Global QCD analysis of polarized PDFs status & prospects

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in collaboration with

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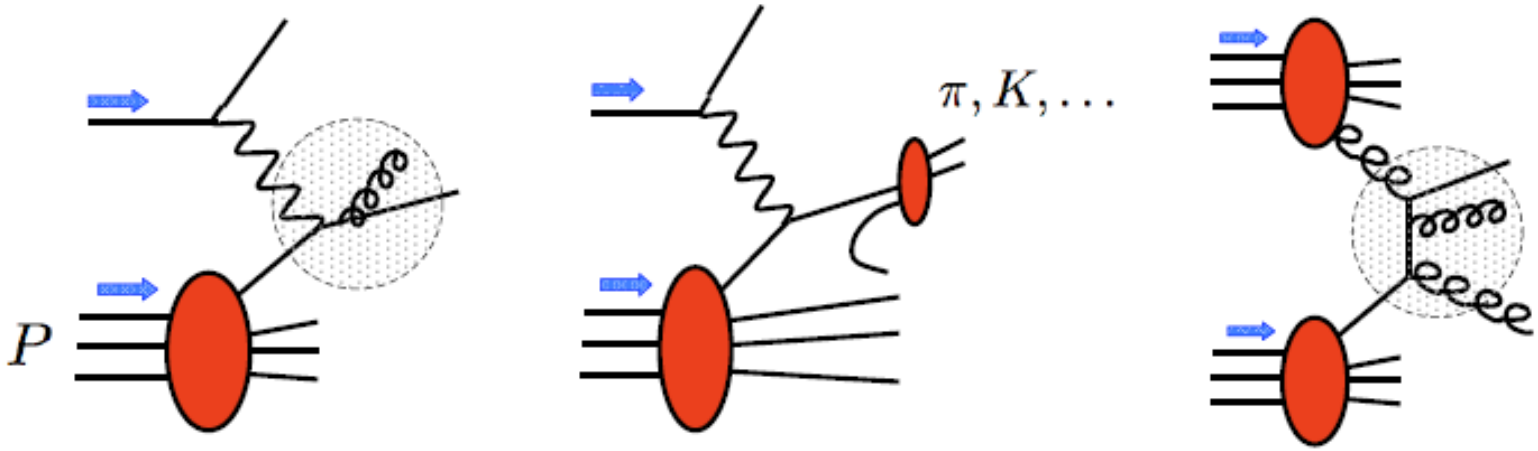
How to determine PDFs from data?

information on nucleon (spin) structure available from

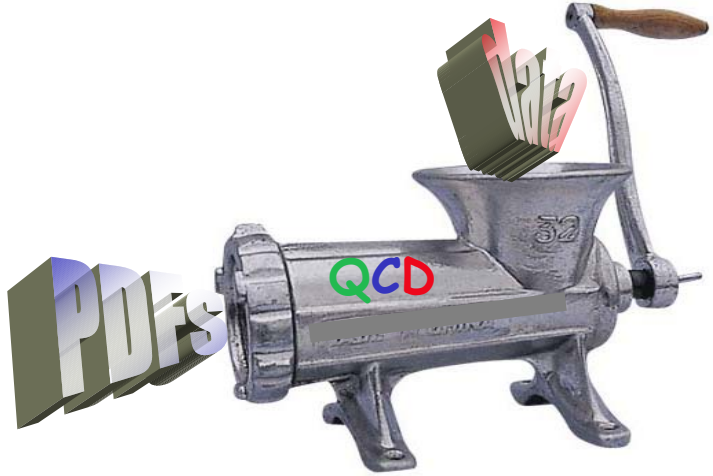


DIS **SIDIS** **hadron-hadron**

task: extract reliable pdfs not just compare some curves to data

- all processes tied together: universality of pdfs & Q^2 - evolution
- each reaction provides insights into *different* aspects and kinematics
- need at least NLO for quantitative analyses; PDFs are not observables!
- information on PDFs "hidden" inside complicated (multi-)convolutions

→ a **"global QCD analysis"** is required



the charge:

analyze a large body of data
from many experiments on different processes
with diverse characteristics and errors
within a theoretical model with many parameters
and hard to quantify uncertainties
without knowing the optimum "ansatz" a priori

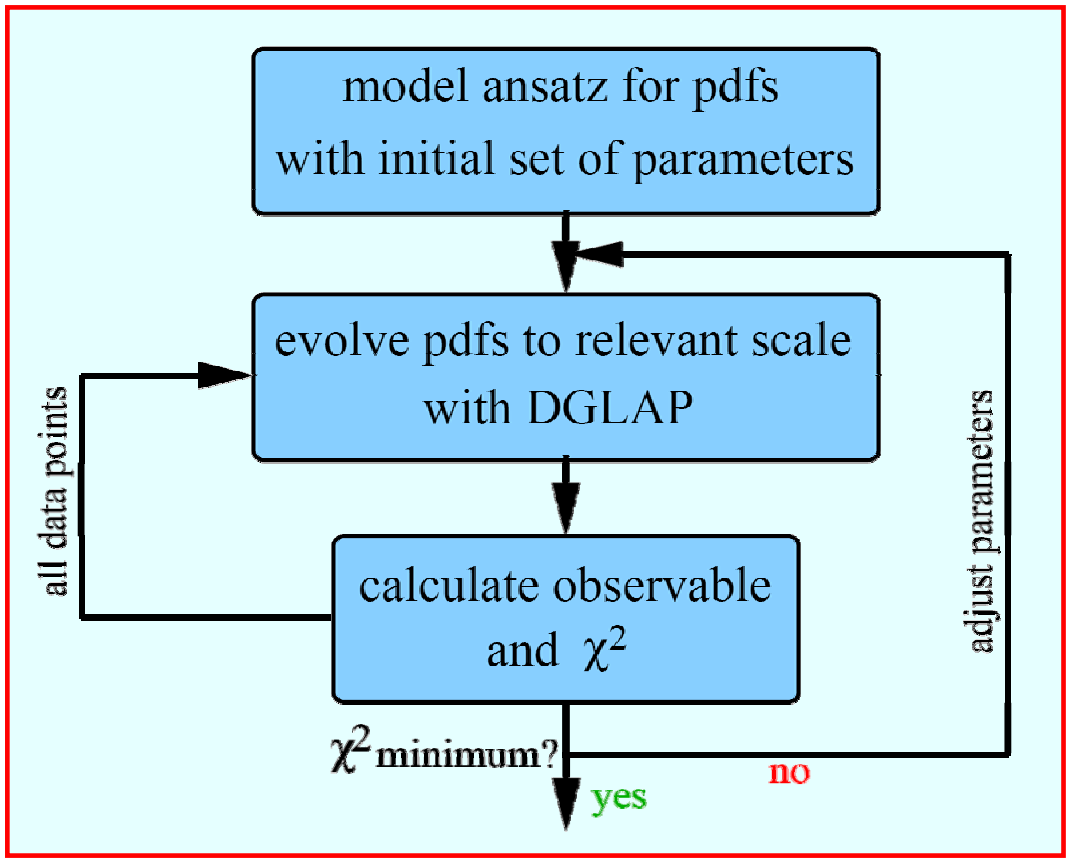
details & results of the DSSV global analysis

- setup
- comparison with data
- uncertainties: Lagrange multipliers vs. Hessian
- emerging picture
- future avenues

Global analysis of helicity parton densities and their uncertainties,
PRL 101 (2008) 072001 (arXiv:0804.0422 [hep-ph])
Extraction of spin-dependent parton densities and their uncertainties,
arXiv:0904.3821 [hep-ph]

outline of a global QCD analysis

start: choose fact. scheme ($\overline{MS}$,...) & pert. order (NLO, ...), select data sets, cuts, ...



parametrize quark and gluon PDFs
a la $\Delta f(x,\mu_0) \simeq x^\alpha (1-x)^\beta$ at
some initial scale $\mu_0 \simeq 1$ GeV

obtain PDFs at any x, $\mu > \mu_0$
relevant for comparing with data

compute DIS, pp, ...cross sections
judge goodness of current fit:

$$\chi^2 = \sum_i \frac{(T_i - E_i)^2}{\delta E_i^2}$$

optimum set of parameters $\{\alpha_i, \beta_i, ...\}$

recent achievement: also quantify **PDF uncertainties** and properly propagate them to any observable of interest

global analysis: a computational challenge

- one has to deal with **O(500) data points** from many processes and experiments
- need to determine **O(20) parameters** describing PDFs at μ_0
- NLO expressions often very complicated $\rightarrow$ computing time becomes excessive
  $\rightarrow$ develop **sophisticated algorithms & techniques**, e.g., based on Mellin moments

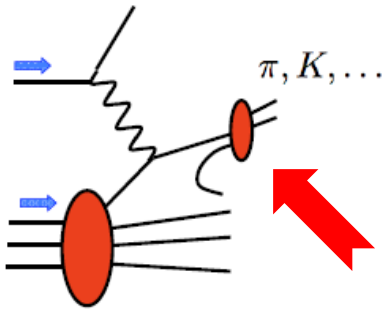
Kosower; Vogt; Vogelsang, MS

DSSV global analysis uses:

	experiment	data type	data point fitted
"classic" inclusive DIS data routinely used in PDF fits $\rightarrow \Delta q + \Delta\bar{q}$	EMC, SMC	DIS	34
	COMPASS	DIS	15
	E142, E143, E154, E155	DIS	123
	HERMES	DIS	39
	HALL-A	DIS	3
	CLAS	DIS	20
semi-inclusive DIS data so far only used in DNS fit $\rightarrow$ flavor separation	SMC	SIDIS, $h^\pm$	48
	HERMES	SIDIS, $h^\pm$	54
		SIDIS, $\pi^\pm$	36
		SIDIS, $K^\pm$	27
	COMPASS	SIDIS, $h^\pm$	24
first RHIC pp data (never used before) $\rightarrow \Delta g$	PHENIX (in part prel.)	200 GeV pp, π^0	20
	PHENIX (prel.)	62 GeV pp, π^0	5
	STAR (in part prel.)	200 GeV pp, jet	19
	TOTAL:		467

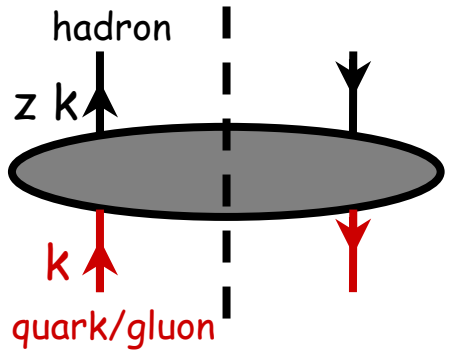
467 data pts in total ($\approx$10% from RHIC)

interlude: fragmentation functions



crucial for pQCD interpretation (factorization!) of data with detected hadrons, e.g., SIDIS (HERMES, COMPASS), $pp \to \pi X$ (PHENIX, ...)

some properties of $D_i^h(z,\mu)$ [very similar to PDFs]:



- non-perturbative but universal; pQCD predicts μ-dep.
- describe the *collinear* transition of a parton "i" into a massless hadron "h" carrying fractional momentum z

global analysis & uncertainty estimates are a recent achievement

DSS fit (de Florian, Sassot, MS)

Phys. Rev. D75 (2007) 114010

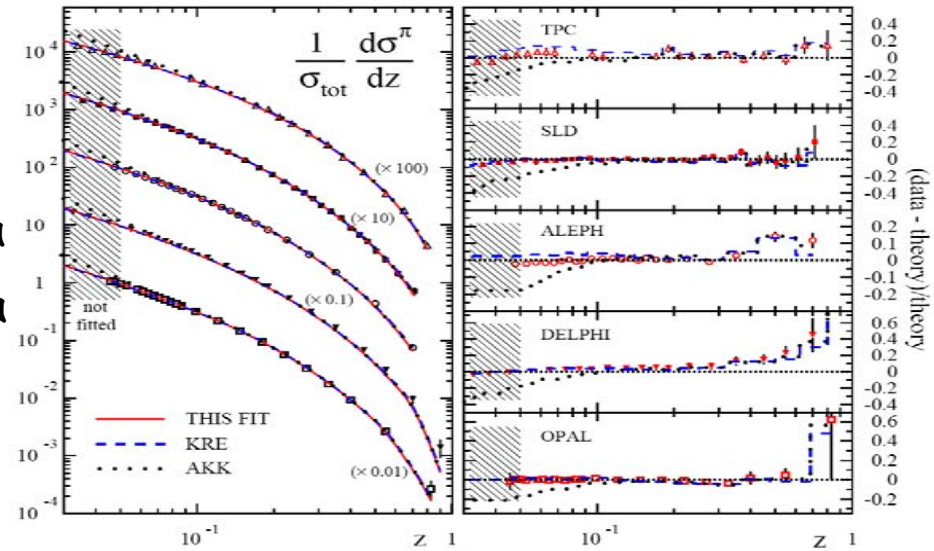
Phys. Rev. D76 (2007) 074033

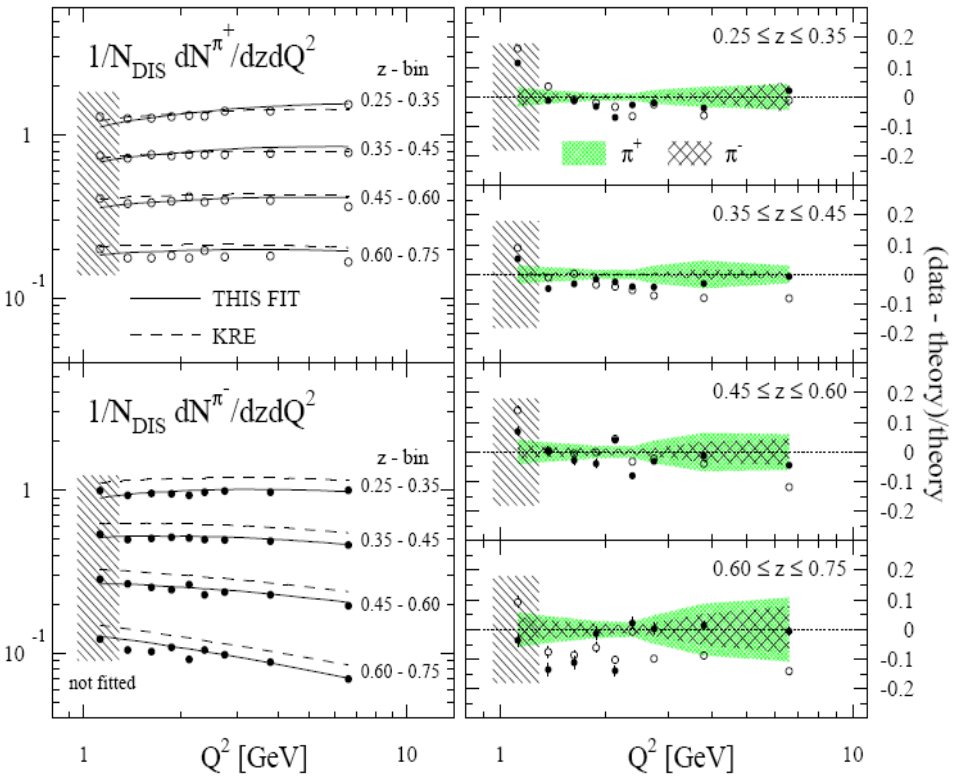
DSS: good global fit of all e^+e^-, ep, and pp hadron data

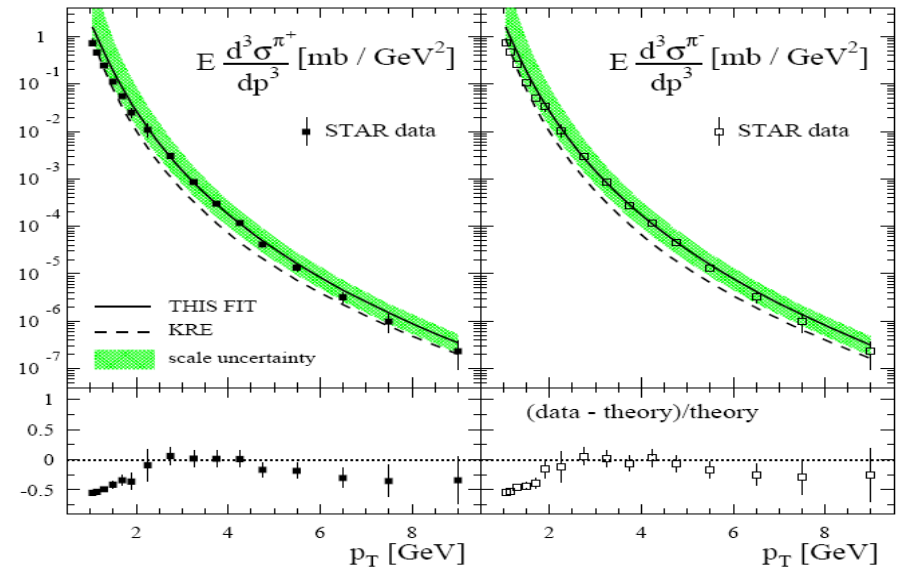
main results:

de Florian, Sassot, MS

- results for $\pi^\pm$, $K^\pm$, chg. hadrons
- full flavor separation for $D_i^H(z)$ and D_g^H
- uncertainties (L.M.) well under control
- fits all LEP, HERMES, SMC, RHIC, … data
- supersede old fits based only on e^+e^- data







setup of DSSV analysis

- flexible, MRST-like input form

possible nodes

$$x\Delta f_j(x, 1\,\text{GeV}) = N_j\, x^{\alpha_j}\,(1-x)^{\beta_j}\left[1 + \kappa_j\sqrt{x} + \gamma_j x\right]$$

input scale

simplified form for sea quarks and Δg: $\kappa_j = 0$

- take α_s from MRST; also use MRST for positivity bounds
- NLO fit, $\overline{\text{MS}}$ scheme
- avoid assumptions on parameters $\{a_j\}$ unless data cannot discriminate

need to impose: $\alpha_{\bar{u}} = \alpha_{u+\bar{u}}$ $\qquad \alpha_{\bar{d}} = \alpha_{\bar{s}} = \alpha_{d+\bar{d}}$

let the fit decide about F,D value constraint on 1st moments:

$$\Delta u_{\text{tot}} - \Delta d_{\text{tot}} = \underbrace{(F+D)}_{1.269\pm0.003}[1 + \varepsilon_{SU(2)}]$$

$$\Delta u_{\text{tot}} + \Delta d_{\text{tot}} - 2\Delta s_{\text{tot}} = \underbrace{(3F-D)}_{0.586\pm0.031}[1 + \varepsilon_{SU(3)}]$$

fitted (end up close to zero)

details & results of the DSSV global analysis

- setup
- comparison with data
- uncertainties: Lagrange multipliers vs. Hessian
- the emerging picture
- future avenues

overall quality of the global fit

very good!

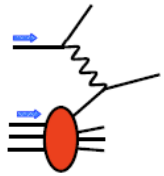
no significant tension
among different data sets

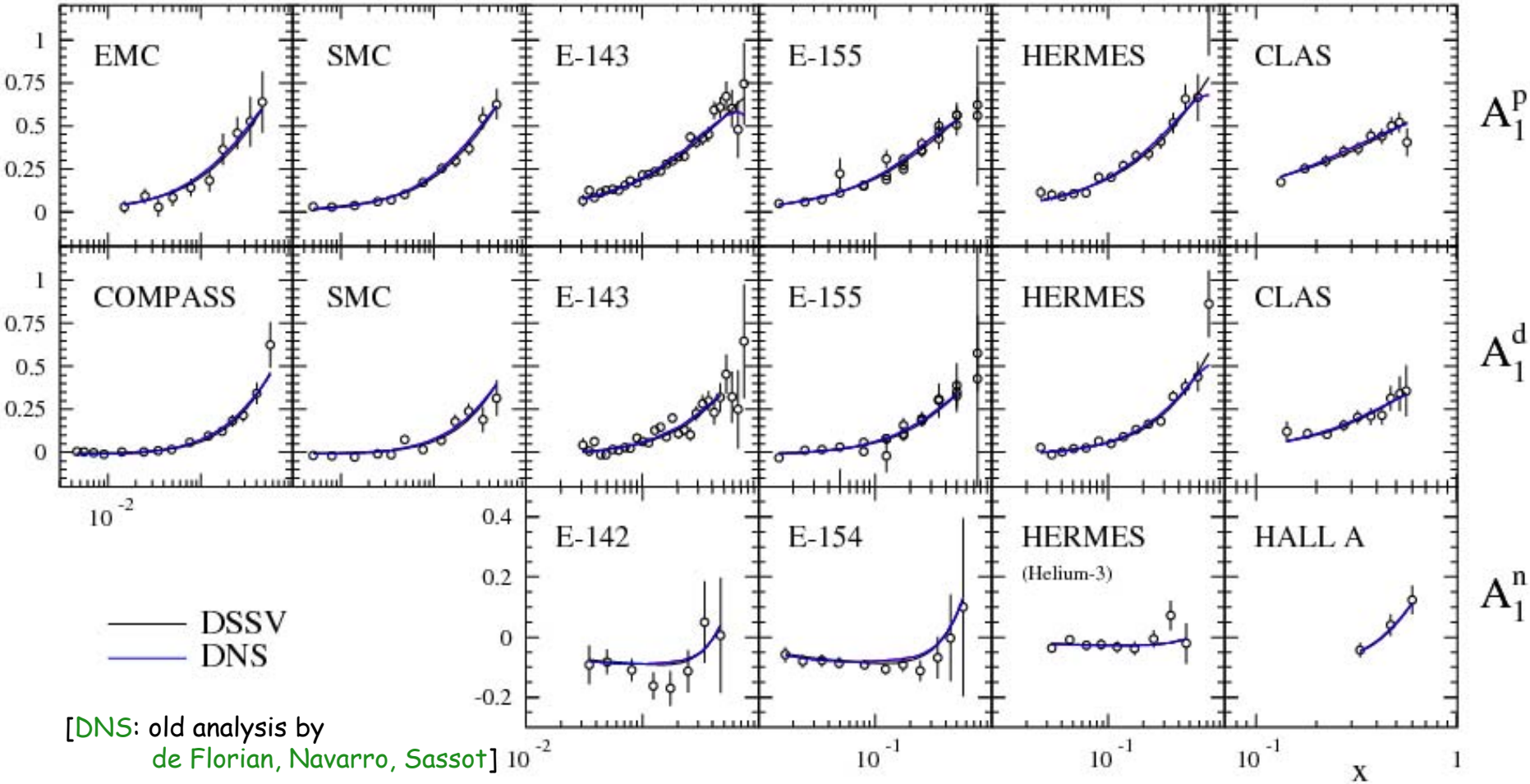
χ^2/d.o.f. $\simeq$ 0.88

note: for the time being, stat. and syst. errors are added in quadrature

experiment	process	$N_{\rm data}$	χ^2
EMC [2]	DIS (p)	10	3.9
SMC [3]	DIS (p)	12	3.4
SMC [3]	DIS (d)	12	18.4
COMPASS [4]	DIS (d)	15	8.1
E142 [5]	DIS (n)	8	5.6
E143 [6]	DIS (p)	28	19.3
E143 [6]	DIS (d)	28	40.8
E154 [7]	DIS (n)	11	4.5
E155 [8]	DIS (p)	24	22.6
E155 [9]	DIS (d)	24	17.1
HERMES [10]	DIS (He)	9	6.3
HERMES [11]	DIS (p)	15	10.5
HERMES [11]	DIS (d)	15	16.9
HALL-A [12]	DIS (n)	3	0.2
CLAS [13]	DIS (p)	10	5.9
CLAS [13]	DIS (d)	10	2.5
SMC [14]	SIDIS (p, h^+)	12	18.7
SMC [14]	SIDIS (p, h^-)	12	10.6
SMC [14]	SIDIS (d, h^+)	12	7.3
SMC [14]	SIDIS (d, h^-)	12	14.1
HERMES [15]	SIDIS (p, h^+)	9	6.4
HERMES [15]	SIDIS (p, h^-)	9	4.9
HERMES [15]	SIDIS (d, h^+)	9	11.4
HERMES [15]	SIDIS (d, h^-)	9	4.5
HERMES [10]	SIDIS (He, h^+)	9	4.7
HERMES [10]	SIDIS (He, h^-)	9	6.9
HERMES [15]	SIDIS (p, π^+)	9	9.6
HERMES [15]	SIDIS (p, π^-)	9	4.9
HERMES [15]	SIDIS (d, π^+)	9	9.4
HERMES [15]	SIDIS (d, π^-)	9	19.5
HERMES [15]	SIDIS (d, K^+)	9	6.2
HERMES [15]	SIDIS (d, K^-)	9	5.8
HERMES [15]	SIDIS (d, $K^+ + K^-$)	9	3.4
COMPASS [16]	SIDIS (d, h^+)	12	6.2
COMPASS [16]	SIDIS (d, h^-)	12	12.0
PHENIX [22]	pp (200 GeV, π^0)	10	14.2
PHENIX [23]	pp (200 GeV, π^0)	10	7.1 [13.8] [a]
PHENIX [24]	pp (62 GeV, π^0)	5	3.1 [2.8] [a]
STAR [25]	pp (200 GeV, jet)	10	8.8
STAR (prel.) [26]	pp (200 GeV, jet)	9	6.9
TOTAL:		467	392.6

spin asymmetries in inclusive DIS

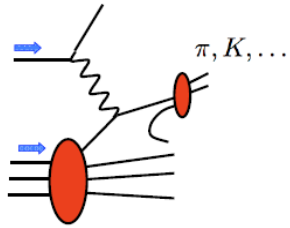


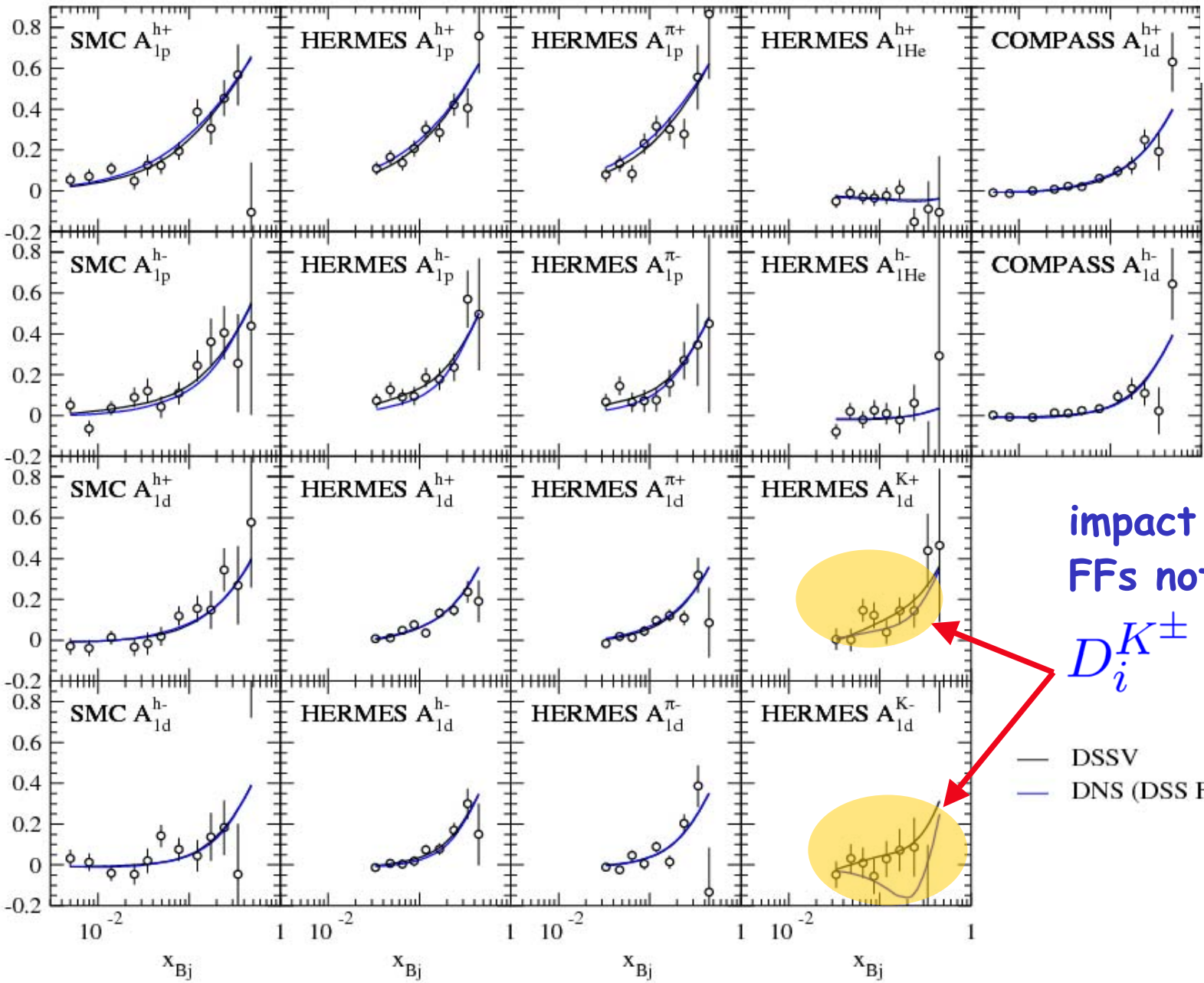


[DNS: old analysis by de Florian, Navarro, Sassot]

- we account for kinematical "mismatches" in $A_1 = (1+\gamma^2)g_1/F_1$
  no need for any dynamical higher twist (contrary to Leader et al.)

spin asymmetries in semi-inclusive DIS



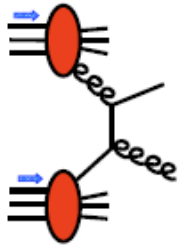


impact of new FFs noticeable!

$D_i^{K^\pm}$

gluons are key players at RHIC

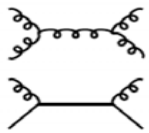
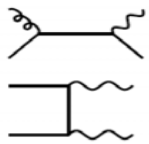
many QCD processes with a **dominant gluon contribution** already at the tree-level:



high-p_T jet, pion, heavy quark, …

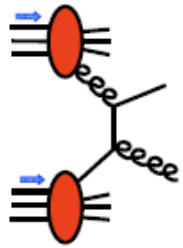
unpolarized "reference data" (π, jets, γ) nicely agree with pQCD

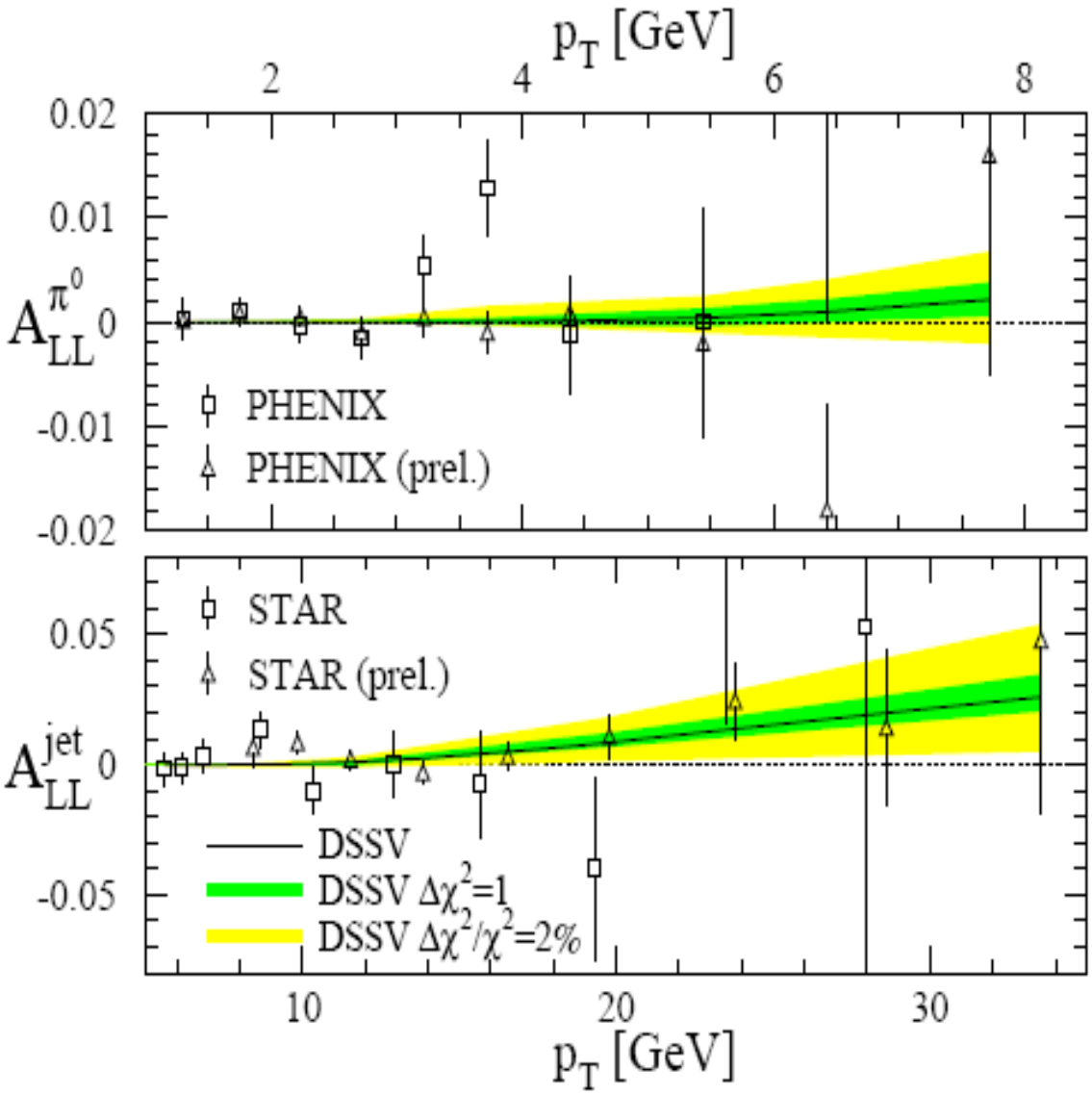
all available at NLO

	Reaction	Dom. partonic process	probes	LO Feynman diagram	
hadrons	$\vec{p}\vec{p} \to \pi + X$ [61, 62]	$\vec{g}\vec{g} \to gg$ $\vec{q}\vec{g} \to qg$	Δg		Jäger,Schäfer,MS, Vogelsang; de Florian
jets	$\vec{p}\vec{p} \to \text{jet(s)} + X$ [71, 72]	$\vec{g}\vec{g} \to gg$ $\vec{q}\vec{g} \to qg$	Δg	(as above)	Jäger,MS,Vogelsang; Signer et al.
photons	$\vec{p}\vec{p} \to \gamma + X$ $\vec{p}\vec{p} \to \gamma + \text{jet} + X$ $\vec{p}\vec{p} \to \gamma\gamma + X$ [67, 73, 74, 75, 76]	$\vec{q}\vec{g} \to \gamma q$ $\vec{q}\vec{g} \to \gamma q$ $\vec{q}\vec{\bar{q}} \to \gamma\gamma$	Δg Δg $\Delta q, \Delta\bar{q}$		Gordon,Vogelsang; Contogouris et al.
heavy flavors	$\vec{p}\vec{p} \to DX, BX$ [77]	$\vec{g}\vec{g} \to c\bar{c}, b\bar{b}$	Δg		Bojak,MS; Riedl,Schäfer,MS

decisive data start to emerge from RHIC …

RHIC pp data (inclusive π^0 or jet)





- good agreement
- important constraint on $\Delta g(x)$ despite large uncertainties

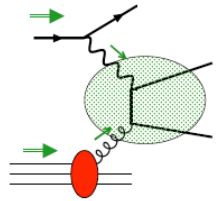
  $\rightarrow$ later

uncertainty bands estimated with Lagrange multipliers by enforcing other values for A_{LL}

Δg in lepton-proton scattering

gluons in DIS: a (small) NLO effect [they don't couple directly to the photon]

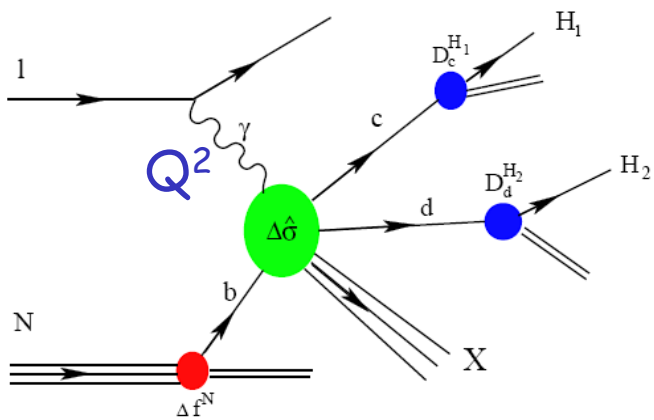
→ study processes sensitive to **photon-gluon-fusion**

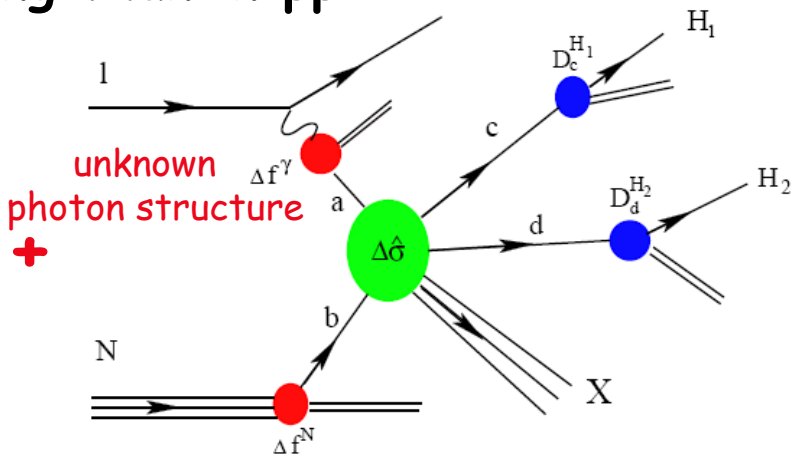


data available for one/two hadron production, charm

COMPASS
HERMES

theory calculations more challenging than in pp:





Q^2 large: "electroproduction"

if $Q^2 \simeq 0$: "photoproduction"

NLO pQCD

nothing for $Q^2 \neq 0$

almost done

1-hadron: ✓ Jäger,MS,Vogelsang

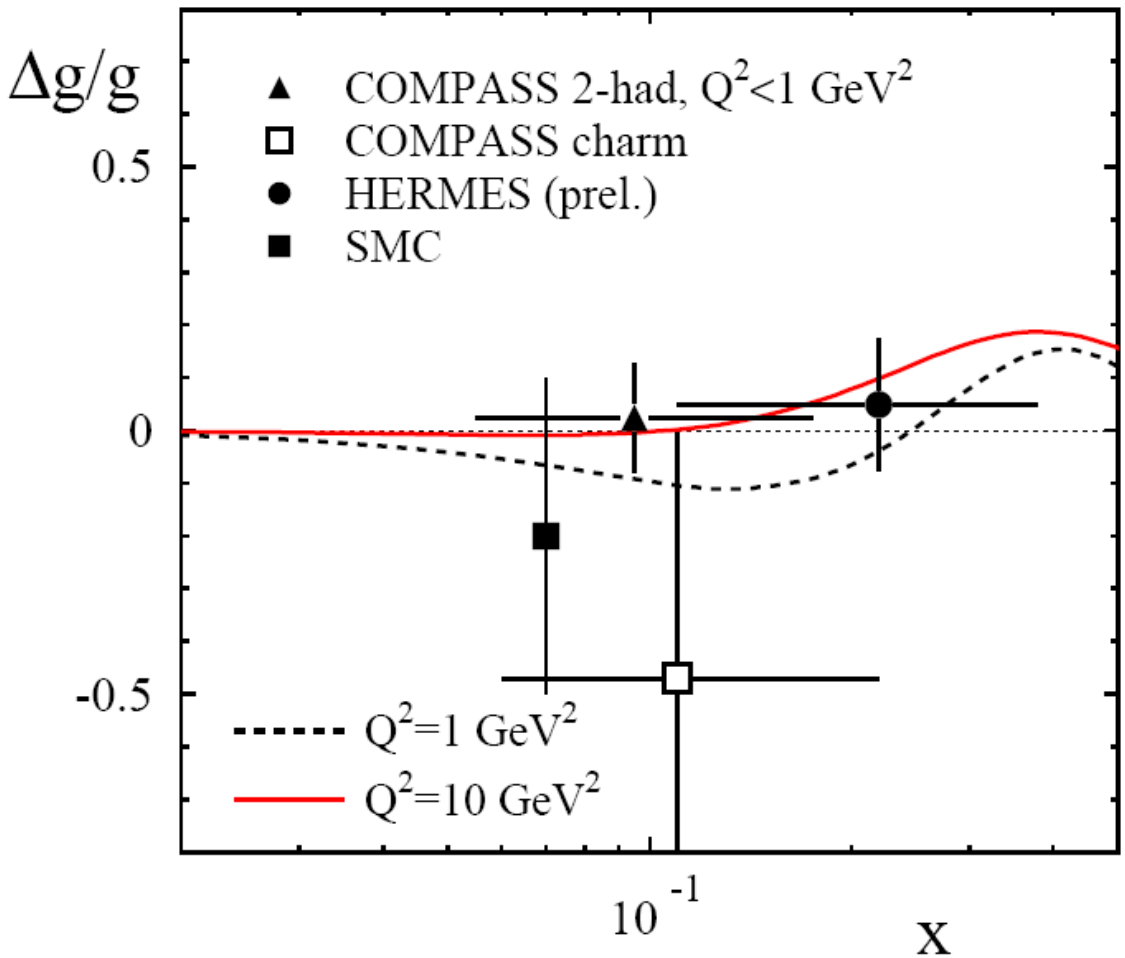
charm: ✓ Bojak, MS (direct γ)

Riedl, Schäfer, MS (resolved γ & MC)

hadron pairs: Hendlmeier, Schäfer, MS (direct γ)

Jäger,Owens,MS,Vogelsang (resolved γ)

DSSV gluon agrees well with model-dependent "LO" extractions of $\Delta g/g$



not in global fit
[NLO not available]

a future global NLO fit will use **measured** A_{LL} not derived $\Delta g/g$
need to check unpolarized cross section as well

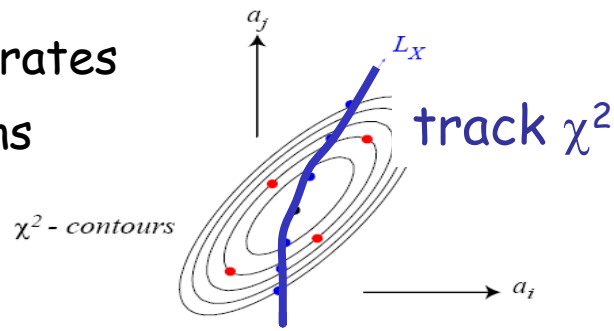
details & results of the DSSV global analysis

- setup
- comparison with data
- **uncertainties:** Lagrange multipliers vs. Hessian
- emerging picture
- future avenues

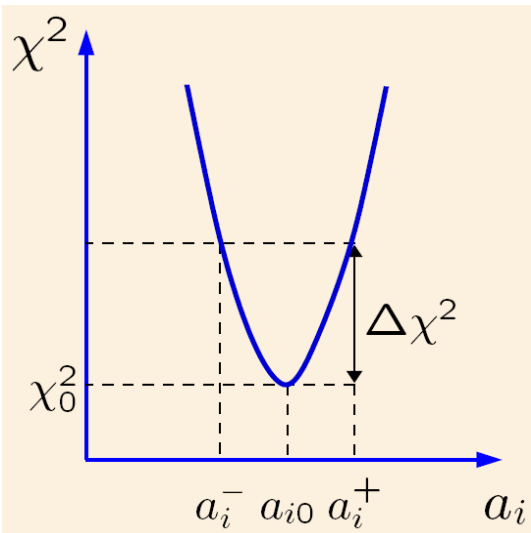
estimating PDF uncertainties

mainly two methods in use: [reshaped for PDF analyses by J. Pumplin and CTEQ]

- **Hessian method:** classic tool, explores vicinity of χ^2-minimum in quadratic approx.; often unstable for multi-parameter PDF analyses
- **Lagrange multiplier:** track how the fit deteriorates when PDFs are forced to give different predictions for selected observables; explores the full paramater space indep. of approximations



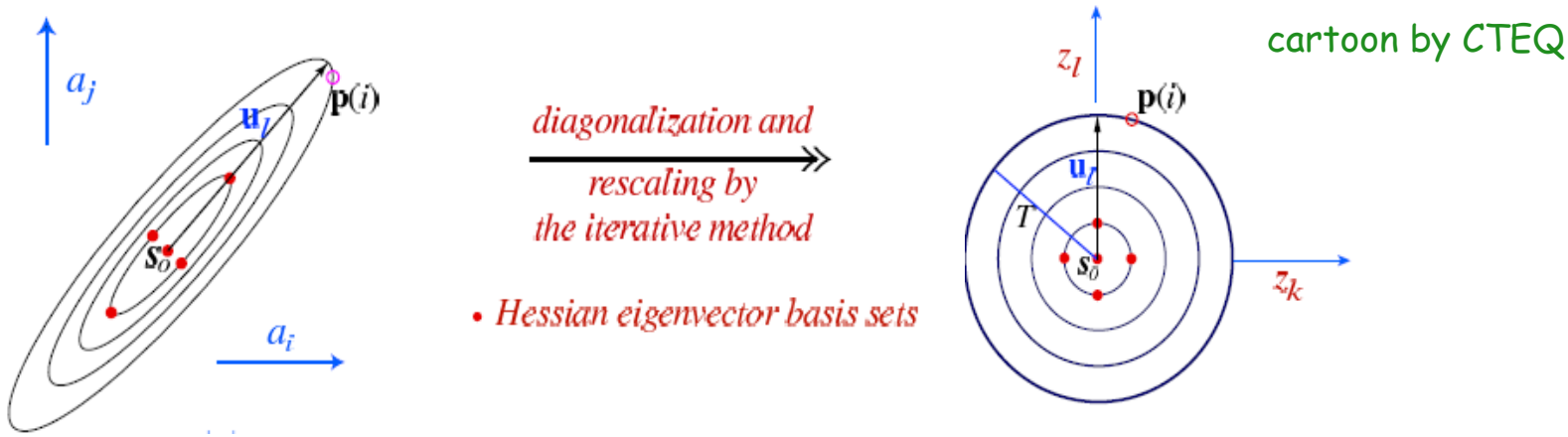
issue: what value of $\Delta\chi^2$ (tolerance) defines a 1-σ error?



- non Gaussian errors, χ^2 "landscape" not parabolic
- uncertainties with diverse characteristics
- theor. errors correlated and poorly known
- data sets often marginally consistent for $\Delta\chi^2=1$

we present uncertainties bands for both $\Delta\chi^2$ = 1 and a more pragmatic 2% increase in χ^2

Hessian eigenvector PDF basis sets



- eigenvectors provide an optimized orthonormal basis near the minimum
- construct $2N_{par}$ eigenvector basis sets $S_k^{\pm}$ by displacing each z_k by ± 1
- the "coordinates" are rescaled such that $\Delta\chi^2 = \sum_k z_k^2$
- sets $S_k^{\pm}$ can be used to calculate uncertainties of observables O_i

$$\Delta\mathcal{O}_i = \frac{1}{2}\left(\sum_{k=1}^{N_{\rm par}}\left[\mathcal{O}_i(S_k^+) - \mathcal{O}_i(S_k^-)\right]^2\right)^{1/2}$$

38 DSSV eigenvector sets are available from ribf.riken.jp/~marco/DSSV

(after DIS'09)

details & results of the DSSV global analysis

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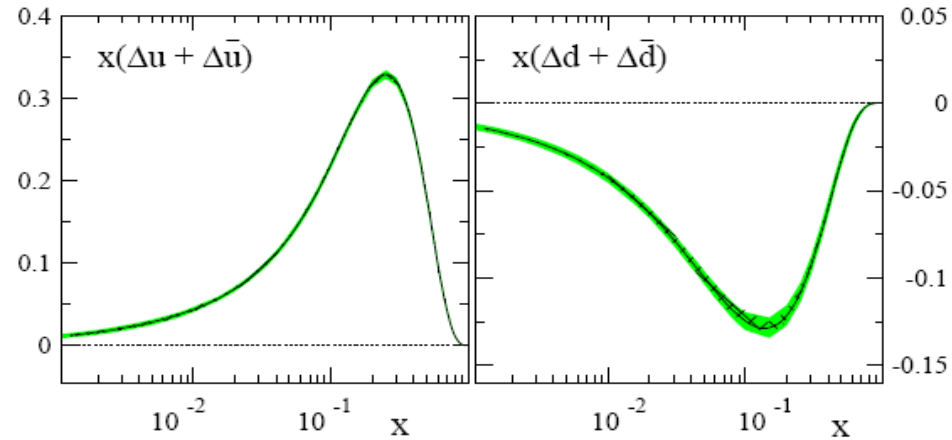
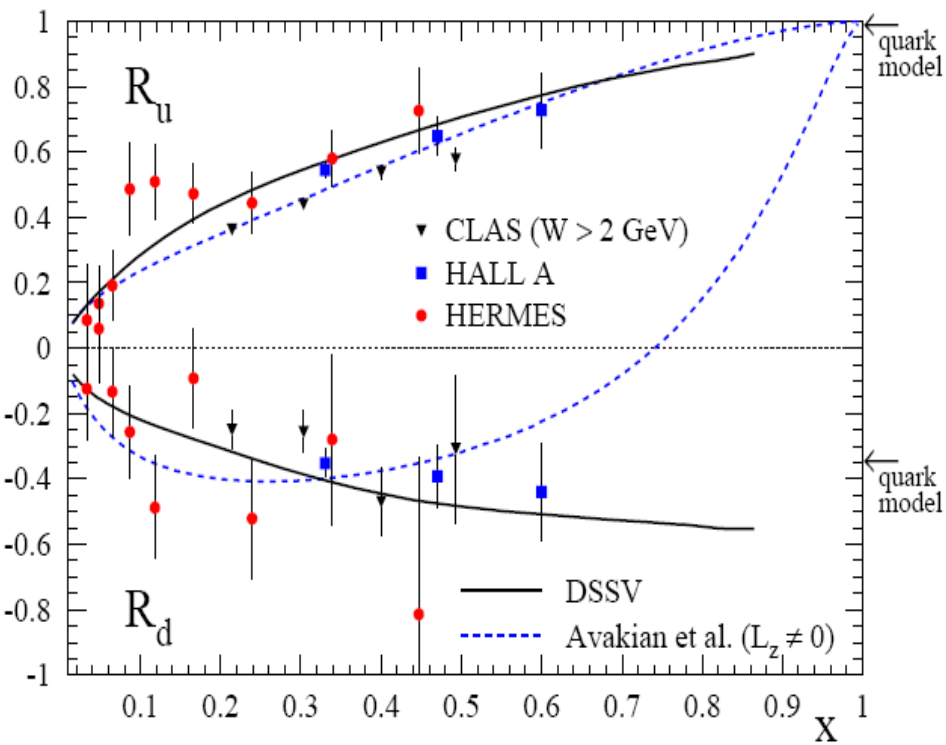
- uncertainties: Lagrange multipliers vs. Hessian
- emerging picture
- future avenues

DSSV valence quark polarizations

- **best determined**
  - uncertainty bands very narrow
  - agrees well with previous "DIS-only" fits





- **large-x region (x→1)**

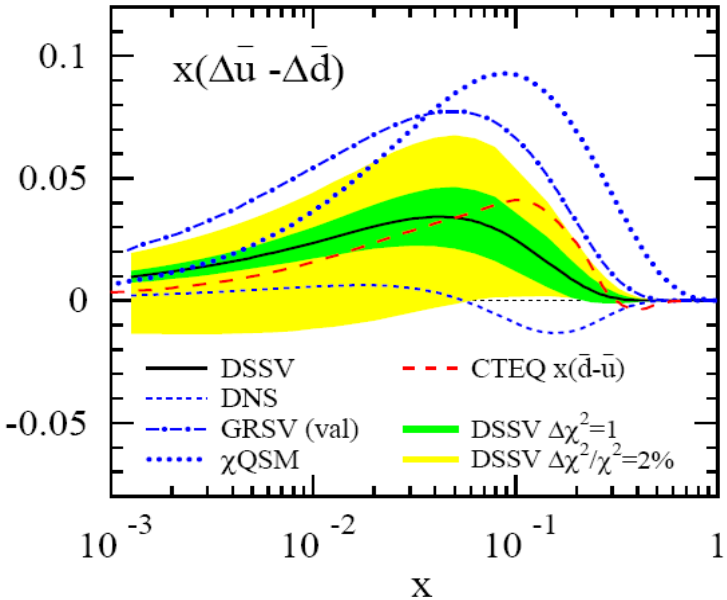
$$R_q(x) \equiv \frac{\Delta q(x) + \Delta \bar{q}(x)}{q(x) + \bar{q}(x)}$$

- R_u (x→ 1) → 1 as expected
- R_d (x→ 1) remains negative
- counting rules + helicity retention + nonzero OAM: expect R_d (x→ 1) → 1

Avakian, Brodsky, Deur, Yuan

DSSV sea polarizations

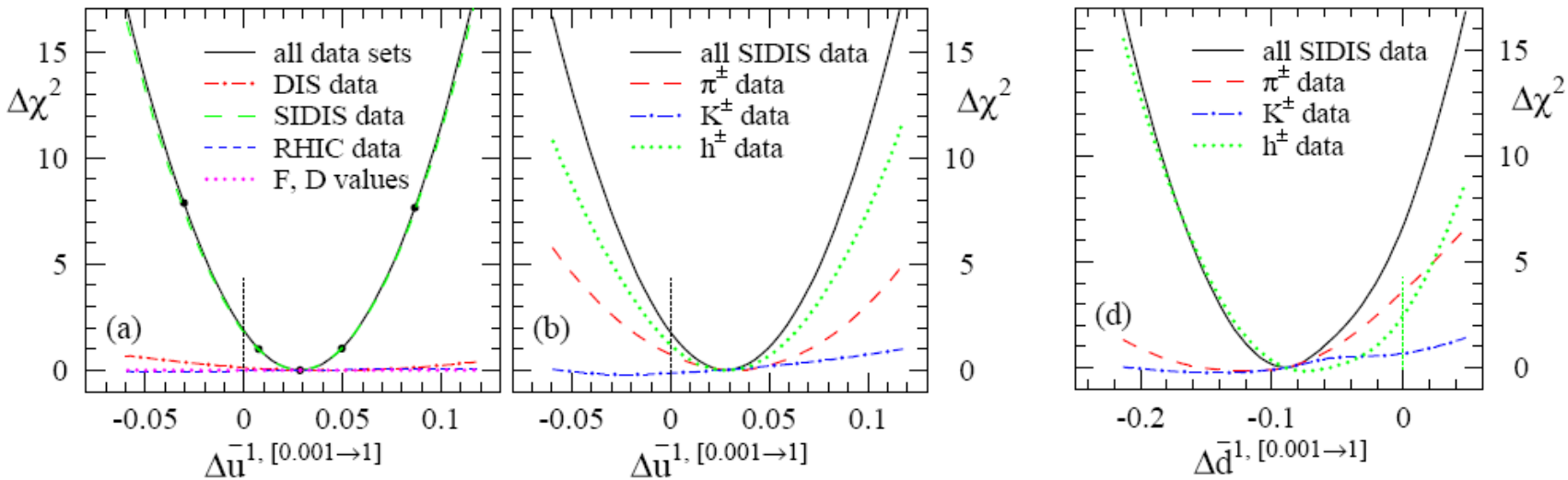
- **indications for an SU(2) breaking of light u,d sea**



- breaking of similar size than in unpol. case
- mainly determined by SIDIS data
- "bands": error estimate from Lagr. mult.
- similar patterns in many models:
  large-N_C, chiral quark soliton, meson cloud

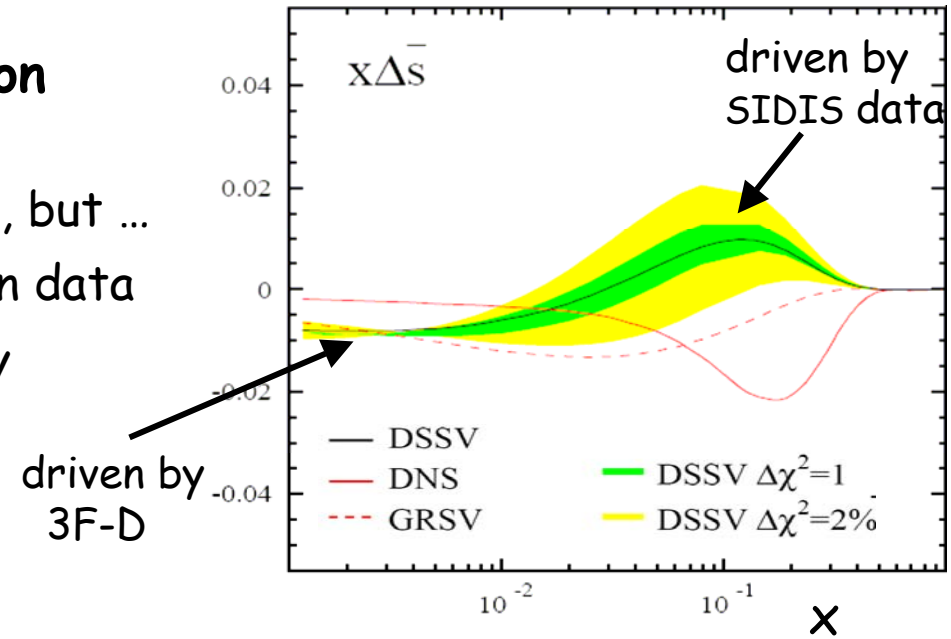
  Thomas, Signal, Cao; Diakonov, Polyakov, Weiss; …

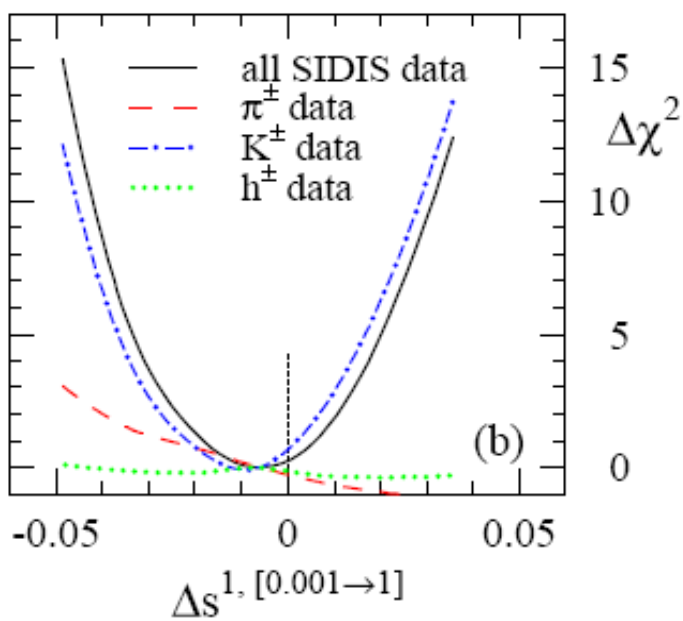
χ^2 profiles for truncated moments:



DSSV sea polarizations – cont'd

- **a strange strangeness polarization**
  - $\Delta s(x)$ always thought to be negative, but …
  - mainly determined from SIDIS kaon data
  - consistent with LO-type analyses by HERMES and COMPASS





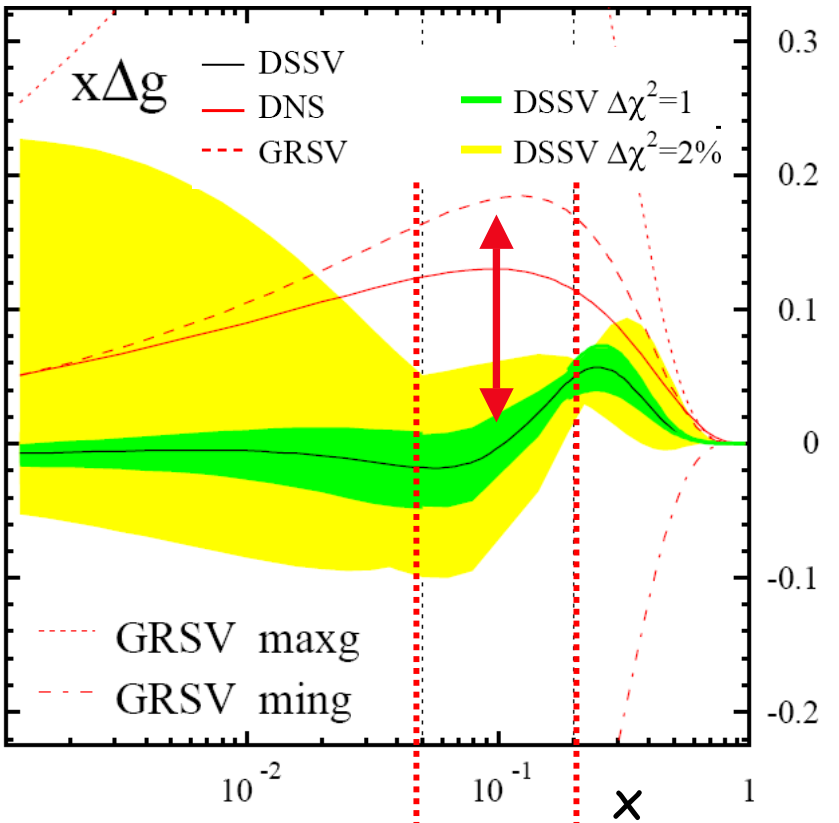
striking result, but relies on

- kaon fragmentation
  more data available soon (BELLE, …)
- unpolarized PDFs
  unpol. strangeness not well determined

needs further studies – exp. & theory !

DSSV gluon polarization

error estimates more delicate: small-x behavior completely unconstrained



small-x
$0.001 \leq x \leq 0.05$

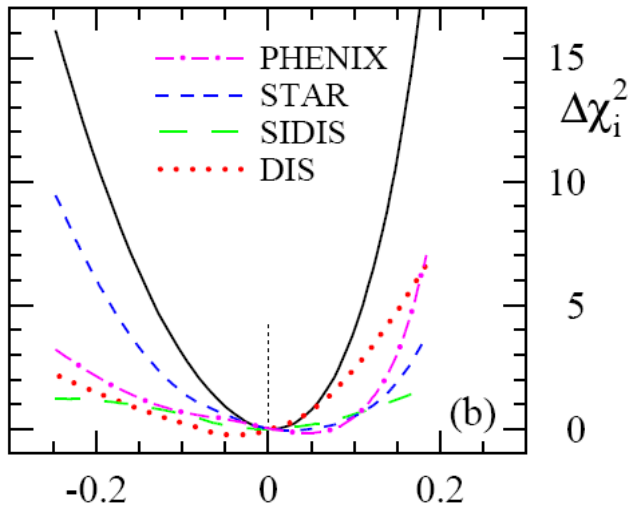
RHIC range
$0.05 \leq x \leq 0.2$

large-x
$x \geq 0.2$

study uncertainties in 3 x-regions

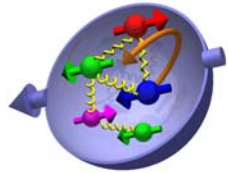
find

- $\Delta g(x)$ very small at medium x (even compared to GRSV or DNS)
- best fit has a node at $x \simeq 0.1$
- huge uncertainties at small x



$$\delta g \equiv \int_{0.05}^{0.2} \Delta g(x, 10\,\mathrm{GeV}^2)\,dx$$

spin audit: 1st moments and the spin of the proton



"helicity sum rule" Jaffe, Manohar; Ji; ...

$$\frac{1}{2}\hbar = \langle P, \frac{1}{2} | J^z_{\rm QCD} | P, \frac{1}{2} \rangle = \sum_q \frac{1}{2} S^z_q + S^z_g + \sum_q L^z_q + L^z_g$$

$A^+ = 0$ gauge, IMF
partonic interpretation

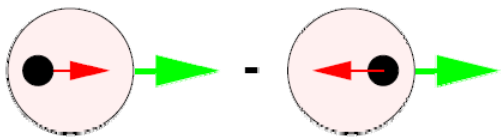
total u+d+s quark spin | gluon spin | angular momentum

"quotable" properties of the nucleon !

momentum fraction

$$\Delta f(x) \equiv f_+^{N_+}(x) - f_-^{N_+}(x)$$

helicity parton densities



x-moment $\xrightarrow{\int_0^1 dx}$ **total spin polarizations S_q and S_g !**

nothing is known yet about L_q and L_g

lattice results for "angular momentum" are for a "different" sum rule (Ji's version) w/o partonic interpretation – they cannot be mixed!

numerical results

$Q^2 = 10\ GeV^2$

$$\Delta f^{1,[x_{\min},1]} \equiv \int_{x_{\min}}^{1} \Delta f(x)dx$$

	$x_{\min} = 0$	$x_{\min} = 0.001$	
	best fit	$\Delta\chi^2 = 1$	$\Delta\chi^2/\chi^2 = 2\%$
$\Delta u + \Delta\bar{u}$	0.813	$0.793\ ^{+0.011}_{-0.012}$	$0.793\ ^{+0.028}_{-0.034}$
$\Delta d + \Delta\bar{d}$	-0.458	$-0.416\ ^{+0.011}_{-0.009}$	$-0.416\ ^{+0.035}_{-0.025}$
$\Delta\bar{u}$	0.036	$0.028\ ^{+0.021}_{-0.020}$	$0.028\ ^{+0.059}_{-0.059}$
$\Delta\bar{d}$	-0.115	$-0.089\ ^{+0.029}_{-0.029}$	$-0.089\ ^{+0.090}_{-0.080}$
$\Delta\bar{s}$	-0.057	$-0.006\ ^{+0.010}_{-0.012}$	$-0.006\ ^{+0.028}_{-0.031}$
Δg	-0.084	$0.013\ ^{+0.106}_{-0.120}$	$0.013\ ^{+0.702}_{-0.314}$
$\Delta\Sigma$	0.242	$0.366\ ^{+0.015}_{-0.018}$	$0.366\ ^{+0.042}_{-0.062}$

- Δs receives a large negative contribution at small x
- Δg: huge uncertainties below $x \simeq 0.01$ → **1st moment still undetermined**

very difficult to give reliable estimates for full moments

both quark and gluons may not contribute much to proton spin but we need to go to smaller x to settle this issue

→ case for a high-energy polarized ep-collider

details & results of the DSSV global analysis

- setup
- comparison with data
- uncertainties: Lagrange multipliers vs. Hessian
- emerging picture
- future avenues

getting ready to analyze new types of data

- **milestone:** RHIC has just completed the 1st 500 GeV run
- hope for O(50pb^{-1}) with 60% pol. from current 200 GeV run

expect:

- significant improvement of existing inclusive jet + π^0 data sample
- charged pion data

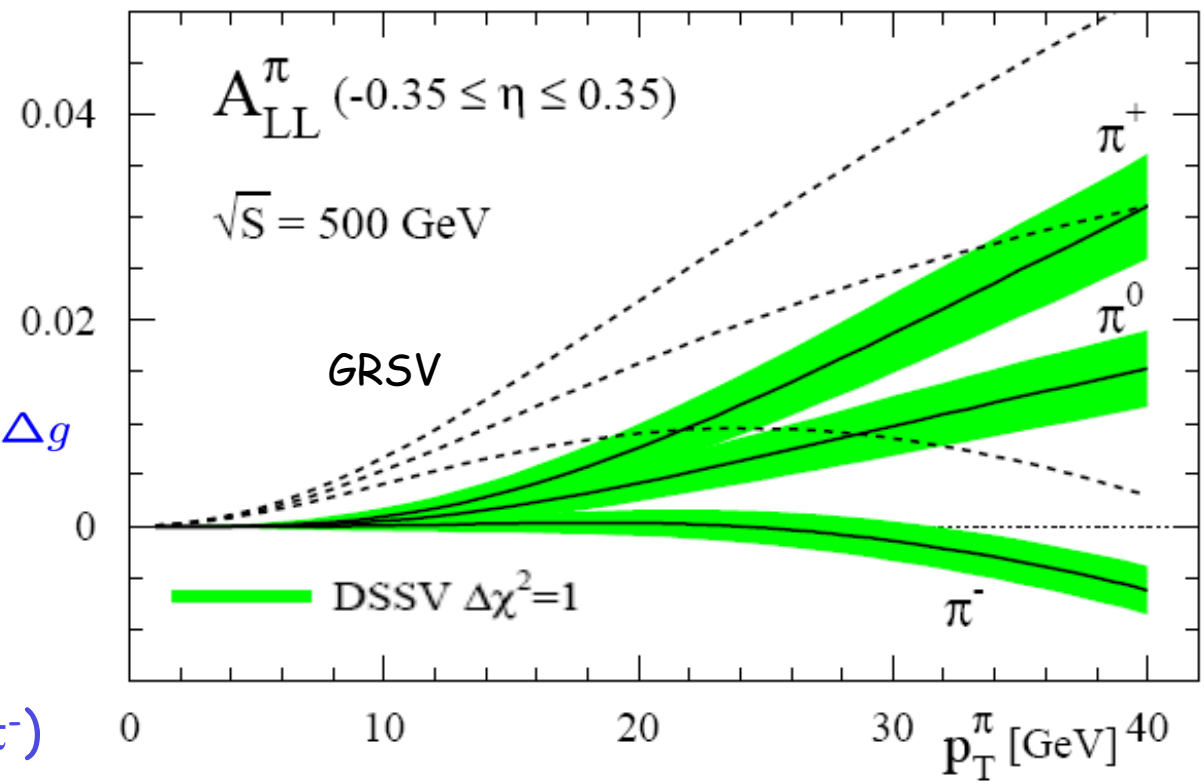
  at large p_T driven by

  - qg scattering

    $[\Delta u \otimes D_u^\pi + \Delta\bar{u} \otimes D_{\bar{u}}^\pi + \Delta d \otimes D_d^\pi + \Delta\bar{d} \otimes D_{\bar{d}}^\pi] \otimes \Delta g$

  - sign of Δg

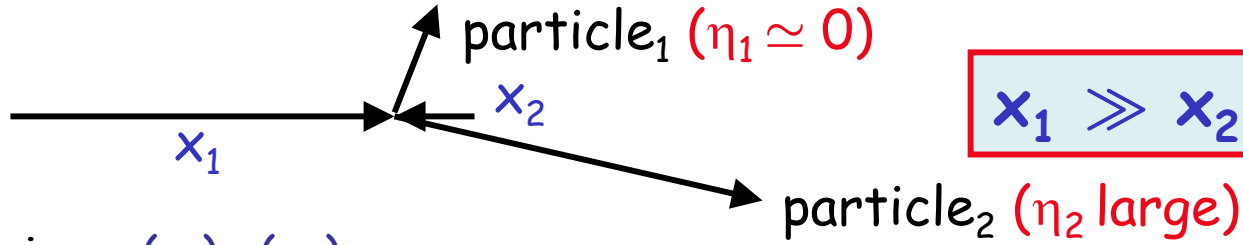
$A_{LL}(\pi^+) > A_{LL}(\pi^0) > A_{LL}(\pi^-)$



o going beyond single-inclusive measurements: **particle correlations**

idea:

forward-central correlations



$x_1 \gg x_2$

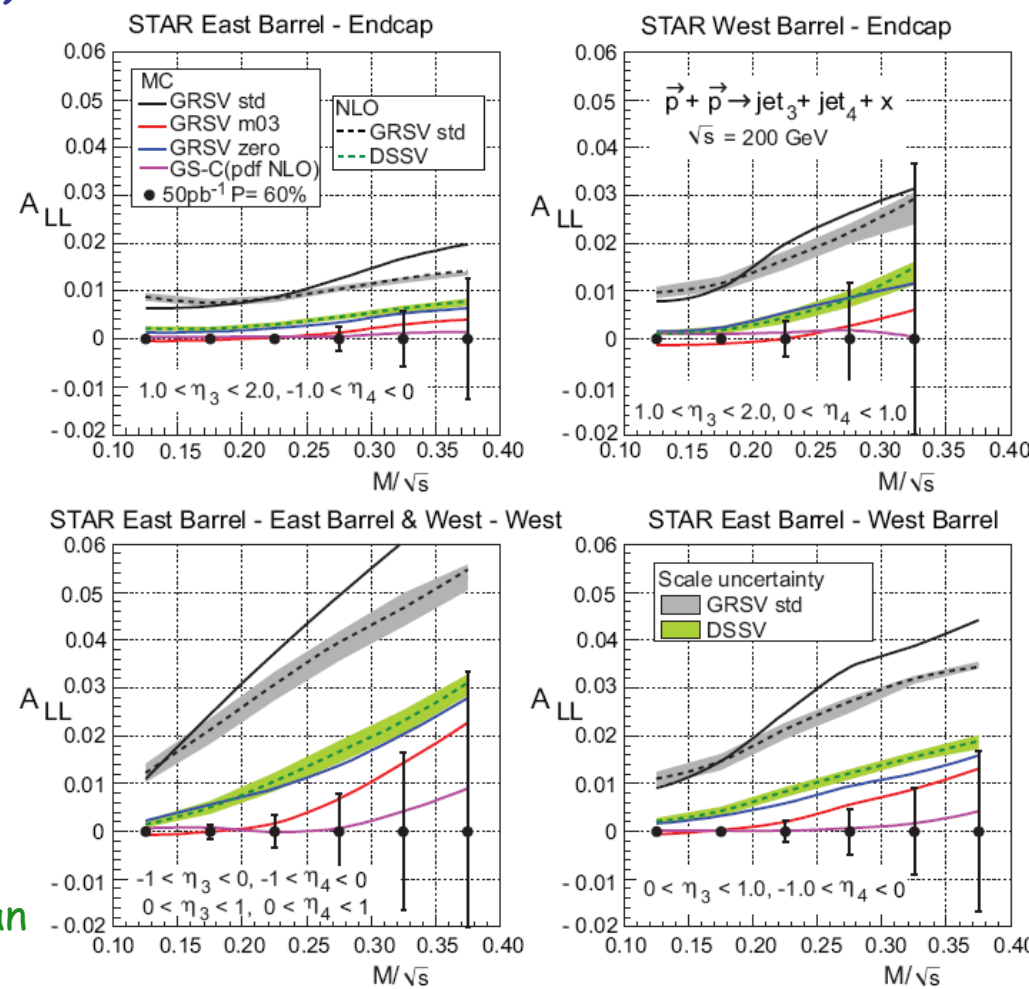
→ mainly qg-scattering: $q(x_1)\, g(x_2)$

di-jet simulation from STAR

→ more precise mapping of $\Delta g(x)$

✓ the Mellin technique is basically in place to analyze also particle correlations

challenge: much slower MC-type codes in NLO than for 1-incl.

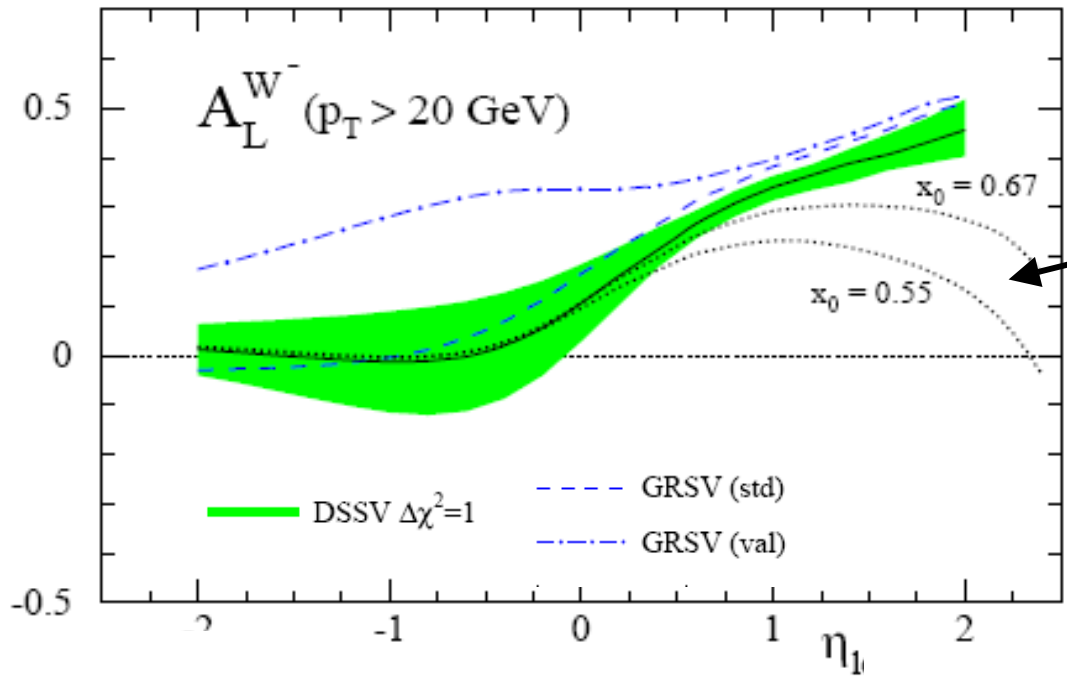


taken from 2008 RHIC spin plan

o **main goal of 500 GeV program:** W boson production

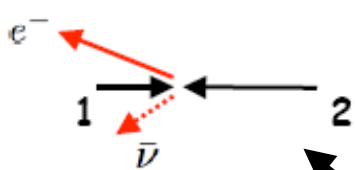
flavor separation from parity-viol. single-spin asymmetry $A_L \equiv \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$

example: A_L for W^-



two sets enforcing $\Delta d/d \to 1$ as $x \to 1$

$\frac{\Delta\bar{u}}{\bar{u}}$

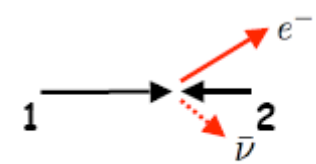


$\theta \to \pi$

$x_2 \gg x_1$

$\theta \to 0$

$x_1 \gg x_2$



$\frac{\Delta d}{d}$

$$\Delta\bar{u}(x_1)d(x_2)(1-\cos\theta)^2 - \Delta d(x_1)\bar{u}(x_2)(1+\cos\theta)^2$$

further improving on uncertainties

- Lagrange multipliers more reliable than Hessian with present data
- Hessian method perhaps useful for $\Delta\chi^2 = 1$ studies, beyond ??
- include experimental error correlations if available
  work started together with help from the RHIC Spin Collaboration
  (aiming at a CTEQ-like collaboration of theory and experiment)