Heavy Flavours Working Group: News from Theory

Ahmed Ali
DESY, Hamburg

DIS, 26 - 30 April, 2009, Madrid

Theoretical talks

I. General-Mass Variable-Flavor-Number Scheme (GM-VFNS)

- Inclusive photoproduction of D^* mesons at NLO (Hubert Spiesberger)
- Charm hadroproduction and the charm content of the proton (Bernd Kniehl)
- Analysis of heavy-quark electroproduction data in VFNS (Sergey Alekhin)

II. SFs, PDFs and HF Cross-sections

- 3-loop corrections to the moments of $F_2^{(c,b)}(x, Q^2)$ for $Q^2 \gg m_Q^2$ (Sebastian Klein)
- Global PDF analysis in an intermediate-mass scheme (Pavel Nadolsky)
- MC @ NLO for heavy quarks at HERA (Tobias Toll)
- Higher-order soft corrections to $t\bar{t}$ production (Ulrich Langenfeld)
- Heavy flavour effects in the virtual photon SF to NLO in QCD (Tsuneo Uematsu)

III. Studies of $(J/\psi, \Upsilon)$ Data

- Theoretical status of J/ψ production at HERA (Pierre Artoisenet)
- J/ψ polarization from fixed-target to collider energies (Pietro Faccioli)
- Production and polarization of Υ in the K_T-factorization approach (Nikolai Zotov)

IV. Spectroscopy of the states X,Y,Z and the b baryons (Antonio Polosa; Marek Karliner)

$ep \to D^{*\pm}X$ at low Q^2 in GM-VFNS

General-Mass Variable-Flavor-Number Scheme

H. Spiesberger

for 1-particle inclusive heavy meson production

- The problem:
  Conventionally, PDFs and FFs are defined in the $\overline{\mathrm{MS}}$ scheme
  $\overline{\mathrm{MS}}$ scheme is based on a massless calculation
  Can not use $\overline{\mathrm{MS}}$ PDFs and FFs in a massive calculation?

- The solution:
  Match massless and massive calculations:

$$d\sigma_{\mathrm{sub}} = \lim_{m\to 0} d\tilde{\sigma}(m) - d\hat{\sigma}_{\overline{\mathrm{MS}}}$$

The subtracted cross section (in a massive calculation)

$$d\hat{\sigma}(m) = d\tilde{\sigma}(m) - d\sigma_{\mathrm{sub}}$$

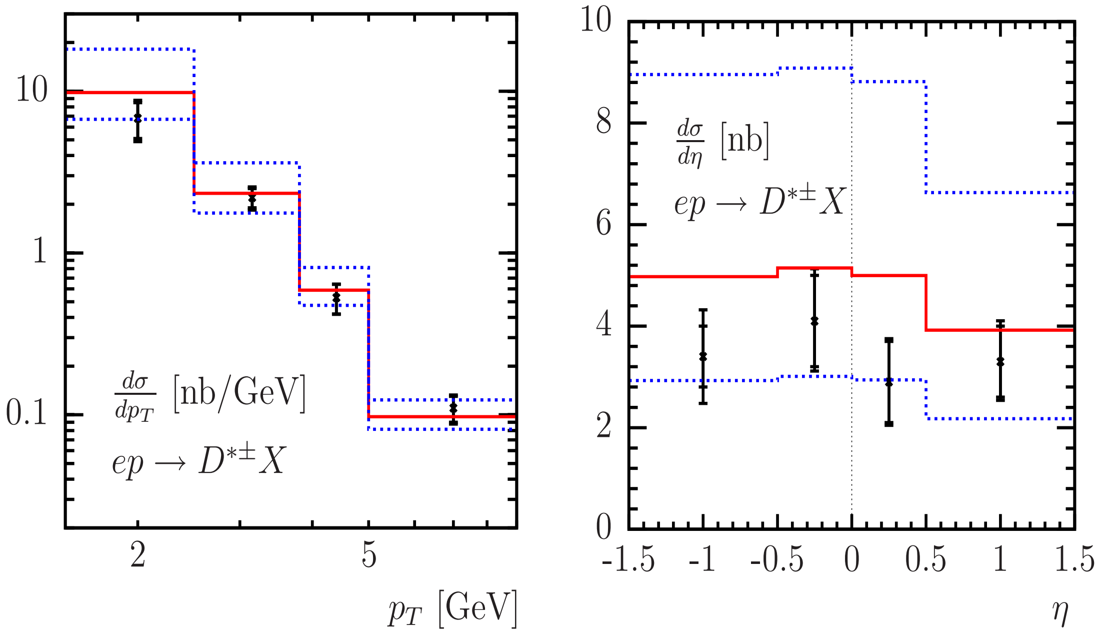
can be used with $\overline{\mathrm{MS}}$ parton distribution and fragmentation functions

➜ The GM-VFNS (general-mass variable flavor number scheme) combines: resummation of large logarithms $\log(\mu/m)$ in PDFs and FFs with finite mass terms in the hard cross section

Work in collaboration with B. A. Kniehl, G. Kramer, I. Schienbein

$ep \to D^{*\pm}X$ at low Q^2 in GM-VFNS

$e^{\pm}p \to D^* + X$ at low Q^2: $0.05 < Q^2 < 0.7$ GeV^2



scale variations by factor 2 around $\mu_{R,F} = \sqrt{p_T^2 + m^2}$, data: ZEUS PLB649

More applications available for: $\gamma + \gamma \to D^{*\pm} + X$: EPJC22, EPJC28; $\gamma^* + p \to D^{*\pm} + X$: EPJC38, arXiv:0902.3166; $p + \bar{p} \to (D^0, D^{*\pm}, D^{\pm}, D_s^{\pm}, \Lambda_c^{\pm}) + X$: PRD71, PRL96, arXiv:0901.4130; $p + \bar{p} \to B + X$: PRD77

Open charm hadroproduction (Kniehl)

OUR THEORETICAL BASIS FOR $p\bar{p} \to D^\star X$

Factorization Formula:

$$d\sigma(p\bar{p} \to D^\star X) = \sum_{i,j,k} \int dx_1\, dx_2\, dz\, f_i^p(x_1)\, f_j^{\bar{p}}(x_2) \times d\hat{\sigma}(ij \to kX)\, D_k^{D^\star}(z) + \mathcal{O}(\alpha_s^{n+1}, (\tfrac{\Lambda}{Q})^p)$$

Q: hard scale, $p = 1, 2$

- $d\hat{\sigma}(\mu_F, \mu'_F, \alpha_s(\mu_R), \frac{m_h}{p_T})$: hard scattering cross sections free of long-distance physics $\to$ m_h kept
- PDFs $f_i^p(x_1, \mu_F)$, $f_j^{\bar{p}}(x_2, \mu_F)$: $i, j = g, q, c$ $\quad [q = u, d, s]$
- FFs $D_k^{D^\star}(z, \mu'_F)$: $k = g, q, c$

$\Rightarrow$ need short distance coefficients including heavy quark masses

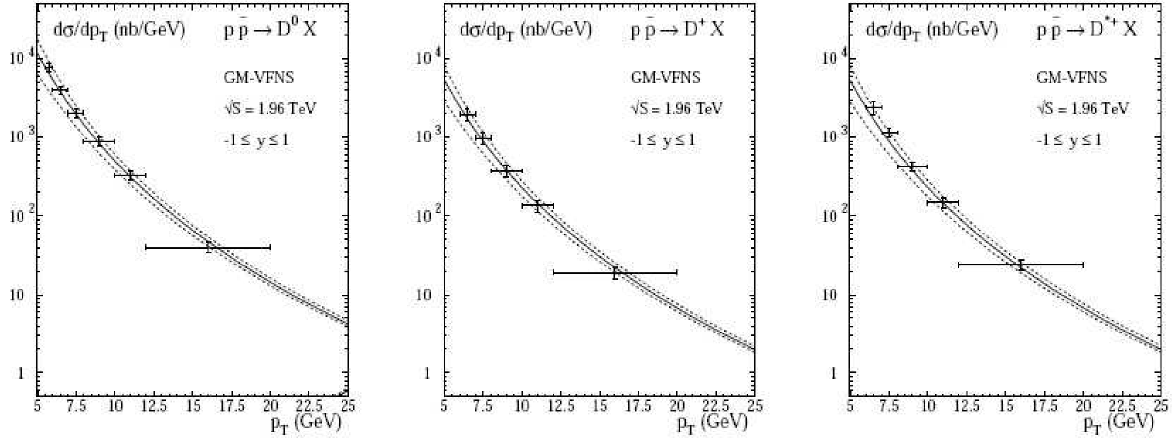
---

[1] J. Collins, 'Hard-scattering factorization with heavy quarks: A general treatment', PRD58(1998)094002

Open charm hadroproduction (Kniehl)

HADROPRODUCTION OF D^0, D^+, D^{*+}, D_s^+

GM-VFNS RESULTS W/ KKKSc FFs [1]



- $d\sigma/dp_T$ [nb/GeV] $|y| \leq 1$ prompt charm
- Uncertainty band: $1/2 \leq \mu_R/m_T, \mu_F/m_T \leq 2$ $\quad (m_T = \sqrt{p_T^2 + m_c^2})$
- CDF data from run II [2]
- GM-VFNS describes data within errors

[1] B.K.,Kramer,Schienbein,Spiesberger, arXiv:0901.4130[hep-ph], PRD(to appear)

[2] Acosta et al., PRL91(2003)241804

HF contr. in $F_2(x,Q^2)$ for $Q^2 \gg m_Q^2$ (Klein)

Moments of the 3-Loop Corrections to the Heavy Flavor Contribution to $F_2(x,Q^2)$ for $Q^2 \gg m^2$ 1

Sebastian Klein (DESY) in collaboration with I. Bierenbaum and J. Blümlein

[0904.3536 [hep-ph]]

- QCD precision analyzes require the description of the heavy quark contributions to 3–loop order to line up with the accuracy reached in the massless case.
  $\Longrightarrow$ QCD analysis and determination of $\alpha_s(M_Z^2)$ from DIS data: $\delta\alpha_s/\alpha_s < 1\,\%$.
- Factorization of heavy flavor Wilson coefficients $H_{j,i}$ for $Q^2 \gg m^2$ [F_2: $Q^2/m^2 \geq 10$]

$$H^{\rm S,NS}_{(2,L),i}\left(\frac{Q^2}{\mu^2},\frac{m^2}{\mu^2}\right) = \underbrace{A^{\rm S,NS}_{ki}\left(\frac{m^2}{\mu^2}\right)}_{\text{massive OMEs}} \otimes \underbrace{C^{\rm S,NS}_{(2,L),k}\left(\frac{Q^2}{\mu^2}\right)}_{\text{light Wilson coefficients}}.$$

[van Neerven, Smith, et. al. 1996]

- Analytic results are known for $Q^2 \gg m^2$ at NLO for $F_2(x,Q^2)$. [$F_L(x,Q^2)$ at NNLO]
- light Wilson coefficients known at NNLO [Vermaseren, Vogt, Moch, 2005.]
  $\Longrightarrow$ massive OMEs needed at $O(a_s^3)$.
- This calculation: **We obtain fixed Mellin moments of all massive OMEs at $O(a_s^3)$**

3-loop corrections to HF contr. in $F_2(x, Q^2)$ for $Q^2 \gg m_Q^2$

- The calculation of fixed moments of the massive operator matrix elements at $O(a_s^3)$ has been finished up to $N = 10, 12, 14$ ($\approx$ 250 days of computer time)

$$A_{Qq}^{(3),\mathrm{PS}}: \ (2,4,..,12); \quad A_{qq,Q}^{(3),\mathrm{PS}}, A_{gq,Q}^{(3)}: \ (2,4,..,14);$$
$$A_{qq,Q}^{(3),\mathrm{NS}\pm}: \ (2,3,..,14); \quad A_{Q(q)g}^{(3)}, A_{gg,Q}^{(3)}: \ (2,4,..,10);$$

- Massive OMEs occur as well as transition functions to define a variable flavor number scheme starting from a fixed flavor number scheme.

- The complete OMEs but the constant terms are derived in terms of renormalization constants known for all N.

- The calculation provides first independent checks on the 3–loop anomalous dimensions $\gamma_{qg}^{(2)}$, $\gamma_{qq}^{(2),\mathrm{PS}}$ and on respective color projections of $\gamma_{qq}^{(2),\mathrm{NS}\pm}$, $\gamma_{gg}^{(2)}$ and $\gamma_{gq}^{(2)}$.
  $\Longrightarrow$ Agreement for the terms $\propto T_F$ with [Larin, Ritbergen, Vermaseren, 1994; Larin, Nogueira, Ritbergen, Vermaseren, 1997; Retey, Vermaseren, 2001; Vogt, Moch, Vermaseren, 2004.]

- We corrected some left outs even at NLO in the literature.

- First phenomenological parameterization to come up soon.

Heavy flavour effects in the virtual photon SF to NLO (Uematsu)

The heavy quark mass effects in parton picture and OPE

$$F_2^\gamma(x,Q^2,P^2) = \vec{q}^\gamma(y,Q^2,P^2,m^2) \otimes \vec{C}\left(\frac{x}{y},\frac{\overline{m}^2}{Q^2},\bar{g}(Q^2)\right)$$

Photon structure function PDF Coefficient function

↖ mass dependence

Parton interpretation of twist-2 operators $\vec{O}_n$

no mass dependence ↙

$$\int_0^1 dx x^{n-1} \vec{q}^\gamma(x,Q^2,P^2,m^2)$$

$$= \vec{A}_n\left(1,\frac{\overline{m}^2(P^2)}{P^2},\bar{g}(P^2)\right) T\exp\left[\int_{\bar{g}(Q^2)}^{\bar{g}(P^2)} dg \frac{\gamma_n(g,\alpha)}{\beta(g)}\right]$$

where

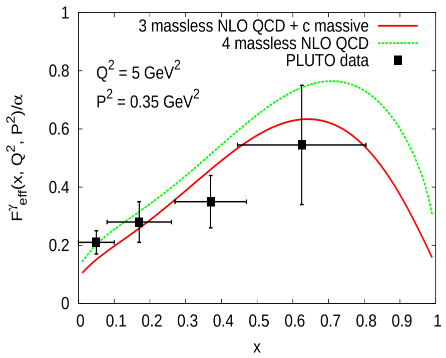
$$\langle\gamma(P^2)|\vec{O}_n(\mu^2)|\gamma(P^2)\rangle = \vec{A}_n\left(\frac{P^2}{\mu^2},\frac{\overline{m}^2(\mu^2)}{\mu^2},\bar{g}(\mu^2)\right)$$

Perturbatively calculable!

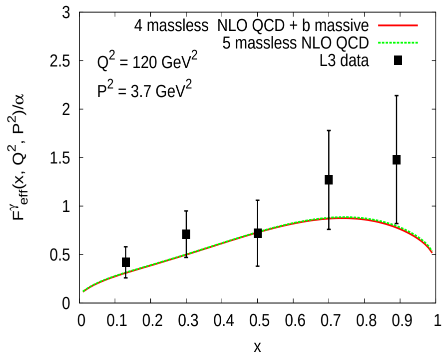
↖ mass dependence

Heavy flavour effects in the virtual photon SF to NLO (Uematsu)

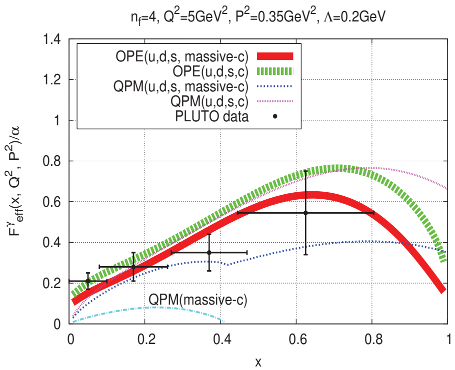
Heavy flavour effects vs. experimental data for $F_{\rm eff}^{\gamma}$



QCD prediction vs. PLUTO data



QCD prediction vs. L3 data



QCD prediction with PM Box contribution

In the massive quark limit

$$\Lambda_{\rm QCD}^2 \ll P^2 \ll m^2 \ll Q^2$$

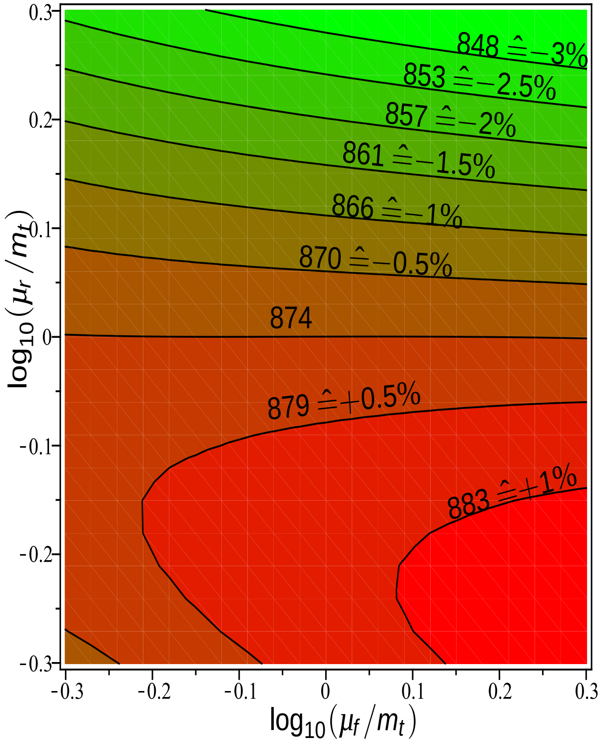
Heavy quark mass threshold effects illustrated by PM Box

$\hookleftarrow$

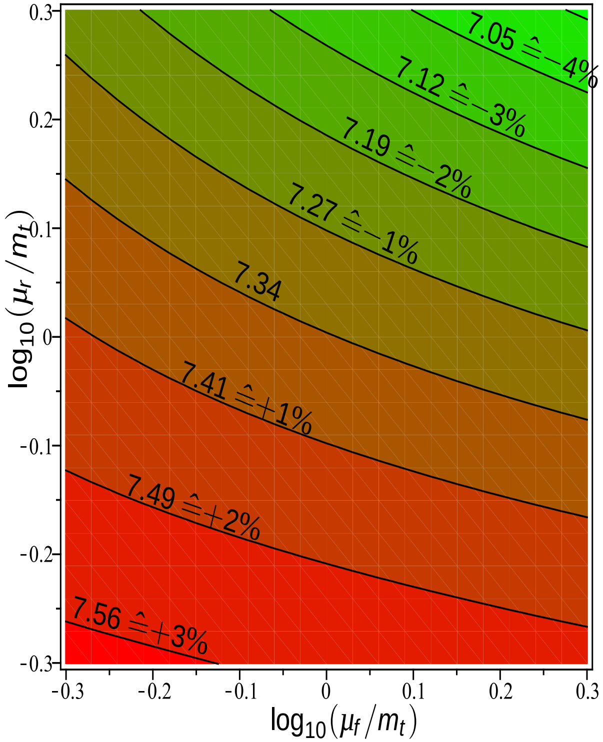
$$x_{\max} = \frac{1}{1+\frac{4m^2}{Q^2}}$$

Scale dependence of the $t\bar{t}$ production cross section

Contour lines of the total hadronic cross section for top pair production in the μ_f - μ_r - plane in pb ($\log_{10}$ - scale)



LHC



Tevatron

1/2

$t\bar{t}$ production cross sections (Langenfeld)

Cross section Predictions

- Total cross section prediction for $m_t = 173\,\text{GeV}$ and PDF set Cteq6.6:

$$\sigma(p\bar{p} \to t\bar{t}) = 7.34^{+0.23}_{-0.38}\,\text{pb @ Tevatron}$$

$$\sigma(pp \to t\bar{t}) = 874^{+14}_{-33}\,\text{pb @ LHC 14 TeV}$$

- Fit functions to the hadronic cross section for different scales ($\mu = 1/2m_t, m_t, 2m_t$) for the two colliders Tevatron and LHC ($10/14\,\text{TeV}$) and for two different PDF sets (Cteq6.6, MSTW 2008 NNLO), w/o PDF errors.
  Ansatz: $\sigma(m_t) = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6$,
  $x = (m_t/\text{GeV} - 173)$.
- Total cross section with combined theoretical and PDF error:

$$\sigma(p\bar{p} \to t\bar{t}) = 7.34^{+0.66}_{-0.76}\,\text{pb @ Tevatron}$$

$$\sigma(pp \to t\bar{t}) = 874^{+26}_{-50}\,\text{pb @ LHC 14 TeV}$$

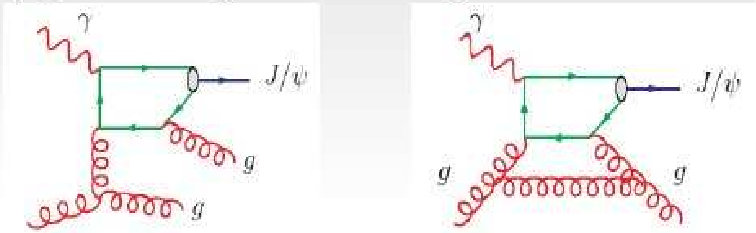
2/2

Theoretical status of J/ψ production at HERA (Artoisenet)

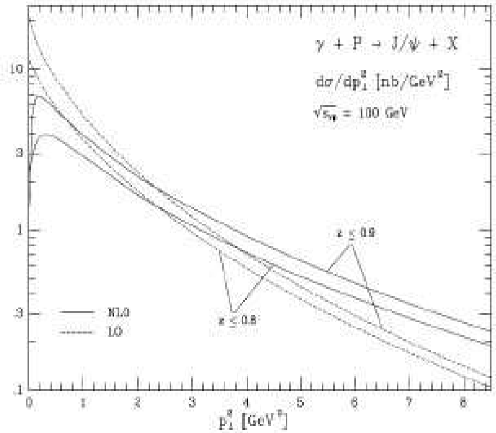
Colour-singlet yield at NLO

M. Kramer (1995)
P. A., J. Campbell, F. Maltoni, F. Tramontano (2009)
C.-H. Chang, R. Li, J.-X. Wang (2009)

- Typical Feynman diagrams:



- Differential cross section:



substantial enhancement at high transverse momentum (new kinematic contributions)

Kramer, Nucl.Phys.B459 :3-50,1996.

Theoretical status of J/ψ production at HERA (Artoisenet)

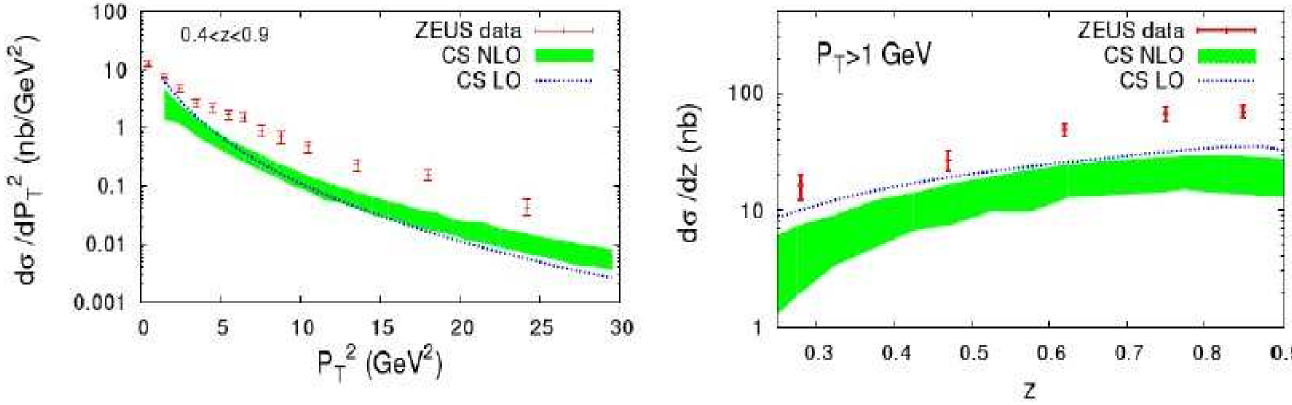
Comparison with the data

- Colour singlet yield at NLO undershoots the data

mass uncertainty: $1.4\ \text{GeV} < m_c < 1.6\ \text{GeV}$

scale uncertainty:

$$\mu_0 = 4m_c, \quad 0.5\mu_0 < \mu_r, \mu_f < 2\mu_0, \quad 0.5 < \frac{\mu_r}{\mu_f} < 2$$



Theoretical status of J/ψ production at HERA (Artoisenet)

J/Ψ Polarization

- J/Ψ polarization deduced from the angular distribution of the produced leptons
- spin quantization axis: $\boldsymbol{z} = -\frac{\boldsymbol{P}_N}{|\boldsymbol{P}_N|}$ (target frame)
- angular distr. of the leptons in the J/Ψ rest frame

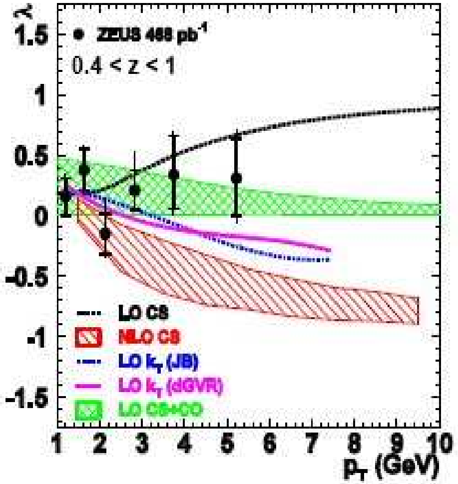
$$\frac{d\sigma}{d\Omega dy} \propto 1 + \lambda(y)\cos^2\theta + \mu(y)\sin 2\theta\cos\phi + \frac{\nu(y)}{2}\sin^2\theta\cos 2\phi$$

with respect to the spin quantization axis, and

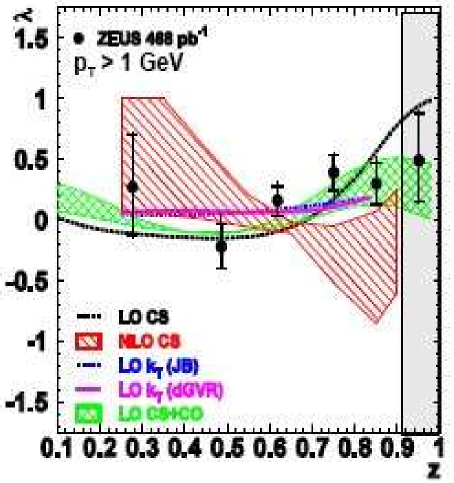
$$\lambda = \frac{\rho_{11} - \rho_{00}}{\rho_{11} + \rho_{00}}, \quad \mu = \frac{\sqrt{2}Re\rho_{10}}{\rho_{11} + \rho_{00}}, \quad \nu = \frac{2\rho_{1,-1}}{\rho_{11} + \rho_{00}}$$

Theory vs. Expt: J/ψ polarization at HERA (Bertolin)

J/ψ helicity at HERA



- LO CS and NLO CS predictions have opposite sign … we initially thought NLO corrections would be small …
- LO k_T CS has the same sign of NLO, parton transverse momentum, k_T, mimics NLO terms
- LO CS+CO is flat
- data are consistent with being flat in the probed p_T range
- proton dissociative background mostly at low p_T
- analysis redone for z < 0.9, effects in the sys. errors

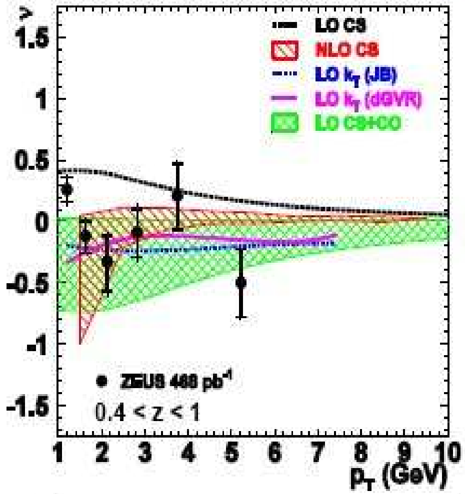


- LO CS describe the data well
- NLO CS has large uncertainties … p_T > 1 GeV may be not enough …
- LO k_T CS not too different from LO
- LO CS+CO is pretty much the same as LO CS
- proton dissociative is at the 60 – 70 % level for 0.9 < z < 1, << 5 % elsewhere

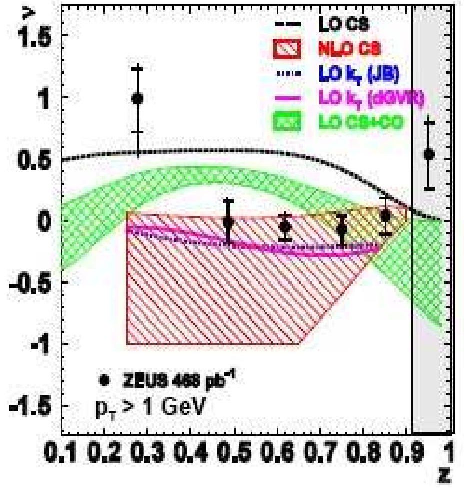
10

Theory vs. Expt: J/ψ polarization at HERA (Bertolin)

J/ψ helicity at HERA (cont.)



- LO CS is positive … all other predictions are negative … and in better agreement with the data
- LO k_T CS is pretty much as NLO CS
- LO CS+CO is flat
- data are consistent with being flat in the probed p_T range
- proton dissociative background mostly at low p_T
- analysis redone for z < 0.9, effects in the sys. errors



- LO CS does not describe the data, positive
- NLO CS has large uncertainties … negative … $p_T > 1$ GeV may be not enough …
- LO k_T CS fine … except at low z
- LO CS+CO does not describe the data, positive
- proton dissociative is at the 60 – 70 % level for 0.9 < z < 1, << 5 % elsewhere

11

Theoretical status of J/ψ production at HERA (Artoisenet)

Conclusion

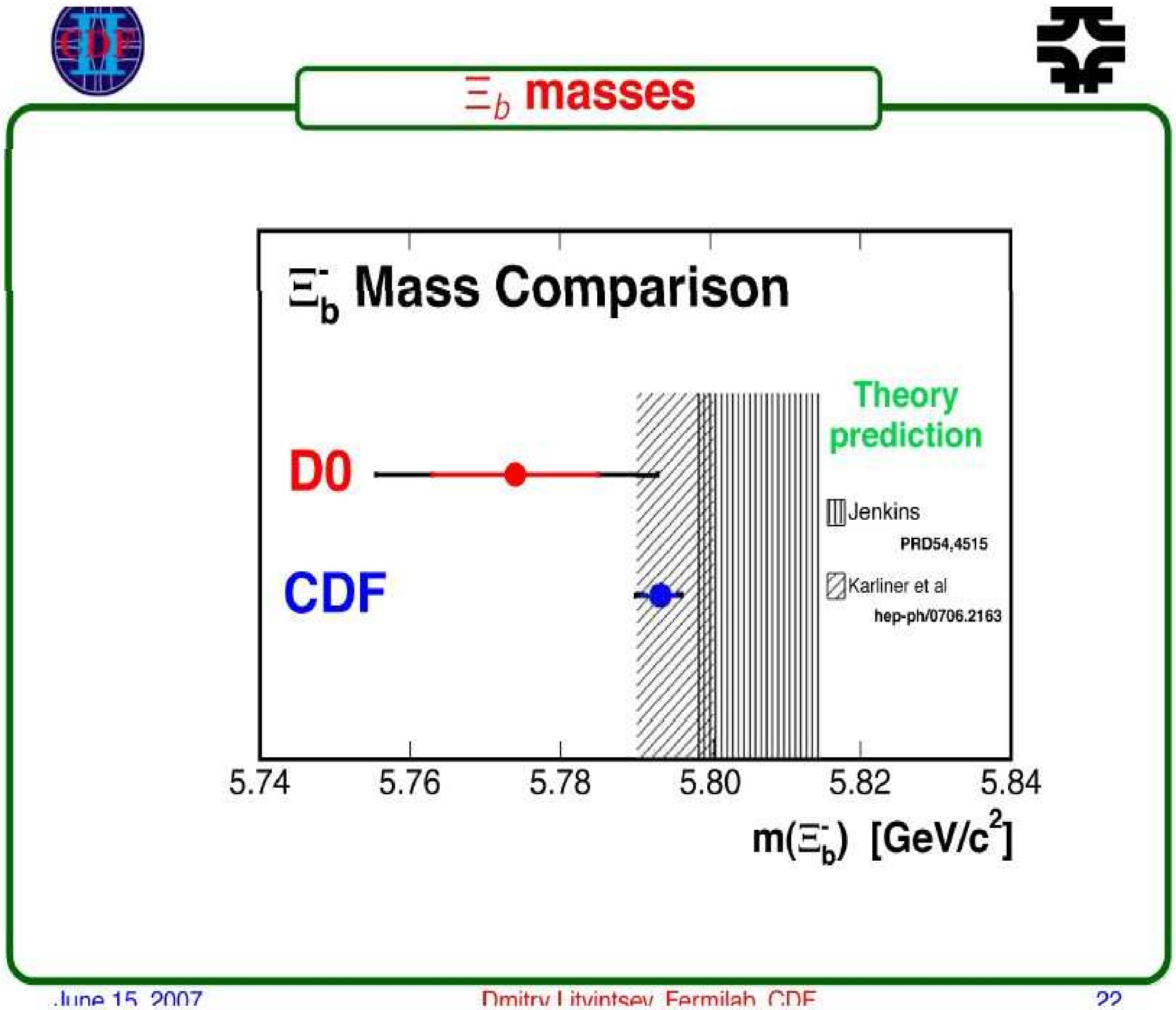
- New results on the polarization of the J/ψ in photoproduction (colour-singlet at NLO)
- Colour-singlet prediction alone does not describe all features of the data collected at HERA
- NLO computation of the polarization associated to the colour-octet P-wave transition may solve the current discrepancy

Quark Model and spectroscopy of the b-baryons (Karliner)

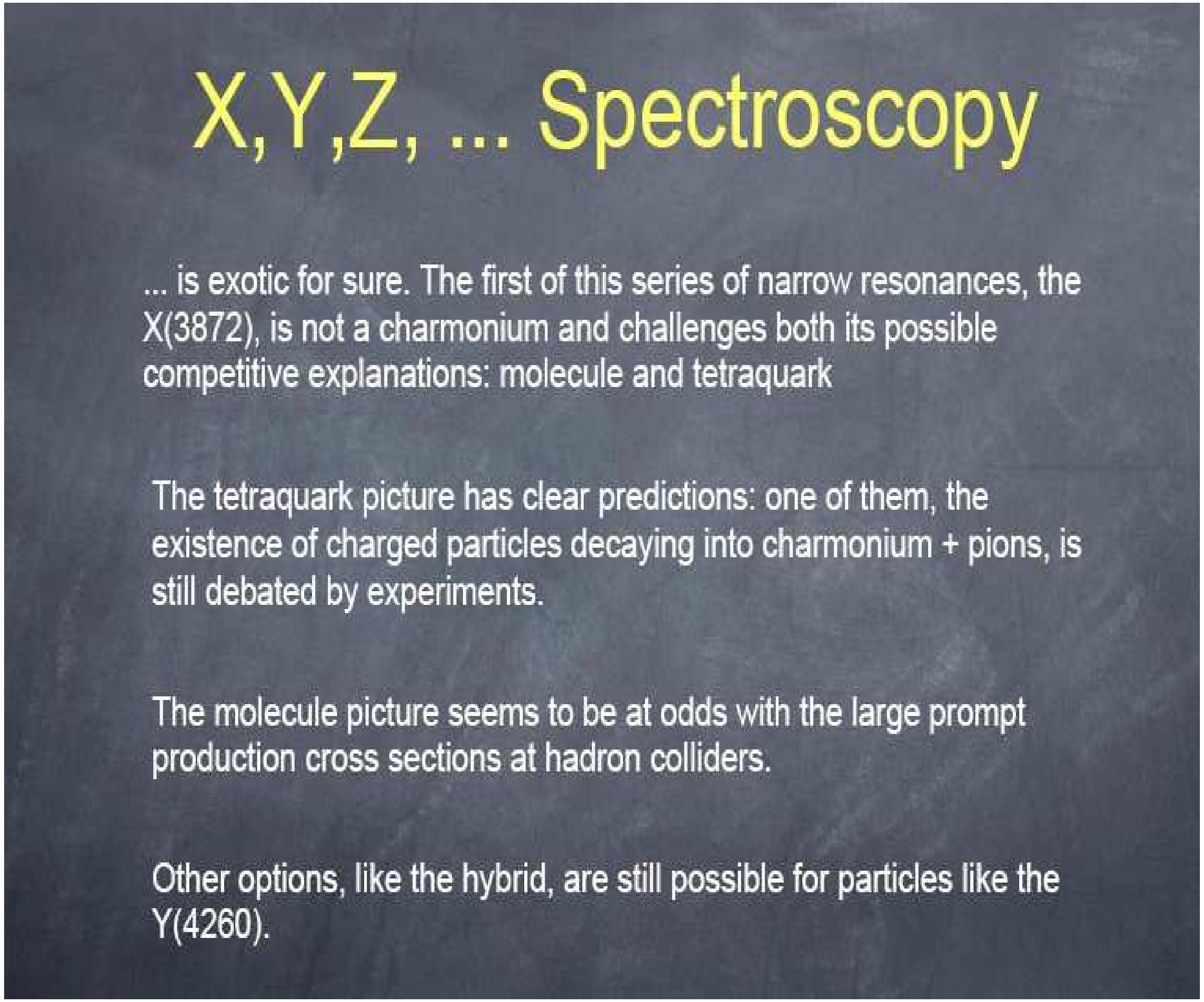
Summary

- Consitituent quark model with color HF interaction
  gives highly accurate predictions for heavy baryon masses
- a challenge for theory: derivation from QCD
- constituent quark masses depend on the spectator quarks
- $M_{\Sigma_b} - M_{\Lambda_b} = 194\,\text{MeV}$ vs 192 in EXP (CDF)
- $M(\Sigma_b^*) - M(\Sigma_b) = 22$ MeV vs 21 MeV in EXP (CDF)
- $\mu_{\Lambda_c} = 0.43\,\text{n.m.}$ $\mu_{\Lambda_b} = -0.067\,\text{n.m.}$
- meson-baryon effective supersymmetry
- meson/baryon HF splitting confirms Cornell potential
- Ξ_b mass prediction: 5795+-5 MeV vs $5793 \pm 2.4 \pm 1.7$ MeV
- puzzle in Υ(5S) decays: $\bar{b}bu\bar{d}$ candidates?

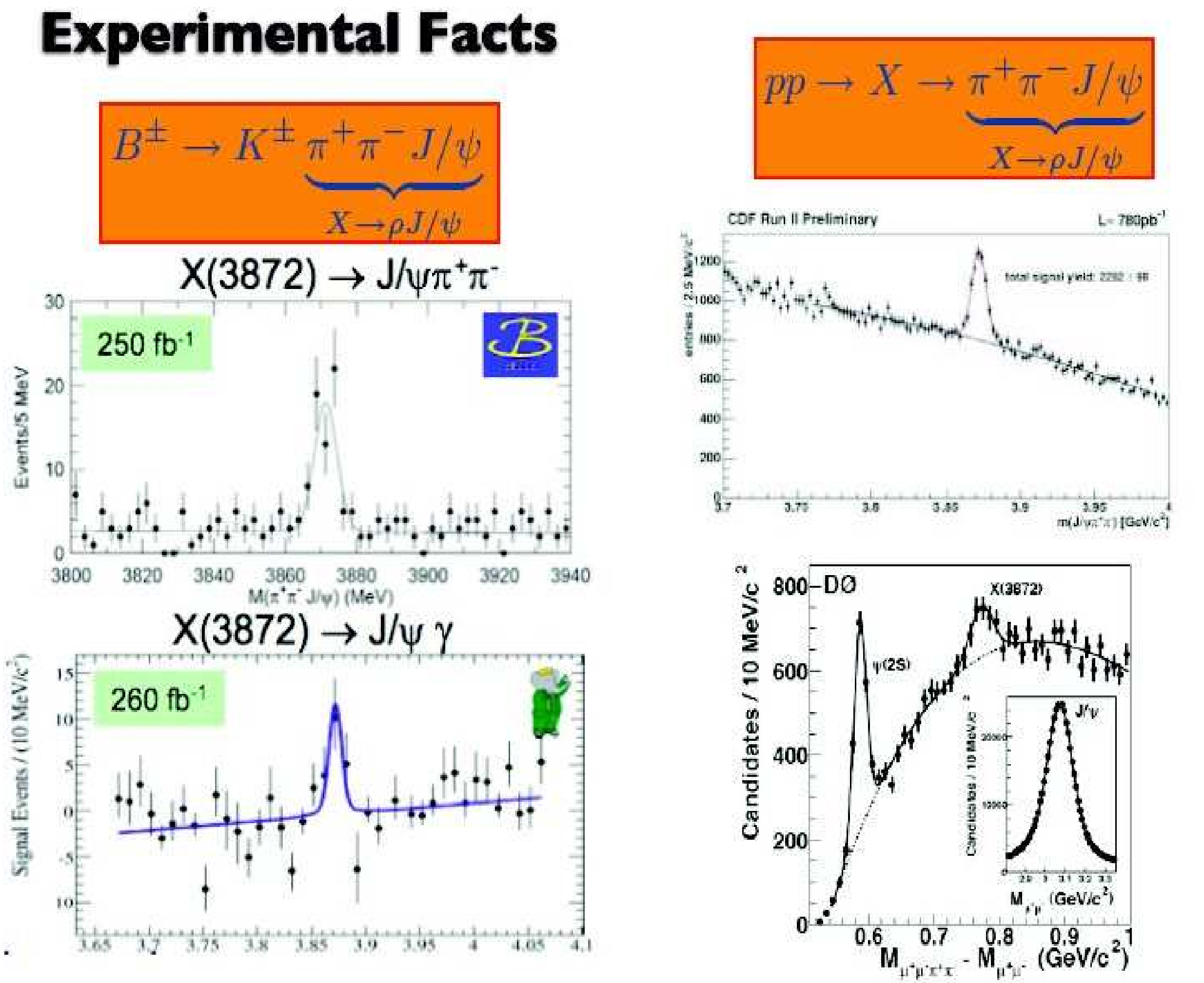
Quark Model and spectroscopy of the b-baryons (Karliner)



$X, Y, Z, \ldots$ spectroscopy (Polosa)

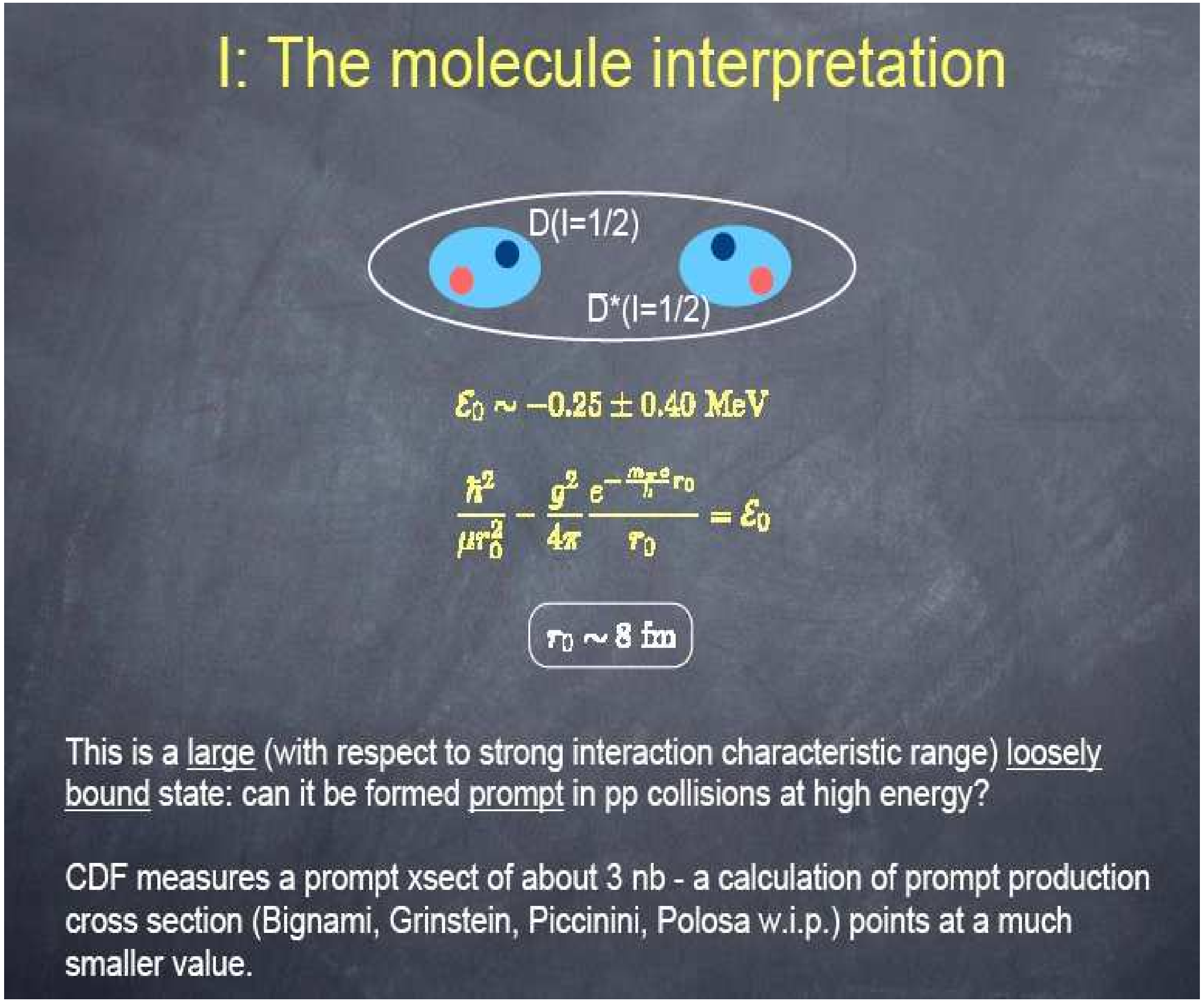


$X, Y, Z, \ldots$ spectroscopy (Polosa)



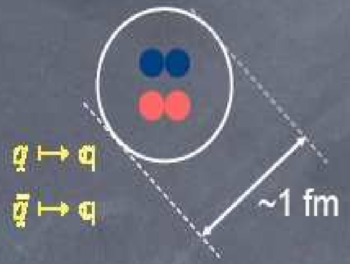
$$X \to \gamma J/\psi \mapsto C = +1 \quad \text{and} \quad X \to \rho^0 J/\psi \to (\pi^+\pi^-)_S J/\psi \mapsto P = 1$$

$X, Y, Z, \ldots$ spectroscopy (Polosa)



$X, Y, Z, \ldots$ spectroscopy (Polosa)

II: The tetraquark interpretation



$\mathfrak{q} = [q \uparrow q \downarrow]_{s_c, s_t}$, or, more precisely, $\mathfrak{q}_{i\alpha} = \epsilon_{ijk}\epsilon_{\alpha\beta\gamma}\bar{q}_C^{j\beta}\gamma_5 q^{k\gamma}$

-This is a compact, tightly bound object.
Drawbacks: Many states are predicted. X^+ and X^- yet to be found. (Some indication by Belle of charged particles decaying into charmonium + charged pions).

-Two neutral X, with a difference in mass of few MeV and with different decay modes are expected in order to explain the observed isospin violation in $X \to \psi\rho$, $\psi\omega$ (which seem to occurr at the same rate).

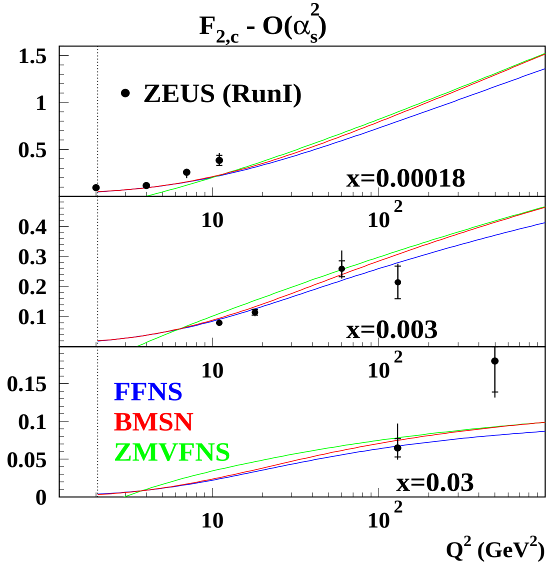
-A doubly charged state W^{++}=[cu][db sb] could exist (very unlikely for a molecule: at 10 fm electromagnetic repulsion starts to compete with strong interaction)

Backups

- VFNS and heavy quark electroproduction data
  Sergey Alekhin
- MC @ NLO for heavy quarks at HERA
  Tobias Toll
- Global PDF analysis in an intermediate-mass scheme
  Pavel Nadolsky
- J/ψ polarization from fixed-target to collider energies
  Pietro Faccioli
- Production and polarization of Upsilon mesons in the K_T-factorization approach
  Nikolai Zotov

VFNS and heavy quark electroproduction data (Alekhin)

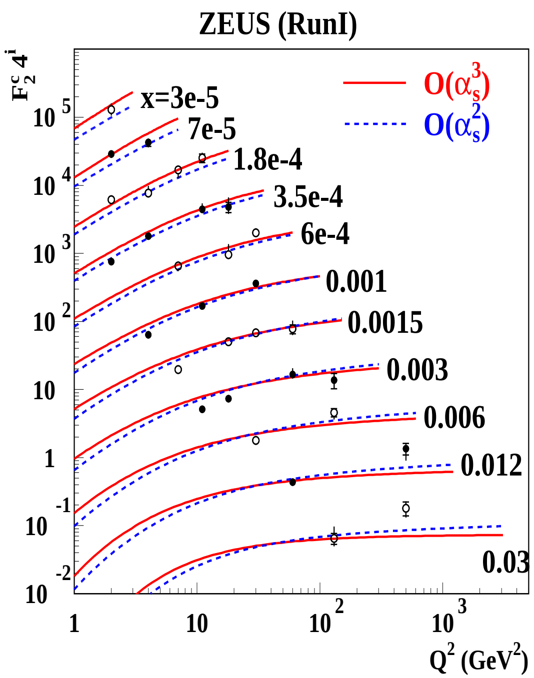
(S. Alekhin, J. Blümlein, S. Klein, S. Moch)



- The Buza-Matiounine-Smith-van Neerven (BMSN) prescription for the variable-flavour-number scheme (VFNS), is not too far from the fixed-flavour-number scheme (FFNS) for the realistic HERA kinematics; it seems to be the case for any smooth matching.
- The remaining discrepancies with the data cannot be cured by the smooth VFNS.

1

VFNS and heavy quark electroproduction data (Alekhin)



- The FFNS with account of the partial $O(\alpha_s^3)$ corrections provides a good description of the HERA data at small/moderate Q^2.
- At large Q^2 it undershoots the data due to negative contribution from $c_{2,g}^{(2,1)}$, the missing contribution from $c_{2,g}^{(2,0)}$ at large η must be positive in order to improve the agreement.

(c.f. S.Klein's talk)

2

MC @ NLO for heavy quarks at HERA (Toll)

Conclusions

- MC@NLO constructed for HERA
- At parton level it behaves as expected
- Where pQCD is expected to be important MC@NLO is doing well

χ^2/ndf combined for all beauty plots shown :

	MC@NLO	FMNR	HERWIG (rescaled)	ndf
χ^2	0.86	4.51	1.31	57

MC @ NLO for heavy quarks at HERA (Toll)

Charm Issues

- NLO calculation unable to describe Charm data
- Shapes are well described
- One analysis too large, one too small
- Charm mass is low m_t=1.5 GeV, also Q^2 is low
- Probing the edge of pQCD
- This leads to large sensitivity to the scales:

χ^2/ndf H1 2006 D* + other jet

Scale	m_t	$2 \cdot m_t$	$4 \cdot m_t$	ndf
$p_t(D^*)$	25.87	20.22	16.34	4
$\eta(D^*)$	3.97	2.09	1.47	5
$\Delta\eta(D^*, jet)$	2.99	1.64	1.07	6
$p_t(jet)$	4.78	2.29	1.53	4
$\eta(jet)$	3.48	1.56	1.12	5
$\Delta\phi(D^*, jet)$	4.20	2.79	2.20	6
Total	6.77	4.50	3.47	30

By increasing the scale, the description improves by > 3 units in χ^2/ndf

Global PDF analysis in an intermediate-mass scheme (Nadolsky)

Rescaling variables in heavy-flavor DIS

P. Nadolsky, W.-K. Tung, arXiv:0903:2667

- **General-mass scheme:** leading heavy-quark mass effects are captured by a **rescaling variable** χ in HQ PDF's:

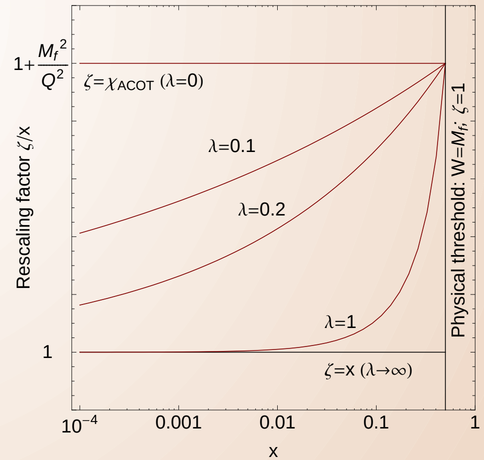
$$x = \chi \Big/ \left(1 + M_f^2/Q^2\right), \text{ with } M_f^2 = \; 4m_c^2\ (m_c^2) \text{ in NC (CC) DIS}$$

Barnett, Haber, Soper; Tung, Kretzer, Schmidt

- **Zero-mass scheme at NLO:** realistic threshold behavior is reproduced by a **generalized rescaling variable** $\zeta(\lambda)$:

$$x = \zeta \Big/ \left(1 + \zeta^\lambda M_f^2/Q^2\right), \text{ with } 0 \le \lambda \lesssim 1$$

- ▶ $\lambda = 0 : \zeta = \chi$ (the ACOT-χ variable)
- ▶ $\lambda \gtrsim 1$: $\zeta \approx x$ (no rescaling)
- ▶ $\zeta \approx x$ for $Q^2 \gg M_f^2$



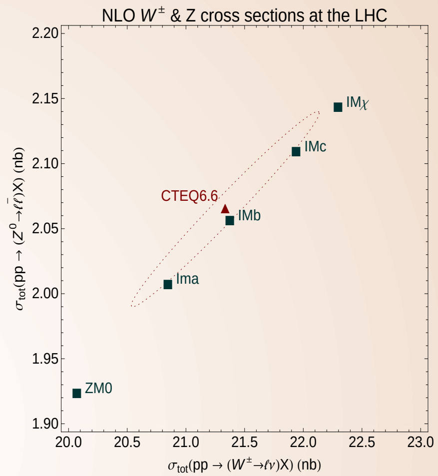
P. Nadolsky (SMU) DIS'2009 workshop, Spain April 2009 1

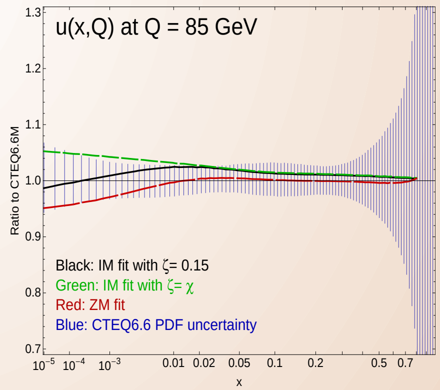
Global PDF analysis in an intermediate-mass scheme (Nadolsky)

Intermediate-mass scheme

PDF fits with ZM Wilson coefficients evaluated at $\zeta(x, M_f/Q, \lambda)$ closely reproduce GM fits for some λ values

This **intermediate-mass** formulation improves the NLO ZM scheme and approximates (more fundamental) GM predictions by simple means



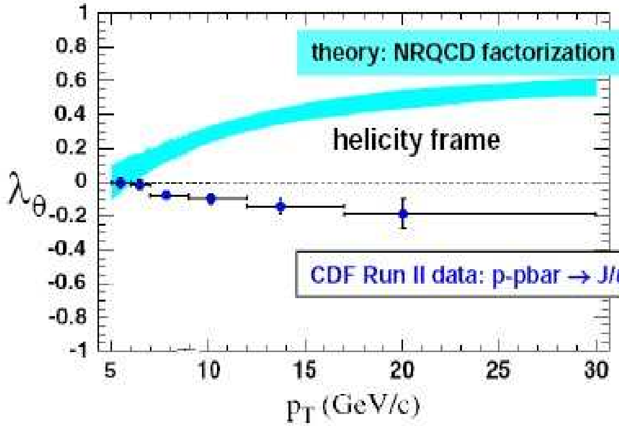


P. Nadolsky (SMU) DIS’2009 workshop, Spain April 2009 2

J/ψ polarization (Faccioli)

The "J/ψ polarization puzzle"

theory vs experiment:

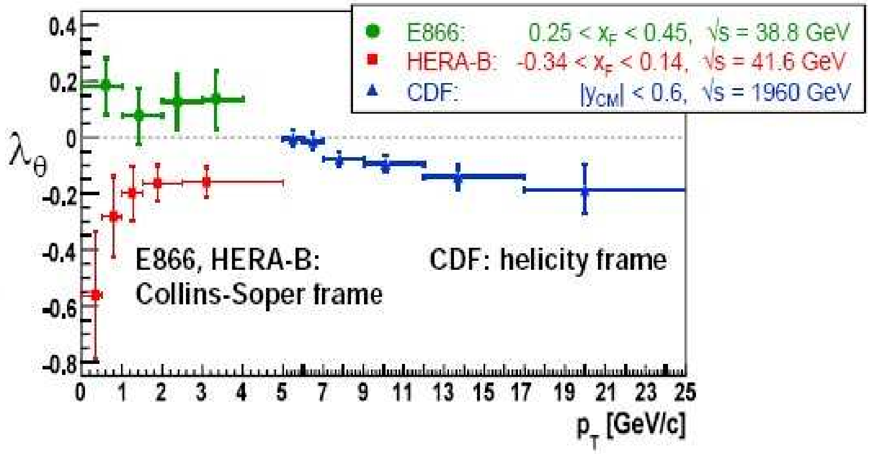


strong "*transverse*" polarization

weak "*longitudinal*" polarization

experiment vs experiment:

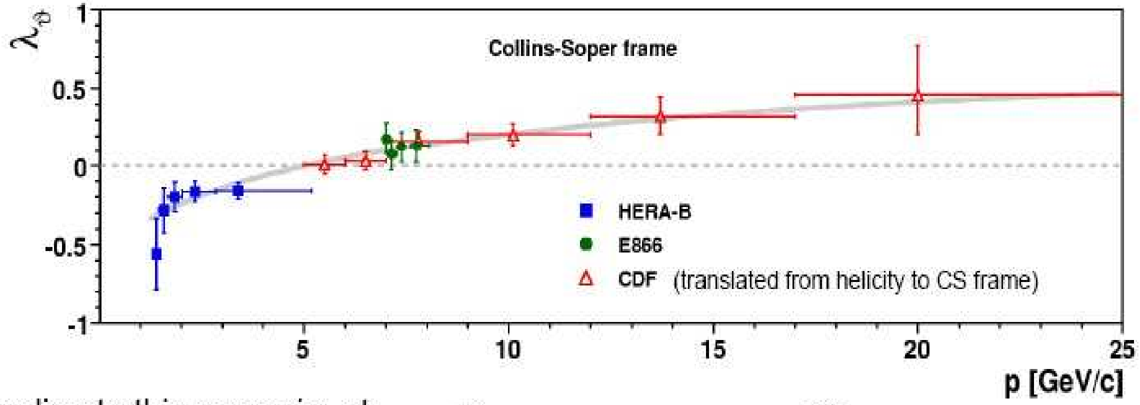
(we address *this* part of the problem)



The "J/ψ polarization puzzle "

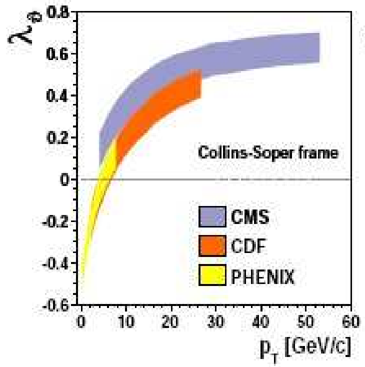
Towards a clearer view of the experimental picture

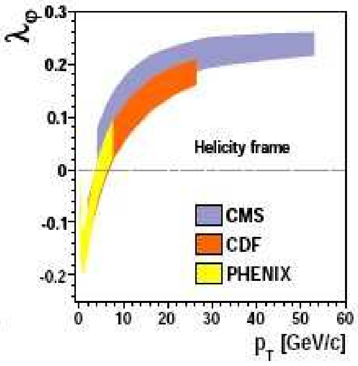
The polarization *depends* on the reference frame. In the Collins-Soper frame (*assumed* to be "the natural frame"), the E866, HERA-B and CDF J/ψ polarization results nicely overlap as a function of the J/ψ cms total momentum, showing that the polarization changes from longitudinal at small momentum to transverse at high momentum:



According to this scenario, at very high-p_T the experiments should observe:

- large transverse polarization in the CS frame
- large azimuthal anisotropy in the helicity frame





Υ polarization in K_T factorization

- Goal: Theoretical predictions on the polarization of the Υ at the Tevatron & LHC
- $p\bar{p}$ and pp X-sections calculated as a convolution of u.g.d. $\mathcal{F}_g(x, k_T^2, \mu^2)$ and partonic X-section $\hat{\sigma}$

$$\sigma_{pp} = \int \mathcal{F}_g(x_1, k_{1T}^2, \mu^2) \cdot \mathcal{F}_g(x_2, k_{2T}^2, \mu^2) \hat{\sigma}_{gg}(x_1, x_2, k_{1T}^2, k_{2T}^2) dx_1 dx_2 dk_{1T}^2 dk_{2T}^2$$

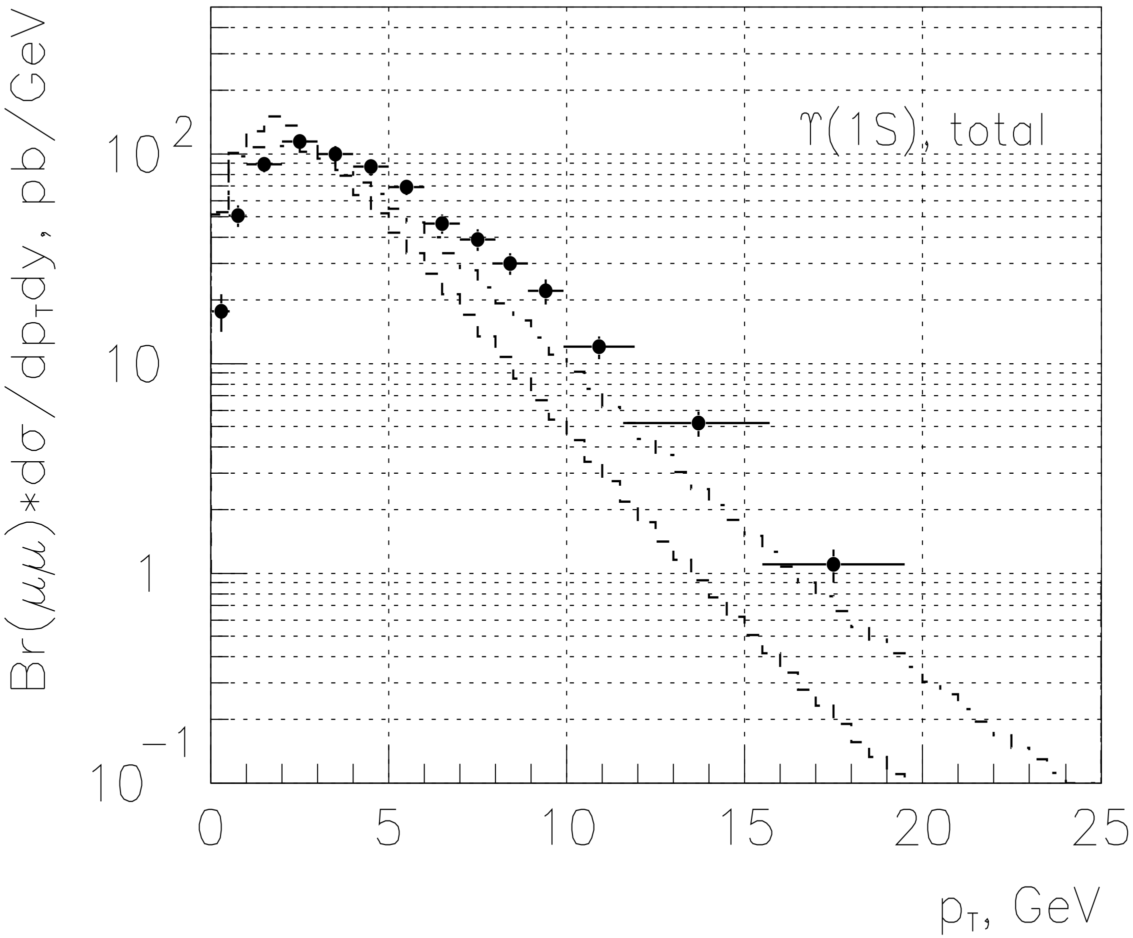
- BFKL prescription for the off-shell gluon spin density matrix:

$$\overline{\epsilon_g^{\mu} \epsilon_g^{*\nu}} = p_p^{\mu} p_p^{\nu} x_g^2 / |k_T|^2 = k_T^{\mu} k_T^{\nu} / |k_T|^2$$

- Choice of u.g.d.:
  - dGRV, derived from the collinear gluon density $G(x, \mu^2)$, differentiating it w.r.t. μ^2 and setting $\mu^2 = k_T^2$
  - JB parametrization [J. Blümlein; J. Phys. G 19 (1993) 1623], solution of leading order BFKL equation in the double logarithmic approximation
- Production Mechanisms:
  - Direct gluon-gluon fusion: $g + g \longrightarrow \Upsilon + g$
  - Production of the P-wave states $g + g \longrightarrow \chi_b \longrightarrow \Upsilon + \gamma$
- Polarization state characterized by the spin alighnment parameter α:

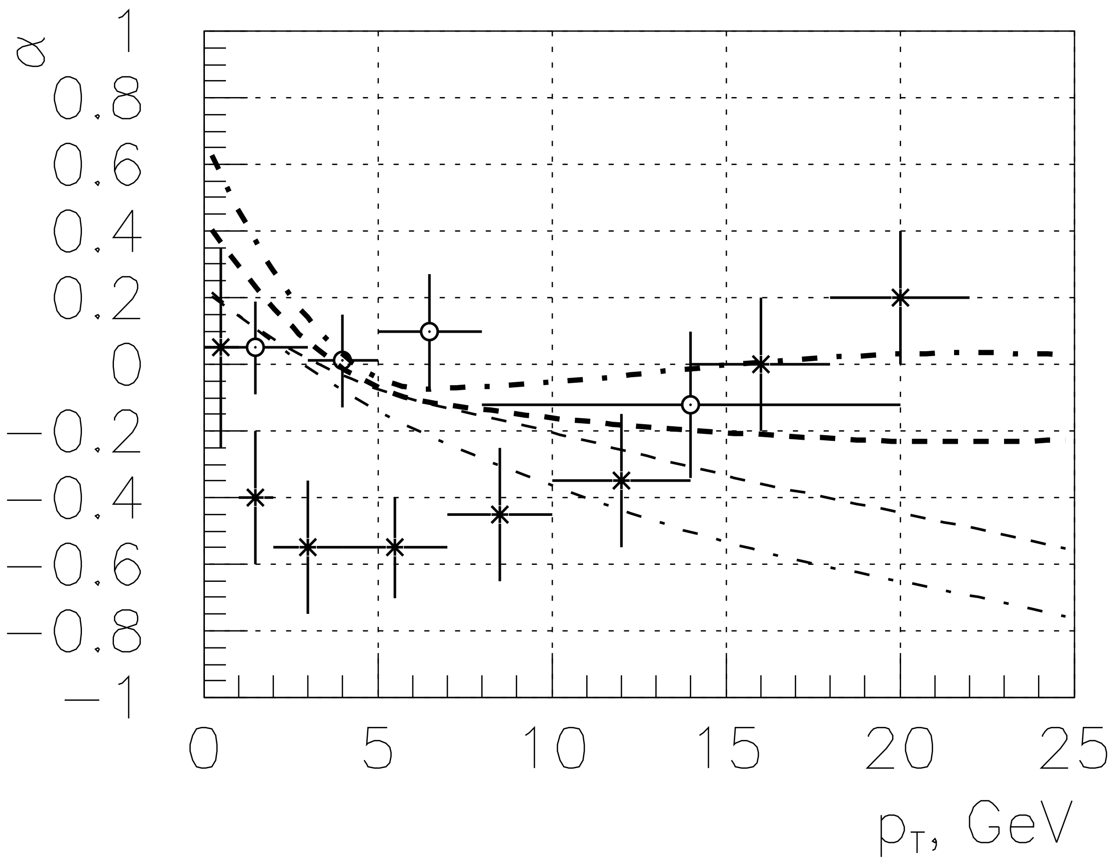
$$\alpha(\mathcal{P}) = (d\sigma/d\mathcal{P} - 3d\sigma_L/d\mathcal{P})/(d\sigma/d\mathcal{P} + d\sigma_L/d\mathcal{P})$$

Υ X-section at the Tevatron & Theory (Zotov)



- Dashed-dotted (upper) histograms: JB Parametrization
- Dashed (lower) dGRV parametrization

Spin alignment parameter $\alpha(\Upsilon)$ vs. p_T (Zotov)



- Upper data points (D0 Run I: Direct Υ and $\chi_b \to \Upsilon + \gamma$) & Theoretical curves (upper: JB, lower: dGRV)
- Lower data points (D0 Run II: Direct Υ) & Theoretical curves (upper: dGRV, lower: JB)
- Conclusion: Need more theoretical studies