Global QCD analysis of polarized PDFs
status & prospects

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in collaboration with
Daniel de Florian, Rodolfo Sassot, Werner Vogelsang
How to determine PDFs from data?

information on nucleon (spin) structure available from

- DIS
- SIDIS
- hadron-hadron

**task**: extract reliable PDFs not just compare some curves to data

- all processes tied together: universality of PDFs & $Q^2$ - evolution
- each reaction provides insights into different aspects and kinematics
- need at least NLO for quantitative analyses; PDFs are not observables!
- information on PDFs “hidden” inside complicated (multi-)convolutions

→ a “global QCD analysis” is required
the charge:

analyze a large body of data from many experiments on different processes with diverse characteristics and errors within a theoretical model with many parameters and hard to quantify uncertainties without knowing the optimum "ansatz" a priori
details & results of
the DSSV global analysis

setup
- comparison with data
- uncertainties: Lagrange multipliers vs. Hessian
- emerging picture
- future avenues

Global analysis of helicity parton densities and their uncertainties,

Extraction of spin-dependent parton densities and their uncertainties,
outline of a global QCD analysis

**start:** choose fact. scheme (\(\overline{\text{MS}},\ldots\)) & pert. order (NLO, \ldots), select data sets, cuts, ...

parametrize quark and gluon PDFs a la \(\Delta f(x, \mu_0) \sim x^\alpha (1-x)^\beta\) at some initial scale \(\mu_0 \approx 1\text{ GeV}\)

obtain PDFs at any \(x, \mu > \mu_0\) relevant for comparing with data

compute DIS, pp, \ldots cross sections

judge goodness of current fit:

\[
\chi^2 = \sum_i \frac{(T_i - E_i)^2}{\delta E_i^2}
\]

optimum set of parameters \(\{\alpha_i, \beta_i, \ldots\}\)

recent achievement: also quantify **PDF uncertainties** and properly propagate them to any observable of interest
global analysis: a computational challenge

• one has to deal with $O(500)$ data points from many processes and experiments

• need to determine $O(20)$ parameters describing PDFs at $\mu_0$

• NLO expressions often very complicated $\rightarrow$ computing time becomes excessive
  $\rightarrow$ develop sophisticated algorithms & techniques, e.g., based on Mellin moments

Kosower; Vogt; Vogelsang, MS

DSSV global analysis uses:

<table>
<thead>
<tr>
<th>experiment</th>
<th>data type</th>
<th>data point fitted</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMC, SMC</td>
<td>DIS</td>
<td>34</td>
</tr>
<tr>
<td>COMPASS</td>
<td>DIS</td>
<td>15</td>
</tr>
<tr>
<td>E142, E143, E154, E155</td>
<td>DIS</td>
<td>123</td>
</tr>
<tr>
<td>HERMES</td>
<td>DIS</td>
<td>39</td>
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<td>HALL-A</td>
<td>DIS</td>
<td>3</td>
</tr>
<tr>
<td>CLAS</td>
<td>DIS</td>
<td>20</td>
</tr>
<tr>
<td>SMC</td>
<td>SIDIS, $h^\pm$</td>
<td>48</td>
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<tr>
<td>HERMES</td>
<td>SIDIS, $h^\pm$</td>
<td>54</td>
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<tr>
<td></td>
<td>SIDIS, $\pi^\pm$</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>SIDIS, $K^\pm$</td>
<td>27</td>
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<tr>
<td>COMPASS</td>
<td>SIDIS, $h^\pm$</td>
<td>24</td>
</tr>
<tr>
<td>PHENIX (in part prel.)</td>
<td>200 GeV pp, $\pi^0$</td>
<td>20</td>
</tr>
<tr>
<td>PHENIX (prel.)</td>
<td>62 GeV pp, $\pi^0$</td>
<td>5</td>
</tr>
<tr>
<td>STAR (in part prel.)</td>
<td>200 GeV pp, jet</td>
<td>19</td>
</tr>
<tr>
<td>TOTAL:</td>
<td></td>
<td>467</td>
</tr>
</tbody>
</table>

“classic” inclusive DIS data
routinely used in PDF fits
$\rightarrow$ $\Delta q + \bar{\Delta q}$

semi-inclusive DIS data
so far only used in DNS fit
$\rightarrow$ flavor separation

first RHIC pp data (never used before)
$\rightarrow$ $\Delta g$

467 data pts in total ($\approx$10% from RHIC)
interlude: fragmentation functions

crucial for pQCD interpretation (factorization!) of data with detected hadrons, e.g.,
SIDIS (HERMES, COMPASS), \( pp \to \pi X \) (PHENIX, ...)

some properties of \( D_i^h(z, \mu) \) [very similar to PDFs]:

- non-perturbative but universal; pQCD predicts \( \mu \)-dep.
- describe the collinear transition of a parton “i” into a massless hadron “h” carrying fractional momentum \( z \)

global analysis & uncertainty estimates are a recent achievement

\textbf{DSS fit} (de Florian, Sassot, MS)

\begin{align*}
\end{align*}
**DSS**: good global fit of all $e^+e^-$, ep, and pp hadron data

**Main results:**
- Results for $\pi^\pm$, $K^\pm$, chg. hadrons
- Full flavor separation for $D_i^H(z)$ and $D_g^H$
- Uncertainties (L.M.) well under control
- Fits all LEP, HERMES, SMC, RHIC, ... data
- Supersede old fits based only on $e^+e^-$ data
setup of **DSSV** analysis

- flexible, MRST-like input form

\[
x \Delta f_j(x, 1 \text{ GeV}) = N_j x^\alpha_j (1 - x)_{\beta_j} \left[ 1 + \kappa_j \sqrt{x} + \gamma_j x \right]
\]

simplified form for *sea quarks* and \( \Delta g \): \( \kappa_j = 0 \)

- take \( \alpha_s \) from MRST; also use MRST for positivity bounds

- NLO fit, MS scheme

- avoid assumptions on parameters \( \{a_j\} \) unless data cannot discriminate

need to impose: \( \alpha_{\bar{u}} = \alpha_u + \bar{u} \) \( \alpha_{\bar{d}} = \alpha_s = \alpha_{d + \bar{d}} \)

let the fit decide about \( F, D \) value constraint on 1st moments:

\[
\Delta u_{\text{tot}} - \Delta d_{\text{tot}} = (F + D)[1 + \varepsilon_{\text{SU}(2)}]^{1.269 \pm 0.003}_{\text{fitted}}
\]

\[
\Delta u_{\text{tot}} + \Delta d_{\text{tot}} - 2\Delta s_{\text{tot}} = (3F - D)[1 + \varepsilon_{\text{SU}(3)}]^{0.586 \pm 0.031}_{\text{fitted}}
\]
details & results of
the DSSV global analysis

- setup
- comparison with data
  - uncertainties: Lagrange multipliers vs. Hessian
  - the emerging picture
  - future avenues
The overall quality of the global fit is very good! There is no significant tension among different data sets. The chi-squared per degree of freedom is approximately 0.88. 

\[
\chi^2/\text{d.o.f.} \approx 0.88
\]

Note: For the time being, statistical and systematic errors are added in quadrature.

<table>
<thead>
<tr>
<th>experiment</th>
<th>process</th>
<th>$N_{\text{data}}$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMC [2]</td>
<td>DIS (p)</td>
<td>10</td>
<td>3.9</td>
</tr>
<tr>
<td>SMC [3]</td>
<td>DIS (p)</td>
<td>12</td>
<td>3.4</td>
</tr>
<tr>
<td>SMC [3]</td>
<td>DIS (d)</td>
<td>12</td>
<td>18.4</td>
</tr>
<tr>
<td>COMPASS [4]</td>
<td>DIS (d)</td>
<td>15</td>
<td>8.1</td>
</tr>
<tr>
<td>E142 [5]</td>
<td>DIS (n)</td>
<td>8</td>
<td>5.6</td>
</tr>
<tr>
<td>E143 [6]</td>
<td>DIS (p)</td>
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<td>19.3</td>
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<td>40.8</td>
</tr>
<tr>
<td>E154 [7]</td>
<td>DIS (n)</td>
<td>11</td>
<td>4.5</td>
</tr>
<tr>
<td>E155 [8]</td>
<td>DIS (p)</td>
<td>24</td>
<td>22.6</td>
</tr>
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<td>E155 [9]</td>
<td>DIS (d)</td>
<td>24</td>
<td>17.1</td>
</tr>
<tr>
<td>HERMES [10]</td>
<td>DIS (He)</td>
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<td>6.3</td>
</tr>
<tr>
<td>HERMES [11]</td>
<td>DIS (d)</td>
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<td>16.9</td>
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<td>HALL-A [12]</td>
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<td>CLAS [13]</td>
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</tr>
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<td>DIS (d)</td>
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<td>SMC [14]</td>
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<td>18.7</td>
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<tr>
<td>SMC [14]</td>
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<td>SMC [14]</td>
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<td>SMC [14]</td>
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<td>14.1</td>
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<td>HERMES [15]</td>
<td>SIDIS (p, $h^+$)</td>
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<td>6.4</td>
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<td>HERMES [15]</td>
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<tr>
<td>HERMES [10]</td>
<td>SIDIS (He, $h^+$)</td>
<td>9</td>
<td>4.7</td>
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<tr>
<td>HERMES [10]</td>
<td>SIDIS (He, $h^-$)</td>
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<td>6.0</td>
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<td>HERMES [15]</td>
<td>SIDIS (p, $\pi^+$)</td>
<td>9</td>
<td>9.6</td>
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<tr>
<td>HERMES [15]</td>
<td>SIDIS (p, $\pi^-$)</td>
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<td>9.6</td>
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<td>0.4</td>
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<tr>
<td>HERMES [15]</td>
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<td>19.5</td>
</tr>
<tr>
<td>HERMES [15]</td>
<td>SIDIS (d, $K^+$)</td>
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<td>6.2</td>
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<td>HERMES [15]</td>
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<td>9</td>
<td>5.8</td>
</tr>
<tr>
<td>HERMES [15]</td>
<td>SIDIS (d, $K^{*+}$)</td>
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<td>3.4</td>
</tr>
<tr>
<td>COMPASS [16]</td>
<td>SIDIS (d, $h^+$)</td>
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<td>6.2</td>
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<tr>
<td>COMPASS [16]</td>
<td>SIDIS (d, $h^-$)</td>
<td>12</td>
<td>12.0</td>
</tr>
<tr>
<td>PHENIX [22]</td>
<td>pp (200 GeV, $\pi^0$)</td>
<td>10</td>
<td>14.2</td>
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<tr>
<td>PHENIX [23]</td>
<td>pp (200 GeV, $\pi^0$)</td>
<td>10</td>
<td>7.1 \cite{13,8} a</td>
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<tr>
<td>PHENIX [24]</td>
<td>pp (62 GeV, $\pi^0$)</td>
<td>5</td>
<td>3.1 \cite{2,8} a</td>
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<tr>
<td>STAR [25]</td>
<td>pp (200 GeV, jet)</td>
<td>10</td>
<td>8.8</td>
</tr>
<tr>
<td>STAR (prel.) [26]</td>
<td>pp (200 GeV, jet)</td>
<td>9</td>
<td>6.9</td>
</tr>
</tbody>
</table>

**Total:** 467 392.6
spin asymmetries in inclusive DIS

- we account for kinematical "mismatches" in \( A_1 = (1 + \gamma^2) g_1 / F_1 \)
- no need for any dynamical higher twist (contrary to Leader et al.)
spin asymmetries in semi-inclusive DIS

<table>
<thead>
<tr>
<th>SMC $A_{1p}^{h_1}$</th>
<th>HERMES $A_{1p}^{h_1}$</th>
<th>HERMES $A_{1p}^{x_1}$</th>
<th>HERMES $A_{1He}^{h_1}$</th>
<th>COMPASS $A_{1d}^{h_1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMC $A_{1d}^{h_1}$</td>
<td>HERMES $A_{1d}^{h_1}$</td>
<td>HERMES $A_{1d}^{x_1}$</td>
<td>HERMES $A_{1He}^{h_1}$</td>
<td>COMPASS $A_{1d}^{h_1}$</td>
</tr>
<tr>
<td>SMC $A_{1d}^{h_1}$</td>
<td>HERMES $A_{1d}^{h_1}$</td>
<td>HERMES $A_{1d}^{x_1}$</td>
<td>HERMES $A_{1He}^{h_1}$</td>
<td>COMPASS $A_{1d}^{h_1}$</td>
</tr>
</tbody>
</table>

impact of new FFs noticeable!

$D_i^{K\pm}$

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DSSV
DNS (DSS FFs)
gluons are key players at RHIC

many QCD processes with a dominant gluon contribution already at the tree-level:

unpolarized “reference data” (π, jets, γ) nicely agree with pQCD

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Dom. partonic process</th>
<th>probes</th>
<th>LO Feynman diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{p}p \to \pi + X )</td>
<td>( \bar{g}g \to gg )</td>
<td>( \Delta g )</td>
<td></td>
</tr>
<tr>
<td>([61, 62])</td>
<td>( q\bar{q} \to qg )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{p}p \to \text{jet(s)} + X )</td>
<td>( \bar{g}g \to gg )</td>
<td>( \Delta g )</td>
<td>(as above)</td>
</tr>
<tr>
<td>([71, 72])</td>
<td>( q\bar{q} \to qg )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \bar{p}p \to \gamma + X )</td>
<td>( \bar{g}g \to \gamma q )</td>
<td>( \Delta g )</td>
<td></td>
</tr>
<tr>
<td>( \bar{p}p \to \gamma + \text{jet} + X )</td>
<td>( q\bar{q} \to \gamma q )</td>
<td>( \Delta g )</td>
<td></td>
</tr>
<tr>
<td>([67, 73, 74, 75, 76])</td>
<td>( \bar{q}q \to \gamma \gamma )</td>
<td>( \Delta q, \Delta \bar{q} )</td>
<td></td>
</tr>
<tr>
<td>( \bar{p}p \to D, B X )</td>
<td>( \bar{g}g \to c\bar{c}, b\bar{b} )</td>
<td>( \Delta g )</td>
<td></td>
</tr>
<tr>
<td>([77])</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

decisive data start to emerge from RHIC ...

Jäger, Schäfer, MS, Vogelsang; de Florian
Jäger, MS, Vogelsang; Signer et al.
Gordon, Vogelsang; Contogouris et al.
Bojak, MS; Riedl, Schäfer, MS
RHIC pp data (inclusive $\pi^0$ or jet)

- good agreement
- important constraint on $\Delta g(x)$ despite large uncertainties

$\rightarrow$ later

uncertainty bands estimated with Lagrange multipliers by enforcing other values for $A_{LL}$
Δg in lepton-proton scattering

**Gluons in DIS:** a (small) NLO effect [they don't couple directly to the photon]

→ study processes sensitive to **photon-gluon-fusion**

data available for one/two hadron production, charm

**Theory calculations more challenging than in pp:**

- **Q**^2 large: “electroproduction”
- **Q**^2 ≈ 0: “photoproduction”

**NLO**  
**pQCD**  
nothing for **Q**^2 ≠ 0

1-hadron: ✓ Jäger, MS, Vogelsang  
charm: ✓ Bojak, MS (direct γ)  
Riedl, Schäfer, MS (resolved γ & MC)  
hadron pairs: Hendlmeier, Schäfer, MS (direct γ)  
Jäger, Owens, MS, Vogelsang (resolved γ)
DSSV gluon agrees well with model-dependent “LO” extractions of $\Delta g/g$

A future global NLO fit will use measured $A_{LL}$ not derived $\Delta g/g$.

Need to check unpolarized cross section as well.
details & results of
the DSSV global analysis

- setup
- comparison with data
  - uncertainties: Lagrange multipliers vs. Hessian
- emerging picture
- future avenues
estimating PDF uncertainties

mainly two methods in use: [reshaped for PDF analyses by J. Pumplin and CTEQ]

- **Hessian method**: classic tool, explores vicinity of $\chi^2$-minimum in quadratic approx.; often unstable for multi-parameter PDF analyses

- **Lagrange multiplier**: track how the fit deteriorates when PDFs are forced to give different predictions for selected observables; explores the full parameter space indep. of approximations

**issue**: what value of $\Delta \chi^2$ (tolerance) defines a 1-\(\sigma\) error?

- non Gaussian errors, $\chi^2$ “landscape” not parabolic
- uncertainties with diverse characteristics
- theor. errors correlated and poorly known
- data sets often marginally consistent for $\Delta \chi^2=1$

we present uncertainties bands for both $\Delta \chi^2 = 1$ and a more pragmatic 2\% increase in $\chi^2$
Hessian eigenvector PDF basis sets

- eigenvectors provide an optimized orthonormal basis near the minimum
- construct $2N_{\text{par}}$ eigenvector basis sets $S_k^\pm$ by displacing each $z_k$ by $\pm 1$
- the “coordinates” are rescaled such that $\Delta \chi^2 = \sum_k z_k^2$
- sets $S_k^\pm$ can be used to calculate uncertainties of observables $O_i$

$$\Delta O_i = \frac{1}{2} \left( \sum_{k=1}^{N_{\text{par}}} \left[ O_i(S_k^+) - O_i(S_k^-) \right]^2 \right)^{1/2}$$

38 DSSV eigenvector sets are available from ribf.riken.jp/~marco/DSSV
(after DIS'09)
details & results of the DSSV global analysis

- setup
- comparison with data
- uncertainties: Lagrange multipliers vs. Hessian

emerging picture
- future avenues
DSSV valence quark polarizations

- best determined
- uncertainty bands very narrow
- agrees well with previous "DIS-only" fits

large-x region ($x \to 1$)

$$R_q(x) \equiv \frac{\Delta q(x) + \Delta \bar{q}(x)}{q(x) + \bar{q}(x)}$$

- $R_u (x \to 1) \to 1$ as expected
- $R_d (x \to 1)$ remains negative
- counting rules + helicity retention + nonzero OAM: expect $R_d (x \to 1) \to 1$

Avakian, Brodsky, Deur, Yuan
DSSV sea polarizations

- indications for an SU(2) breaking of light u,d sea
  - breaking of similar size than in unpol. case
  - mainly determined by SIDIS data
  - “bands”: error estimate from Lagr. mult.
  - similar patterns in many models:
    - large-$N_c$, chiral quark soliton, meson cloud
    - Thomas, Signal, Cao; Diakonov, Polyakov, Weiss; ...

\[ \chi^2 \text{ profiles for truncated moments:} \]

- \( \Delta u^1, [0.001 \to 1] \)
- \( \Delta d^1, [0.001 \to 1] \)
DSSV sea polarizations – cont’d

- a strange strangeness polarization
  - $\Delta s(x)$ always thought to be negative, but ...
  - mainly determined from SIDIS kaon data
  - consistent with LO-type analyses by HERMES and COMPASS

![Graph]

striking result, but relies on
- kaon fragmentation
- more data available soon (BELLE, ...)
- unpolarized PDFs
  - unpol. strangeness not well determined

needs further studies – exp. & theory!
**DSSV gluon polarization**

- Error estimates more delicate: small-\(x\) behavior completely unconstrained.

- Study uncertainties in 3 \(x\)-regions:
  - \(\Delta g(x)\) very small at medium \(x\) (even compared to GRSV or DNS).
  - Best fit has a node at \(x \approx 0.1\).
  - Huge uncertainties at small \(x\).

- DSSV gluon polarization

\[
\delta g \equiv \int_{0.05}^{0.2} \Delta g(x, 10 \text{ GeV}^2) dx
\]
spin audit: 1st moments and the spin of the proton

“helicity sum rule”

\[
\frac{1}{2} \mathcal{H} = \langle P, \frac{1}{2} | J_{QCD}^z | P, \frac{1}{2} \rangle = \sum_q \frac{1}{2} S_q^z + S_g^z + \sum_q L_q^z + L_g^z
\]

Jaffe, Manohar; Ji; …

A^+ = 0 gauge, IMF
partonic interpretation
total u+d+s quark spin gluon spin angular momentum

“quotable” properties of the nucleon!

\[
\Delta f(x) \equiv f_{N_+}^{N_+}(x) - f_{N_-}^{N_-}(x)
\]

momentum fraction

helicity parton densities

nothing is known yet about \( L_q \) and \( L_g \)

lattice results for “angular momentum” are for a “different” sum rule (Ji’s version) w/o partonic interpretation - they cannot be mixed!
numerical results

\[ \Delta f^{1,[x_{\text{min}},1]} \equiv \int_{x_{\text{min}}}^{1} \Delta f(x) \, dx \]

- $\Delta s$ receives a large negative contribution at small $x$
- $\Delta g$: huge uncertainties below $x \approx 0.01 \rightarrow 1\textsuperscript{st} \text{ moment still undetermined}$

<table>
<thead>
<tr>
<th></th>
<th>$x_{\text{min}} = 0$ best fit</th>
<th>$x_{\text{min}} = 0.001$</th>
<th>$\Delta \chi^2 = 1$</th>
<th>$\Delta \chi^2 / \chi^2 = 2%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u + \Delta \bar{u}$</td>
<td>0.813</td>
<td>0.793 $^{+0.011}_{-0.012}$</td>
<td>0.793 $^{+0.028}_{-0.034}$</td>
<td></td>
</tr>
<tr>
<td>$\Delta d + \Delta \bar{d}$</td>
<td>-0.458</td>
<td>-0.416 $^{+0.011}_{-0.009}$</td>
<td>-0.416 $^{+0.035}_{-0.025}$</td>
<td></td>
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<tr>
<td>$\Delta \bar{u}$</td>
<td>0.036</td>
<td>0.028 $^{+0.021}_{-0.020}$</td>
<td>0.028 $^{+0.059}_{-0.059}$</td>
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<tr>
<td>$\Delta \bar{d}$</td>
<td>-0.115</td>
<td>-0.089 $^{+0.029}_{-0.029}$</td>
<td>-0.089 $^{+0.090}_{-0.080}$</td>
<td></td>
</tr>
<tr>
<td>$\Delta \bar{s}$</td>
<td>-0.057 $^{+0.010}_{-0.012}$</td>
<td>-0.006 $^{+0.010}_{-0.012}$</td>
<td>-0.006 $^{+0.028}_{-0.031}$</td>
<td></td>
</tr>
<tr>
<td>$\Delta g$</td>
<td>-0.084</td>
<td>0.013 $^{+0.106}_{-0.120}$</td>
<td>0.013 $^{+0.702}_{-0.314}$</td>
<td></td>
</tr>
<tr>
<td>$\Delta \Sigma$</td>
<td>0.242</td>
<td>0.366 $^{+0.015}_{-0.018}$</td>
<td>0.366 $^{+0.042}_{-0.062}$</td>
<td></td>
</tr>
</tbody>
</table>

very difficult to give reliable estimates for full moments

both quark and gluons may not contribute much to proton spin
but we need to go to smaller $x$ to settle this issue
→ case for a high-energy polarized ep-collider
details & results of the DSSV global analysis

- setup
- comparison with data
- uncertainties: Lagrange multipliers vs. Hessian
- emerging picture
- future avenues
- getting ready to analyze new types of data
  - milestone: RHIC has just completed the 1st 500 GeV run
  - hope for $O(50 \text{pb}^{-1})$ with 60% pol. from current 200 GeV run

**expect:**
- significant improvement of existing inclusive jet $+ \pi^0$ data sample
- charged pion data
  - at large $p_T$ driven by
    - $qg$ scattering
    - $\Delta g$

$$A_{LL}(\pi^+) > A_{LL}(\pi^0) > A_{LL}(\pi^-)$$

![Graph showing $A_{LL}$ for different hadrons and $p_T$ values with legends indicating $\pi^+$, $\pi^0$, and $\pi^-$, and a note $\sqrt{S} = 500$ GeV.]
going beyond single-inclusive measurements: **particle correlations**

**idea:**
forward-central correlations

→ mainly qg-scattering: \( q(x_1) g(x_2) \)

**di-jet simulation from STAR**

→ more precise mapping of \( \Delta g(x) \)

✓ the Mellin technique is basically in place to analyze also particle correlations

**challenge:** much slower MC-type codes in NLO than for 1-incl.

taken from 2008 RHIC spin plan
The main goal of the 500 GeV program: W boson production

Flavor separation from parity-violent single-spin asymmetry $A_L \equiv \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$

Example: $A_L$ for $W^-$

\[ \Delta \bar{u}(x_1)d(x_2)(1 - \cos \theta)^2 - \Delta d(x_1)\bar{u}(x_2)(1 + \cos \theta)^2 \]
■ **further improving on uncertainties**

■ Lagrange multipliers more reliable than Hessian with present data

■ Hessian method perhaps useful for $\Delta \chi^2 = 1$ studies, beyond ??

■ include experimental error correlations if available
  work started together with help from the RHIC Spin Collaboration
  (aiming at a CTEQ-like collaboration of theory and experiment)