Recent developments in QCD

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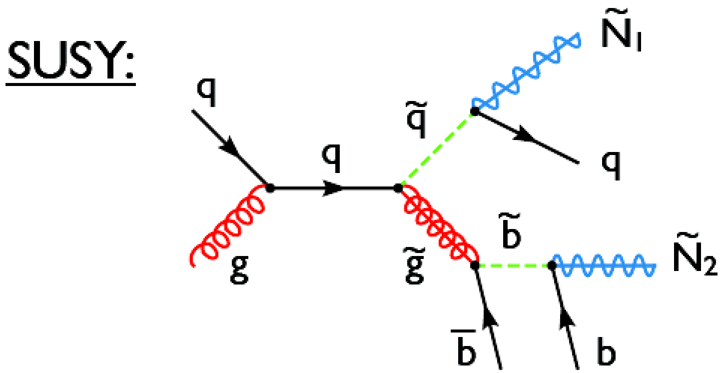
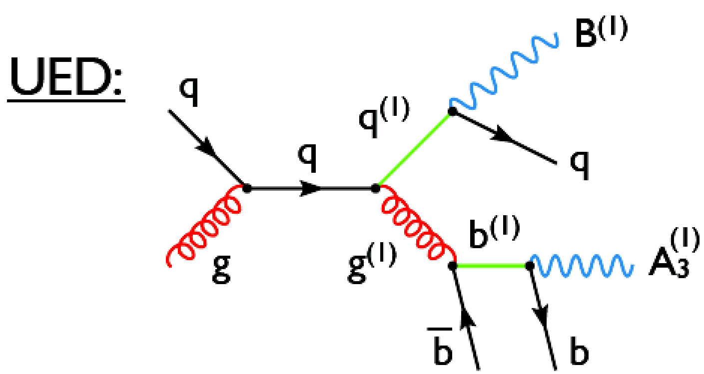
Present Status of QCD

- ✓ Thanks to LEP, HERA and the TEVATRON
  QCD now firmly established theory of strong interactions
- ✓ We have gained a lot of confidence in comparing theoretical predictions with experimental data
- ✓ No major areas of discrepancies
- ?? But LHC brings new frontiers in energy and luminosity
- ?? typical SM process is accompanied by multiple radiation to form multi-jet events
- ?? most BSM signals involve pair-production and subsequent chain decays

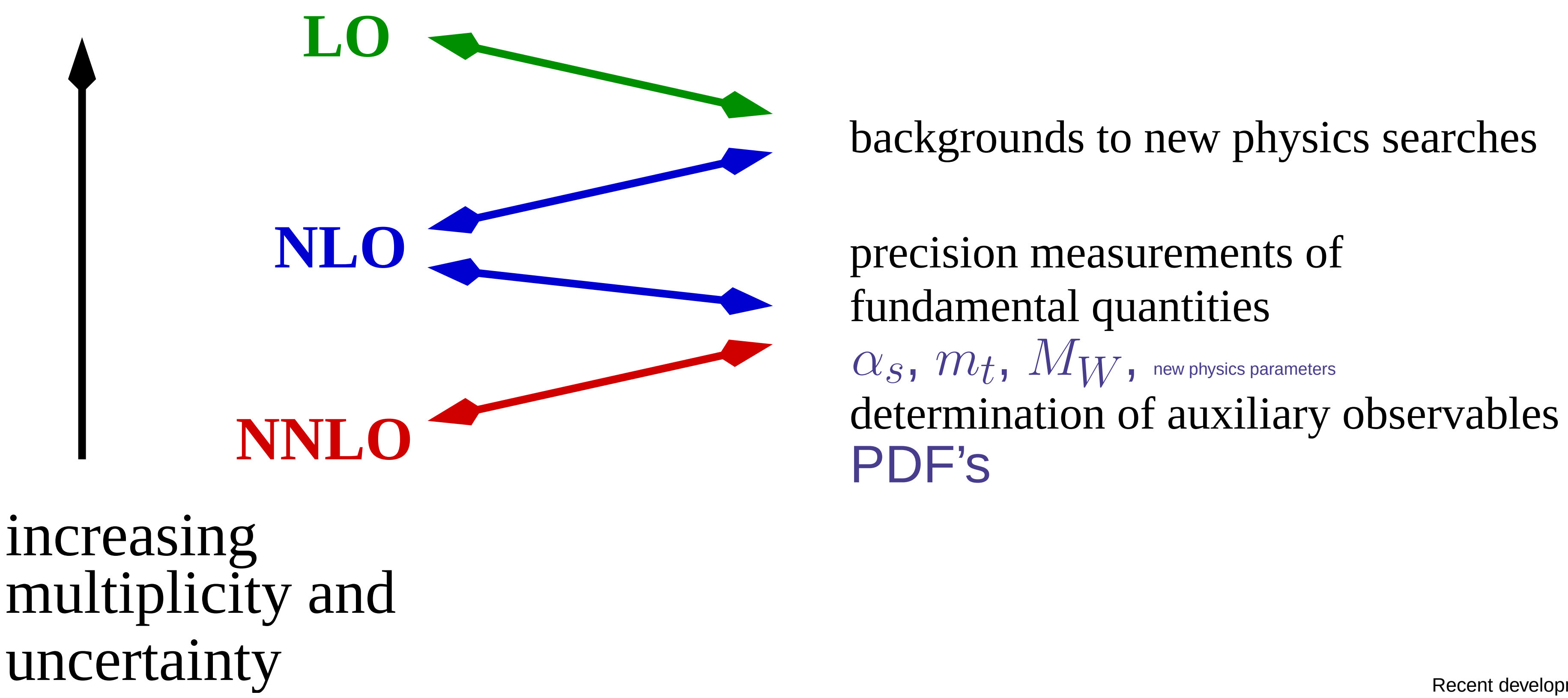




Matching onto Physics Goals

Twin Goals:

1. Identification and study of New Physics
2. Precision measurements (e.g. α_s, PDF's) leading to improved theoretical predictions



What is covered in this talk

1. Overview
2. NLO multiparticle production
3. Jets
4. NNLO
5. Beyond NLO

State of the Art - at a glance

Relative Order	$2 \to 1$	$2 \to 2$	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
1	LO					
α_s	NLO	LO				
α_s^2	NNLO	NLO	LO			
α_s^3	NNNLO	NNLO	NLO	LO		
α_s^4				NLO	LO	
α_s^5					NLO	LO

LO Automated and under control, even for multiparticle final states

NLO Well understood for $2 \to 1$ and $2 \to 2$ in SM and beyond

NLO Many new $2 \to 3$ calculations from Les Houches wish list since 2007

NLO Very first $2 \to 4$ LHC cross section in 2008 $q\bar{q} \to t\bar{t}b\bar{b}$

NLO Important developments in automation, $W + 3$ jets (2009)

NNLO Inclusive and exclusive Drell-Yan and Higgs cross sections

NNLO $e^+e^- \to 3$ jets, but still waiting for $2 \to 2$

NNNLO F_2, F_3 and form-factors

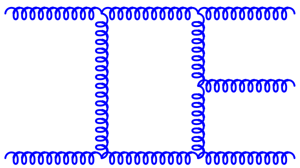
2. NLO multiparticle production

Limitations of LO

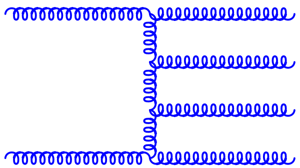
Very large uncertainty for multiparticle final states

- ✗ Large renormalisation scale uncertainty, magnified by the large amount of radiation e.g. a $\pm 10\%$ uncertainty in α_s leads to a $\pm 30\%$ uncertainty for $W + 3$ jets
- ✗ Large factorisation scale uncertainty
  higher scales deplete partons at large x - may increase or decrease cross section
- ✗ Both of these effects change the shapes of distributions
- ✓ Partly stabilised by going to NLO
- ✓ New channels open up at higher orders qg + large gluon PDF
- ✓ Increased phase space allows more radiation
- ✓ Large π^2 coefficients in s-channel $\Rightarrow$ large NLO corrections 30% - 100%

Anatomy of a NLO calculation



✓ one-loop $2 \to 3$ process
looks like 3 jets in final state



✓ tree-level $2 \to 4$ process
looks like 3 or 4 jets in final state

✓ plus method for combining the infrared divergent parts - dipole subtraction

Catani, Seymour; Dittmaier, Trocsanyi, Weinzierl, Phaf

✓ automated dipole subtraction

Gleisberg, Krauss (SHERPA); Hasegawa, Moch, Uwer; Frederix, Gehrmann, Greiner (MadDipole); Seymour, Tevlin

Bottleneck: one-loop matrix elements

LHC priority NLO wish list, Les Houches 2005/7*

process	background	status - mostly from Feynman diagram approach
$pp \to VV + 1$ jet	WBF $H \to VV$	WWj (07)
$pp \to t\bar{t} + b\bar{b}$	$t\bar{t}H$	$q\bar{q} \to t\bar{t}b\bar{b}$ (08)
$pp \to t\bar{t} + 2$ jets	$t\bar{t}H$	$t\bar{t}j$ (07)
$pp \to VV + b\bar{b}$	WBF $H \to VV$, $t\bar{t}H$, NP	
$pp \to VV + 2$ jets	WBF $H \to VV$	WBF $pp \to VVjj$ (07)
$pp \to V + 3$ jets	NP	$W + 3$ jets (09)
$pp \to VVV$	SUSY trilepton	ZZZ (07), WWZ (07), WWW (08), ZZW (08)
$pp \to b\bar{b}b\bar{b}^*$	Higgs and NP	

✓ $pp \to H + 2$ jets via gluon fusion — Campbell, Ellis, Zanderighi, (06)

✓ $pp \to H + 2$ jets via WBF, electroweak and QCD corrections — Ciccolini, Denner, Dittmaier, (07)

✓ $pp \to H + 3$ jets via WBF, — Figy, Hankele, Zeppenfeld, (07)

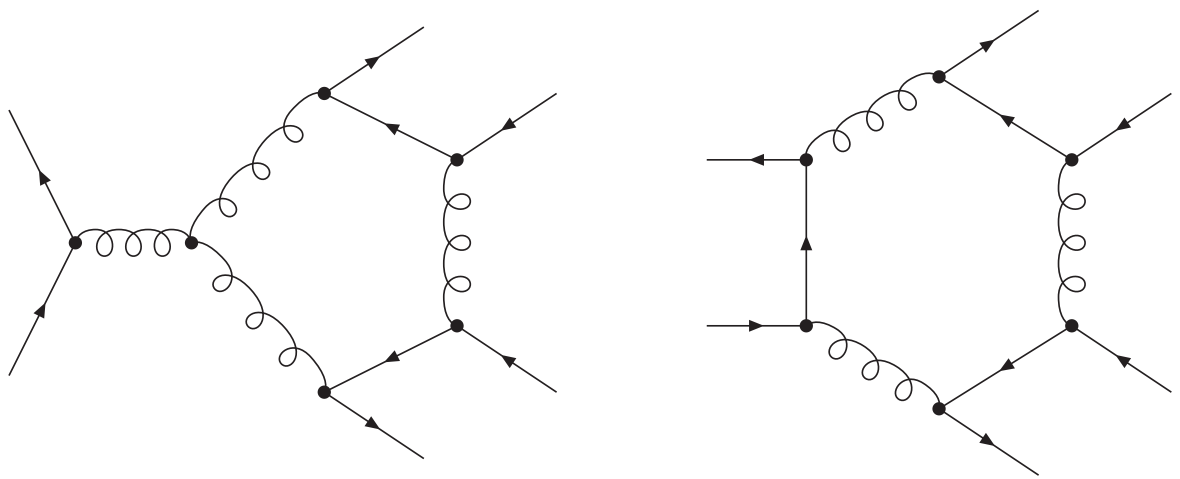
✓ ....

NLO example: Top pair plus bottom pair production

QCD corrections to $q\bar{q} \to t\bar{t}b\bar{b} + X$

Bredenstein, Denner, Dittmaier, Pozzorini, (08)

✓ Background to the Higgs signal in $t\bar{t}H$ production where the Higgs decays into a bottom pair

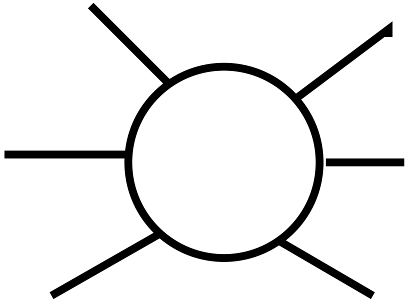
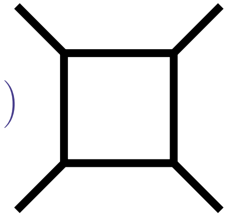
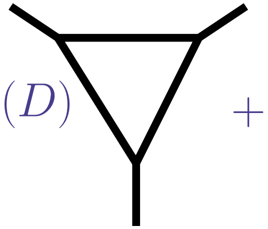
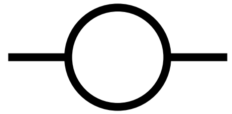


✓ First successful demonstration of Feynman diagrammatic evaluation of $2 \to 4$ process at LHC

✓ Dominant $gg \to t\bar{t}b\bar{b} + X$ process underway

The one-loop problem

Any (massless) one-loop integral can be written as

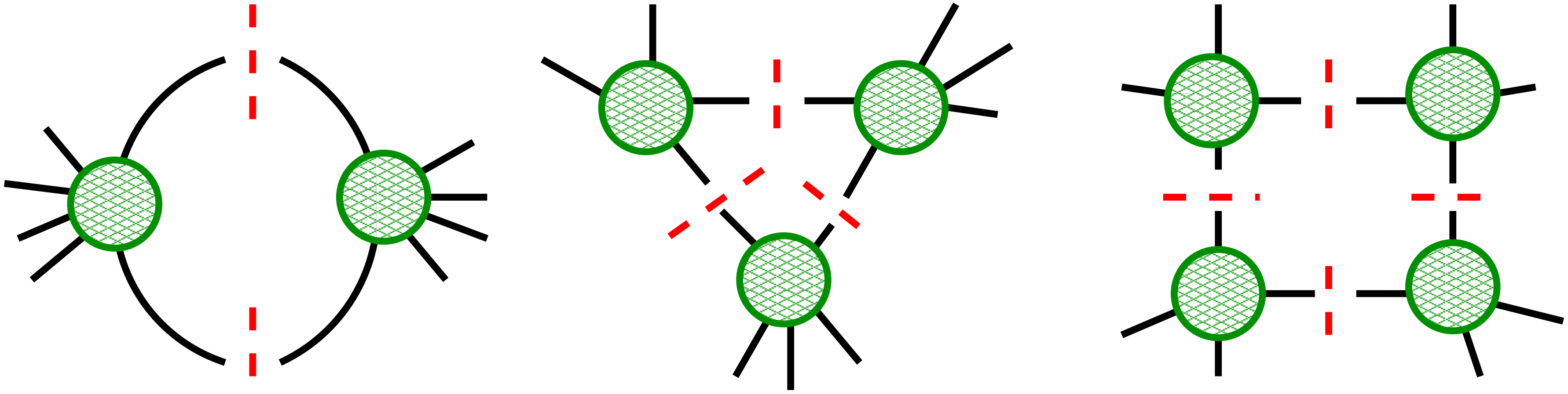
 $= \sum_i d_i(D)$  $+ \sum_i c_i(D)$  $+ \sum_i b_i(D)$ 

$$\mathcal{M} = \sum d(D)\text{boxes(D)} + \sum c(D)\text{triangles(D)} + \sum b(D)\text{bubbles(D)}$$

- ✓ higher polygon contributions drop out
- ✓ scalar loop integrals are known analytically around $D = 4$ Ellis, Zanderighi (08)
- ✓ need to compute the D-dimensional coefficients $a(D)$ etc.

The problem is complexity - the number of terms generated is too large to deal with, even with computer algebra systems, and there can be very large cancellations.

Breakthough idea - Generalised Unitarity

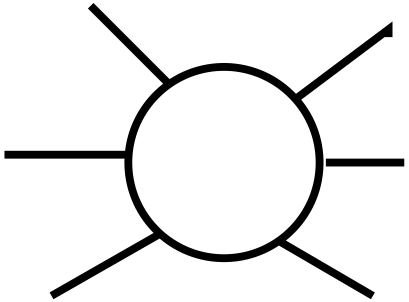


Bern, Dixon, Dunbar, Kosower (94); Britto, Cachazo, Feng (04)

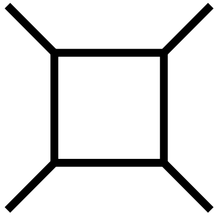
- ✓ put internal propagators on-shell $\frac{1}{p^2+i0} \to -i\delta^+(p^2)$
- ✓ coefficient is product of tree-amplitudes with loop-momentum frozen
- ✓ can recycle tree-amplitudes in 4-D
- ✓ tree three-vertices do not vanish - complex momentum
- ✓ two-cut sensitive to box, triangle and bubble

4-dimensional unitarity

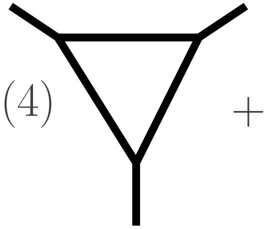
With 4-dimensional cuts - loop momentum in 4-dimensions and using 4-dimensional tree vertices



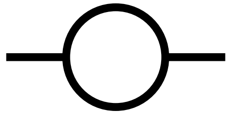
$= \sum_i d_i(4)$



$+ \sum_i c_i(4)$



$+ \sum_i b_i(4)$



$+\mathcal{R}$

- ✓ $\mathcal{R}$ is a rational part that is generated by the D dependence of the coefficients $d_i(D)$ etc
- ✓ dimensionality of the loop momentum
- ✓ number of polarisation states of internal particles
- ✓ $\mathcal{R}$ can be computed with on-shell recursion (as for tree-diagrams)

Berger, Bern, Dixon, Forde, Kosower (06)

Analytic one-loop six gluon amplitude

$$A^{QCD} = A^{[1]} + \frac{n_f}{N} A^{[1/2]},$$

$$A^{[1]} = A^{\mathcal{N}=4} - 4A^{\mathcal{N}=1} + A^{[0]}, A^{[1/2]} = A^{\mathcal{N}=1} - A^{[0]}$$

Amplitude	$\mathcal{N} = 4$	$\mathcal{N} = 1$	scalar(cut)	scalar (rat)
$--++++$	BDDK (94)	BDDK (94)	BDDK (94)	BDK (94)
$-+-+++$	BDDK (94)	BDDK (94)	BBST (04)	BBDFK (06), XYZ (06)
$-++-++$	BDDK (94)	BDDK (94)	BBST (04)	BBDFK (06), XYZ (06)
$---+++$	BDDK (94)	BDDK (94)	BBDI (05), BFM (06)	BBDFK (06), XYZ (06)
$--+-++$	BDDK (94)	BBDP (05), BBCF (05)	BFM (06)	XYZ (06)
$-+-+-+$	BDDK (94)	BBDP (05), BBCF (05)	BFM (06)	XYZ (06)

✓ Analytic computation

Bedford, Berger, Bern, Bidder, Bjerrum-Bohr, Brandhuber, Britto, Buchbinder, Cachazo, Dixon, Dunbar, Feng, Forde, Kosower, Mastrolia, Perkins, Spence, Travaglini, Xiao, Yang, Zhu

A second breakthrough - OPP

Reducing full one-loop amplitudes to scalar integrals at the integrand level

Ossola, Papadopoulos, Pittau (06)

- ✓ systematic algebraic reduction at the integrand level
- ✓ integrand is decomposed by partial fractioning into linear combination of terms with 4-,3-,2,-1 denominator factors

$$\mathcal{A}(\ell) = \sum_{i_1,\dots i_4} \frac{\overline{d_{i_1 i_2 i_3 i_4}}}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{i_1,\dots i_3} \frac{\overline{c_{i_1 i_2 i_3}}}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{i_1,\dots i_2} \frac{\overline{b_{i_1 i_2}}}{d_{i_1} d_{i_2}}$$

- ✓ obtain numerators by taking residues; i.e. set inverse propagator $= 0$

$$\overline{d_{i_1 i_2 i_3 i_4}} = d_{i_1 i_2 i_3 i_4} + \tilde{d}_{i_1 i_2 i_3 i_4}, \qquad \text{etc.}$$

where $\tilde{d}_{i_1 i_2 i_3 i_4}$ integrates to zero

- ✓ Very algorithmic, can be automated.

NLO automation: HELAC/CutTools

Cafarella, van Hameren, Kanaki, Ossola, Papadopoulos, Pittau, Worek

- ✓ HELAC: off-shell recursion for the full Standard Model
- ✓ CutTools: fortan90 implementation of OPP recursion
- ✓ Automatic 1-loop computation of amplitude at single phase-space point all 2→4 wish-list processes

$$
\begin{aligned}
q\bar{q}, gg &\rightarrow t\bar{t}b\bar{b}, b\bar{b}b\bar{b}, W^+W^-b\bar{b}, t\bar{t}gg \\
q\bar{q}' &\rightarrow Wggg, Zggg
\end{aligned}
$$

Ossola, Papadopoulos, Pittau (09)

- ✓ all masses, colours and helicities treated exactly
- ✓ still need to combine with LO 2→ 5 processes, subtraction terms and efficient MC integration

NLO automation: BlackHat

Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre

- ✓ C++ implementation of on-shell technqiues for 1-loop amplitudes
- ✓ based on $D = 4$ unitarity - to generate all of the coefficients of loop integrals
- ✓ and on-shell recursion for the rational parts
- ✓ up to 8 gluon amplitudes numerically — Berger et al, (08)
- ✓ leading colour $Vq\bar{q}ggg$ numerically — Berger et al, (08)
- ✓ interfaced with SHERPA Monte Carlo for real radiation and infrared subtraction terms to produce (leading colour) $W + 3$ jet cross sections

$$\sigma_n^{NLO} = \int_n \sigma_n^{tree} + \int_n \left(\sigma_n^{virt} + \Sigma_n^{sub}\right) + \int_{n+1} \left(\sigma_{n+1}^{real} - \sigma_{n+1}^{sub}\right)$$

Berger et al, (09)

NLO automation: Rocket

Ellis, Giele, Kunzst, Melnikov, Zanderighi

a Fortran 90 package which fully automates the calculation of virtual amplitudes via tree level recursion + D-unitarity

- ✓ based on OPP and two different values of D
- ✓ off-shell recursion for tree-input
- ✓ up to 20 gluon amplitudes numerically

Giele, Zanderighi, (08)

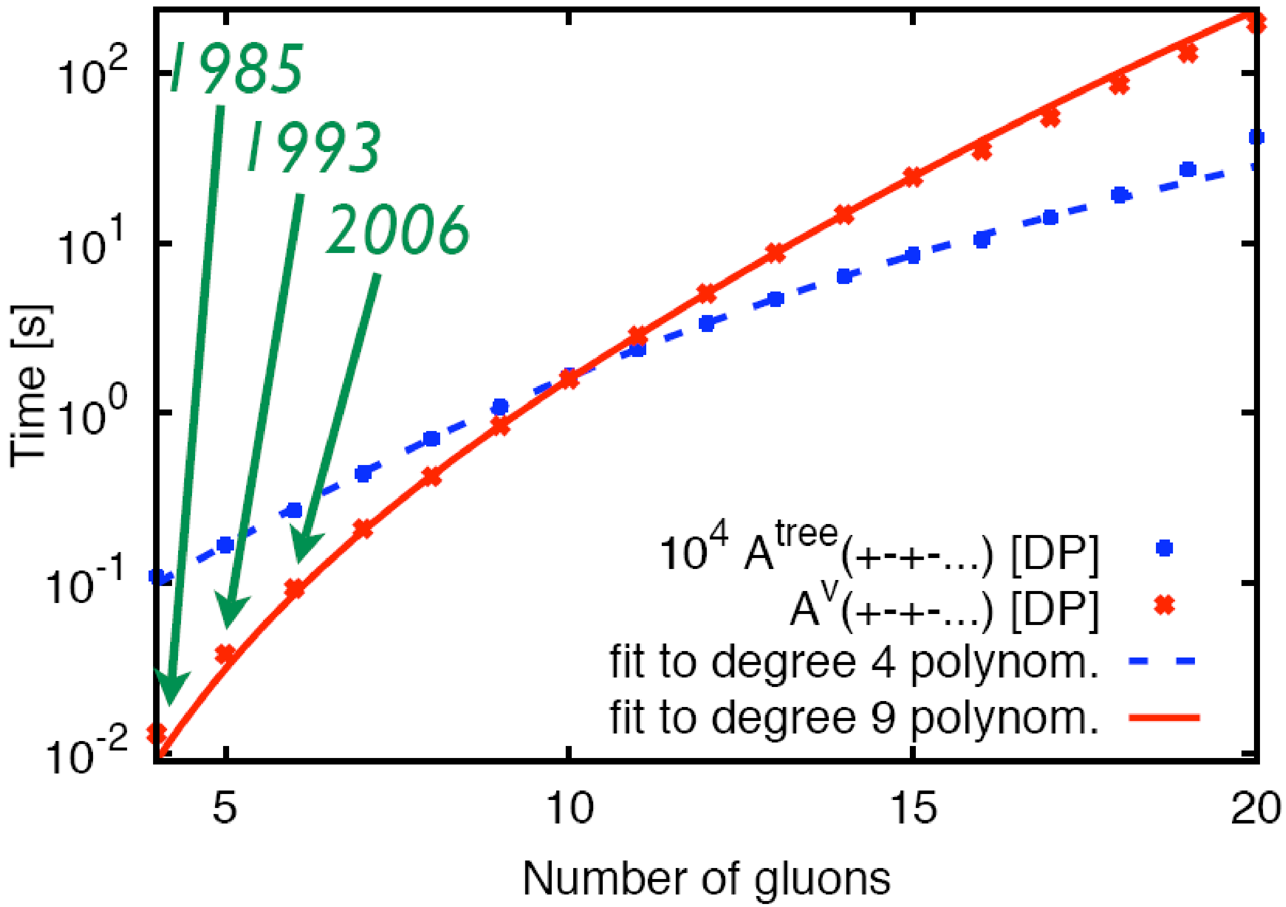
- ✓ all vector boson plus five parton processes numerically at single phase-space points

Ellis, Giele, Zanderighi, (08)

- ✓ physical $W + 3$ jet cross section

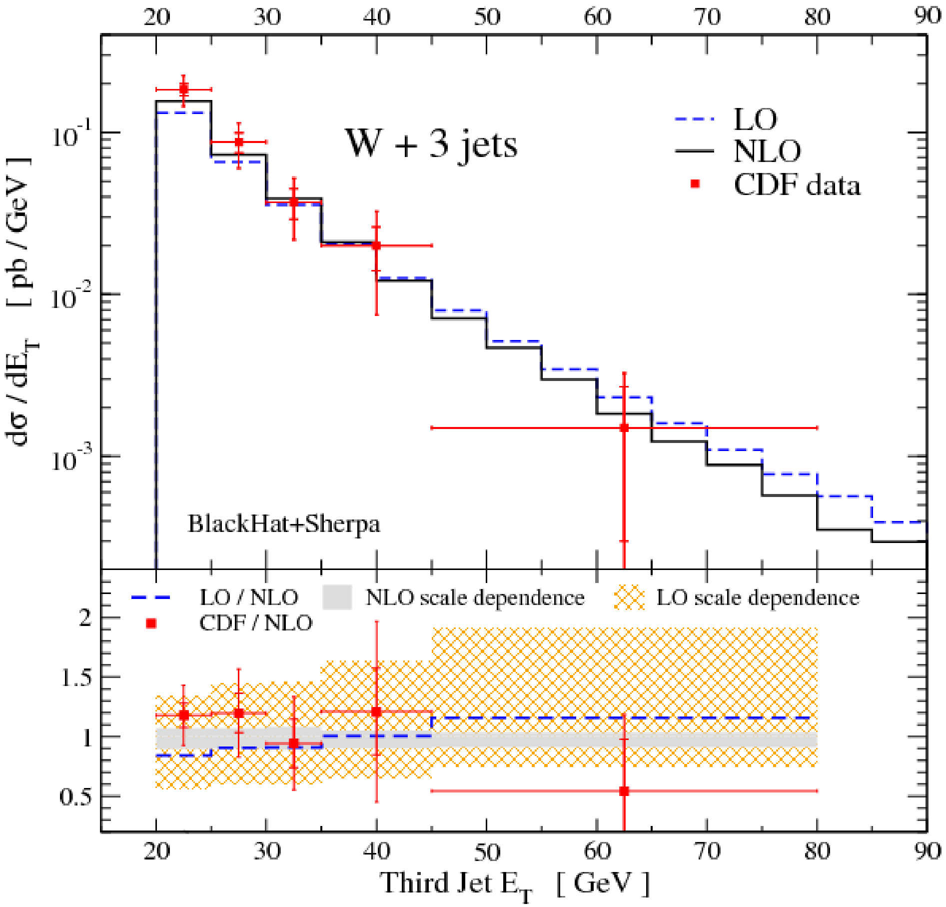
Ellis, Giele, Zanderighi, (09)

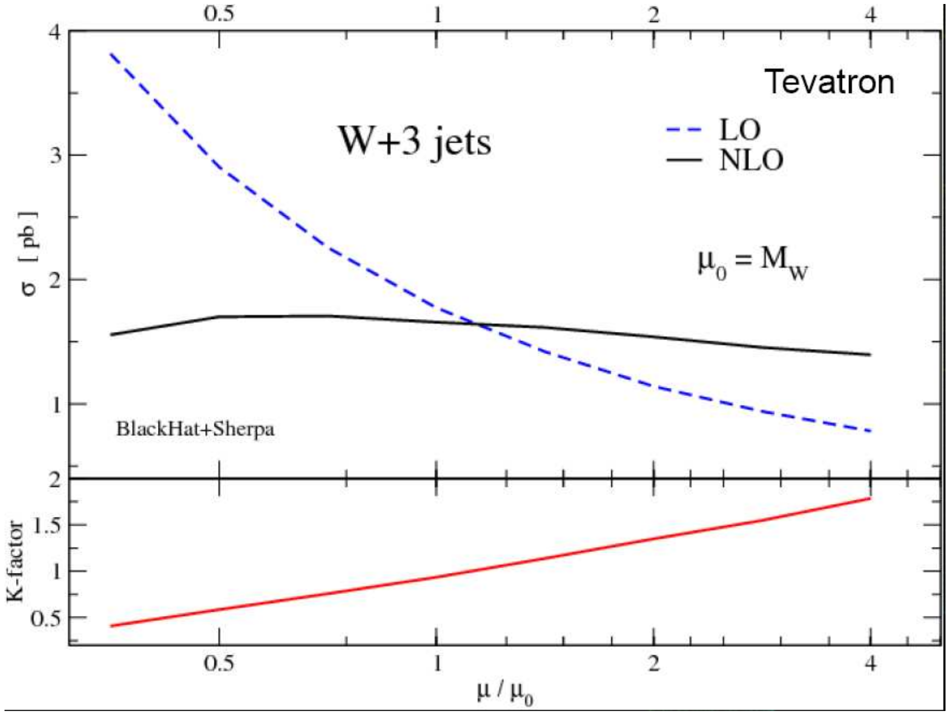
gg → (N-2)g at 1-loop



single colour ordering, single phase space point Giele, Zanderighi (08)
other numerical programs by Lazopoulos (08) and Giele, Winter (09)

W+3 jet at NLO





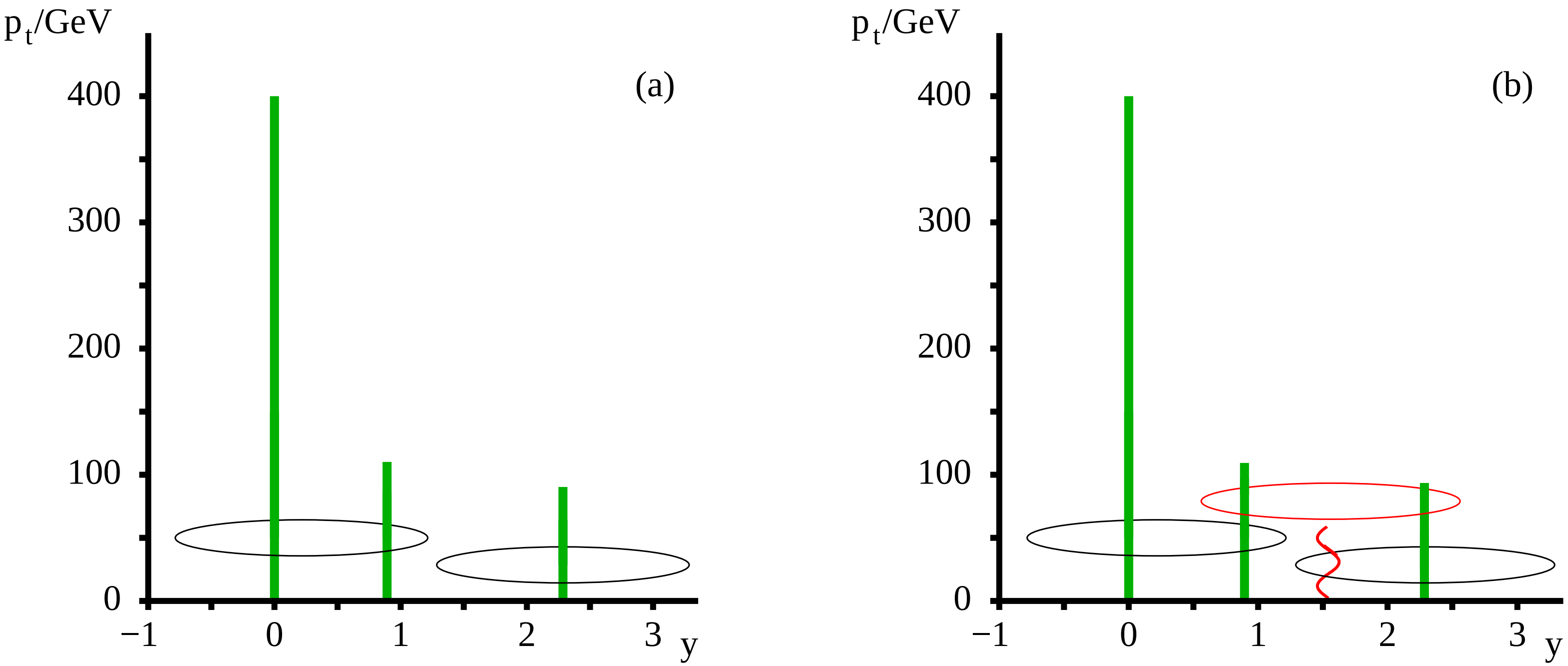
Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maitre, arXiv:0902.2760

3. Jets

Jets: Cones vs Recombination

- ✓ Cone algorithms
  - ✓ Intutitive, clear jet structure
  - ✗ Complicated; problems with IR safety
  - ✓ Solved by SiSCone — Salam, Soyez, (07)
- ✓ Recombination algorithms (k_T etc)
  - ✓ Simple, IR safe
  - ✗ Messy jet structure
  - ✓ Solved by anti-k_T — Cacciari, Salam, Soyez, (08)

Cone algorithms and Infrared Safety



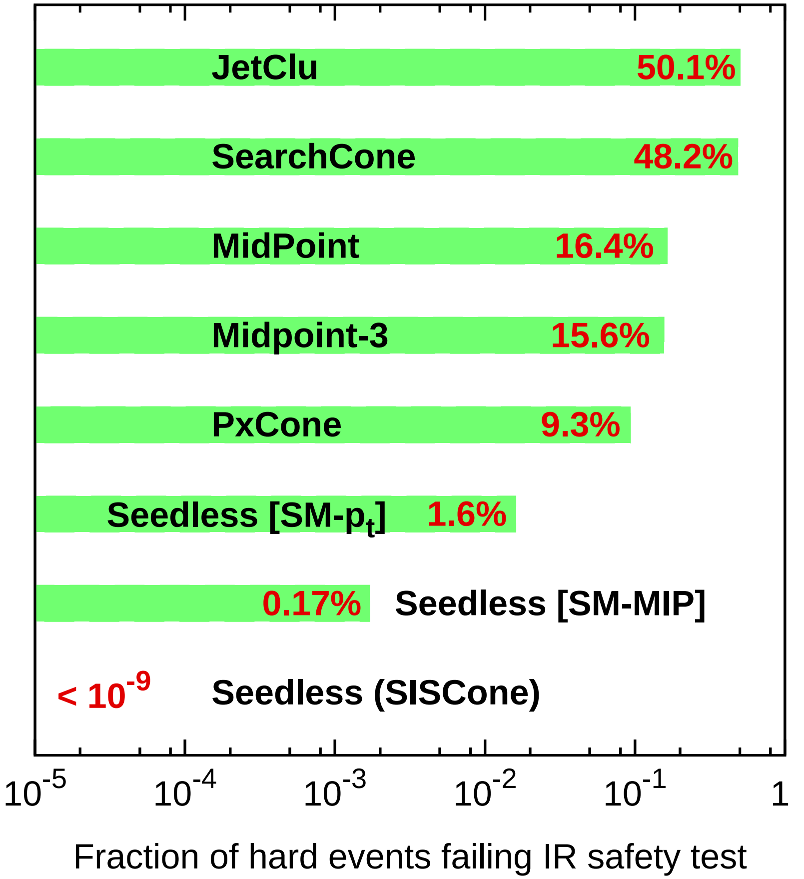
- ✗ Adding one extra soft particle changes the number of jets
- ✗ Soft emission changes the hard jets ⇒ algorithm is unsafe
- ✓ Solution: use seedless algorithm to find stable jet cones
  SISCone Salam, Soyez, (07)

Cone algorithms

Will find discrepancies between theory and experiment using the midpoint cone when more partons allowed in the event

Observable	problem at
Inclusive jet	NNLO
$V + 1$ jet	NNLO
3 jets	NLO
$V + 2$ jets	NLO
jet masses in 3 jets	LO
jet masses, V+2 jets	LO

Will have an impact on LHC physics



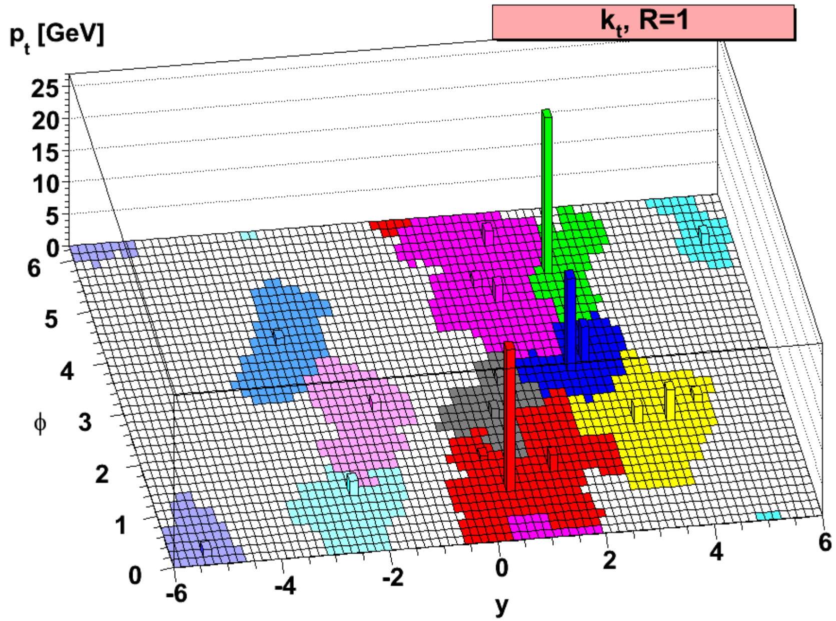
SISCone is IRC safe, similar complexity to midpoint algorithms

Recombination algorithms

$$d_{ij} = \min\{k_{Ti}^p, k_{Tj}^p\}\Delta R_{ij}/R, \qquad d_{iB} = k_{Ti}^p$$

$$\Delta R^2 = \Delta\eta^2 + \Delta\phi^2$$

- ✓ $p > 0$ $\mathrm{k_T}$/Durham
- ✓ $p = 0$ Cambridge/Aachen
- ✓ $p < 0$ anti-$\mathrm{k_T}$



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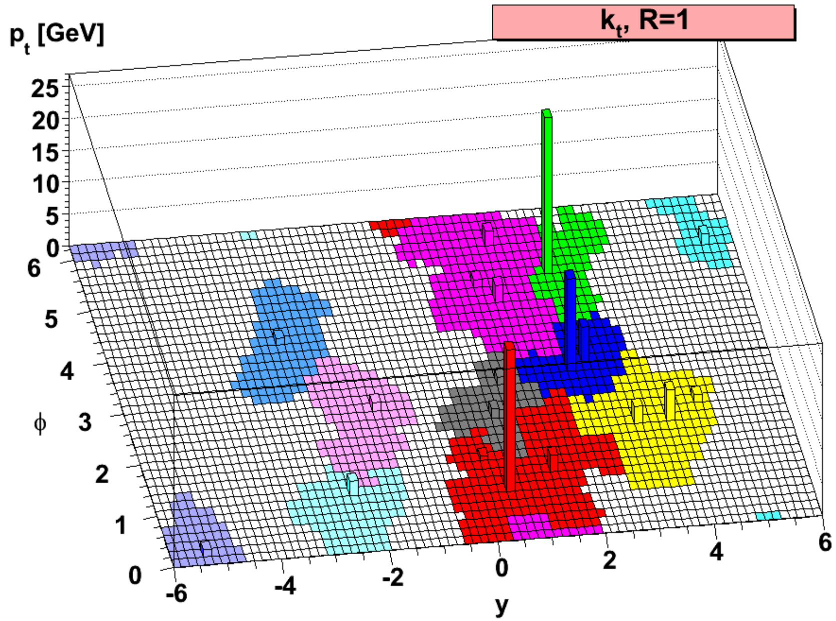
$p > 0$ $\mathrm{k_T}$/Durham

✓ clusters softest particles first

✓ leads to very irregular jets

✓ includes a lot of underlying event

✓ hard to get jet energy scale right



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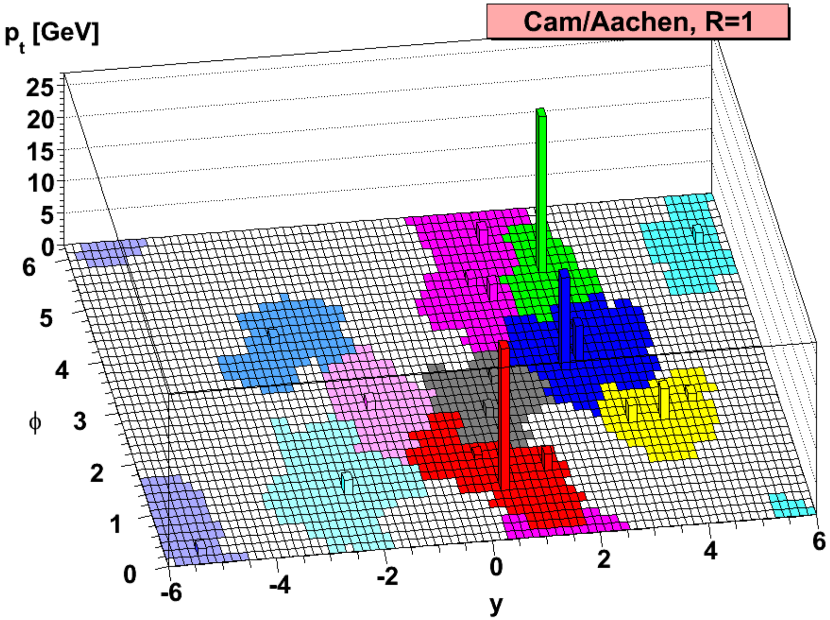
$p = 0$ Cambridge/Aachen

- ✓ clusters closest particles first
- ✓ still leads to very irregular jets
- ✓ similar problems to kT algorithm



Recombination algorithms

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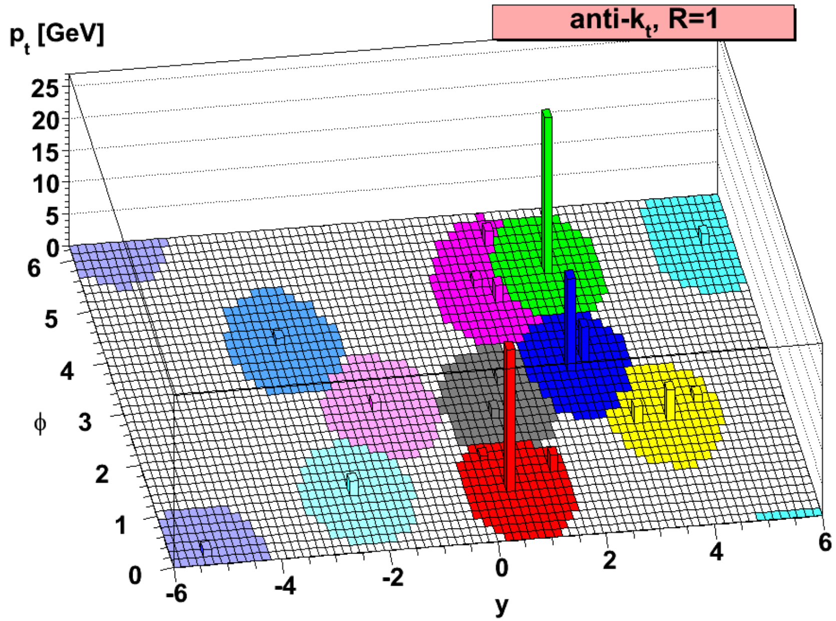
$$\Delta R^2 = \Delta\eta^2 + \Delta\phi^2$$

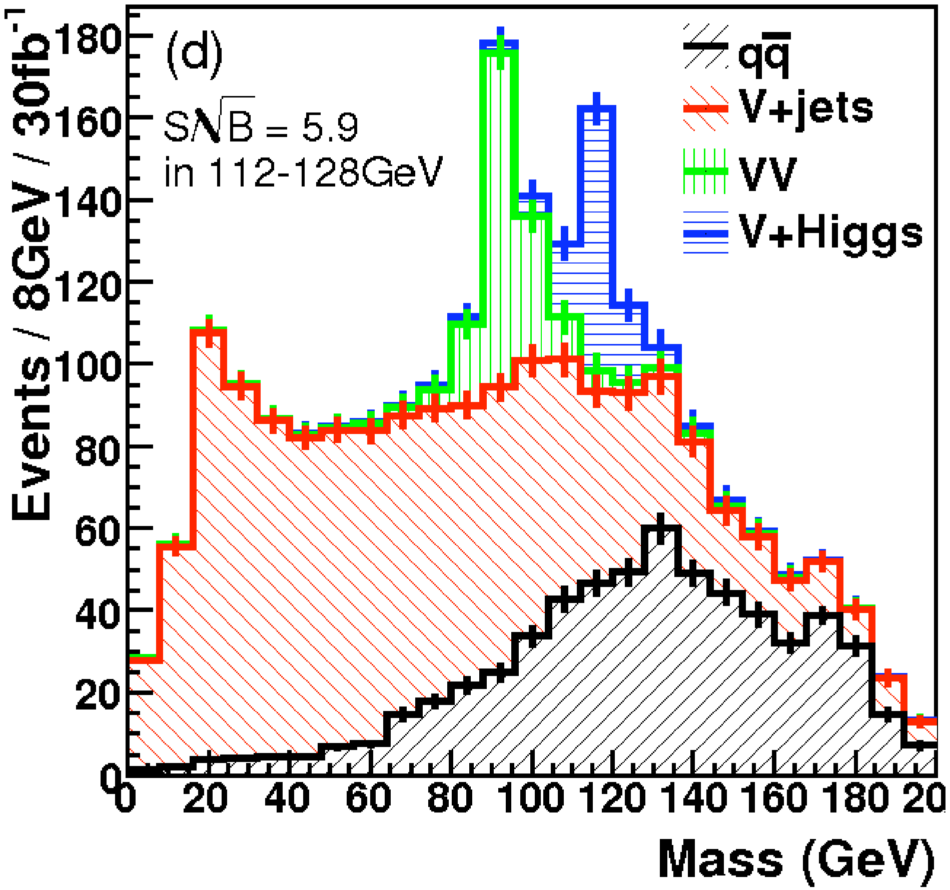
$p < 0$ anti-$\mathrm{k_T}$

- ✓ clusters hardest particles first
- ✓ shape of jet insensitive to soft particles
- ✓ cone-like jets
- ✓ may be easier to get jet energy scale right



Z/W + H $(\to b\bar{b})$ rescued

- ✓ Boosted Higgs at high pt: central decay products $\to$ single massive jet
- ✓ Jet-finding adapted to identify the characteristic structure of Higgs decay into $b\bar{b}$ with small angular separation



5.9σ at 30 fb^{-1}: VH with H $\to$ bb recovered as one of the best discovery channels for light Higgs

Butterworth, Davison, Rubin, Salam (08)

4. NNLO

NNLO

When is NNLO needed?

- ✓ When corrections are large - e.g. H production
- ✓ For benchmark measurements where experimental errors are small

What is known so far?

- ✓ Inclusive cross sections for W, Z and H production

  van Neerven, Harlander, Kilgore, Anastasiou, Melnikov, Ravindran, Smith;

- ✓ Semi-inclusive $2 \to 1$ distributions - W, Z and H rapidity distributions

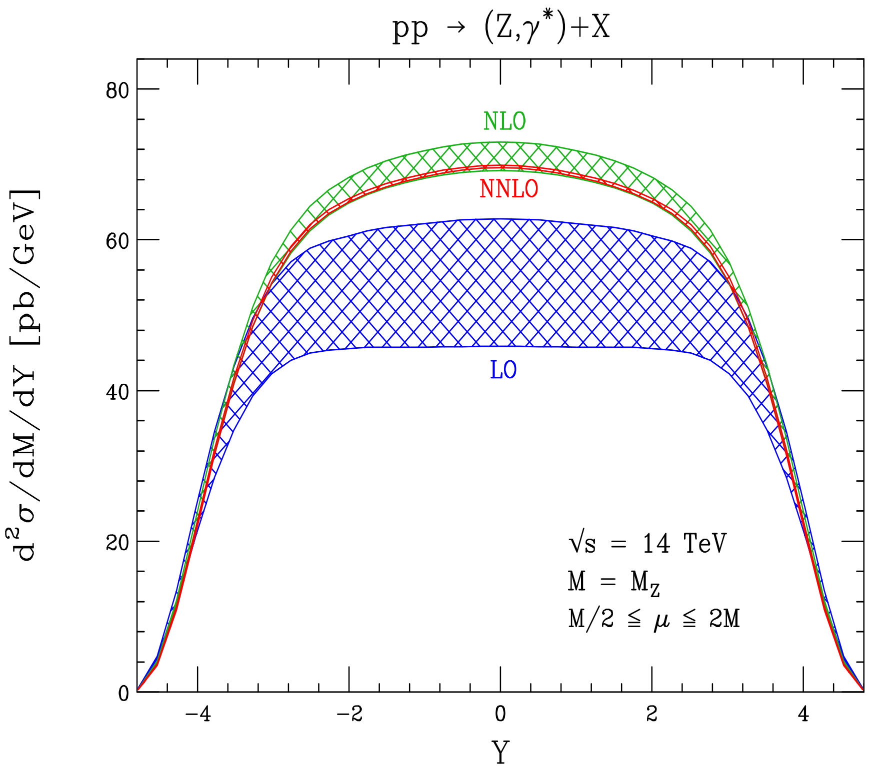
  Anastasiou, Dixon, Melnikov, Petriello

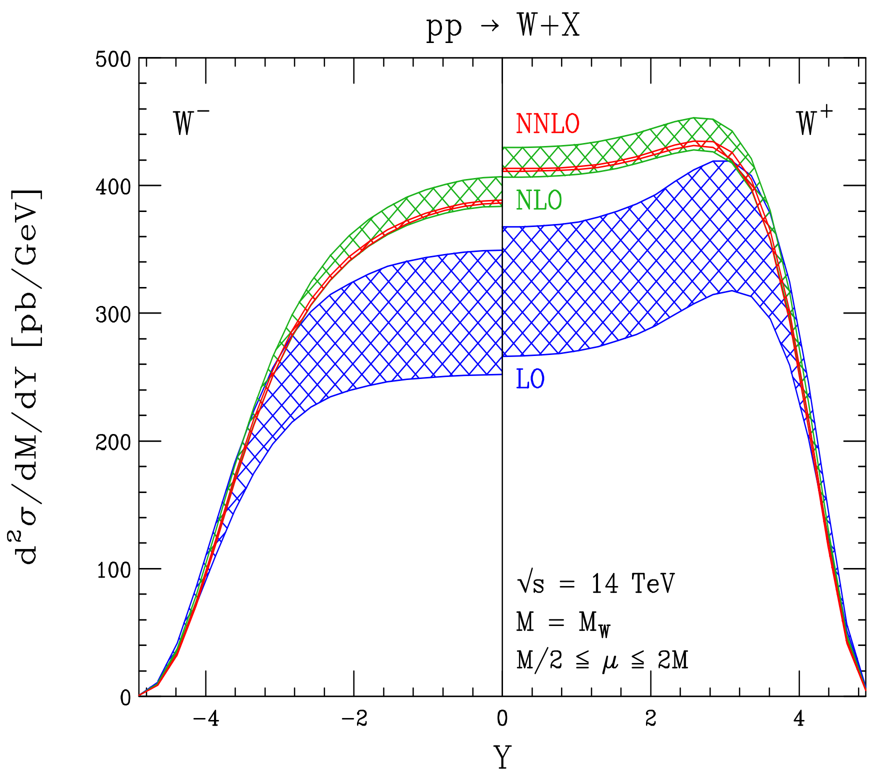
- ✓ Fully differential $pp \to H, W, Z + X$

  Anastasiou, Melnikov, Petriello; Catani, Cieri, de Florian, Grazzini

- ✓ DGLAP splitting kernels — Moch, Vermaseren, Vogt
- ✓ NNLO parton distributions — Martin, Stirling, Thorne, Watt

Gauge boson production at the LHC





Gold-plated process

Anastasiou, Dixon, Melnikov, Petriello (03)

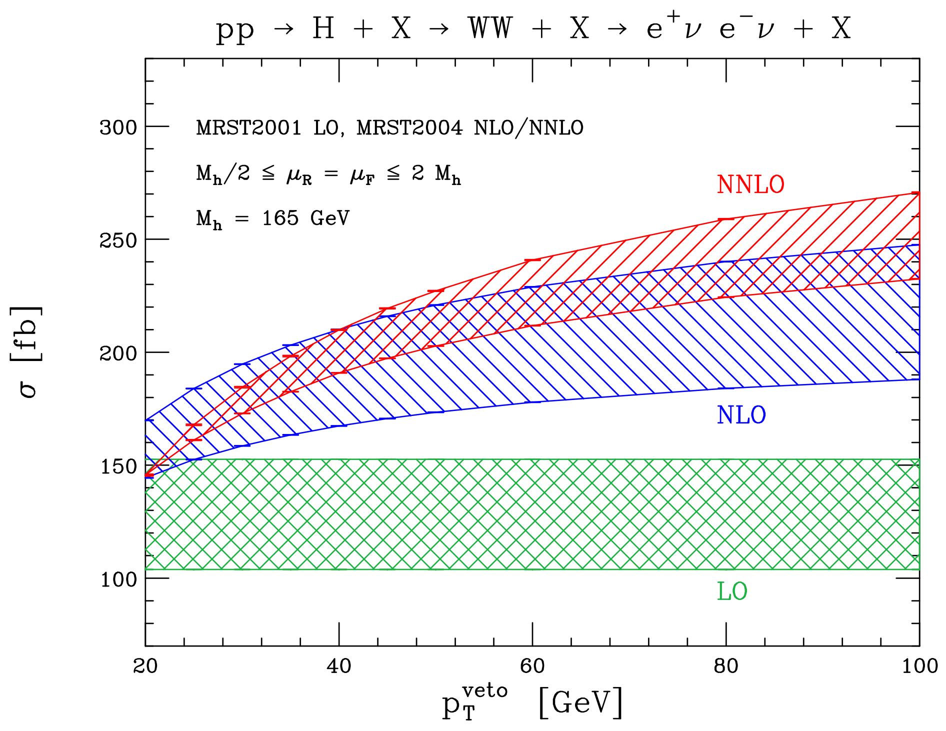
At LHC NNLO perturbative accuracy better than 1%

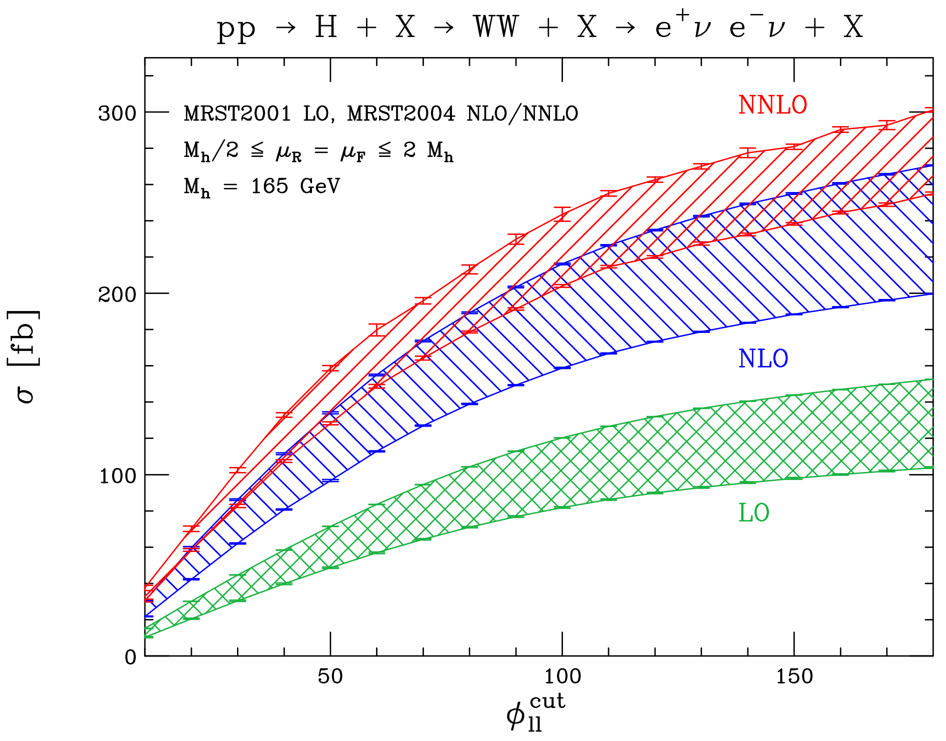
$\Rightarrow$ could use to determine parton-parton luminosities at the LHC

Higgs boson production at the LHC

- ✓ First study of fully exclusive $pp \to H \to WW \to \ell\nu\ell\nu$ with $m_H \sim 165$ GeV Anastasiou, Dissertori, Stöckli, (07) Catani, Grazzini (07)
- ✓ Experimental cuts to reduce backgrounds affect LO/NLO/NNLO cross sections differently e.g. jet-veto suppresses additional radiation,

$\Longrightarrow$ Absolutely vital to include cuts and decays in realistic studies





NNLO 3-jets in e^+e^-

✓ Motivation: error on α_s from jet-observables

$$\alpha_s(M_Z) = 0.121 \pm 0.001(\mathrm{exp}) \pm 0.005(\mathrm{th})$$

Bethke (06)

⇒ dominated by theoretical uncertainty

✓ First NNLO results for 3-jet event shapes in 2007

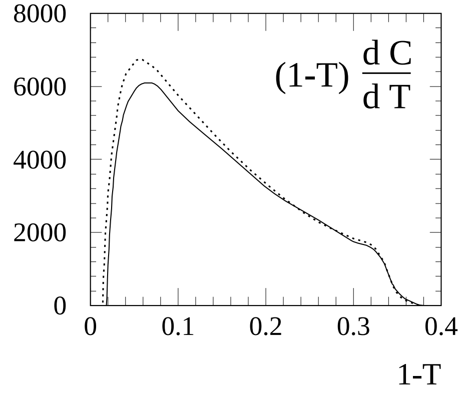
Gehrmann, Gehrmann-De Ridder, NG, Heinrich (07)

✓ Problem in the two-jet region identified in two colour structures

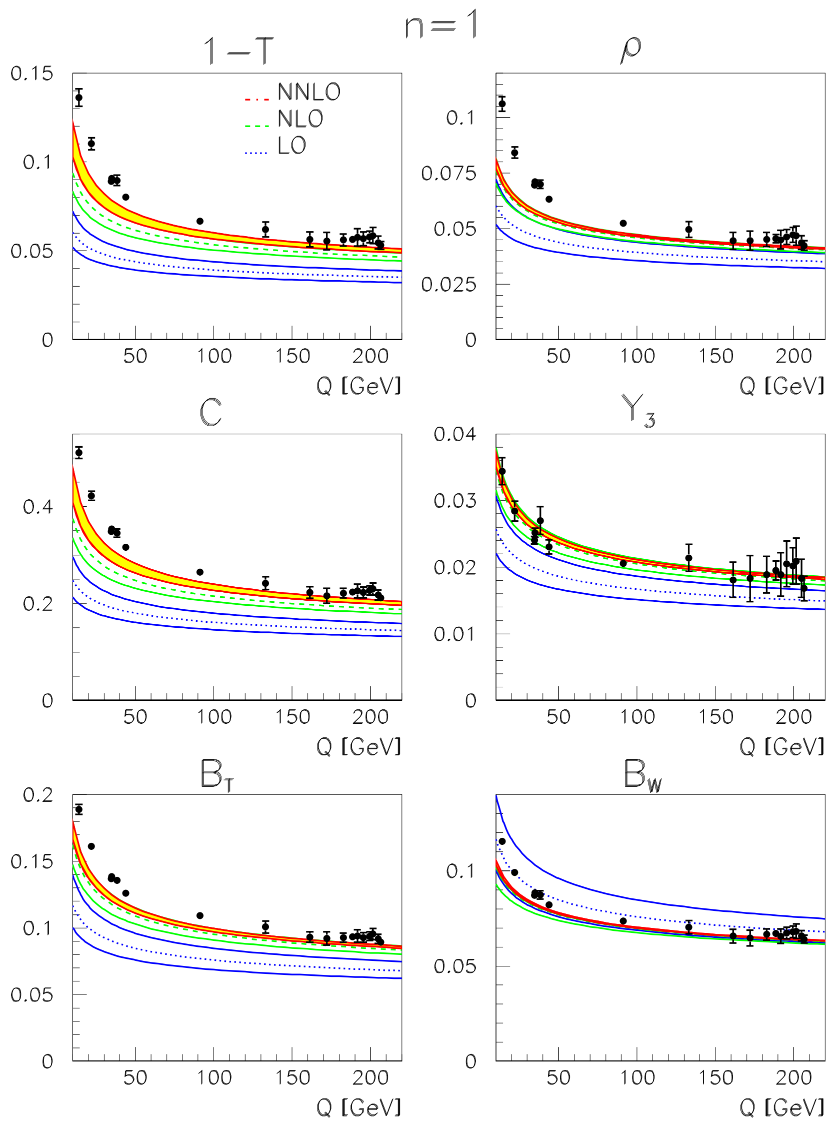
Becher, Schwartz (08); Weinzierl (08)

✓ over-subtraction of wide angle soft emission

✓ now fixed - minor correction in three-jet region



NNLO 3-jets in e^+e^-



Moments of event shapes

Gehrmann, Gehrmann-De Ridder, NG, Heinrich (09)

- ✓ Agreement with independent calculation of Weinzierl (09)
- ✓ NNLO corrections are moderate for all event shapes
- ✓ NNLO corrections result in a substantial reduction of the theoretical uncertainty on these predictions
- ✓ size of power corrections appears to be reduced
- ✓ needs full analysis

Other NNLO calculations on horizon

✓ $pp \to jet$ +X

- needed to constrain PDF's and fix strong coupling
- matrix elements known for some time — Anastasiou et al, Bern et al
- antenna subtraction terms worked out — Daleo, Gehrmann, Maitre

✓ $pp \to t\bar{t}$

- necessary for precise m_t determination
- matrix elements partially known

Czakon, Mitov, Moch; Bonciani, Ferroglia, Gehrmann, Studerus, Maitre

✓ $pp \to VV$

- signal: to study the gauge structure of the Standard Model
- background: for Higgs boson production and decay in the intermediate mass range
- large NLO corrections — Chachamis, Czakon, Eiras

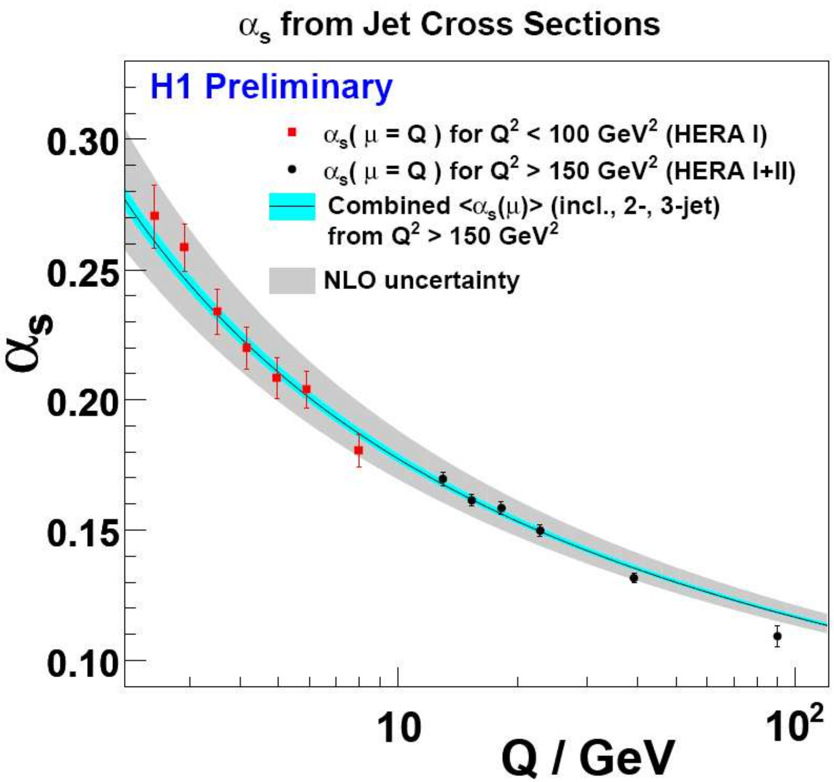
Other NNLO calculations on horizon

$ep \to 2 + 1$ jets

✓ needed to constrain gluon PDF and fix strong coupling

✓ NLO uncertainty dominates experimental error Gouzevitch (08)

✓ two-loop $\gamma^* g \to q\bar{q}$ and $\gamma^* q \to qg$ helicity amplitudes recently worked out Gehrmann, NG (09)



5. Beyond NNLO

Infrared singularities in QCD amplitudes

- ✓ Infrared singularities open a window into the all-order structure of perturbation theory and beyond
- ✓ For massless amplitudes IR $\leftrightarrow$ UV singularities
- ✓ singularities controlled by anomalous dimension

$$\mathbf{\Gamma} = -\frac{1}{\mathbf{Z}}\frac{d\mathbf{Z}}{d\log\mu}, \qquad\qquad \mathbf{Z}^{-1} \leftrightarrow \text{singularities}$$

- ✓ simplest (Dipole) all orders solution for soft anomalous dimension evolution equation Becher, Neubert (08,09); Gardi, Magnea (09)

$$\mathbf{\Gamma} = \sum_{i,j} \mathbf{T}_i \cdot \mathbf{T}_j \Gamma_{\rm cusp}(\alpha_s) \log\left(\frac{\mu^2}{-s_{ij}}\right) + \sum_i \gamma^i(\alpha_s)$$

γ^i : quark/gluon anomalous dimension

- ✓ triple correlations do not contribute (three-loops/all-orders)

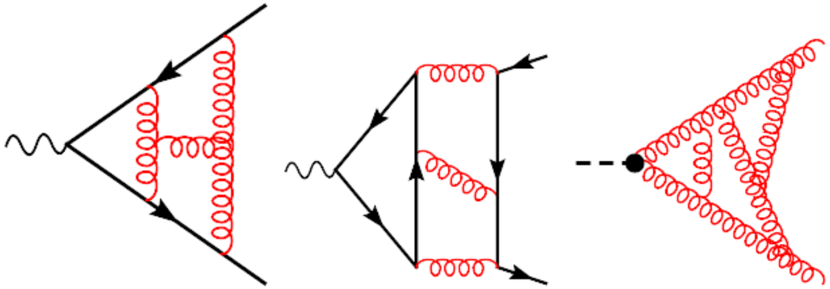
Infrared singularities in QCD amplitudes

- ✓ reproduces Catani singularity structure for two-loops - with prediction for structure of $\mathbf{H}_2$
- ✓ reproduces pole structure of $e^+e^- \to q\bar{q}g$ two-loop amplitude
- ✓ reproduces pole structure of $gg \to gg$ two-loop amplitude
- ✓ reproduces pole structure of three-loop quark and gluon form factor

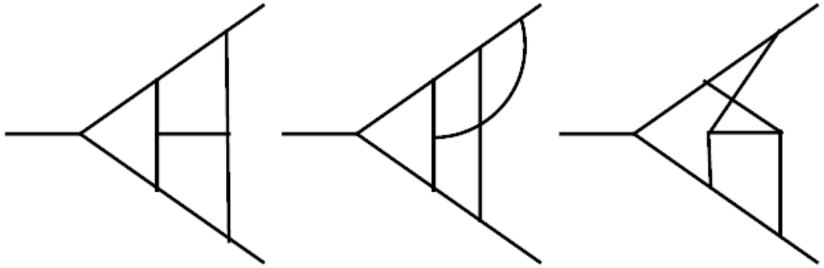
Moch, Vermaseren, Vogt (05)

- ✓ reproduces pole structure of four-loop four-gluon amplitude in $\mathcal{N} = 4$ theory in planar limit Bern, Czakon, Dixon, Kosower, Smirnov (06)
- ⇒ The full beauty of gauge theory amplitudes is not yet revealed...

Three-loop form factor



(a) (b) (c)



$A_{9,1}$ $A_{9,2}$ $A_{9,4}$

$$
\begin{aligned}
A_{9,1} = {} & \frac{1}{18\epsilon^5} - \frac{1}{2\epsilon^4} + \frac{1}{\epsilon^3}\left(\frac{53}{18} + \frac{29\zeta(2)}{36}\right) + \frac{1}{\epsilon^2}\left(-\frac{29}{2}\right. \\
& \left. - \frac{149\zeta(2)}{36} + \frac{35\zeta(3)}{18}\right) + \frac{1}{\epsilon}\left(\frac{129}{2} + \frac{139\zeta(2)}{12}\right. \\
& \left. - \frac{307\zeta(3)}{18} + \frac{5473\zeta(4)}{288}\right) - \frac{537}{2} - \frac{57\zeta(2)}{4} \\
& + \frac{1103\zeta(3)}{18} - \frac{15625\zeta(4)}{288} + \frac{871\zeta(2)\zeta(3)}{36} \\
& + \frac{793\zeta(5)}{10} + \epsilon X_{9,1} + \mathcal{O}\left(\epsilon^2\right) , \qquad (2)
\end{aligned}
$$

Baikov, Chetyrkin, Steinhauser, Smirnov, Smirnov (09); Heinrich, Huber, Kosower, Smirnov (09)

Three most difficult nine-propagator master integrals evaluated using Mellin-Barnes methods and checked between the two groups

Three-loop form factor

Finite parts of the three-loop quark form-factor

$$F_q^{(3),g+n_f}\Big|_{\rm fin} = C_F^3\left(\frac{26871}{8} - \frac{95137\zeta(2)}{60} + \frac{5569\zeta(3)}{5} + \frac{95375\zeta(4)}{48} + \frac{30883\zeta(2)\zeta(3)}{15} - \frac{16642\zeta(5)}{5} + \frac{2669(\zeta(3))^2}{3}\right.$$
$$\left. + \frac{1961387\zeta(6)}{2880} - \frac{24X_{9,1}}{5} + \frac{24X_{9,2}}{5} + \frac{6X_{9,4}}{5}\right) + C_A C_F^2\left(\frac{20003431}{29160} + \frac{4239679\zeta(2)}{1620} - \frac{121753\zeta(3)}{30}\right.$$
$$ - \frac{11155817\zeta(4)}{4320} - \frac{92554\zeta(2)\zeta(3)}{45} + \frac{610462\zeta(5)}{225} - \frac{36743(\zeta(3))^2}{30} - \frac{1118529\zeta(6)}{640} + \frac{24X_{9,1}}{5}$$
$$\left. - \frac{16X_{9,2}}{5} - \frac{9X_{9,4}}{5}\right) + C_A^2 C_F\left(-\frac{88822328}{32805} - \frac{3486997\zeta(2)}{2916} + \frac{3062512\zeta(3)}{1215} + \frac{4042277\zeta(4)}{4320}\right.$$
$$\left. + \frac{5233\zeta(2)\zeta(3)}{12} - \frac{202279\zeta(5)}{450} + \frac{63043(\zeta(3))^2}{180} + \frac{4741699\zeta(6)}{11520} - X_{9,1} + \frac{2X_{9,2}}{5} + \frac{3X_{9,4}}{5}\right)$$
$$ + C_F^2 n_f T\left(-\frac{2732173}{1458} - \frac{45235\zeta(2)}{81} + \frac{102010\zeta(3)}{81} + \frac{40745\zeta(4)}{216} - \frac{686\zeta(3)\zeta(2)}{9} + \frac{556\zeta(5)}{45}\right)$$
$$ + C_A C_F n_f T\left(\frac{17120104}{6561} + \frac{442961\zeta(2)}{729} - \frac{90148\zeta(3)}{81} - \frac{5465\zeta(4)}{27} + \frac{736\zeta(3)\zeta(2)}{9} - \frac{416\zeta(5)}{3}\right)$$
$$ + C_F n_f^2 T^2\left(-\frac{2710864}{6561} - \frac{248\zeta(2)}{3} + \frac{12784\zeta(3)}{243} - \frac{166\zeta(4)}{27}\right), \qquad (8)$$

Baikov, Chetyrkin, Steinhauser, Smirnov, Smirnov (09)

- ✓ $X_{9,1}$, $X_{9,2}$ and $X_{9,4}$ known numerically
- ✓ gluon form-factor also computed

Summary

remarkable development pace in QCD for higher order calculations in past few years

- ✓ first signs of automated mutiparticle NLO cross sections
- ✓ many new ideas for sophisticated jet definitions
- ✓ high precision NNLO calculations for standard candle processes on the way
- ✓ glimpses of more structure in higher loop gauge theory amplitudes

  link to massive progress in multiparticle multi-loop N=4 Super Yang Mills

- ✓ . . . apologies to those whose important work I have not (sufficiently) discussed