# Variable-flavour-number scheme in analysis of heavy-quark electro-production data

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*in collaboration with*

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The heavy-quark electroproduction contributes up to 30% to the inclusive structure functions measured at HERA. In the LO of QCD at large $Q^2$ the structure function $F_{2,c} \sim \alpha_\mathrm{s} \ln(Q^2/m_c^2)$ (Witten 76) and it must be resumed (Shifman-Vainshten-Zakharov 78). This resummation is performed in the evolution equation that naturally leads to a concept of the heavy-quarks parton distributions.

(Aivasiz-Collins-Olness-Tung 94)

In this LO scheme the heavy-quark parton distribution $h_i = 0$ at a certain scale (typically $Q^2 = m_c^2$) and is evolved to bigger values of $Q^2$ through the QCD evolution equations together with the light partons.

The accurate QCD formalism of the variable-flavour-number scheme (VFNS) in the DIS requires that at $Q^2 \gg m_c^2$ the heavy-quark fixed-flavour-number scheme (FFNS) coefficient functions are factorized by the convolution of the operator matrix elements (OMEs) and the light parton coefficient functions. The OMEs are process independent and its convolution with the light-parton PDFs $p_i$ is considered as a heavy-quark PDF $h = A_{H,i} \otimes p_i$.

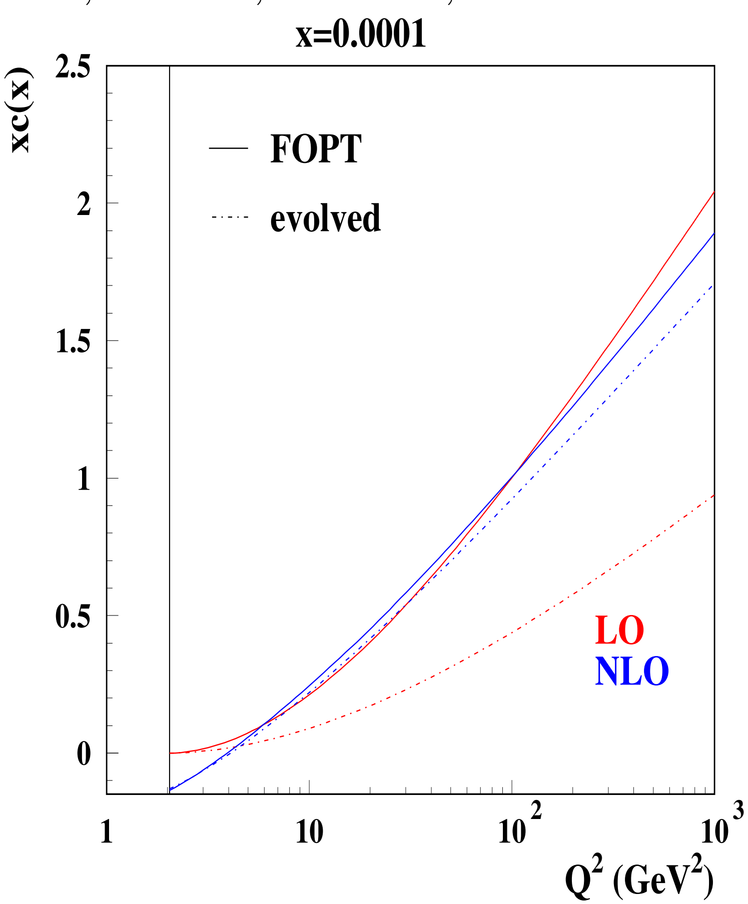
(Buza-Matiounine-Smith-van Neerven)

The light-partons are modified accordingly $p_j^H = A_{j,i} \otimes p_i$ and this completes matching condition for the $(N_f + 1)$- and $N_f$-flavour PDFs so that momentum is conserved

$$\int_0^1 dx[h(x) + \sum_i p_i^h(x)] = 1.$$

$$A_{H,i}^{(1)} = a_{H,i}^{(1,1)} \ln(\mu^2/m_H^2) \qquad O(\alpha_s)$$

$$A_{H,i}^{(2)} = a_{H,i}^{(2,0)} + a_{H,i}^{(2,1)} \ln(\mu^2/m_H^2) + a_{H,i}^{(2,2)} \ln^2(\mu^2/m_H^2) \qquad O(\alpha_s^2)$$




- In the $O(\alpha_\mathrm{s})$ the difference between the evolved and fixed-order-perturbative theory (FOPT) PDFs is sizable, however in the $O(\alpha_\mathrm{s}^2)$ it is greatly reduced, particularly for the realistic kinematics.

(Glück-Reya-Stratmann 94)

- With the NNLO OME's the difference should become even smaller.

(Bierenbaum-Blümlein-Klein [arxiv:0904.3563])

Convolution of the massless coefficient functions with the light-parton and heavy-quark PDFs gives $F_{2,c}$ in the zero-mass variable-flavour-number scheme, $F_{2,c}^{\text{ZMVFNS}}$. At $Q^2 \gg m_c^2$ this scheme reproduces asymptotic FFNS value $F_{2,c}^{\text{ASYMP}}$,

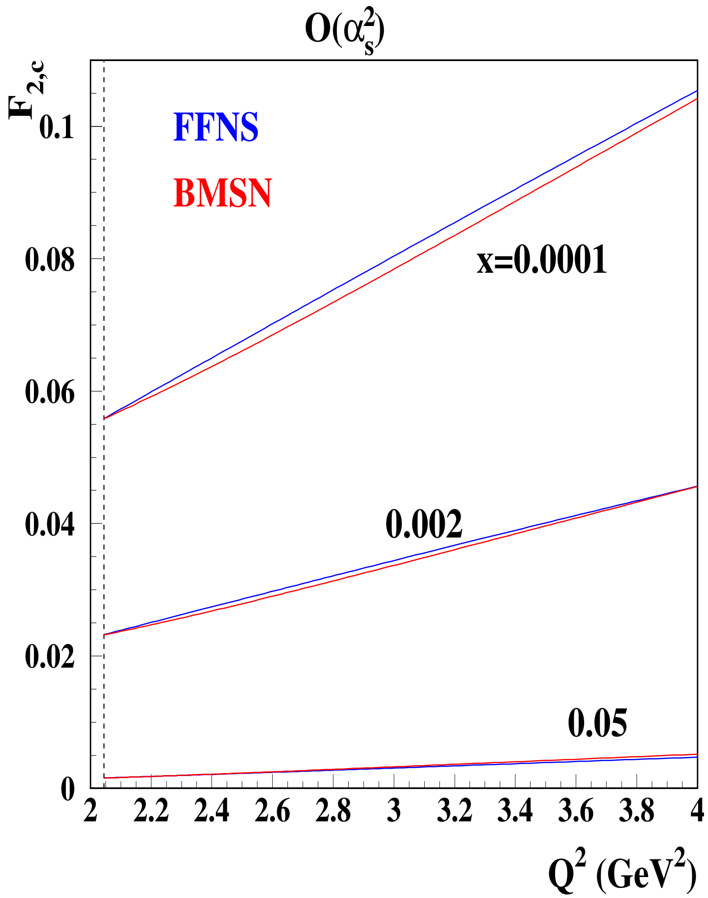
At $Q^2 \sim m_c$ ZMVFNS is clearly irrelevant since the concept of heavy-quark PDFs is irrelevant due to the power corrections in $F_{2,c}^{\text{FFNS}}$ spoil the collinear factorization.

A complete definition of the VFNS should include a matching between $F_{2,c}^{\text{FFNS}}$ at small $Q^2$ and $F_{2,c}^{\text{ZMVFNS}}$ at large $Q^2$. This matching cannot be derived from the first principles and must be modeled, with a natural requirement of the smooth transition between the large- and small-$Q^2$ regions.

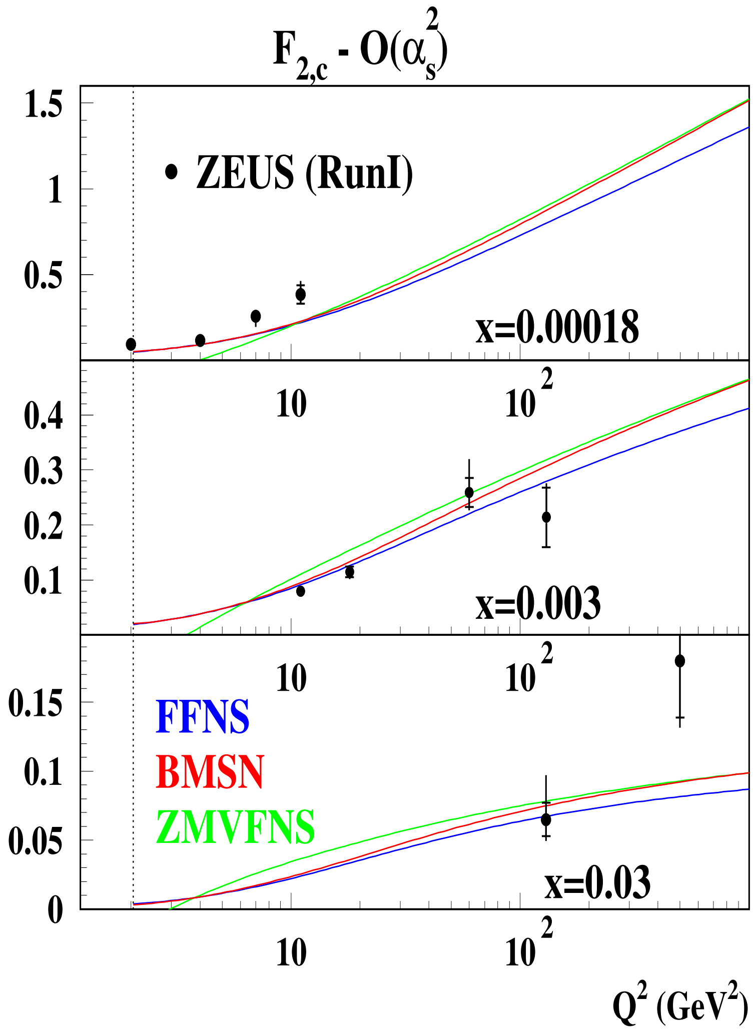
Number of VFNS variants were suggested last years (Thorne-Roberts, Thorne, ACOT($\chi$), and modifications of these three) for the use in global PDFs fits including the DIS data.

The Buza-Matiounine-Smith-van Neerven (BMSN) prescription is

$$F_{2,c}^{\rm BMSN} = F_{2,c}^{\rm FFNS}(N_f = 3) + F_{2,c}^{\rm ZMVFNS}(N_f = 4) - F_{2,c}^{\rm ASYMP}(N_f = 3)$$

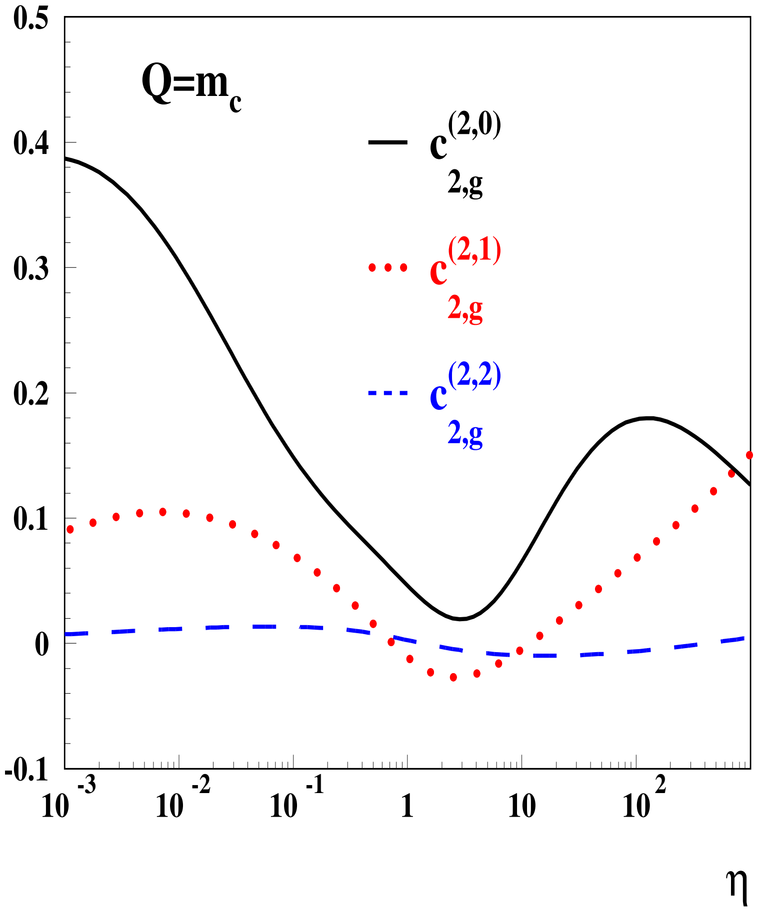



- The non-singlet term in $F_{2,c}^{\rm FFNS}$ at large $Q^2$ is $\sim\ \alpha_{\rm s}^2 \ln^3(Q^2/m_c^2)$ with no corresponding compensation term in $F_{2,c}^{\rm ASYMP}$, however numerically it is very small even at very large $Q^2$.
- The convolution of $C_{2,i} \otimes h$ in $F_{2,c}^{\rm ZMVFNS}$ contains terms $\sim \alpha_{\rm s}^3, \alpha_{\rm s}^4$, which do not appear in $F_{2,c}^{\rm ASYMP}$, one has to generate $h$ in the LO, NLO, and NNLO to rid of these extra terms.

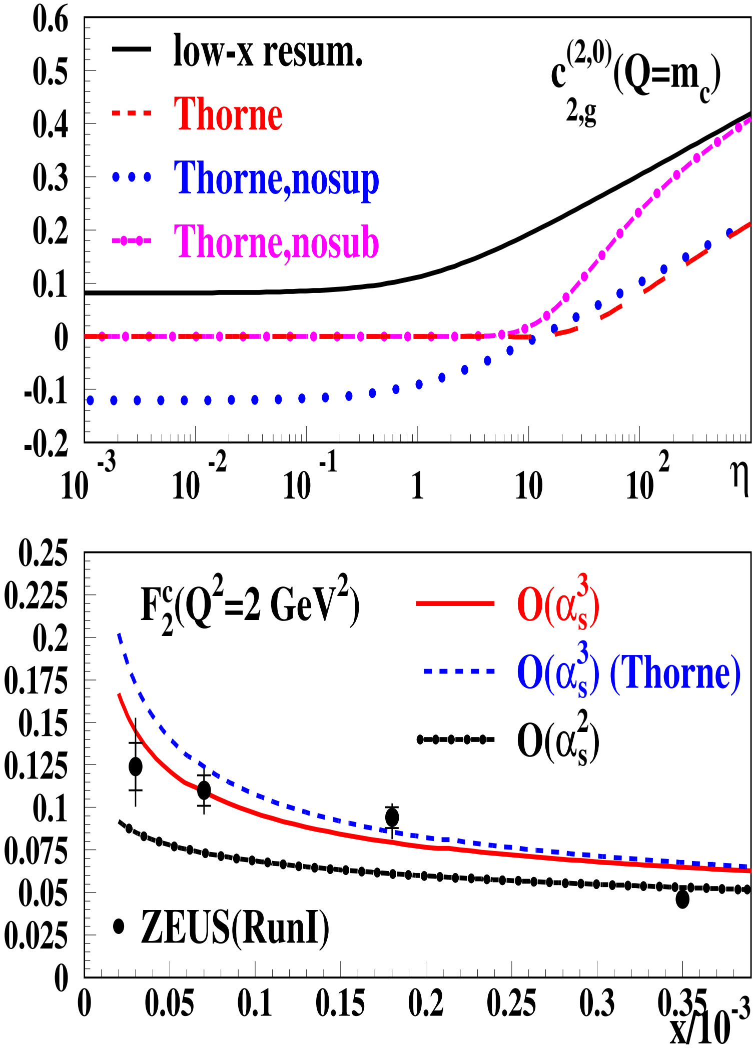



- The BMSN prescription, which provides a smooth transition between the FFNS and ZMVFNS, is not too far from the FFNS for the realistic HERA kinematics; it seems to be the case for any smooth matching.
- The remaining discrepancies with the data cannot be cured by the smooth VFNS.

$$C_{2,g}^{\rm NNLO} = c_{2,g}^{(2,0)} + c_{2,g}^{(2,1)} \ln(\mu^2/m_c^2) + c_{2,g}^{(2,2)} \ln^2(\mu^2/m_c^2)$$

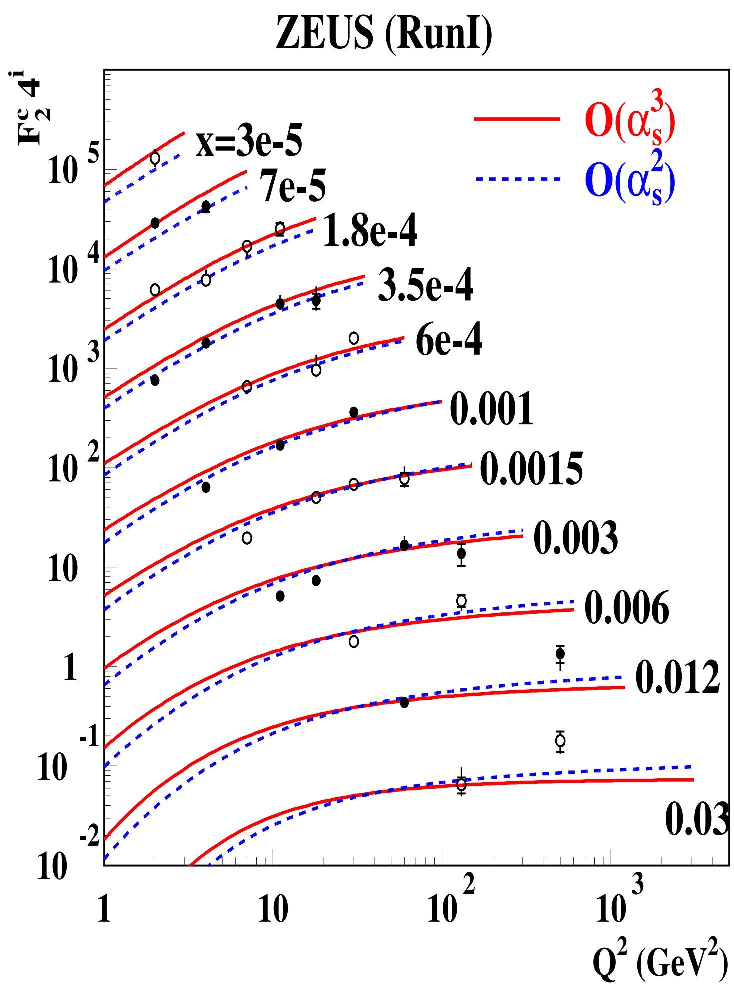



- The coefficients $c_{2,g}^{(2,1)}$ and $c_{2,g}^{(2,2)}$ are known exactly.
- The coefficient $c_{2,g}^{(2,0)}$ can be estimated from the soft-gluon threshold resummation (Laenen-Moch 99). At $\eta = \hat{s}/4m_c^2 - 1 > 1$ this approximation is out of control and $c_{2,g}^{(2,0)}$ is set to 0. The full NNLO calculations are necessary in order to get $c_{2,g}^{(2,0)}$ in the full range of $\eta$.



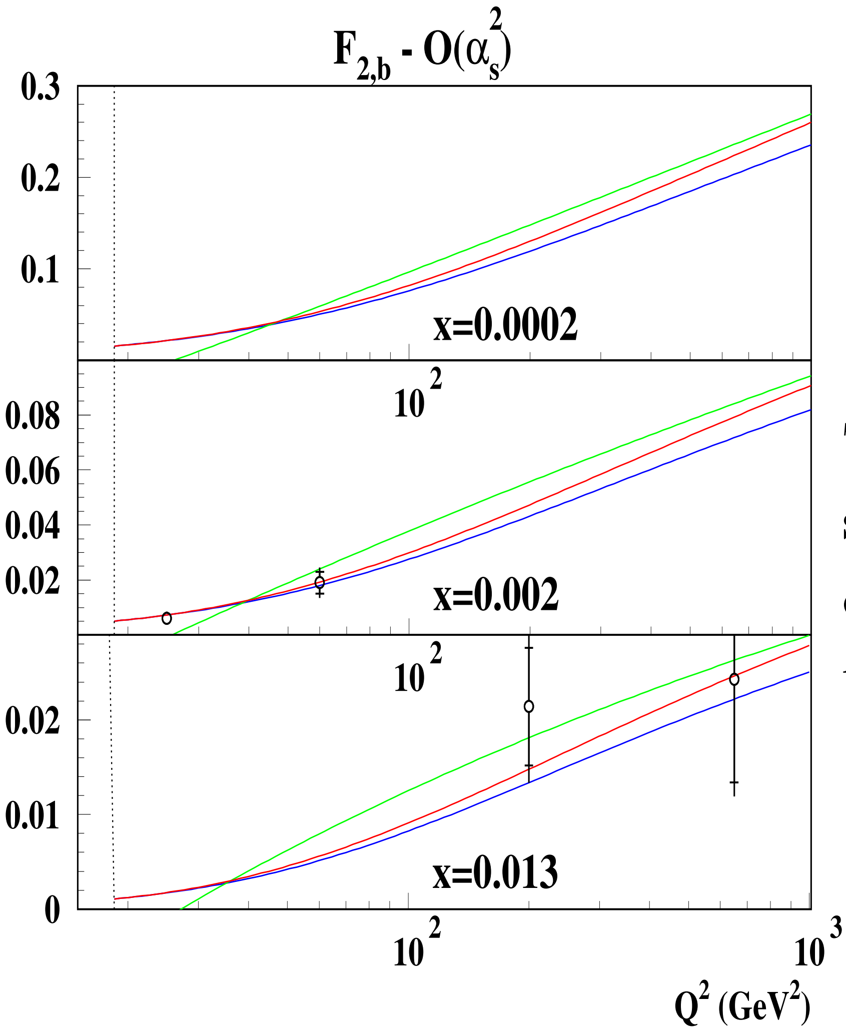

- The coefficient $c_{2,g}^{(2,0)}$ was modeled by Thorne using the Catani-Hautmann small-$x$ resummation results, however uncertainty in the model is quite big.
- Meanwhile at small $Q^2$ impact of the high-$\eta$ tail of $c_{2,g}^{(2,0)}$ is suppressed

(Vogt 96)

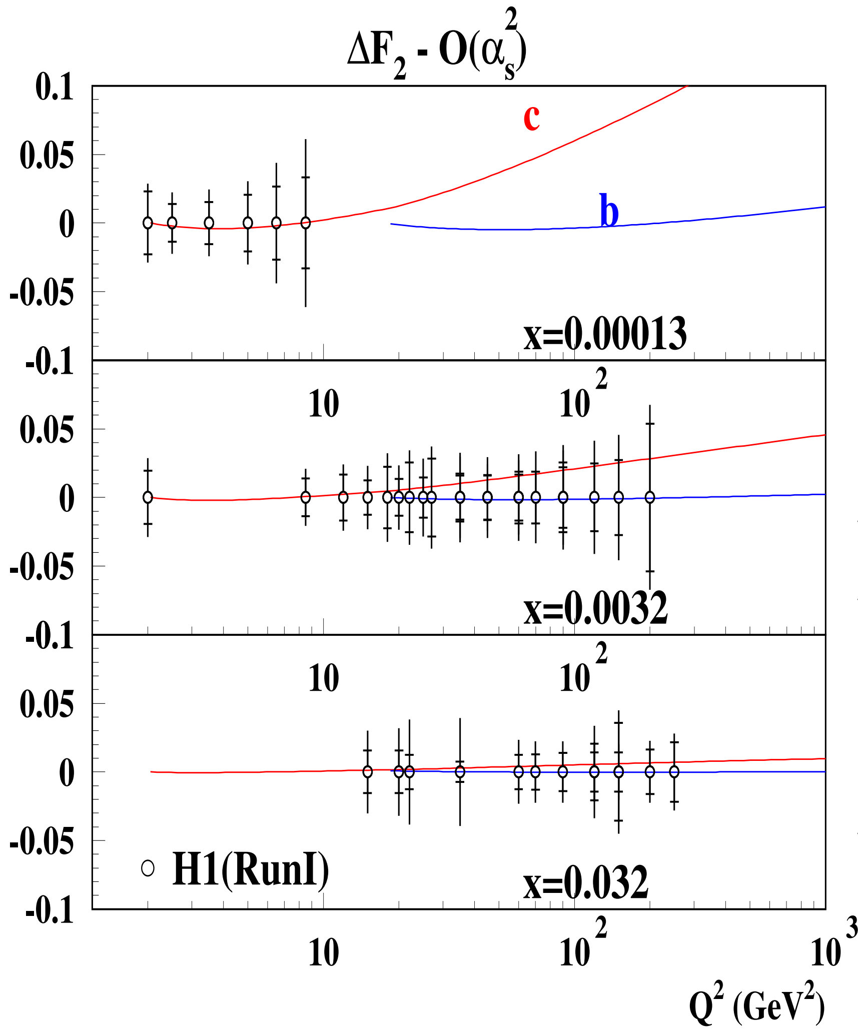



- The FFNS with account of the partial $O(\alpha_s^3)$ corrections provides a good description of the HERA data at small/moderate $Q^2$.
- At large $Q^2$ it undershoots the data due to negative contribution from $c_{2,g}^{(2,1)}$, the missing contribution from $c_{2,g}^{(2,0)}$ at large $\eta$ must be positive in order to improve the agreement.

(c.f. S.Klein's talk)




The $b$-quark production is less sensitive to the scheme choice due to to the shift of the asymptotic region to bigger $Q^2$.

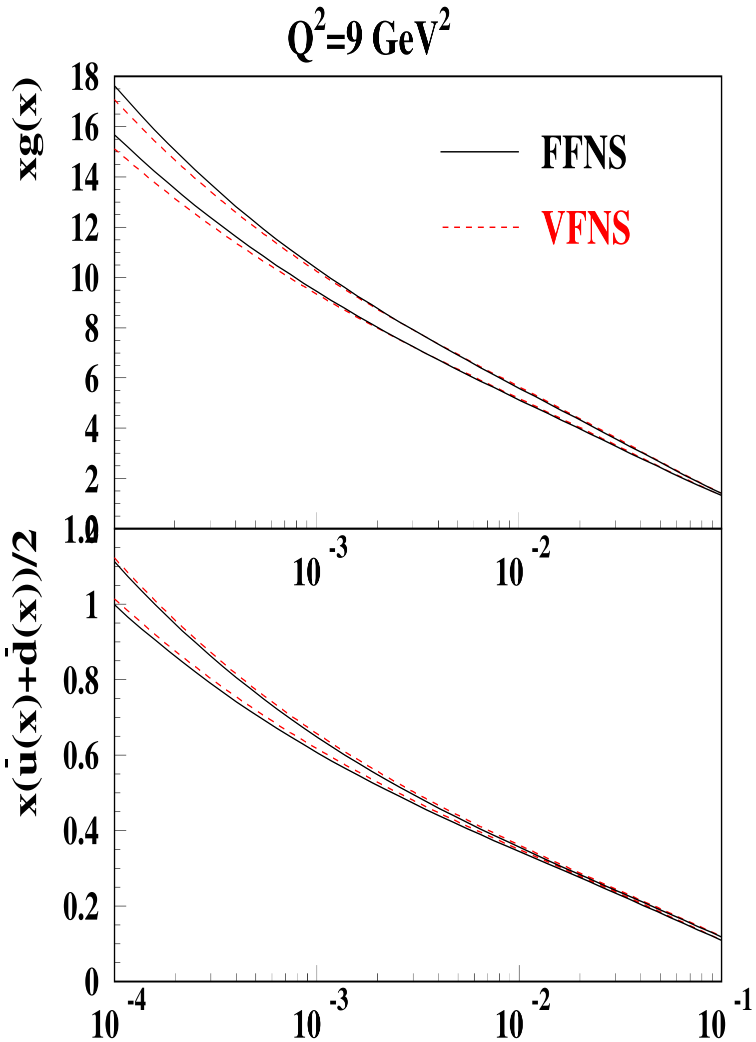



The inclusive SF measurements are more sensitive to the choice of scheme for $c$-quark production, however even in this case the kinematic region of the sensitivity is limited and the shift is comparable to uncertainties in the data.

We perform analysis of

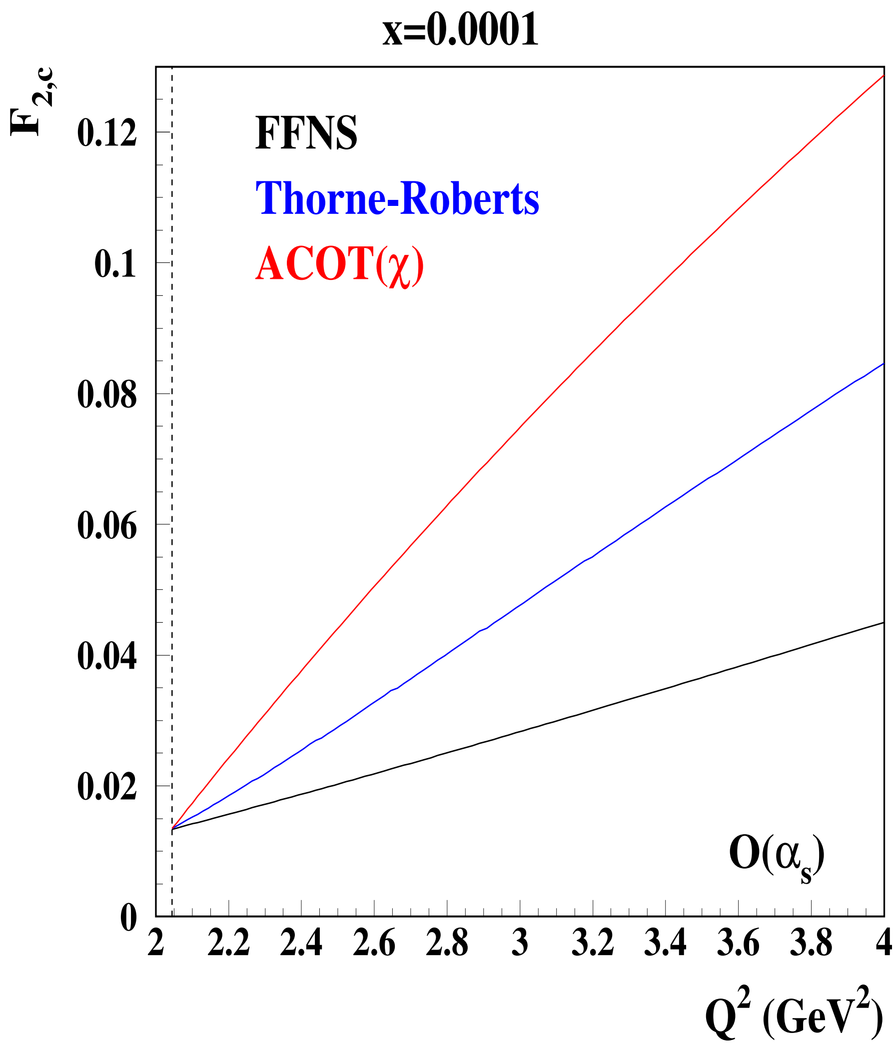
- the inclusive DIS data with the transferred momentum $Q^2 > 2.5$ GeV$^2$ (SLAC-BCDMS-NMC-H1-ZEUS).
- the fixed target Drell-Yan data by FNAL-E-605 (p Cu) and FNAL-E-866 (pp/pD).
- data on dimuon production in the $\nu N$ interactions by the CCFR and NuTeV collaborations

in the NNLO approximation for the PDFs evolution and the light-parton coefficient functions. The heavy quark contribution to the charged-lepton DIS is calculated in $O(\alpha_s^2)$ and two variants for $F_{2,c}$ are considered: FFNS and VFNS with the BMSN prescription.

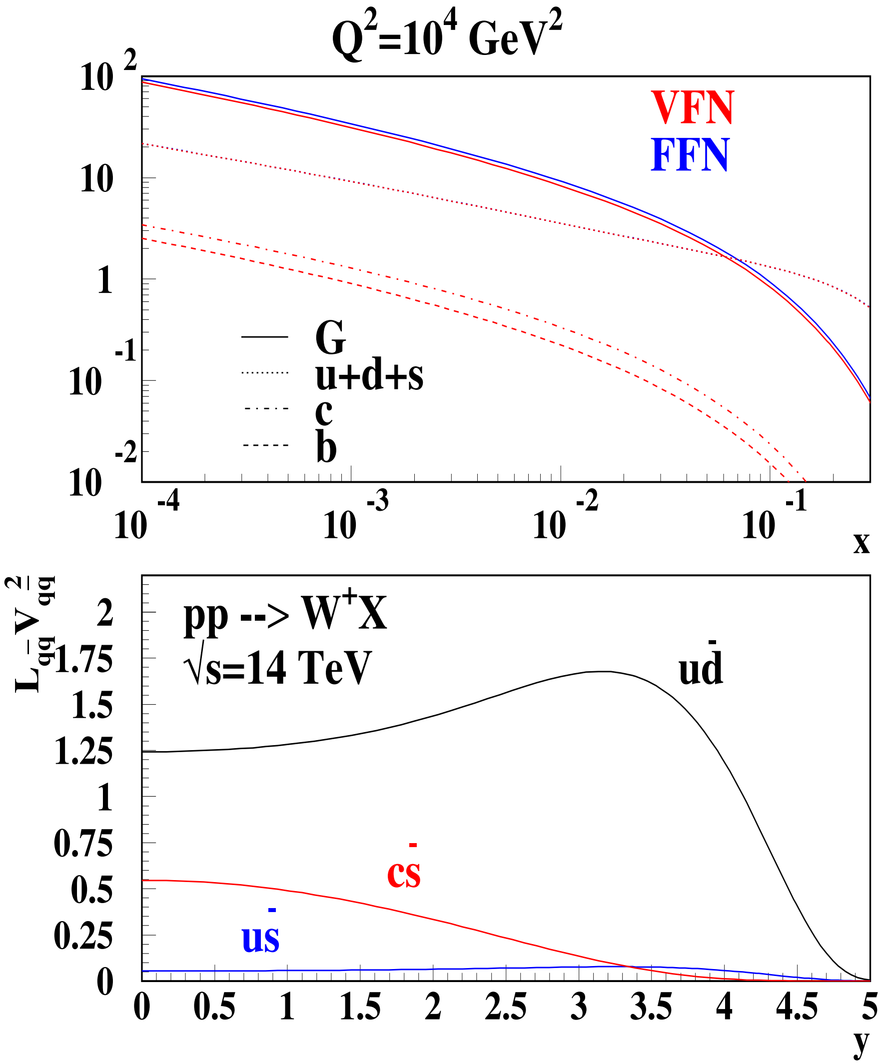



- Impact of the scheme choice on the PDFs is marginal. For the sea and gluon distribution at small $x$ effect is well within $1\sigma$; other PDFs are practically the same.
- In some other fits the difference between the FFNS and popular VFNS variants is not significant too, however is somewhat bigger, this apparently happens due to different behaviour of the latter in the matching region.

(Cooper-Sarkar 07)




Different variants of the GM-VFNS used in the global PDFs fits demonstrate a kink in the matching region. It cannot be attributed to the large-log effects and just reflects uncertainty in the ingredients of these models. On a practical side this leads to overestimation of the heavy-quark contribution and corresponding suppression of the other PDFs.




- The VFNS and FFNS light quarks are practically the same; the gluons are some 5% off at $Q^2 \sim M^2_{W,Z}$.
- At $Q^2 \sim M^2_{W,Z}$ the $c$-quark distribution is $0.1 \div 0.2$ of the total sea and its contribution to the $W$ production is sizable.

$\sigma^{NNLO}(nb)$

| | $W^\pm$ | $Z$ |
|---|---|---|
| $p\bar{p}(1.96)$ | 26.3 | 7.8 |
| $pp(14)$ | 206.9 | 60.9 |
| $pp(10)$ | 145.4 | 42.5 |

# Conclusions

- For the analysis of the existing HERA DIS data the VFNS is not necessary, in the case of smooth matching with the FFNS it is very similar to the latter at realistic kinematics and does not change/improve a PDFs fit.

- Instead of VFNS the partial $O(\alpha_s^3)$ corrections do improve agreement to the data at small $Q^2$. We need the complete NNLO corrections to clarify interpretation of the data at large $Q^2$; for the planned high-energy *ep* facility this would be even more important due to the wider range of $Q^2$.

- In the global fits including the DIS and hadron collider data the VFNS is necessary, in this case it can be efficiently combined with the FFNS using the BMSN matching conditions.