CCFM Evolution with Unitarity bound

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Introduction

- What do we want to do and why do we want it?
- Why should we work with CCFM?
- The way of doing it: The absorptive boundary

CCFM

Results

Experimental and Theoretical situation

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- LHC jets: $Q \gtrsim 10$ GeV. Not necessarily DGLAP physics. BFKL type physics important when $Y = \ln s \gtrsim \ln Q^2$.

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- No detailed knowledge of saturation mechanism necessary. Q_s determined fully by linear evolution, *if* the linear evolution is endowed by an absorptive boundary restoring unitarity.

Saturation on final states

The boundary method opens possibility to study effects of saturation on formalism whose non-linear generalization not known yet, e.g. CCFM or BFKL beyond LL.

Saturation effects studied so far mostly for inclusive observables. A major improvement would be to study effects of saturation on exclusive final states.

CCFM suitable for this task since BFKL not appropriate formalism, even though strong similarities between the two.

Moreover, already existing event generators based on CCFM.

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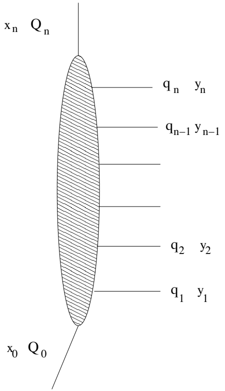
- In arXiv:0901.2873 (PLB 673:24-29,2009) we demonstrated that this method is completely equivalent to solving full BK for all energies and also running coupling.

CCFM: Gluon ladder and kinematics



y_k energy fraction, ξ_k squared angle: $\xi_k = q_k^2/(y_k^2 E^2)$.

Real emissions and Virtual form factors

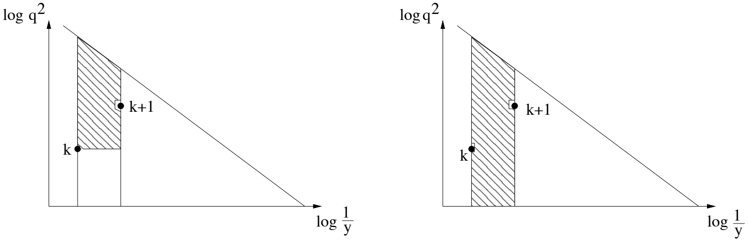
Real emission density in CCFM: $\bar{\alpha}_s \frac{dy_k}{y_k} \frac{d\xi_k}{\xi_k}$.

Virtual form factors S_{eik} and S_{ne}:

$$
\begin{aligned}
S_{eik}^2 &= \exp\left(-\bar{\alpha}_s \int_{y_{k+1}}^{y_k} \frac{dy}{y} \int^{\bar{\xi}} \frac{d\xi}{\xi}\right), \\
S_{ne}^2 &= \exp\left(+\bar{\alpha}_s \int_{y_{k+1}}^{y_k} \frac{dy}{y} \int_{\xi(Q_k)}^{\bar{\xi}} \frac{d\xi}{\xi}\right),
\end{aligned}
$$

$\bar{\xi}$: Maximal angle allowed by coherence.

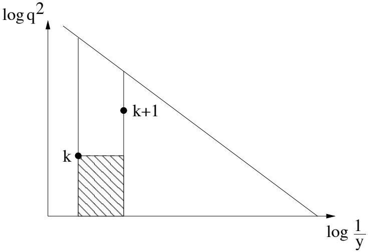
Virtual form factors



Left: $S^2_{ne}(k)$, Right: $S^2_{eik}(k)$, $Q_k = -\sum_{i=1}^{k} q_i$. Notice different signs $\Rightarrow$ Cancellation below Q_k.

Relation to BFKL

Then left with:



$$S_{ne}^2(k) * S_{eik}^2(k) = \exp\left(-\bar{\alpha}_s \int_{y_{k+1}}^{y_k} \frac{dy}{y} \int^{Q_k^2} \frac{dq^2}{q^2}\right)$$
$$= \Delta_{ne}^{(BFKL)}(k)$$

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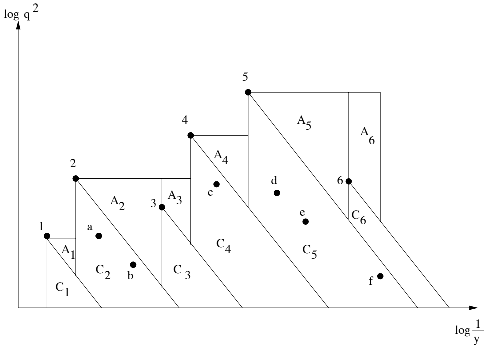
- Hard: $1/z_k$, Soft: $1/(1-z_k)$, where $y_k = (1-z_k)x_{k-1}$.
- Virtual form factors split into two new form factors: "Sudakov" Δ_s and "non-Sudakov" Δ_{ns}: $S_{eik}(k) \cdot S_{ne}(k) = \Delta_{ns}(k) \cdot \Delta_s(k)$.
  Δ_s defined such as to compensate soft emissions.

Graphical representation



$\Delta_s(k) = \exp(-\bar{\alpha}_s C_k)$, and $\Delta_{ns}(k) = \exp(-\bar{\alpha}_s A_k)$. However, A_k can be "negative"! In litterature, unfortunately not correct Δ_{ns} used.

Simplify more

After cancellation of soft emissions, one is left with

$$\mathcal{A}(x,k,\bar{p}) = \bar{\alpha}_s \int_x^1 \frac{dz}{z} \int \frac{d^2p}{\pi p^2} \theta(\bar{p} - zp) \Delta_{ns} \mathcal{A}(\frac{x}{z}, |k + (1-z)p|, p)$$

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  Why not use same strategy to also cancel emissions in A_k as well?
- Indeed one can do this. First realized by Lund group (LDC model)

Some Comments

For $\bar{q} \geq k$, gluon distrb. $\mathcal{A}(x, k, \bar{q}) \Rightarrow \mathcal{A}(x, k)$, great simplification. Much faster numerical solution to integral eq.

However, eq. to be derived not exactly unique, and different eq. $\Rightarrow$ different intercepts.

Compared to BFKL, higher intercept. This can be fixed by treating real-virtual cancellations more carefully.

For $\bar{q} \leq k$, Q_s in CCFM will also depend on $\bar{q}$, *i.e.* $Q_s = Q_s(x, \bar{q})$. We have not yet studied this case. However, implementation of saturation boundary exactly the same also in that case.

Explicit equation to be solved

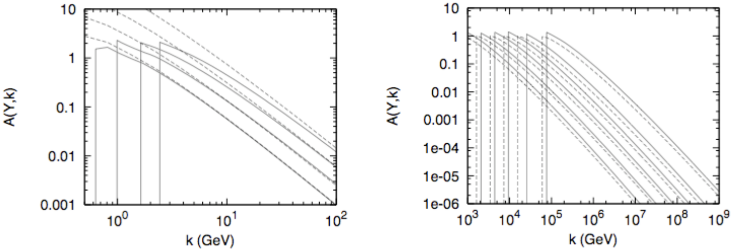
We implement saturation boundary on following eq. obtained after cancellation of Δ_{ns}:

$$\partial_Y \mathcal{A}(Y,k) = \bar{\alpha}_s \int \frac{dk'^2}{|k^2 - k'^2|} h(\kappa) \Big(\theta(k^2 - k'^2)\mathcal{A}(Y,k') + \theta(k'^2 - k^2)\theta(Y - \ln(k'^2/k^2))\mathcal{A}(Y - \ln(k'^2/k^2), k') \Big).$$

where $Y = \ln 1/x$, and $\kappa \equiv \min(k^2, k'^2)/\max(k^2, k'^2)$ and

$$h(\kappa) = 1 - \frac{2}{\pi} \arctan \left(\frac{1+\sqrt{\kappa}}{1-\sqrt{\kappa}} \sqrt{\frac{2\sqrt{\kappa}-1}{2\sqrt{\kappa}+1}} \right) \theta(\kappa - 1/4).$$

Results



Left: Solution with and without saturation boundary for $Y = 8, 10, 12, 14$ and for running coupling. Effects of saturation clearly visible.
Right: Solution vs BFKL with saturation boundary. Same energy dependence for Q_s up to extremely high $Y = 120$.

Saturation momentum

Saturation momentum Q_s can now be constructed. For running coupling Q_s can be parametrized as

$$Q_s^2 = Q_0^2 \exp(\lambda_r \sqrt{Y})$$

For both CCFM and BFKL we find the value $\lambda_r \approx 3.0$ over very large interval in Y.

If additional kin. constraint included in BFKL, then structure function much lower but energy dependence of Q_s same (only Q_0 changes).

Final comments

- Another possibility is to use boundary which is not completely absorptive. For example $\mathcal{A}$ set to constant $\neq 0$ behind saturation front.

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- Important that this procedure gives Q_s consistent with the evolution. Q_s not enforced by hand, but determined by evolution.
- This method only to be used for $k_\perp > Q_s$. Below Q_s nonlinear physics must be dealt with exactly.
- Big motivation is to look for saturation effects in exclusive final states. Method can easily be implemented in event generators, ex: CASCADE.