Small-x resummed DIS phenomenology

Juan Rojo

In collaboration with G. Altarelli, R. Ball and S. Forte

INFN, Sezione di Milano

DIS 2009
PDF and low-x Session
28/04/2009, Madrid

MOTIVATION

Small-x resummation and phenomenology

- Any realistic small-x resummation procedure should:
    1. Include a stable solution of BFKL with running coupling at NLL
    2. Match smoothly to DGLAP at moderate and large x
    3. Provide the complete set of splitting functions and (at least) DIS coefficient functions in a well defined factorization scheme
- Full set of small-x resummed DIS splitting functions and coefficient functions recently available (Altarelli, Ball and Forte, NPB799:199-240,2008) where all above requirements met $\longrightarrow$ Next step is to study its phenomenological implications
- See also S. Marzani's talk for discussion on the small-x resummation of the Drell-Yan process
- Important related work by the Ciafaloni-Colferai-Salam-Stasto (see A. Stasto's talk) and Thorne-White groups
- Not a review talk $\longrightarrow$ Apologies in advance for lack of references!

Small-x resummation and phenomenology

- Any realistic small-x resummation procedure should:
  1. Include a stable solution of BFKL with running coupling at NLL
  2. Match smoothly to DGLAP at moderate and large x
  3. Provide the complete set of splitting functions and (at least) DIS coefficient functions in a well defined factorization scheme
- Full set of small-x resummed DIS splitting functions and coefficient functions recently available (Altarelli, Ball and Forte, NPB799:199-240,2008) where all above requirements met $\longrightarrow$ Next step is to study its phenomenological implications
- See also S. Marzani's talk for discussion on the small-x resummation of the Drell-Yan process
- Important related work by the Ciafaloni-Colferai-Salam-Stasto (see A. Stasto's talk) and Thorne-White groups
- Not a review talk $\longrightarrow$ Apologies in advance for lack of references!

Small-x resummation and phenomenology

- Any realistic small-x resummation procedure should:
  1. Include a stable solution of BFKL with running coupling at NLL
  2. Match smoothly to DGLAP at moderate and large x
  3. Provide the complete set of splitting functions and (at least) DIS coefficient functions in a well defined factorization scheme
- Full set of small-x resummed DIS splitting functions and coefficient functions recently available (Altarelli, Ball and Forte, NPB799:199-240,2008) where all above requirements met → Next step is to study its phenomenological implications
- See also S. Marzani's talk for discussion on the small-x resummation of the Drell-Yan process
- Important related work by the Ciafaloni-Colferai-Salam-Stasto (see A. Stasto's talk) and Thorne-White groups
- Not a review talk → Apologies in advance for lack of references!

Small-x resummation and phenomenology

- Any realistic small-x resummation procedure should:
  1. Include a stable solution of BFKL with running coupling at NLL
  2. Match smoothly to DGLAP at moderate and large x
  3. Provide the complete set of splitting functions and (at least) DIS coefficient functions in a well defined factorization scheme
- Full set of small-x resummed DIS splitting functions and coefficient functions recently available (Altarelli, Ball and Forte, NPB799:199-240,2008) where all above requirements met → Next step is to study its phenomenological implications
- See also S. Marzani's talk for discussion on the small-x resummation of the Drell-Yan process
- Important related work by the Ciafaloni-Colferai-Salam-Stasto (see A. Stasto's talk) and Thorne-White groups
- Not a review talk → Apologies in advance for lack of references!

Small-x resummation and phenomenology

- Any realistic small-x resummation procedure should:
  1. Include a stable solution of BFKL with running coupling at NLL
  2. Match smoothly to DGLAP at moderate and large x
  3. Provide the complete set of splitting functions and (at least) DIS coefficient functions in a well defined factorization scheme
- Full set of small-x resummed DIS splitting functions and coefficient functions recently available (Altarelli, Ball and Forte, NPB799:199-240,2008) where all above requirements met → Next step is to study its phenomenological implications
- See also S. Marzani's talk for discussion on the small-x resummation of the Drell-Yan process
- Important related work by the Ciafaloni-Colferai-Salam-Stasto (see A. Stasto's talk) and Thorne-White groups
- Not a review talk → Apologies in advance for lack of references!

Small-x resummation and phenomenology

- Any realistic small-x resummation procedure should:
  1. Include a stable solution of BFKL with running coupling at NLL
  2. Match smoothly to DGLAP at moderate and large x
  3. Provide the complete set of splitting functions and (at least) DIS coefficient functions in a well defined factorization scheme
- Full set of small-x resummed DIS splitting functions and coefficient functions recently available (Altarelli, Ball and Forte, NPB799:199-240,2008) where all above requirements met → Next step is to study its phenomenological implications
- See also S. Marzani's talk for discussion on the small-x resummation of the Drell-Yan process
- Important related work by the Ciafaloni-Colferai-Salam-Stasto (see A. Stasto's talk) and Thorne-White groups
- Not a review talk → Apologies in advance for lack of references!

Small-x resummation and phenomenology

- Any realistic small-x resummation procedure should:
  1. Include a stable solution of BFKL with running coupling at NLL
  2. Match smoothly to DGLAP at moderate and large x
  3. Provide the complete set of splitting functions and (at least) DIS coefficient functions in a well defined factorization scheme
- Full set of small-x resummed DIS splitting functions and coefficient functions recently available (Altarelli, Ball and Forte, NPB799:199-240,2008) where all above requirements met → Next step is to study its phenomenological implications
- See also S. Marzani's talk for discussion on the small-x resummation of the Drell-Yan process
- Important related work by the Ciafaloni-Colferai-Salam-Stasto (see A. Stasto's talk) and Thorne-White groups
- Not a review talk → Apologies in advance for lack of references!

Small-x resummation and phenomenology

- Any realistic small-x resummation procedure should:
  1. Include a stable solution of BFKL with running coupling at NLL
  2. Match smoothly to DGLAP at moderate and large x
  3. Provide the complete set of splitting functions and (at least) DIS coefficient functions in a well defined factorization scheme
- Full set of small-x resummed DIS splitting functions and coefficient functions recently available (Altarelli, Ball and Forte, NPB799:199-240,2008) where all above requirements met → Next step is to study its phenomenological implications
- See also S. Marzani's talk for discussion on the small-x resummation of the Drell-Yan process
- Important related work by the Ciafaloni-Colferai-Salam-Stasto (see A. Stasto's talk) and Thorne-White groups
- Not a review talk → Apologies in advance for lack of references!

Small-x resummation and phenomenology

- Any realistic small-x resummation procedure should:
  1. Include a stable solution of BFKL with running coupling at NLL
  2. Match smoothly to DGLAP at moderate and large x
  3. Provide the complete set of splitting functions and (at least) DIS coefficient functions in a well defined factorization scheme
- Full set of small-x resummed DIS splitting functions and coefficient functions recently available (Altarelli, Ball and Forte, NPB799:199-240,2008) where all above requirements met $\rightarrow$ Next step is to study its phenomenological implications
- See also S. Marzani's talk for discussion on the small-x resummation of the Drell-Yan process
- Important related work by the Ciafaloni-Colferai-Salam-Stasto (see A. Stasto's talk) and Thorne-White groups
- Not a review talk $\rightarrow$ Apologies in advance for lack of references!

SMALL-X RESUMMATION
& PRACTICAL IMPLEMENTATION

ABF approach to small-x resummation

- Duality between DGLAP kernel $\gamma(N, \alpha_s)$ and BFKL kernel $\chi(M, \alpha_s)$

$$\chi(\gamma(N, \alpha_s), \alpha_s) = N$$

  Solution of BFKL and DGLAP equations coincide at leading twist (fixed α_s) $\to$ Double leading (logs of x and Q^2) expansion
  Automatic resummation of collinear poles of BFKL kernel χ which makes its perturbative expansion not reliable

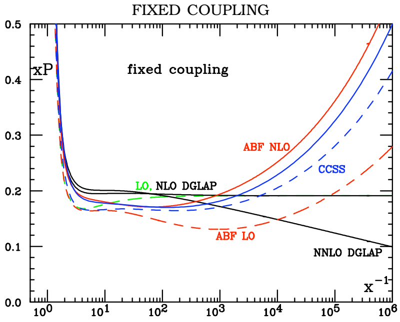
$$\gamma(1, \alpha_s) = 0 \,, \quad \to \quad \chi(0, \alpha_s) = 1$$

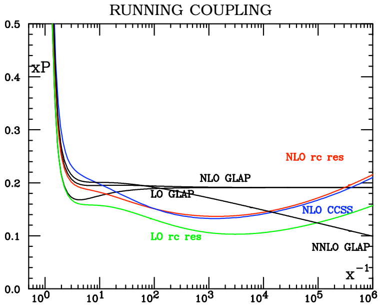
- Symmetry of BFKL kernel upon gluon exchange: at fixed α_s

$$\chi(M) = \chi(1 - M)$$

  Use collinear resummation for $M \sim 0$ in DL expansion in anti-collinear region $M \sim 1 \to$ Stable perturbative expansion at resummed level
- Running coupling resummation of BFKL solution
- Factorization scheme defined at the resummed level, whose relation with large-x schemes is known exactly

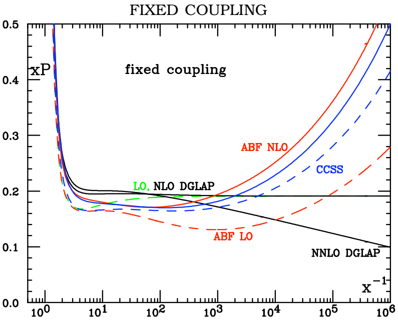
Example: Resummed $xP_{gg}(x, \alpha_s)$ for $N_f = 0$

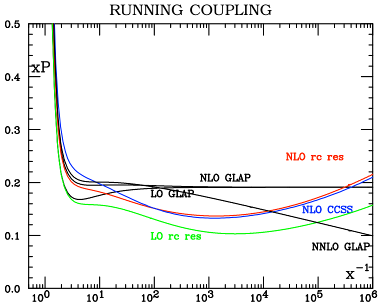




- Poor convergence of unresummed expansions
- Fast convergence of resummed expansions
- Match to large-x DGLAP expansion for $x \leq 10^{-2}$
- Small-x rise tamed by running coupling effects
- Phenomenology dominated by the *dip*

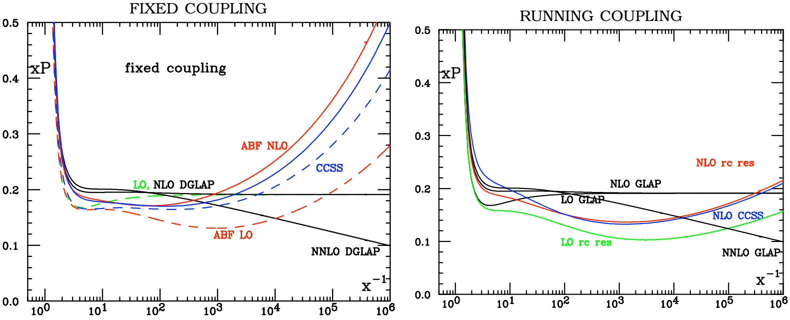
Example: Resummed $xP_{gg}(x, \alpha_s)$ for $N_f = 0$





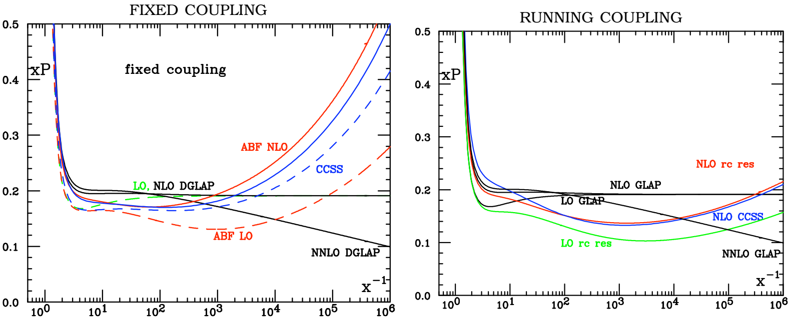
- Poor convergence of unresummed expansions
- Fast convergence of resummed expansions
- Match to large-x DGLAP expansion for $x \lesssim 10^{-2}$
- Small-x rise tamed by running coupling effects
- Phenomenology dominated by the *dip*

Example: Resummed $xP_{gg}(x, \alpha_s)$ for $N_f = 0$



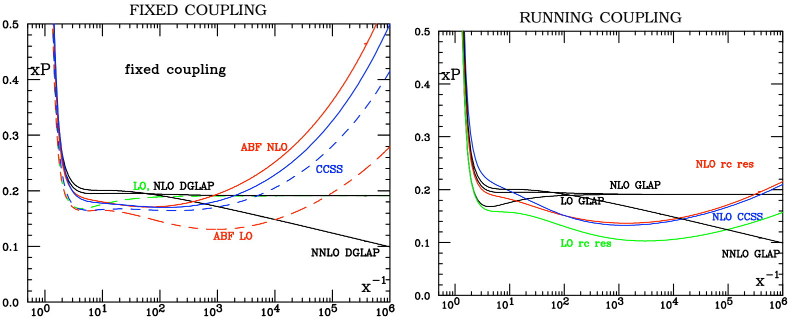
- Poor convergence of unresummed expansions
- Fast convergence of resummed expansions
- Match to large-x DGLAP expansion for $x \leq 10^{-2}$
- Small-x rise tamed by running coupling effects
- Phenomenology dominated by the *dip*

Example: Resummed $xP_{gg}(x, \alpha_s)$ for $N_f = 0$



- Poor convergence of unresummed expansions
- Fast convergence of resummed expansions
- Match to large-x DGLAP expansion for $x \leq 10^{-2}$
- Small-x rise tamed by running coupling effects
- Phenomenology dominated by the *dip*

Example: Resummed $xP_{gg}(x, \alpha_s)$ for $N_f = 0$



- Poor convergence of unresummed expansions
- Fast convergence of resummed expansions
- Match to large-x DGLAP expansion for $x \leq 10^{-2}$
- Small-x rise tamed by running coupling effects
- Phenomenology dominated by the *dip*

Resummed K-factors

For realistic phenomenology → Compute small-x resummed K-factors with NNPDF1.0 parton distributions (see my talk and M. Ubiali's one this morning)

$$K_{F_2}^{\rm res}(x,Q^2) \equiv F_2^{\Sigma}(x,Q^2)\Big|_{\rm NLOres} \Big/ F_2^{\Sigma}(x,Q^2)\Big|_{\rm NLO}$$

K-factors obtained fixing structure functions at *HERA* matching scale $t_0 \equiv \ln Q^2_{\rm match}$:

$$F_{\rm NLO}(N,t_0) = c_{\rm NLO}(N,\alpha_s(t_0)) f_{\rm NLO}(N,t_0).$$

redefining the input PDFs

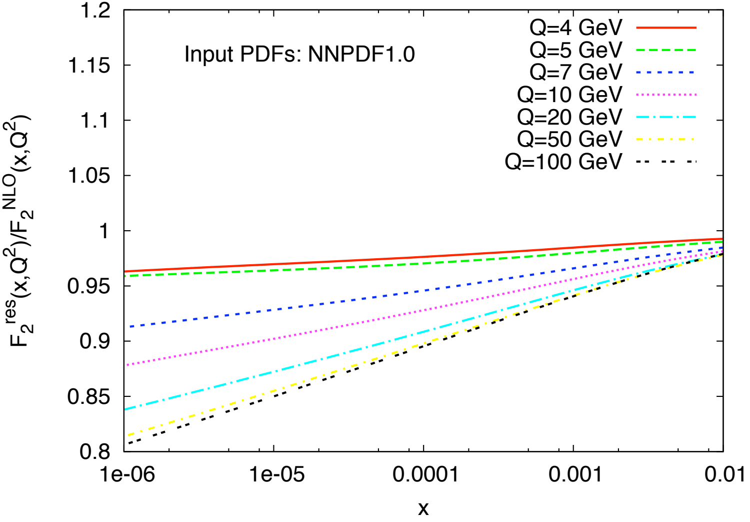
$$f_{\rm res}(N,t_0) = C^{-1}_{\rm res}(N,\alpha_s(t_0)) F_{\rm NLO}(N,t_0).$$

then evolve with appropriate kernels - closer to a real PDF fit!

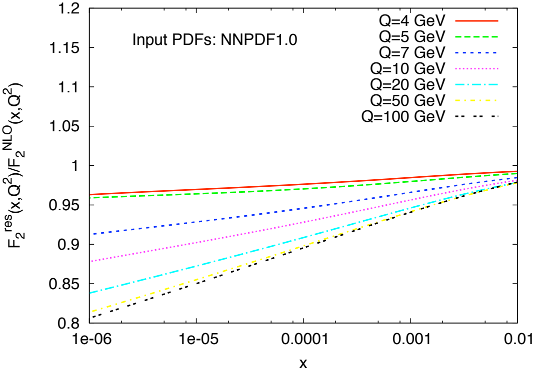
$$\begin{aligned} F_{\rm res/nlo/nnlo}(N,t) &= C_{\rm res/nlo/nnlo}(N,\alpha_s(t))\Gamma_{\rm res/nlo/nnlo}(t,t_0) f_{\rm res/nlo/nnlo}(N,t_0) \\ F_{\rm res}(N,t_0) &= F_{\rm NNLO}(N,t_0) = F_{\rm NLO}(N,t_0) \end{aligned}$$

Results shown here assume $Q_{\rm match} = 3$ GeV

Resummed K-factors



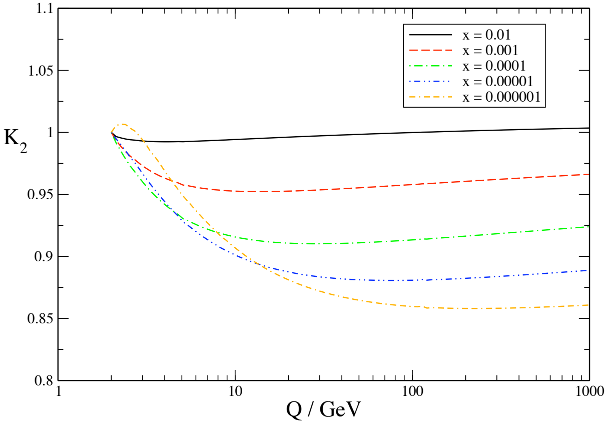
Resummed K-factors



- Smooth matching with DGLAP region $\lim_{x\to 1} K^{\rm res}(x,Q^2) = 1$
- Resummed corrections similar in size to NNLO, but opposite direction
- Resummation effects small for $Q^2 \sim Q^2_{\rm match}$ (which fixes the PDFs), differences seen in Q^2 evolution

Resummed K-factors

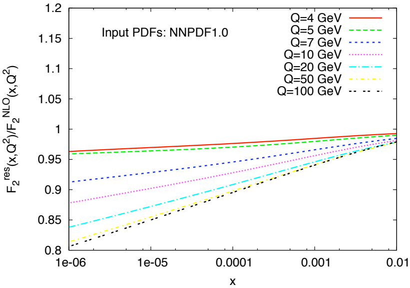
Smooth large Q^2 limit (Toy PDFs, arxiv:0901.2504)
→ Asymptotic freedom at work!

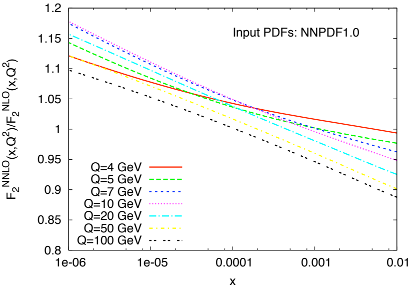


Resummed K-factors for HERA

Compare NLL and NNLO K-factors in HERA region

For $F_2(x, Q^2)$



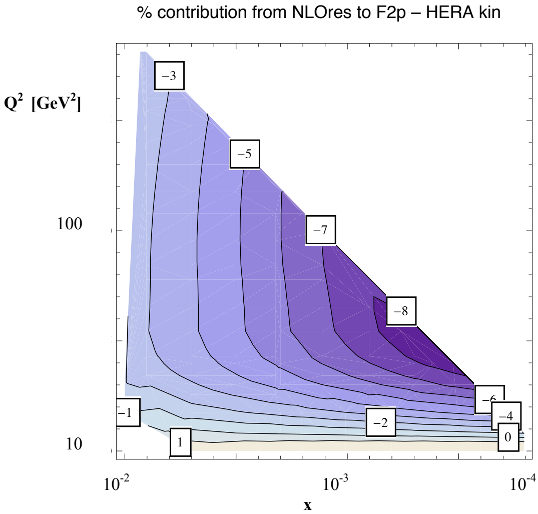


- NLL small-x corrections have similar size but opposite sign as NNLO

SMALL-X RESUMMATION
DIS PHENOMENOLOGY

Small-x resummation at HERA

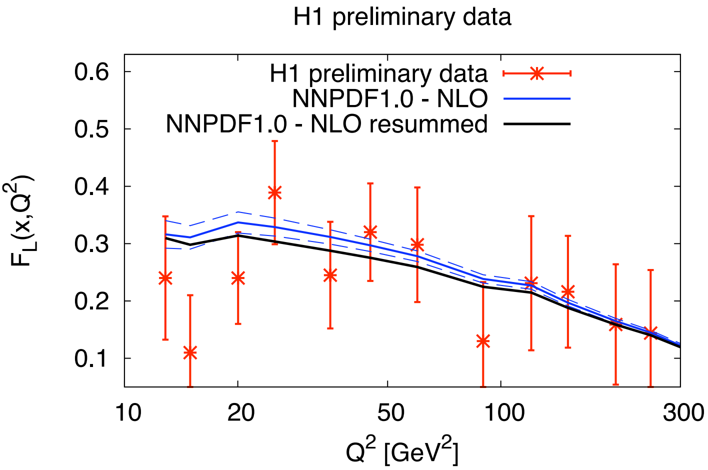
Size of small-x resummation effects in HERA kinematics



- Moderate size of corrections explain why DGLAP works so well at HERA ....
- ... but size large enough to be discriminated if ultimate HERA precision achieved!

Small-x resummed F_L

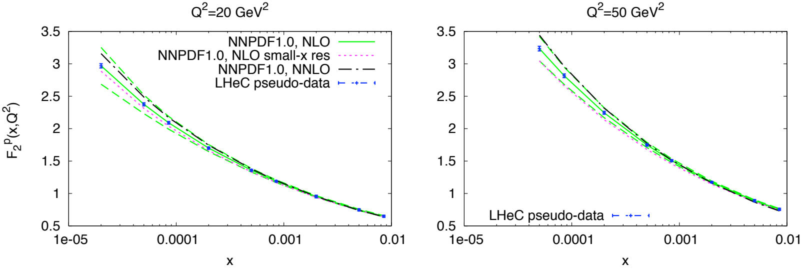
Small-x resummation effects in F_L larger than PDF uncertainties, but too large experimental errors to discriminate scenarios



Small-x resummation at the LHeC

Small-x resummation effects larger for the LHeC
(see my talk at the *Future Facilities* session)

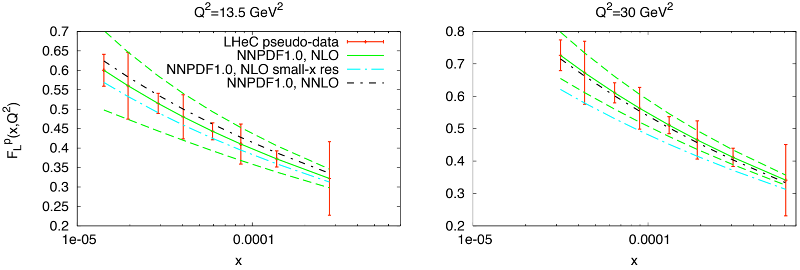
Again notice opposite trend of NLOres and NNLO corrections



Small-x resummation at the LHeC

Small-x resummation effects larger for the LHeC
(see my talk at the *Future Facilities* session)

Again notice opposite trend of NLOres and NNLO corrections



ULTRA HIGH ENERGY NEUTRINO INTERACTIONS

Ultra-High Energy (UHE) Neutrinos

- Neutrino telescopes (*e.g.* ICECUBE) and cosmic ray observatories (*e.g.* AUGER) $\rightarrow$ potential for detecting UHE neutrinos up to $E_\nu = 10^{12}$ GeV ($\sqrt{s} \simeq 10^2 \sqrt{s}_{\mathrm{LHC}}$)
- Detection of cosmic neutrinos $\rightarrow$ New window to the universe
- Detection of cosmic neutrinos $\rightarrow$ Probe the Standard Model in a completely unexplored kinematical regime
- In this talk $\rightarrow$ Implications oh UHE neutrinos for Quantum Chromodynamics (But also much to learn for New Physics!)

Ultra-High Energy (UHE) Neutrinos

- Neutrino telescopes (*e.g.* ICECUBE) and cosmic ray observatories (*e.g.* AUGER) $\longrightarrow$ potential for detecting UHE neutrinos up to $E_\nu = 10^{12}$ GeV ($\sqrt{s} \simeq 10^2 \sqrt{s}_{\rm LHC}$)
- Detection of cosmic neutrinos $\rightarrow$ New window to the universe
- Detection of cosmic neutrinos $\rightarrow$ Probe the Standard Model in a completely unexplored kinematical regime
- In this talk $\rightarrow$ Implications oh UHE neutrinos for Quantum Chromodynamics (But also much to learn for New Physics!)

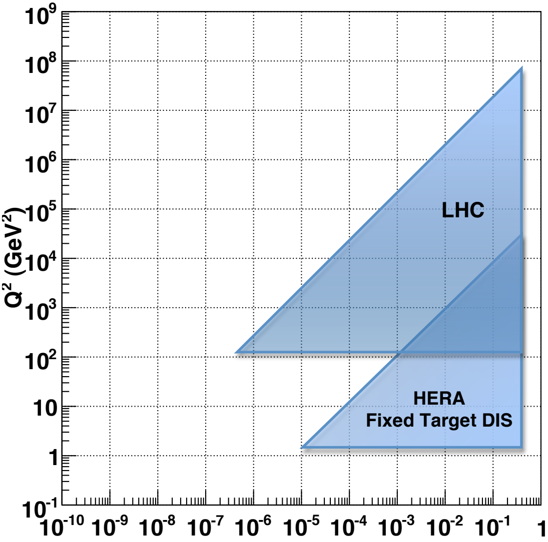
Ultra-High Energy (UHE) Neutrinos

- Neutrino telescopes (*e.g.* ICECUBE) and cosmic ray observatories (*e.g.* AUGER) $\longrightarrow$ potential for detecting UHE neutrinos up to $E_\nu = 10^{12}$ GeV ($\sqrt{s} \simeq 10^2\sqrt{s}_{\rm LHC}$)
- Detection of cosmic neutrinos $\rightarrow$ New window to the universe
- Detection of cosmic neutrinos $\rightarrow$ Probe the Standard Model in a completely unexplored kinematical regime
- In this talk $\rightarrow$ Implications oh UHE neutrinos for Quantum Chromodynamics (But also much to learn for New Physics!)

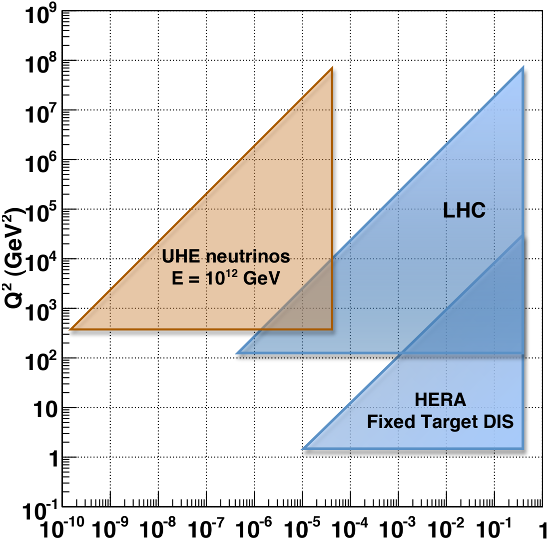
Ultra-High Energy (UHE) Neutrinos

- Neutrino telescopes (*e.g.* ICECUBE) and cosmic ray observatories (*e.g.* AUGER) $\rightarrow$ potential for detecting UHE neutrinos up to $E_\nu = 10^{12}$ GeV ($\sqrt{s} \simeq 10^2 \sqrt{s}_{\rm LHC}$)
- Detection of cosmic neutrinos $\rightarrow$ New window to the universe
- Detection of cosmic neutrinos $\rightarrow$ Probe the Standard Model in a completely unexplored kinematical regime
- In this talk $\rightarrow$ Implications oh UHE neutrinos for Quantum Chromodynamics (But also much to learn for New Physics!)

UHE Kinematics



UHE Kinematics



UHE Kinematics

The total neutrino-nucleus scattering cross section:

$$\sigma^{\nu N}(E_\nu) = \int_{Q_0^2/s}^{1} dx \int_{Q_0^2}^{xs} dQ^2 \frac{d^2\sigma^{\nu N}}{dx dQ^2}(x, Q^2) \ , \quad s = 2 M_N E_\nu$$

can be decomposed in terms of structure functions

$$\frac{d^2\sigma^{\nu(\bar{\nu})p}}{dx dQ^2}\left(x, y, Q^2\right) = \frac{G_F^2}{4\pi x}\left(\frac{M_W^2}{Q^2 + M_W^2}\right)^2$$
$$\times \left[Y_+ F_2^{\nu(\bar{\nu})p}(x, Q^2) - y^2 F_L^{\nu(\bar{\nu})p}(x, Q^2) \pm Y_- x F_3^{\nu(\bar{\nu})p}(x, Q^2)\right]$$

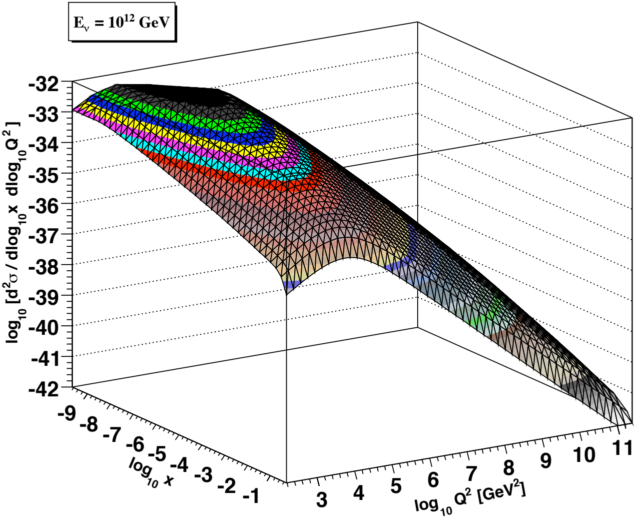
Integration dominated by $Q^2 \sim M_W^2$, $x \ll 1$ region

$$\langle x \rangle \sim \frac{M_W^2}{2 M_N E_\nu} \ll 1$$

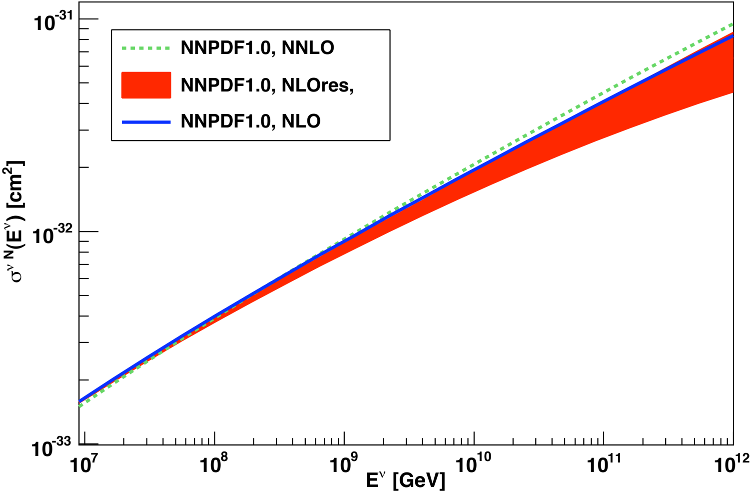
$\rightarrow$ UHE interations very sensitive to

- Small-x Parton Distributions
- Small-x (BFKL) effects

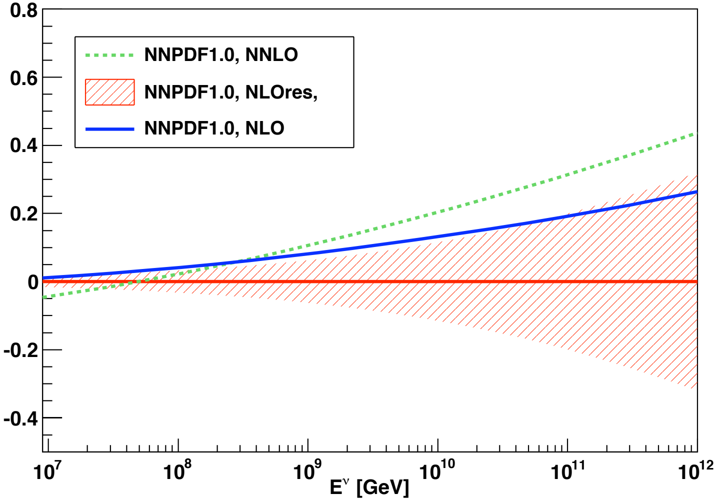
UHE Kinematics



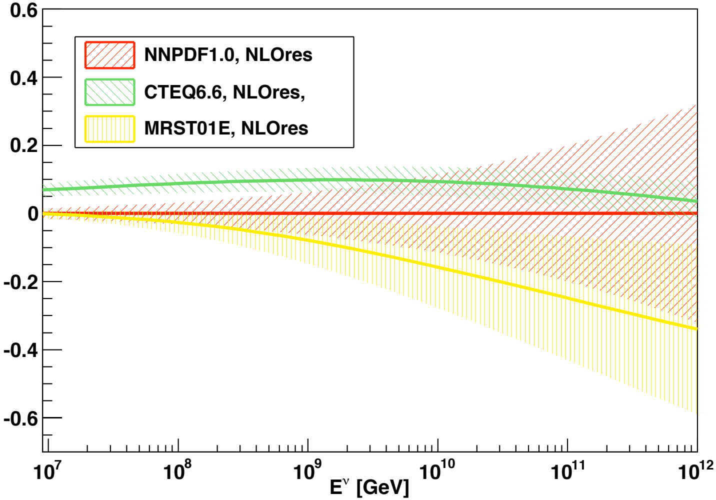
Small-x resummation of UHE neutrinos



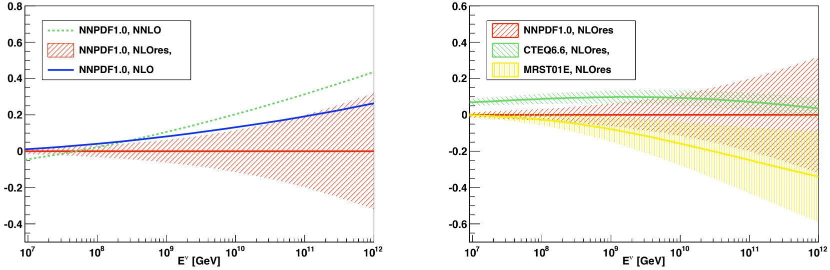
Small-x resummation of UHE neutrinos



Small-x resummation of UHE neutrinos



Small-x resummation of UHE neutrinos



- Small-x resummation decreases $\sigma^{\nu}(E_{\nu})$ with respect NNLO by a factor 2 at $E_{\nu} = 10^{12}$ GeV (Similar trend in *saturation models*)
- NNLO-NLOres differences outside PDF uncertainty band
- PDF uncertainties sizable in extrapolation region → Should be reduced in near future with final HERA and LHC data for small-x PDFs

OUTLOOK

Outlook

- BFKL small-x resummed DIS structure functions (with associated resummed K-factors) available after several years of continuous efforts
- Small-x corrections have a moderate effect in HERA region, but could be discriminated if precision high enough, even more at the LHeC
- Cosmic neutrinos provide a useful testing ground for small-x QCD scenarios
- The K-factor approach provides realistic qualitative insights, but accuracy limited by matching scale choice $\longrightarrow$ A fully resummed small-x PDF analysis is ultimately unavoidable
- Work in progress in this direction in the frameworks of the CTEQ (P. Nadolsky) and NNPDF global PDF analysis

Thanks for your attention!

Outlook

- BFKL small-x resummed DIS structure functions (with associated resummed K-factors) available after several years of continuous efforts
- Small-x corrections have a moderate effect in HERA region, but could be discriminated if precision high enough, even more at the LHeC
- Cosmic neutrinos provide a useful testing ground for small-x QCD scenarios
- The K-factor approach provides realistic qualitative insights, but accuracy limited by matching scale choice $\rightarrow$ A fully resummed small-x PDF analysis is ultimately unavoidable
- Work in progress in this direction in the frameworks of the CTEQ (P. Nadolsky) and NNPDF global PDF analysis

Thanks for your attention!

Outlook

- BFKL small-x resummed DIS structure functions (with associated resummed K-factors) available after several years of continuous efforts
- Small-x corrections have a moderate effect in HERA region, but could be discriminated if precision high enough, even more at the LHeC
- Cosmic neutrinos provide a useful testing ground for small-x QCD scenarios
- The K-factor approach provides realistic qualitative insights, but accuracy limited by matching scale choice $\rightarrow$ A fully resummed small-x PDF analysis is ultimately unavoidable
- Work in progress in this direction in the frameworks of the CTEQ (P. Nadolsky) and NNPDF global PDF analysis

Thanks for your attention!

Outlook

- BFKL small-x resummed DIS structure functions (with associated resummed K-factors) available after several years of continuous efforts
- Small-x corrections have a moderate effect in HERA region, but could be discriminated if precision high enough, even more at the LHeC
- Cosmic neutrinos provide a useful testing ground for small-x QCD scenarios
- The K-factor approach provides realistic qualitative insights, but accuracy limited by matching scale choice $\rightarrow$ A fully resummed small-x PDF analysis is ultimately unavoidable
- Work in progress in this direction in the frameworks of the CTEQ (P. Nadolsky) and NNPDF global PDF analysis

Thanks for your attention!

Outlook

- BFKL small-x resummed DIS structure functions (with associated resummed K-factors) available after several years of continuous efforts
- Small-x corrections have a moderate effect in HERA region, but could be discriminated if precision high enough, even more at the LHeC
- Cosmic neutrinos provide a useful testing ground for small-x QCD scenarios
- The K-factor approach provides realistic qualitative insights, but accuracy limited by matching scale choice $\rightarrow$ A fully resummed small-x PDF analysis is ultimately unavoidable
- Work in progress in this direction in the frameworks of the CTEQ (P. Nadolsky) and NNPDF global PDF analysis

Thanks for your attention!

Outlook

- BFKL small-x resummed DIS structure functions (with associated resummed K-factors) available after several years of continuous efforts
- Small-x corrections have a moderate effect in HERA region, but could be discriminated if precision high enough, even more at the LHeC
- Cosmic neutrinos provide a useful testing ground for small-x QCD scenarios
- The K-factor approach provides realistic qualitative insights, but accuracy limited by matching scale choice $\rightarrow$ A fully resummed small-x PDF analysis is ultimately unavoidable
- Work in progress in this direction in the frameworks of the CTEQ (P. Nadolsky) and NNPDF global PDF analysis

Thanks for your attention!

Outlook

- BFKL small-x resummed DIS structure functions (with associated resummed K-factors) available after several years of continuous efforts
- Small-x corrections have a moderate effect in HERA region, but could be discriminated if precision high enough, even more at the LHeC
- Cosmic neutrinos provide a useful testing ground for small-x QCD scenarios
- The K-factor approach provides realistic qualitative insights, but accuracy limited by matching scale choice $\rightarrow$ A fully resummed small-x PDF analysis is ultimately unavoidable
- Work in progress in this direction in the frameworks of the CTEQ (P. Nadolsky) and NNPDF global PDF analysis

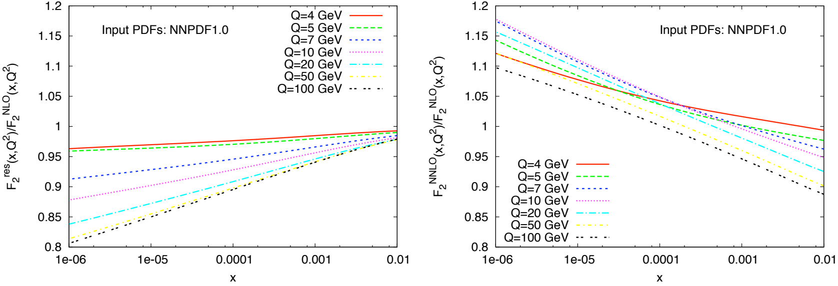
Thanks for your attention!

EXTRA MATERIAL

Resummed K-factors for HERA

Compare NLL and NNLO K-factors in HERA region

For $F_2(x, Q^2)$

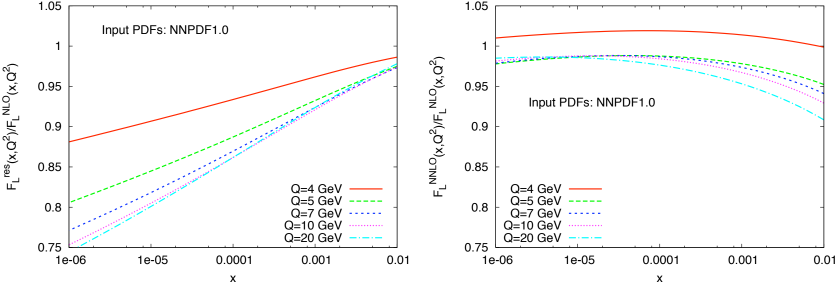


NLL small-x corrections have similar size but opposite sign as NNLO

Resummed K-factors for HERA

Compare NLL and NNLO K-factors in HERA region

For $F_L(x, Q^2)$

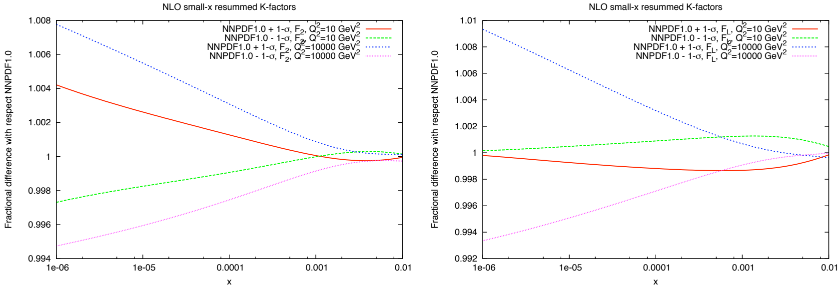


Note again that small-x resummation effects increase with Q^2 (as would be obtained from a resummed PDF analysis)

Resummed K-factors for HERA

Compare NLL and NNLO K-factors in HERA region

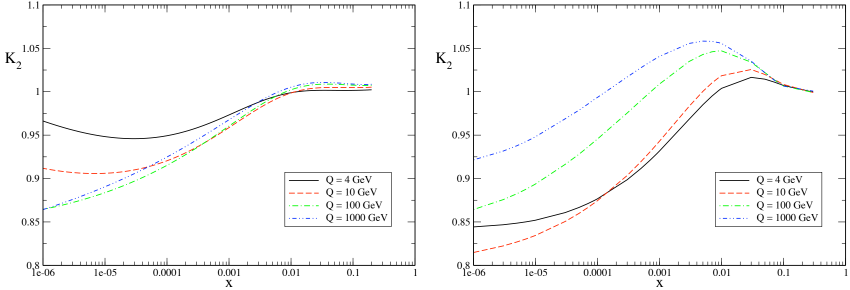
Accuracy of the K-factor approach not limited by choice of input PDF set



K-factors variation $\leq 1\%$ if the 1-σ NNPDF envelope instead of central PDFs used

Comparison between ABF and TW

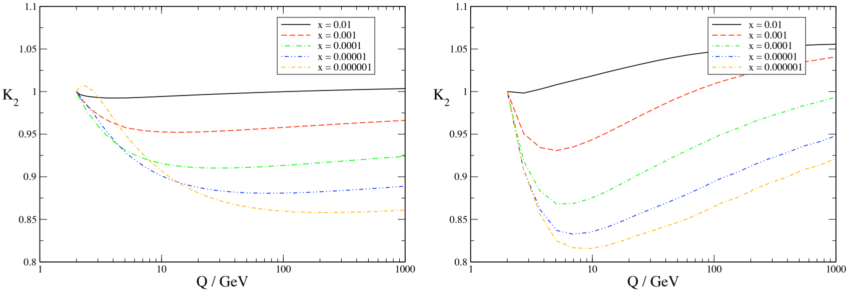
Compare $F_2^{\rm NLL}/F_2^{\rm NL0}$ obtained from toy PDFs at $Q_0^2 = 4\ \mathrm{GeV}^2$



- Qualitative agreement: $F_2^{\rm NLL}$ smaller than $F_2^{\rm NL0}$
- Quantitative difference I: TW missing smooth matching at large-x, while for ABF $F_2^{\rm NLL}/F_2^{\rm NL0} \to 1$

Comparison between ABF and TW

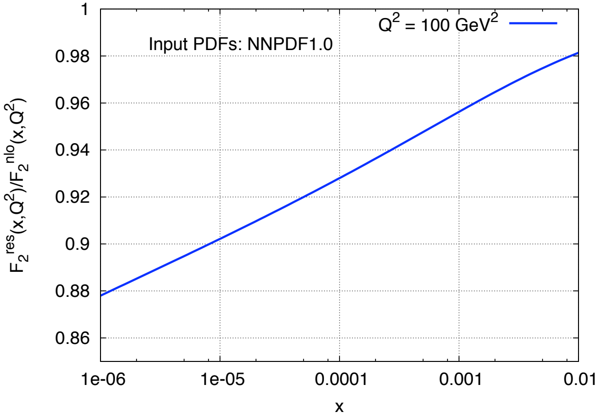
Compare $F_2^{\rm NLL}/F_2^{\rm NL0}$ obtained from toy PDFs at $Q_0^2 = 4$ GeV^2



- Qualitative agreement: $F_2^{\rm NLL}$ smaller than $F_2^{\rm NL0}$
- Quantitative difference II: ABF resummation stabilized at large-Q^2 (asymptotic freedom) while TW not $\rightarrow$ Collinear resummation of BFKL kernel missing in TW

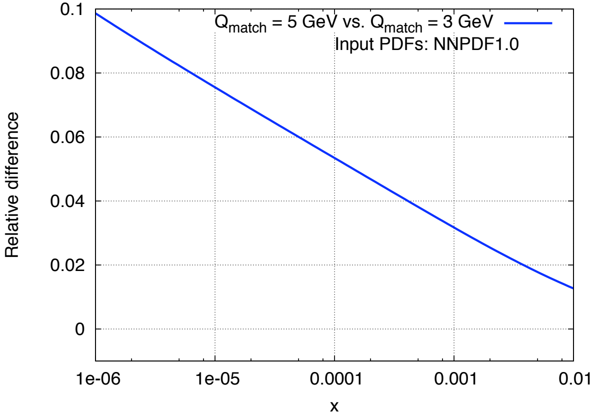
Dependence with matching scale

Compare $Q_{\rm match} = 3$ GeV^2 with $Q_{\rm match} = 5$ GeV^2



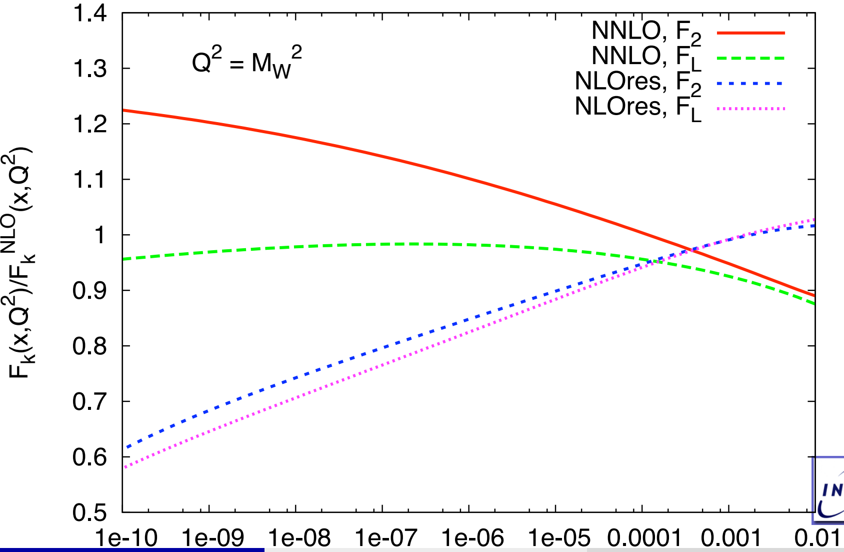
Dependence with matching scale

Compare $Q_{\rm match} = 3\ \mathrm{GeV}^2$ with $Q_{\rm match} = 5\ \mathrm{GeV}^2$



Matching scale dominant uncertainty in the K-factor approach to small-x resummation
Need for a fully small-x resummed global PDF fit

K-factors for UHE neutrinos

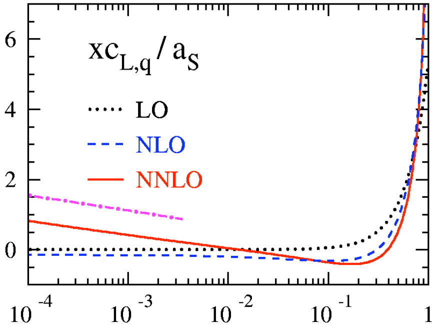


Why to resum small-x terms?

Instability of perturbation theory (I)

THE COEFFICIENT FUNCTION C_L

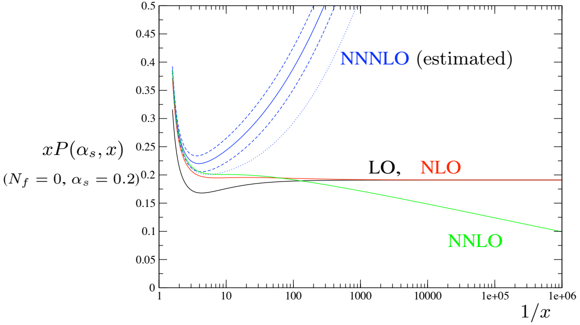
(Moch, Vermaseren, Vogt 2005)



Why to resum small-x terms?

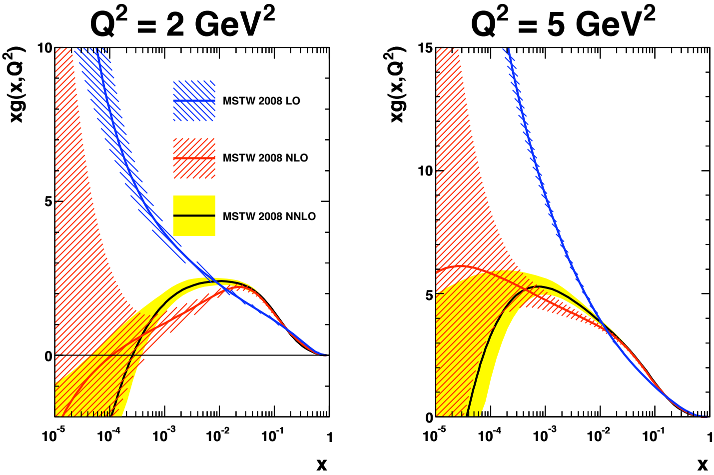
Instability of perturbation theory (II)

$$xP(\alpha_s, x) \underset{x\to 0}{\sim} \alpha_s c_1^{(1)} + \alpha_s^2 c_2^{(1)} + \alpha_s^3 \left(c_3^{(2)} \ln x + c_3^{(1)} \right) + \alpha_s^4 \left(c_4^{(4)} \ln^3 x + c_4^{(3)} \ln^2 x + c_4^{(2)} \ln x + c_4^{(1)} \right) + \dots$$



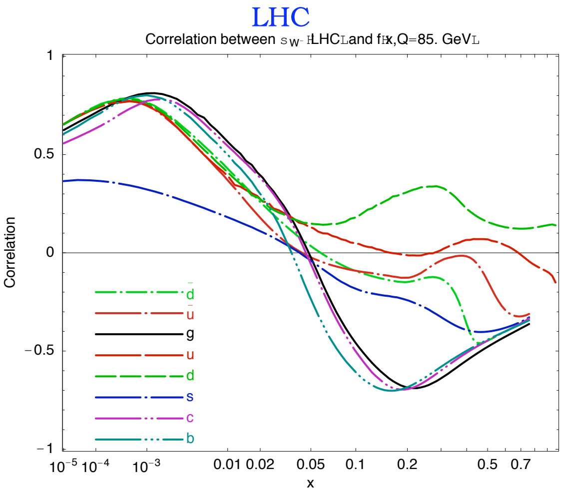
Why to resum small-x terms?

Instability of NNLO global PDF determinations



Why to resum small-x terms?

Small-x PDFs propagate to LHC inclusive observables (CTEQ6.6 study)



Small-x resummation FAQ

- Q: can one resum large small x corrections to all orders?
  A: yes, both at the leading and subleading level for anomalous dimensions and deep-inelastic and HQ hard coefficients
  BFKL 75-76, FL 98; Ciafaloni, Catani and Hautmann, 91-94
- Q: can one combine small x resummation with standard perturbative evolution?
  A: yes, by performing a suitable double BFKL+GLAP expansion
  Ball, Forte 95, + Altarelli (ABF) 2000
- Q: can one obtain a stable perturbative expansion at the resummed level?
  A: yes, provided one exploits suitable physical constraints: momentum conservation & underlying gluon symmetry
  Salam 98, Ciafaloni, Colferai, Salam 99 + Stasto (CCSS) 02, ABF 06
- Q: can one understand the success of NLO perturbation theory despite large small x terms?
  A: yes, if one resums at the running coupling level
  Ciafaloni, Colferai 99, ABF 01, Thorne 01

Small-x resummation FAQ

- Q: can one resum large small x corrections to all orders?
  A: yes, both at the leading and subleading level for anomalous dimensions and deep-inelastic and HQ hard coefficients
  BFKL 75-76, FL 98; Ciafaloni, Catani and Hautmann, 91-94
- Q: can one combine small x resummation with standard perturbative evolution?
  A: yes, by performing a suitable double BFKL+GLAP expansion
  Ball, Forte 95, + Altarelli (ABF) 2000
- Q: can one obtain a stable perturbative expansion at the resummed level?
  A: yes, provided one exploits suitable physical constraints: momentum conservation & underlying gluon symmetry
  Salam 98, Ciafaloni, Colferai, Salam 99 + Stasto (CCSS) 02, ABF 06
- Q: can one understand the success of NLO perturbation theory despite large small x terms?
  A: yes, if one resums at the running coupling level
  Ciafaloni, Colferai 99, ABF 01, Thorne 01

Small-x resummation FAQ

- Q: can one resum large small x corrections to all orders?
  A: yes, both at the leading and subleading level for anomalous dimensions and deep-inelastic and HQ hard coefficients
  BFKL 75-76, FL 98; Ciafaloni, Catani and Hautmann, 91-94
- Q: can one combine small x resummation with standard perturbative evolution?
  A: yes, by performing a suitable double BFKL+GLAP expansion
  Ball, Forte 95, + Altarelli (ABF) 2000
- Q: can one obtain a stable perturbative expansion at the resummed level?
  A: yes, provided one exploits suitable physical constraints: momentum conservation & underlying gluon symmetry
  Salam 98, Ciafaloni, Colferai, Salam 99 + Stasto (CCSS) 02, ABF 06
- Q: can one understand the success of NLO perturbation theory despite large small x terms?
  A: yes, if one resums at the running coupling level
  Ciafaloni, Colferai 99, ABF 01, Thorne 01

Small-x resummation FAQ

- Q: can one resum large small x corrections to all orders?
  A: yes, both at the leading and subleading level for anomalous dimensions and deep-inelastic and HQ hard coefficients
  BFKL 75-76, FL 98; Ciafaloni, Catani and Hautmann, 91-94
- Q: can one combine small x resummation with standard perturbative evolution?
  A: yes, by performing a suitable double BFKL+GLAP expansion
  Ball, Forte 95, + Altarelli (ABF) 2000
- Q: can one obtain a stable perturbative expansion at the resummed level?
  A: yes, provided one exploits suitable physical constraints: momentum conservation & underlying gluon symmetry
  Salam 98, Ciafaloni, Colferai, Salam 99 + Stasto (CCSS) 02, ABF 06
- Q: can one understand the success of NLO perturbation theory despite large small x terms?
  A: yes, if one resums at the running coupling level
  Ciafaloni, Colferai 99, ABF 01, Thorne 01

Small-x resummation FAQ

- Q: can one resum large small x corrections to all orders?
  A: yes, both at the leading and subleading level for anomalous dimensions and deep-inelastic and HQ hard coefficients
  BFKL 75-76, FL 98; Ciafaloni, Catani and Hautmann, 91-94
- Q: can one combine small x resummation with standard perturbative evolution?
  A: yes, by performing a suitable double BFKL+GLAP expansion
  Ball, Forte 95, + Altarelli (ABF) 2000
- Q: can one obtain a stable perturbative expansion at the resummed level?
  A: yes, provided one exploits suitable physical constraints: momentum conservation & underlying gluon symmetry
  Salam 98, Ciafaloni, Colferai, Salam 99 + Stasto (CCSS) 02, ABF 06
- Q: can one understand the success of NLO perturbation theory despite large small x terms?
  A: yes, if one resums at the running coupling level
  Ciafaloni, Colferai 99, ABF 01, Thorne 01

Small-x resummation FAQ II

- Q: does the resummation interfere with the choice of factorization scheme?
  A: no: BFKL resummation is compatible with collinear factorization; resummation can be performed in any scheme
  ABF 01, CCSS 02, ABF 08
- Q: can one estimate the ambiguities in the resummation and how large are they?
  A: ambiguities are small due to physical constraints. Can be reduced by careful matching, and estimated by scale variation to be smaller than the impact of the resummation
  CCSS 03, ABF 06, ABF 08
- Q: are resummed hard coefficients available for hadroproduction processes?
  A: yes, for Heavy quark production, Higgs production in Gluon fusion and Drell-Yan
  K. Ellis, Ball 01; Marzani, Ball, Del Duca, Forte, Vicini 08; Marzani, Ball 08
  $\rightarrow$ LHC Small-x resummed phenomenology around the corner!

Small-x resummation FAQ II

- Q: does the resummation interfere with the choice of factorization scheme?
  A: no: BFKL resummation is compatible with collinear factorization; resummation can be performed in any scheme
  ABF 01, CCSS 02, ABF 08
- Q: can one estimate the ambiguities in the resummation and how large are they?
  A: ambiguities are small due to physical constraints. Can be reduced by careful matching, and estimated by scale variation to be smaller than the impact of the resummation
  CCSS 03, ABF 06, ABF 08
- Q: are resummed hard coefficients available for hadroproduction processes?
  A: yes, for Heavy quark production, Higgs production in Gluon fusion and Drell-Yan
  K. Ellis, Ball 01; Marzani, Ball, Del Duca, Forte, Vicini 08; Marzani, Ball 08
  $\rightarrow$ LHC Small-x resummed phenomenology around the corner!

Small-x resummation FAQ II

- Q: does the resummation interfere with the choice of factorization scheme?
  A: no: BFKL resummation is compatible with collinear factorization; resummation can be performed in any scheme
  ABF 01, CCSS 02, ABF 08
- Q: can one estimate the ambiguities in the resummation and how large are they?
  A: ambiguities are small due to physical constraints. Can be reduced by careful matching, and estimated by scale variation to be smaller than the impact of the resummation
  CCSS 03, ABF 06, ABF 08
- Q: are resummed hard coefficients available for hadroproduction processes?
  A: yes, for Heavy quark production, Higgs production in Gluon fusion and Drell-Yan
  K. Ellis, Ball 01; Marzani, Ball, Del Duca, Forte, Vicini 08; Marzani, Ball 08
  → LHC Small-x resummed phenomenology around the corner!

Small-x resummation FAQ II

- Q: does the resummation interfere with the choice of factorization scheme?
  A: no: BFKL resummation is compatible with collinear factorization; resummation can be performed in any scheme
  ABF 01, CCSS 02, ABF 08
- Q: can one estimate the ambiguities in the resummation and how large are they?
  A: ambiguities are small due to physical constraints. Can be reduced by careful matching, and estimated by scale variation to be smaller than the impact of the resummation
  CCSS 03, ABF 06, ABF 08
- Q: are resummed hard coefficients available for hadroproduction processes?
  A: yes, for Heavy quark production, Higgs production in Gluon fusion and Drell-Yan
  K. Ellis, Ball 01; Marzani, Ball, Del Duca, Forte, Vicini 08; Marzani, Ball 08
  → LHC Small-x resummed phenomenology around the corner!

Small-x resummation FAQ II

- Q: does the resummation interfere with the choice of factorization scheme?
  A: no: BFKL resummation is compatible with collinear factorization; resummation can be performed in any scheme
  ABF 01, CCSS 02, ABF 08
- Q: can one estimate the ambiguities in the resummation and how large are they?
  A: ambiguities are small due to physical constraints. Can be reduced by careful matching, and estimated by scale variation to be smaller than the impact of the resummation
  CCSS 03, ABF 06, ABF 08
- Q: are resummed hard coefficients available for hadroproduction processes?
  A: yes, for Heavy quark production, Higgs production in Gluon fusion and Drell-Yan
  K. Ellis, Ball 01; Marzani, Ball, Del Duca, Forte, Vicini 08; Marzani, Ball 08
  $\longrightarrow$ LHC Small-x resummed phenomenology around the corner!