 n_f^2 \right\} L_x^4 \\
&+ O(L_x^3)
\end{aligned}
$$

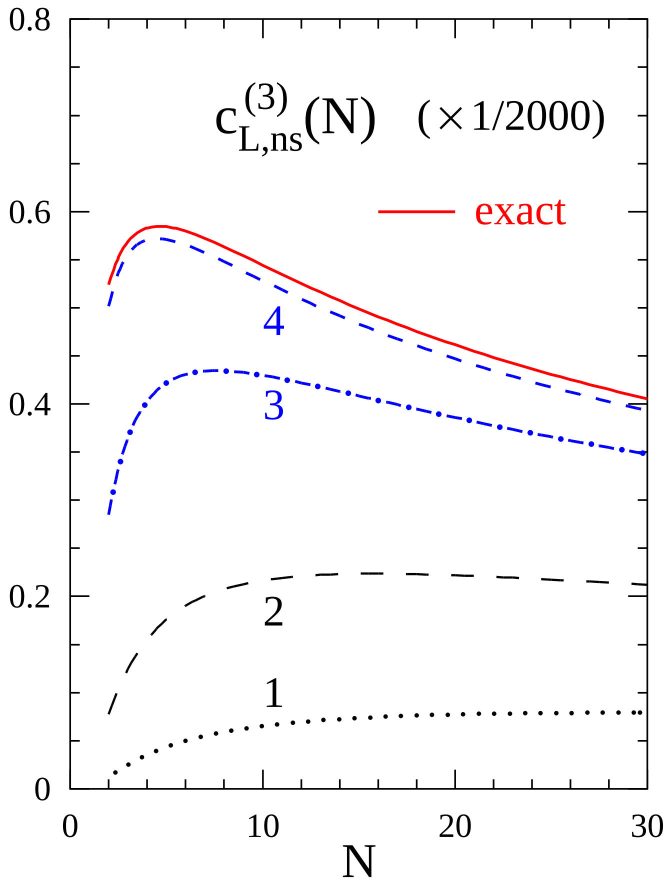
L_x^6 as predicted by us before (04), rest new

Exp. form $C_{L,\mathrm{ns}}(N, a_s) \;=\; \frac{1}{N}\, g_0(a_s) \exp[L g_1(a_s L) + g_2(a_s L) + \ldots]$ **with**

$$
\begin{aligned}
g_{11} &= 2C_F \;, \quad g_{12} = \frac{2}{3}\beta_0 C_F \;, \quad g_{13} = \frac{1}{3}\beta_0^2 C_F \\
g_{21} &= \beta_0 + 4\gamma_e C_F - C_F + (4 - 4\zeta_2)(C_A - 2C_F) \\
g_{22} &= \frac{1}{2}(\beta_0\, g_{21} + A_2) - 8\,(C_A - 2C_F)^2 (1 - 3\zeta_2 + \zeta_3 + \zeta_2^2)
\end{aligned}
$$

A_2: 2-loop cusp anom. dim., $g_{21} \Leftrightarrow \gamma_{J'}$, Akhoury, Sotiropoulos, Sterman (98)

Numerics at the third and fourth order



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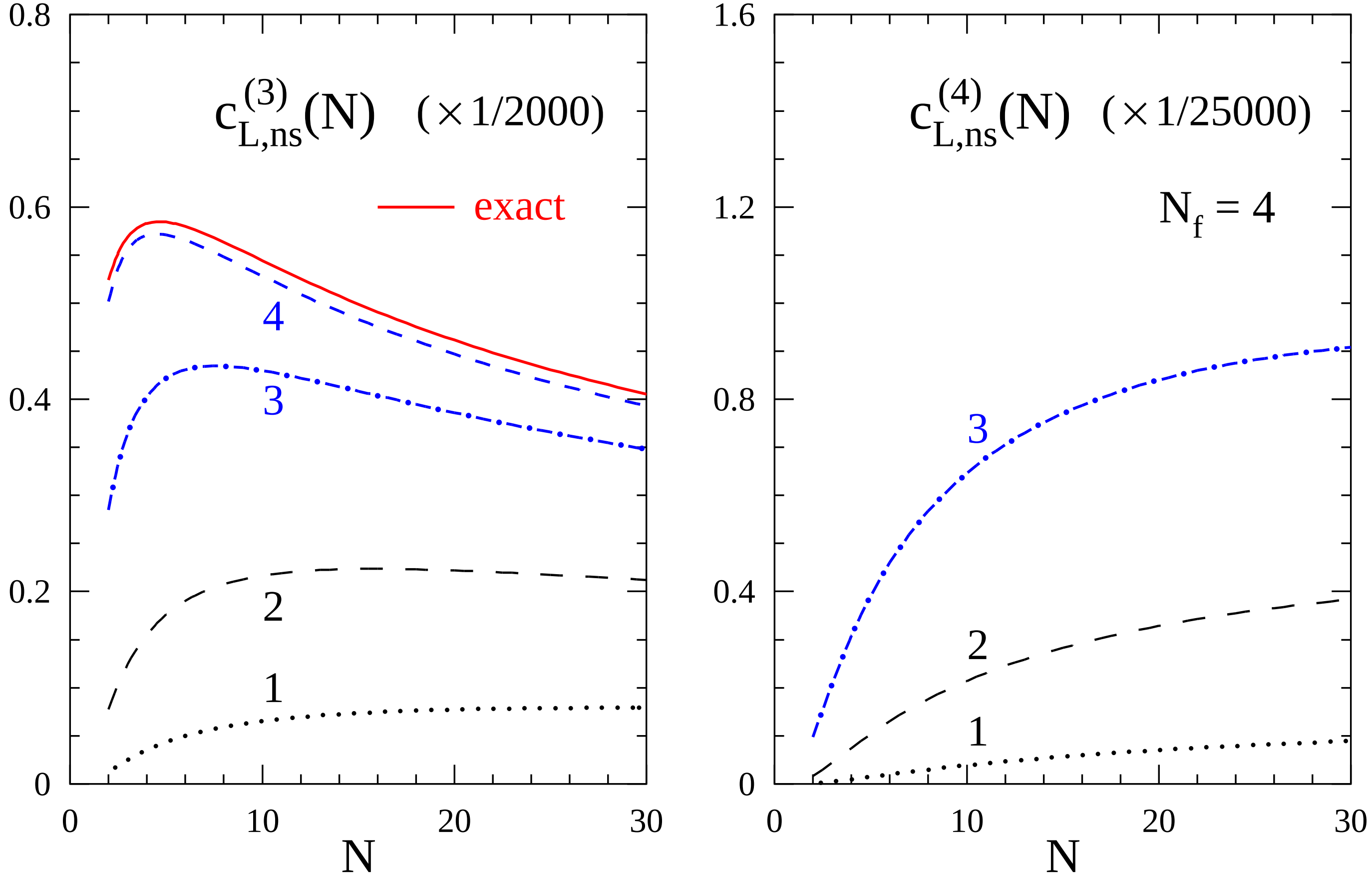

$c_L^{(3)}$: very good approx. by all four $\ln N$ terms. Estimate by highest three

$c_L^{(4)}$: known terms insufficient for estimate. Padé predicts 2.0 at $N = 20$.

Useful with future four-loop fixed-N calculations: fewer moments needed

Present and forthcoming extensions

Gluon coefficient function

$$C_{L,g}(\alpha_s, N) = \sum_{n=1} a_s^n \left\{ 8 n_f \frac{(2C_A)^{n-1}}{(n-1)!} \frac{1}{N^2} \ln^{2n-2} N + O\left(\frac{1}{N^2} \ln^{2n-3} N \right) \right\}$$

Also: $C_A^k n_f^{n-k}$ contributions to next two logs $\Leftarrow$ 'ns' gluonic physical kernel

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$$K_a^{(n)}(x)\Big|_{\ln^k(1-x)} \overset{\text{M-trf}}{=} K_a^{(n)}(N)\Big|_{N^{-1}\ln^k N} \;=\; 0 \quad \text{for} \quad k \geq n$$

Again established to $n = 4$ for $F_{2,3}$ and $n = 3$ for F_L by our 3-loop results

All-n generalization $\Rightarrow$ $(1-x)\ln^{2n-1-k}(1-x)$ of $c_{L,q}^{(n)}$ for $k = 1, 2, 3$

So far: MV, 0902.2342

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Next step: integrable large-x logs for $F_{2,3}$, timelike $F_{T,L}$, Drell-Yan $d\sigma/dM^2$

MV, 0905.abcd

No double-log found so far to a (ns) physical kernel at any order in $1-x$ . . .

Summary and outlook

Structure function $F_3^{W^+ + W^-}$: N^{3}LO coefficient function computed

- Truncation of perturbation series not dominating $\Delta\alpha_s(M_Z^2)$ any more
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- Extension to other (ns) observables? Yes, with similar ‘problem’ for g_{a2}