Spin Response of the Proton in the Resonance Region

XVII International Workshop on Deep-Inelastic Scattering and Related Subjects

Karl J. Slifer
University of New Hampshire
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This Talk

Existing g1p data

Burkhardt–Cottingham Sum Sum Rule
  What does the JLab data tell us?
  Is it enough to make a definitive statement?

Higher Twist Measurements
  Target Mass Corrections

Spin Polarizabilities

Future Experiments
Inclusive Scattering

Inclusive Cross Section

Deviation from point-like behavior characterized by the Structure Functions

\( Q^2 \) : 4-momentum transfer
\( X \) : Bjorken Scaling var
\( W \) : Invariant mass of target
Inclusive Scattering

Inclusive Cross Section

\[ \frac{d^2 \sigma}{d \Omega dE'} = \sigma_{Mott} \left[ \frac{1}{\nu} F_2(x, Q^2) + \frac{2}{M} F_1(x, Q^2) \tan^2 \frac{\theta}{2} \right] \]

Kinematics
- \( Q^2 \): 4-momentum transfer
- \( X \): Bjorken Scaling var
- \( W \): Invariant mass of target

deviation from point-like behavior
characterized by the Structure Functions
Inclusive Scattering

When we add spin degrees of freedom to the target and beam, 2 additional SF needed.

**Inclusive Polarized XS Differences**

\[
\frac{d^2 \sigma_{\uparrow\uparrow}}{d\Omega dE'} - \frac{d^2 \sigma_{\downarrow\uparrow}}{d\Omega dE'} = \frac{4\alpha^2}{\nu Q^2} \frac{E'}{E} \left[ (E + E' \cos \theta) g_1 - 2M x g_2 \right]
\]

\[
\frac{d^2 \sigma_{\uparrow\Rightarrow}}{d\Omega dE'} - \frac{d^2 \sigma_{\downarrow\Rightarrow}}{d\Omega dE'} = \frac{4\alpha^2}{\nu Q^2} \frac{E'}{E} \sin \theta \left[ g_1 + \frac{2ME}{\nu} g_2 \right]
\]
Generalized Sum Rules

Unsubtracted Dispersion Relation + Optical Theorem:

\[ S_1(\nu, Q^2) = 4 \int_0^\infty \frac{d\nu' \nu'}{\nu'^2 - \nu^2} G_1(\nu', Q^2) \]

\[ S_2(\nu, Q^2) = 4 \int_0^\infty \frac{d\nu' \nu}{\nu'^2 - \nu^2} G_2(\nu', Q^2) \]

Extended GDH Sum

\[ \Gamma_1 = \int g_1 \, dx = \frac{Q^2}{8} S_1(0, Q^2) \]

GDH Sum Rule at \( Q^2=0 \)

Bjorken Sum Rule at \( Q^2=\infty \)

BC Sum Rule

\[ \Gamma_2 = \int_0^1 g_2(x, Q^2) \, dx = 0 \]

Superconvergence relation valid at any \( Q^2 \)


Relies on the virtual Compton scattering amplitude \( S_2 \)
falling to zero faster than \( 1/\nu \) as \( \nu \to \infty \)

Generalized Forward Spin Polarizabilities

\[ g_{TT}(\nu, Q^2) = \frac{\nu}{2\pi^2} \mathcal{P} \int_{\nu_0}^{\infty} \frac{d\nu'K}{\nu'^2 - \nu^2} \sigma_{TT}(\nu', Q^2) \]

\[ g_{LT}(\nu, Q^2) = \frac{1}{2\pi^2} \mathcal{P} \int_{\nu_0}^{\infty} \frac{d\nu'\nu'K}{\nu'^2 - \nu^2} \sigma_{LT}(\nu', Q^2) \]

LEX of \( g_{TT} \) and \( g_{LT} \) lead to the Generalized Forward Spin Polarizabilities

\[ \gamma_0(Q^2) = \left( \frac{1}{2\pi^2} \right) \int_{\nu_0}^{\infty} \frac{K(\nu, Q^2) \sigma_{TT}(\nu, Q^2)}{\nu^3} d\nu \]

\[ = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1(x, Q^2) - \frac{4M^2}{Q^2} x^2 g_2(x, Q^2) \right] \]

\[ \delta_{LT}(Q^2) = \left( \frac{1}{2\pi^2} \right) \int_{\nu_0}^{\infty} \frac{K(\nu, Q^2) \sigma_{LT}(\nu, Q^2)}{\nu Q\nu^2} d\nu \]

\[ = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1(x, Q^2) + g_2(x, Q^2) \right] \]
Status of World Data

Very Well Known

\[ F_2(x) = 2xF_1(x) \]
Status of World Data

Very Well Known

\[ F_2(x) = 2xF_1(x) \]

\( g_1 \) pretty well known
First moments of g1p

Extended GDH Sum Rule

Bjorken Sum Rule

GDH sum rule
Bjorken sum rule
higher twist effects

talk of Sebastian Kuhn
World g2p Data

SLAC: $\langle Q^2 \rangle = 5 \text{ GeV}^2$

Only a single $Q^2$ point!
World g2p Data

SLAC: $\langle Q^2 \rangle = 5 \text{ GeV}^2$

JLAB: $\langle Q^2 \rangle = 1.3 \text{ GeV}^2$


Doubles the World $Q^2$ Coverage 😊
New Data From JLab

Thanks to the spokesmen of these experiments!

RSS
Mark Jones, Oscar Rondon

E01–012
Nilanga Liyanage, J.P. Chen, Seonho Choi

SaGDH
J.P. Chen, A. Deur, F. Garabaldi
BC Sum Rule

Existing World Data on $\Gamma_2$

$$\int_0^1 g_2(x, Q^2) dx = 0$$

**BLACK :** E94010. (Hall A, $^3$He)

**BROWN :** E155. (SLAC NH3, $^6$LiD)

Note:

SLAC “Measured” = 0.02 < x < 0.8

JLAB “Measured” ≈ Resonance Region

W < 2 GeV
BRAND NEW DATA!

Very Preliminary

**RED**: RSS. (Hall C, NH₃, ND₃)


**BLUE**: E01-012. (Hall A, ³He)

P. Solvignon et al in preparation

**GREEN**: E97-110. (Hall A, ³He)

Courtesy of V. Sulkosky
BC Sum Rule

$$\int_0^1 g_2(x, Q^2) dx = 0$$

$$BC = RES + DIS + ELASTIC$$

“RES”: Here refers to measured x-range

“DIS”: refers to unmeasured low x part of the integral. Not strictly Deep Inelastic Scattering due to low $Q^2$

Assume Leading Twist Behaviour

Elastic: From well known FFs (<5%)
BC Sum Rule

BC satisfied w/in errors for JLab Proton
2.8σ violation seen in SLAC data

BC satisfied w/in errors for Neutron
(But just barely in vicinity of \(Q^2=1\!\))
Proton g2p still relatively unknown for such a fundamental quantity.

Future

Sane: Just completed!

2.3 < Q^2 < 6 GeV^2

“g2p” in Hall A, 2011

0.015 < Q^2 < 0.4 GeV^2
Higher Twists
Most analysis of SSF historically performed in terms of the CN moments.
Operator Product Expansion (OPE)
Expansion of SF moments in powers of $1/Q^2$ ("twist")

$$\Gamma_1(Q^2) = \int_0^1 g_1(x,Q^2)dx = \sum_{\tau=2,4,...} \frac{\mu_\tau(Q^2)}{Q^{\tau-2}}$$
Operator Product Expansion (OPE)
Expansion of SF moments in powers of $1/Q^2$ ("twist")

\[
\Gamma_1(Q^2) = \int_0^1 g_1(x,Q^2) dx = \sum_{\tau=2,4,...} \frac{\mu_\tau(Q^2)}{Q^{\tau-2}}
\]

**Leading Twist**: maps to the reliable predictions of the parton model.

\[
\mu_2(Q^2) = \left[ 1 - \left( \frac{\alpha_s}{\pi} \right) - 3.58 \left( \frac{\alpha_s}{\pi} \right)^2 - 20.22 \left( \frac{\alpha_s}{\pi} \right)^3 \right] \left( \pm \frac{1}{12} g_A + \frac{1}{36} \alpha_s \right) + \left[ 1 - \frac{1}{3} \left( \frac{\alpha_s}{\pi} \right) - 0.55 \left( \frac{\alpha_s}{\pi} \right)^2 - 4.45 \left( \frac{\alpha_s}{\pi} \right)^3 \right] \frac{1}{9} \Delta \Sigma
\]
Operator Product Expansion (OPE)
Expansion of SF moments in powers of $1/Q^2$ ("twist")

$$\Gamma_1(Q^2) = \int_0^1 g_1(x,Q^2) dx = \sum_{\tau=2,4,...} \frac{\mu_\tau(Q^2)}{Q^{\tau-2}}$$

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$$\mu_2(Q^2) = \left[ 1 - \left( \frac{\alpha_s}{\pi} \right) - 3.58 \left( \frac{\alpha_s}{\pi} \right)^2 - 20.22 \left( \frac{\alpha_s}{\pi} \right)^3 \right] \left( \pm \frac{1}{12} g_A + \frac{1}{36} a_8 \right)$$

$$+ \left[ 1 - \frac{1}{3} \left( \frac{\alpha_s}{\pi} \right) - 0.55 \left( \frac{\alpha_s}{\pi} \right)^2 - 4.45 \left( \frac{\alpha_s}{\pi} \right)^3 \right] \frac{1}{9} \Delta \Sigma$$

**Higher Twists**: non-perturbative multiparton interaction and non-zero quark masses.

$$\mu_4 = \frac{1}{9} M^2 \left( \tilde{a}_2 + 4 \tilde{d}_2 + 4 \tilde{f}_2 \right)$$

**Confinement**
Accessing Higher Twists

Cornwall Norton Higher Moments

\[ \int_0^1 x^2 g_1(x, Q^2) \, dx = \frac{1}{2} \tilde{a}_2 \]

\[ \int_0^1 x^2 g_2(x, Q^2) \, dx = \frac{1}{3} (\tilde{d}_2 - \tilde{a}_2) \]

\[ I(Q^2) = \int_0^1 x^2 \left[ 2g_1(x, Q^2) + 3g_2(x, Q^2) \right] \, dx = \tilde{d}_2(Q^2) \]

\[ I(Q^2) \neq \text{the twist-3 matrix element. Ignores terms of order } M^2/Q^2 \]

Y.B. Dong PRC 77, 015201 (2008)
Existing World Data on $I(Q^2)$

**BLACK**: E94010

**BROWN**: E155

**RED**: RSS.

**Magenta**: E99-117

**PROTON**

**NEUTRON**

$Q^2 (GeV^2)$
I(Q^2)

BRAND NEW DATA!

Very Preliminary

RED: RSS. (Hall C, NH$_3$, ND$_3$)

BLUE: E01-012. (Hall A, $^3$He)
P. Solvignon et al in preparation

GREEN: E97-110. (Hall A, $^3$He)
Courtesy of V. Sulkosky
Shaded Region: Estimate of $I(Q^2)$

Large uncertainty due to lack of knowledge of $g_{2p}$
Accessing Higher Twists

Cornwall Norton Higher Moments

\[ \int_0^1 x^2 g_1(x, Q^2) \, dx = \frac{1}{2} \tilde{a}_2 \]

\[ \int_0^1 x^2 g_2(x, Q^2) \, dx = \frac{1}{3} (\tilde{d}_2 - \tilde{a}_2) \]

\[
I(Q^2) = \int_0^1 x^2 \left[ 2g_1(x, Q^2) + 3g_2(x, Q^2) \right] \, dx \\
= \tilde{d}_2(Q^2)
\]
Accessing Higher Twists

Cornwall Norton Higher Moments

\[ \int_{0}^{1} x^2 g_1(x, Q^2) dx = \frac{1}{2} \tilde{a}_2 + \mathcal{O} \left( \frac{M^2}{Q^2} \right) \]

\[ \int_{0}^{1} x^2 g_2(x, Q^2) dx = \frac{1}{3} (\tilde{d}_2 - \tilde{a}_2) + \mathcal{O} \left( \frac{M^2}{Q^2} \right) \]

\[ I(Q^2) = \int_{0}^{1} x^2 \left[ 2g_1(x, Q^2) + 3g_2(x, Q^2) \right] dx \]

\[ = \tilde{d}_2(Q^2) + \mathcal{O} \left( \frac{M^2}{Q^2} \right) \]

\[ I(Q^2) \neq \text{the twist-3 matrix element. Ignores terms of order } M^2/Q^2 \]

Very significant below \( Q^2 \approx 5 \)

Y.B. Dong PRC 77(2008) 015201

Y.B.Dong PLB 653,(2007)18
Nachtmann Moments

Nachtmann Moments:

\[ M_2^3(Q^2) = \int_0^1 dx \frac{x^4}{x^2} \left[ \frac{x}{\xi} g_1 + \left[ \frac{3}{2} \left( \frac{x}{\xi} \right)^2 - \frac{3}{4} \frac{M^2}{Q^2} x^2 \right] g_2 \right] \]

\[ = \frac{\tilde{d}_2}{2} + O \left( \frac{M^8}{Q^8} \right) \]

Y.B. Dong PRC 77(2008) 015201

\[ \xi = \frac{2x}{1 + \sqrt{1 + 4 \frac{M^2}{Q^2} \frac{1}{x^2}}} \]
Nachtmann Moments:

\[ M_2^3(Q^2) = \int_0^1 dx \frac{x^4}{x^2} \left[ xg_1 + \left( \frac{3}{2} \left( \frac{x}{\xi} \right)^2 - \frac{3}{4} \frac{M^2}{Q^2} x^2 \right) g_2 \right] \]

\[ = \frac{\bar{d}_2}{2} + O \left( \frac{M^8}{Q^8} \right) \]

Y.B. Dong PRC 77(2008) 015201

\[ \xi = \frac{2x}{1 + \sqrt{1 + 4 \frac{M^2}{Q^2} \frac{1}{x^2}}} \]

Generalization of CN moments to protect from the TMC

\[ \frac{M^2}{Q^2} \rightarrow 0 \quad M_2^3 \rightarrow \int x_2 (2g_1 + 3g_2) dx \quad \text{Reduces to familiar form} \]

Not a new idea, but difficult to implement unless g_2 measured simultaneously with g_1
Measured $g_1$ and $g_2$ at $Q^2 \approx 1.3$ GeV$^2$

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<th>$2M_2^3$</th>
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Nachtmann moment reveals twist-3 at more than 3 sigma significance.
Measured $g_1$ and $g_2$ at $Q^2 \approx 1.3$ GeV$^2$

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Nachtmann moment reveals twist-3 at more than 3 sigma significance.

CN moment overestimates twist-3 by about 50%!
RSS Experiment

Measured $g_1$ and $g_2$ at $Q^2 \approx 1.3$ GeV$^2$

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Nachtmann moment reveals twist-3 at more than 3 sigma significance.

CN moment overestimates twist-3 by about 50%!

$R(Q^2) = \frac{2M_2^3(Q^2)}{I(Q^2)}$

$R \rightarrow 1$ in case of vanishing nucleon mass

$R$ always less than 1 => $I(Q^2)$ overestimates twist-3
Spin Polarizabilities
Forward Spin Polarizabilities

\( \gamma_0 = \frac{16\alpha M^2}{Q^6} \int_{0}^{x_0} x^2 \left[ g_1 - \frac{4M^2}{Q^2}x^2 g_2 \right] \)

Major failure of chiPT calcs.

Forward Spin Polarizabilities

Similar problem for Neutron

\[
\gamma_0 = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1 - \frac{4M^2}{Q^2} x^2 g_2 \right]
\]

\[
\delta_{LT} = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 [g_1 + g_2]
\]

Add \( \Delta \) by hand:
major effect for \( \gamma_0 \) but not for \( \delta_{LT} \)

Heavy Baryon \( \chi \)PT Calculation
Kao, Spitzenberg, Vanderhaeghen

Relativistic Baryon \( \chi \)PT
Bernard, Hemmert, Meissner
Future JLab Proton Data

The Hall B Eg4 Experiment

The Hall A g2p Experiment

The Hall C SANE Experiment
Hall B: Proton and Deuteron
Ran in 2006: Under analysis

Hall A: Proton
Will Run in 2011
Extended GDH Sum Rule

\[ \int g_1(x, Q^2)dx \]
Extended GDH Sum Rule

\[ \int g_1(x, Q^2) dx \]

Burkhardt-Cottingham Sum Rule

\[ \int g_2(x, Q^2) dx \]
Twist-3 Matrix Element

\[ d_2(Q^2) = 3 \int x^2 [g_2(x, Q^2) - g_2^{WW}(x, Q^2)] \, dx \]
\[ = \int x^2 [2g_1(x, Q^2) + 3g_2(x, Q^2)] \, dx \]
Twist-3 Matrix Element

\[ d_2(Q^2) = 3 \int x^2 \left[ g_2(x, Q^2) - g_2^{WW}(x, Q^2) \right] dx \]

\[ = \int x^2 \left[ 2g_1(x, Q^2) + 3g_2(x, Q^2) \right] dx \]

LT Spin Polarizability

\[ \delta_{LT} = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1(x, Q^2) + g_2(x, Q^2) \right] \]
Summary

Burkhardt—Cottingham Sum Rule
Good coverage for Neutron. Proton g2p is still relatively unknown.

Data seems to validate BC, but at the 2.5σ level around Q²=1
Important to update the systematics of the old experiments

Assuming BC holds, we can use JLab data to say something about low-x.

Target Mass Effects
TMC are significant at JLab kinematics

Nachtmann moments protect the SSF from TMC

Must use Nachtmann Moments in order to cleanly extract Higher twists

JLab 6 GeV Program
Still lots of good Spin Physics to be completed before the upgrade.

SANE
d2n
g2p: E08-027
EG4
Efremov, Leader and Teryaev (ELT) Sum Rule

for the valence quark contribution to the SSF

\[ I^{ELT}(Q^2) = \int_0^1 dx \, x \left[ g_{1}^{val} + 2g_{2}^{val} \right] \]

\[ = \int_0^1 dx \, x \left[ g_{1}^{p} + 2g_{2}^{p} - g_{1}^{n} - 2g_{2}^{n} \right] = 0 \]

2\textsuperscript{nd} equality assumes that the sea quark distributions are isospin independent

RSS experiment result at \( Q^2 = 1.28 \text{ GeV}^2 \):

\[ I^{ELT}_{RES}(Q^2 = 1.28) = 0.0008 \pm 0.0050 \]

Resonance Region only

Total Integral:

\[ I^{ELT}(Q^2 = 1.28) = 0.0057 \pm 0.0077 \]

DIS contribution dominates
**g_2 Structure Function**

**Wandzura-Wilczek relation**

\[
g_2^{WW}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{g_1(y, Q^2)}{y} dy
\]

PLB 72 (1977) 195

Leading twist determined entirely by g_1

\[
g_2 = g_2^{WW} + \overline{g}_2
\]

Higher twist

g_2 doesn’t exist in Parton Model.
Good quantity to study higher twist
Neutron Extraction

For Integrated Quantities:

\[ D = \gamma_d (\text{Proton} + \text{Neutron}) \]

\[ \gamma_d = 0.925 \pm 0.015 \]

Good to a few percent at low \( Q^2 \)

Kulagin and Melnitchouk
PRC 77, 015210 (2008).

\[ ^3\text{He} = P_N \text{Neutron} + 2P_P \text{Proton} \]

\[ P_N = 0.86 \pm 0.02 \]
\[ P_P = -0.028 \pm 0.004 \]

Good to 5% down to \( Q^2 = 0.5 \text{ GeV}^2 \)

C. Ciofi degli Atti and S. Scopetta,

Need for more sophisticated method at very low \( Q^2 \)
Neutron results around $Q^2=1.3$ GeV$^2$ from 2 very different experiments:

- **RSS** in Hall C: Neutron from ND$_3$ & NH$_3$
- **E01-012** in Hall A: Neutron from $^3$He

Excellent agreement!

for neutron from ND$_3$ and $^3$He
Existing DIS $g_2$ Data

SLAC: $<Q^2> = 5$ GeV$^2$

Jlab Hall A: $x \approx 0.2$

Proton

Deuteron

Neutron

![Graph showing $xg_2$ versus $x$ for Proton and Deuteron, and $g_2^n$ versus $Q^2$ for Neutron data.]
Existing Resonance $g_2$ Data

Lowest $Q^2$ Existing Proton Data

(Jlab Hall C : RSS)

$Q^2 = 1.3$
**Low-X Estimate**

Assume $g_2 = g_2^{ww}$ at low $x$.

Supported by RSS data

15% variation seen depending on choice of $g_1$ used.

**Unmeasured Contributions**

**ELASTIC: $x=1$**

$$g_1^{el}(x,Q^2) = \delta(x - 1)G_M(Q^2) \frac{G_E(Q^2) + \tau G_M(Q^2)}{2(1 + \tau)}$$

$$g_2^{el}(x,Q^2) = \delta(x - 1)\tau G_M(Q^2) \frac{G_E(Q^2) - G_M(Q^2)}{2(1 + \tau)}$$

(Form Factor uncertainties less than 5%)
Alternatively, if we assume BC holds we can learn something about the unmeasured part of Integral

\[ \int_0^1 g_2(x, Q^2) dx = 0 \]

BC = 0 \implies \text{Res} + \text{Ela} + "DIS" 

Unmeasured Low-x part

"DIS" = -(RES+ELAS)
What can BC tell us about Low-X?

BC Sum Rule

Measured \( x \)  \( 0 < x < 1 \)

Unmeasured Low-X

\( \text{DIS} = -(\text{RES} + \text{ELAS}) \)

very prelim

Unmeasured Low-X

\( = -(\text{RES} + \text{ELAS}) \)
Hydrogen Hyperfine Structure

\[ \Delta E = 1420.405 \, 751 \, 766 \, 7(9) \, \text{MHz} = (1 + \delta) E_F \]

\[ \delta = (\delta_{QED} + \delta_R + \delta_{small}) + \Delta_S \]

\[ \Delta_S = \Delta_Z + \Delta_{POL} \]

\[ \Delta_{POL} = \frac{\alpha m_e}{\pi g_p m_p} (\Delta_1 + \Delta_2) \]

\[ \Delta_2 = -24m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} B_2(Q^2) \]

\[ B_2(Q^2) = \int_0^{x_{th}} dx \beta_2(\tau) g_2(x, Q^2) \]

\[ \beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau + 1)} \]
Hydrogen Hyperfine Structure

This experiment

\[ \Delta_2 = -24m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} B_2(Q^2) \]

\[ = -0.57 \pm 0.57 \]

NCG 2006: Used CLAS model assuming 100% error

Assuming this uncertainty is realistic, we will improve this by order of magnitude

But, \( g_2^P \) unknown in this region:

\[ \Delta_2 = -1.98 \quad \text{MAID Model} \]

\[ \Delta_2 = -1.86 \quad \text{Simula Model} \]

So 100% error probably too optimistic

We will provide first real constraint on \( \Delta_2 \)