Gluon and dipole cascades on the light-front

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Outline

Modified kernel for dipole evolution.

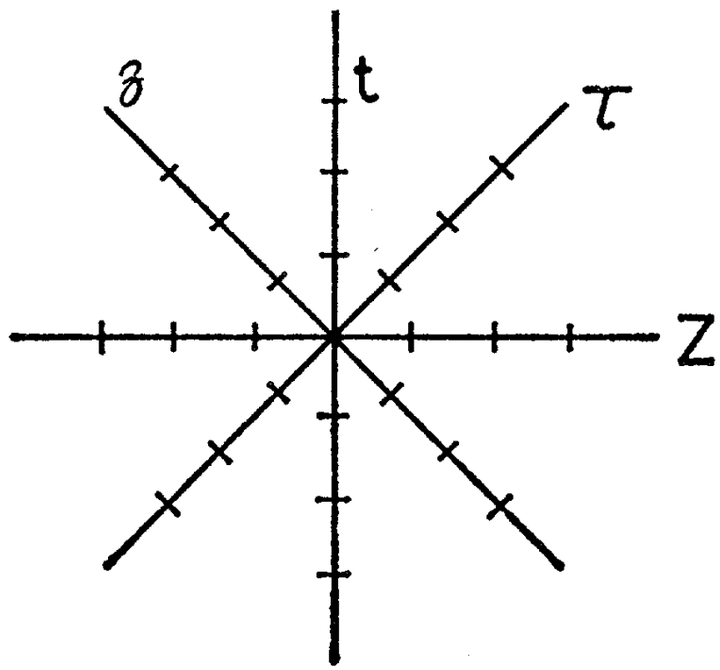
Light-front wave-functions with exact kinematics.

Gluon fragmentation amplitudes.

Relation with the maximally helicity violating (MHV) amplitudes.

Dirac

Light-front formalism



Kogut,Soper

Infinite momentum frame: a limit of a Lorentz frame moving in the -z direction with a (nearly) the speed of light.

Susskind

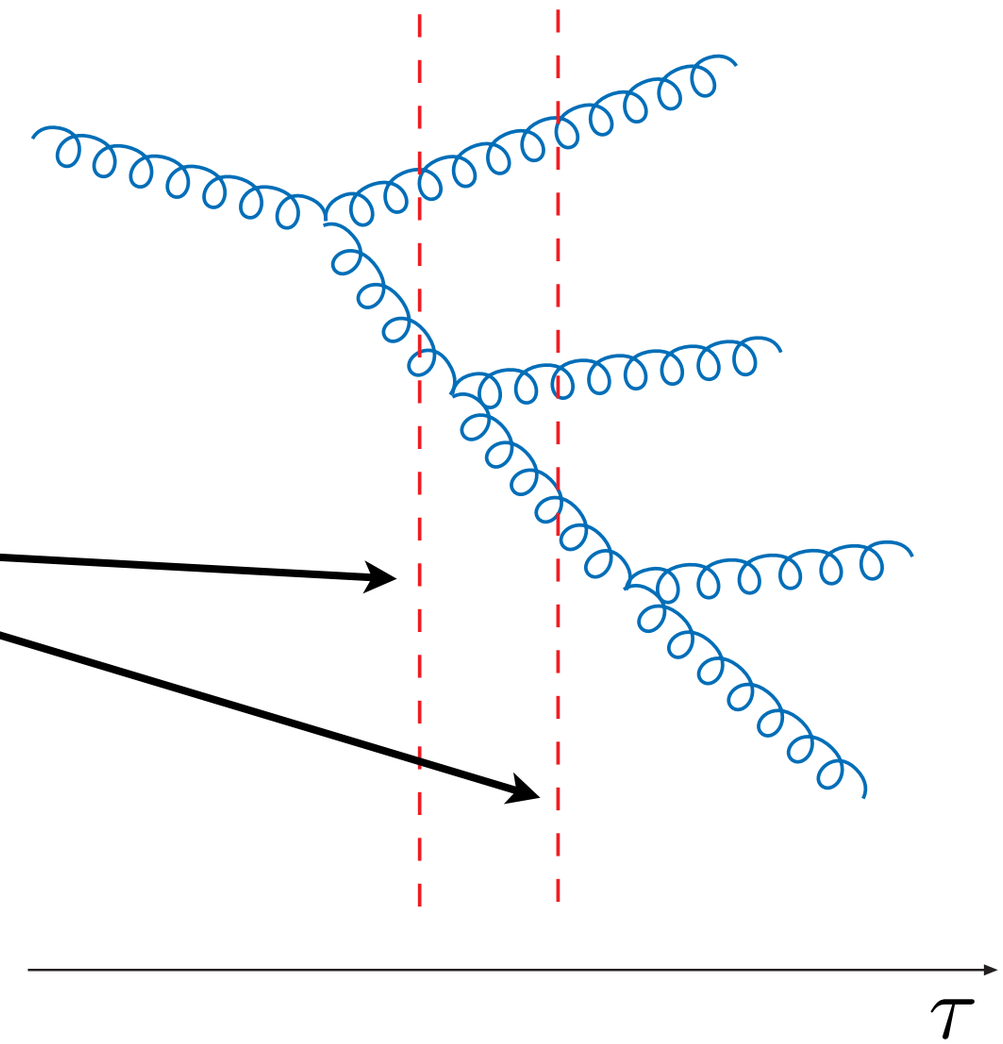
Isomorphism with the Galilean dynamics in 2 dimensions:

$P^- \longrightarrow$ Hamiltonian

$P^+ \longrightarrow$ Mass

$P_T \longrightarrow$ 2-dim. momentum

Non-covariant (light-front) time ordered diagram



Energy denominators

Difference of light - cone energies:

$$D_n = P^- - \sum_i k_i^-$$

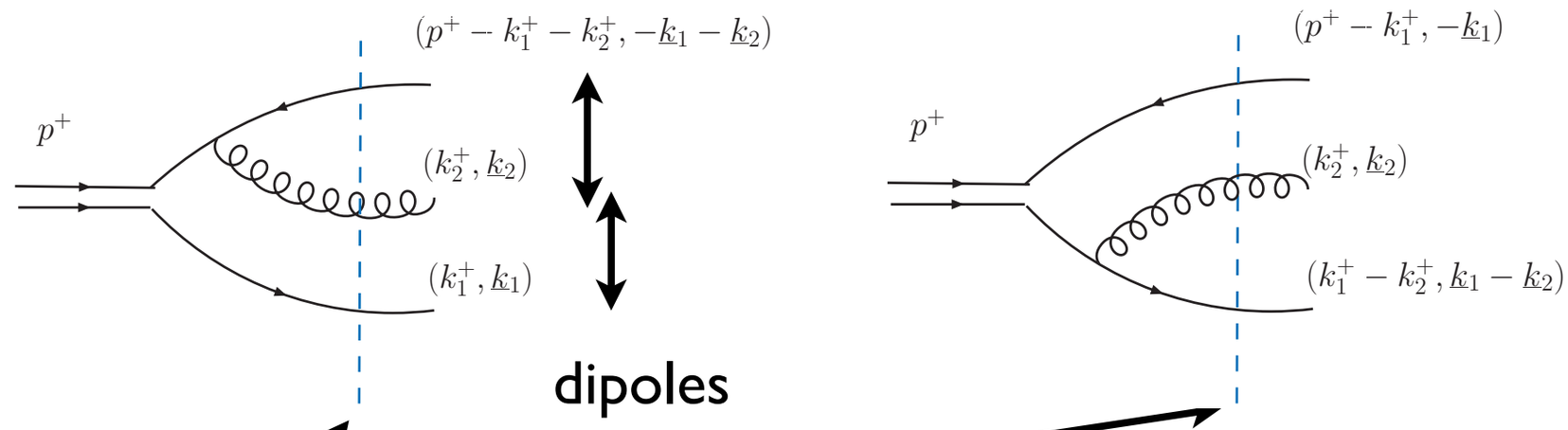
Initial state

Intermediate states

Dipole evolution at high energy

A.Mueller

Quark-antiquark pair emitting longitudinally soft gluon. Dipole evolution.



Energy denominators in light cone perturbation theory

High energy limit: strong ordering in longitudinal momenta

$$\bar{D}_1 = \frac{1}{P^- - [(P - k_1 - k_2)^- + k_1^- + k_2^-]} \simeq \frac{1}{k_2^-}$$

$$k_2^+ \ll k_1^+, P^+ \qquad k_2^- \equiv \frac{\underline{k}_2^2}{2k_2^+}$$

$$\Psi^{(1)}(\underline{k}_1, \underline{k}_2, z_1, z_2) = 2gt_a \frac{\underline{\epsilon}_2 \cdot \underline{k}_2}{\underline{k}_2^2} [\Psi^{(0)}(\underline{k}_1, z_1) - \Psi^{(0)}(\underline{k}_1 + \underline{k}_2, z_1)]$$

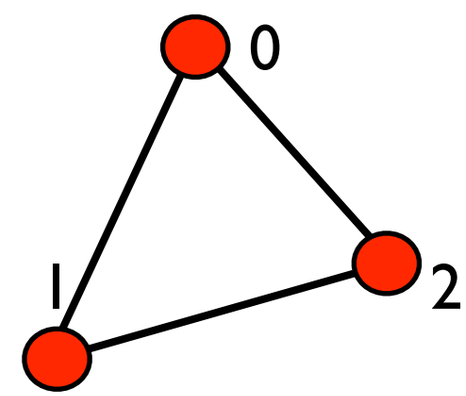
Wave function with 1 gluon

Wave function without gluons

In transverse coordinate space

$$\Phi^{(1)}(\underline{x}_{01}, \underline{x}_{02}; z_1, z_2) = -\frac{igt_a}{\pi}\left(\frac{\underline{x}_{20}}{x_{20}^2} - \frac{\underline{x}_{21}}{x_{21}^2}\right) \cdot \underline{\epsilon}_2 \, \Psi^{(0)}(\underline{x}_{01}; z_1)$$

Soft gluons factorize in the transverse spae



Dipole kernel in the limit of high energy:

$$\frac{d^2\underline{x}_{02}\, \underline{x}_{01}^2}{\underline{x}_{02}^2\, \underline{x}_{12}^2}$$

Dipole evolution in rapidity:

$$\frac{\partial N_{01}}{\partial Y} = \bar{\alpha}_s \int \frac{d^2 x_2}{2\pi} \frac{x_{01}^2}{x_{02}^2 x_{12}^2} [N_{02} + N_{12} - N_{01}]$$

Y rapidity

N_{01} dipole scattering amplitude (related to the gluon density)

No restrictions on the transverse coordinates (or momenta).

In the high energy limit:

$$\bar{D}_1 = \frac{1}{P^- - [(P-k_1-k_2)^- + k_1^- + k_2^-]} \simeq \frac{1}{k_2^-}$$

$$k_2^+ \ll k_1^+, P^+ \qquad k_2^- \equiv \frac{\underline{k}_2^2}{2k_2^+}$$

For the consistency of the calculation we should take:

$$\frac{\underline{k}_2^2}{k_2^+} > \frac{\underline{k}_1^2}{k_1^+}$$

For more emissions

$$\ldots \frac{\underline{k}_4^2}{k_4^+} > \frac{\underline{k}_3^2}{k_3^+} > \frac{\underline{k}_2^2}{k_2^+} > \frac{\underline{k}_1^2}{k_1^+}$$

Ordering in the fluctuation time: Dokshitzer, Marchesini, Salam

$$\tau \sim \frac{k^+}{\underline{k}^2}$$

Longitudinal and transverse momenta tied together.

Modified dipole kernel

Quasi-local approximation $$D_1 \simeq \frac{1}{k_1^- + k_2^-}$$

Keep the energy of the parent emitter

$$\Psi^{(1)}(\underline{k}_1, \underline{k}_2, z_1, z_2) = 2gt_a \frac{\underline{\epsilon}_2 \cdot \underline{k}_2}{\underline{k}_2^2 + \underline{k}_1^2 \frac{k_2^+}{k_1^+}} [\Psi^{(0)}(\underline{k}_1, z_1) - \Psi^{(0)}(\underline{k}_1 + \underline{k}_2, z_1)]$$

Approximate Fourier transform

$$\Phi^{(1)}(\underline{x}_{02}, \underline{x}_{12}; z) \sim gt^a \left(\bar{Q}_{01} K_1(\bar{Q}_{01} x_{02}) \frac{\underline{\epsilon}_2 \cdot \underline{x}_{02}}{x_{02}} - \bar{Q}_{01} K_1(\bar{Q}_{01} x_{12}) \frac{\underline{\epsilon}_2 \cdot \underline{x}_{12}}{x_{12}} \right) \Phi^{(0)}(\underline{x}_{01}; z)$$

Modified Bessel functions of the second kind.

$$\overline{Q}_{01} \simeq \frac{1}{x_{01}} \sqrt{\frac{k_2^+}{k_1^+}} = \frac{1}{x_{01}} \sqrt{z}$$

z longitudinal momentum fraction

$\ln 1/z \sim y$ rapidity

Modified dipole kernel

$$d^2\underline{x}_2\left(\overline{Q}_{01}K_1(\overline{Q}_{01}x_{02})\frac{\underline{\epsilon}_2\cdot\underline{x}_{02}}{x_{02}}-\overline{Q}_{01}K_1(\overline{Q}_{01}x_{12})\frac{\underline{\epsilon}_2\cdot\underline{x}_{12}}{x_{12}}\right)^2 \xrightarrow[\frac{x_{02}}{x_{01}}\sqrt{z}\to 0]{} \frac{d^2\underline{x}_{02}\,\underline{x}_{01}^2}{\underline{x}_{02}^2\,\underline{x}_{12}^2}$$

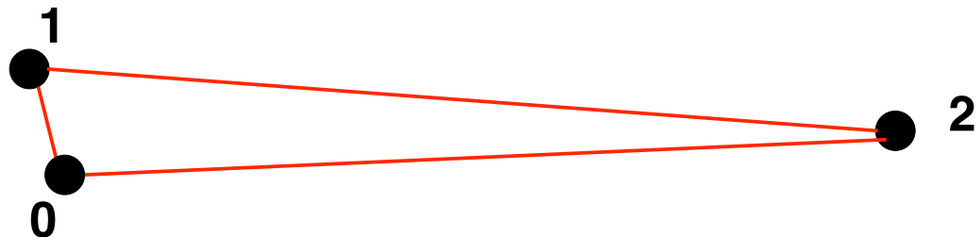
$$\overline{Q}_{01}\simeq\frac{1}{x_{01}}\sqrt{\frac{k_2^+}{k_1^+}}=\frac{1}{x_{01}}\sqrt{z}$$

Dipole kernel with Bessel-Macdonald functions:

- Energy dependent cutoff in impact parameter: exponential tails, range depends on the energy.
- Violation of conformal invariance in 2-dimensions.
- Recovering original dipole kernel in the high energy limit.

Impact parameter and NLL correction

Cutoff on configuration of large dipoles



$$x_{02} \gg x_{01} \;\&\; x_{12} \sim x_{02}$$

exponential behavior

$$\mathcal{K} \sim \exp\left(-\frac{x_{02}}{x_{01}}\sqrt{z}\right)$$

Recovering part of NLL contribution from explicit calculation by Balitsky and Chirilli (non-conformal part).

$$\mathcal{K}^{\rm NLO}_{\rm non-conf.} \otimes N_Y \;\rightarrow\; -\frac{\bar{\alpha}_s^2}{\pi}\int \frac{d^2\underline{x}_2\, x_{01}^2}{x_{02}^4}\log^2\!\left(\frac{x_{02}}{x_{01}}\right)[\ldots]$$

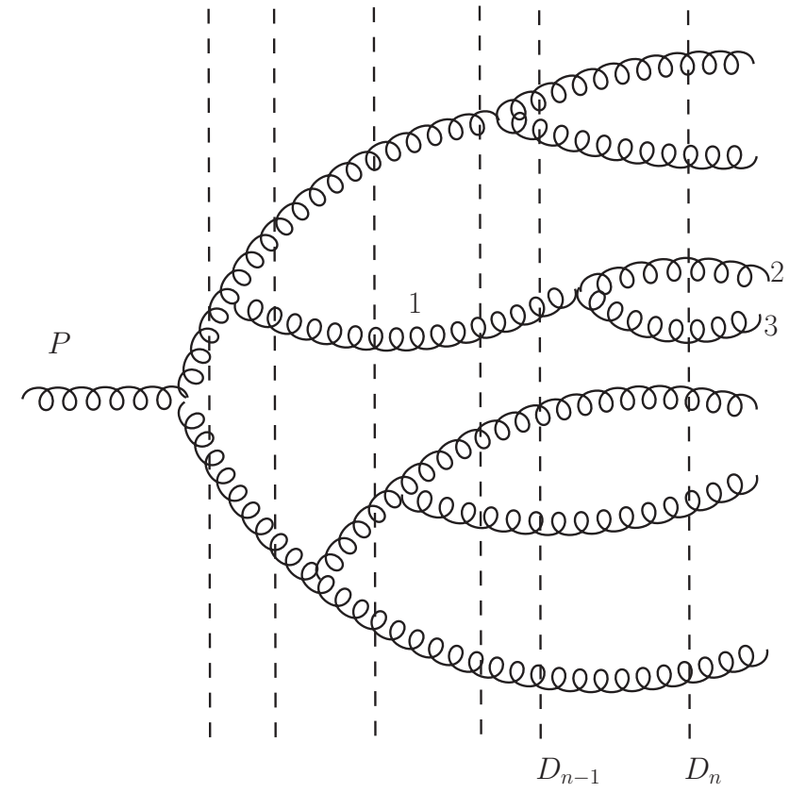
Light cone wave functions

✳ Previously: modified kernel, only some corrections in the energy denominators. Still eikonal vertices.

✳ Keep kinematics exact through the complete evolution: both vertices and energy denominators kept exact.

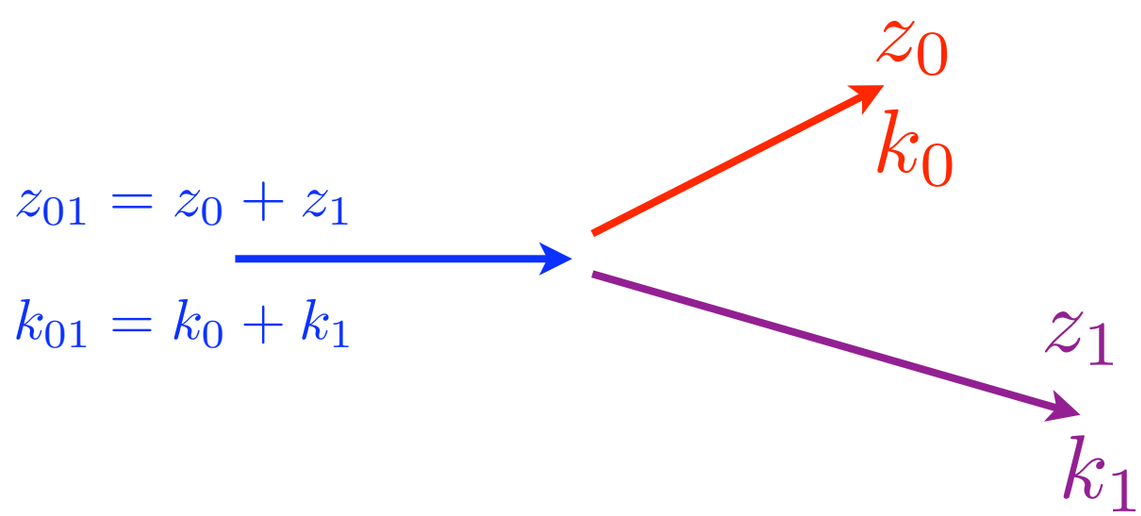
Gluon in the initial state. Dynamics similar to the dipole model.

Helicity conserved through the whole cascade.



Recurrence relations between wave functions

$$\Psi_{n+1}(k_0, k_1, \ldots, k_n) = \frac{g}{\sqrt{\xi_{01}}} \frac{\underline{\epsilon}^{(-)} \underline{v}_{01}}{D_n + \xi_{01}\, \underline{v}_{01}^2} \Psi_n(k_{01}, k_2, \ldots, k_n)$$



reduced mass:

$$\xi_{01} = \frac{z_0 z_1}{z_0 + z_1}$$

relative velocity:

$$\underline{v}_{01} = \frac{\underline{k}_0}{z_0} - \frac{\underline{k}_1}{z_1}$$

Isomorphism with non-relativistic dynamics apparent in the case of the exact kinematics.

Light cone wave function

Case of the on-shell incoming gluon.

Can resum the wave function completely.

$$-D_{n+1}\,\Psi_{n+1}(1,2,\ldots,n+1) \;=\; g\sum_{i=1}^{n}\frac{v^*_{(i,i+1)}}{\sqrt{\xi_{(i,i+1)}}}\Psi_n(1,2,\ldots,(i\,i+1),\ldots,n+1) \qquad n\to n+1$$

$$-D_n\Psi_n(1,2,\ldots,n) \;=\; g\sum_{k=1}^{n-1}\frac{v^*_{(k,k+1)}}{\sqrt{\xi_{(k,k+1)}}}\Psi_{n-1}(1,2,\ldots,(k\,k+1),\ldots,n) \qquad n-1\to n$$

...

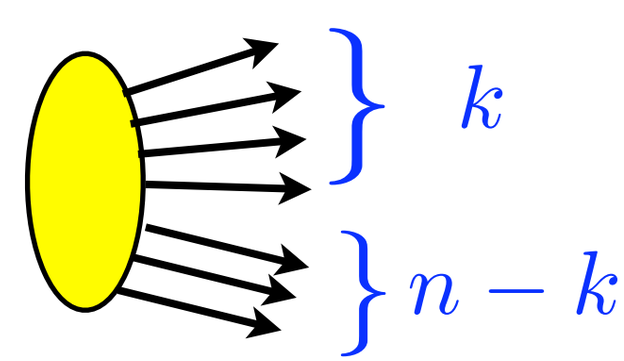
Tree-level gluon wave function with exact kinematics

$$\Psi_n(1,2,\ldots,n) \;=\; (-1)^{n-1}g^{n-1}\,\Delta^{(n)}\,\frac{1}{\sqrt{z_1 z_2\cdots z_n}}\,\frac{1}{\xi_{(12\ldots n-1)n}\,\xi_{(12\ldots n-2)(n-1\,n)}\,\cdots\,\xi_{1(2\ldots n)}}$$

$$\times\,\frac{1}{v_{(12\ldots n-1)n}\,v_{(12\ldots n-2)(n-1\,n)}\,\cdots\,v_{1(2\ldots n)}}\,.$$

$$v_{(i_1 i_2\ldots i_p)(j_1 j_2\ldots j_q)} = \frac{k_{i_1}+k_{i_2}+\ldots+k_{i_p}}{z_{i_1}+z_{i_2}+\ldots+z_{i_p}} - \frac{k_{j_1}+k_{j_2}+\ldots+k_{j_q}}{z_{j_1}+z_{j_2}+\ldots+z_{j_q}},$$

$$\xi_{(i_1 i_2\ldots i_p)(j_1 j_2\ldots j_q)} = \frac{(z_{i_1}+z_{i_2}+\ldots+z_{i_p})(z_{j_1}+z_{j_2}+\ldots+z_{j_q})}{z_{i_1}+z_{i_2}+\ldots+z_{i_p}+z_{j_1}+z_{j_2}+\ldots+z_{j_q}}\,,$$



Relation to Parke-Taylor amplitudes

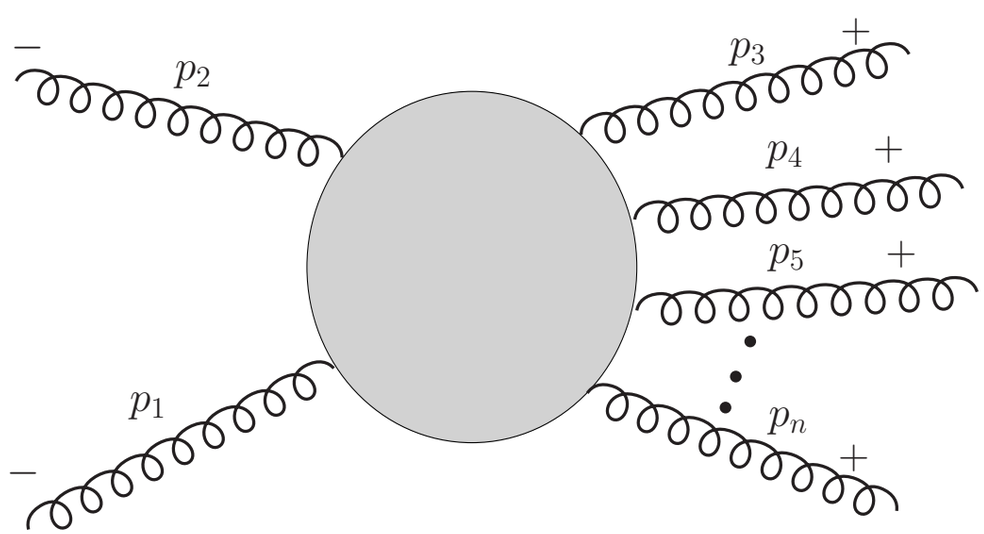
$$\mathcal{M}_n = \sum_{\{1,\ldots,n\}} \mathrm{tr}(t^{a_1} t^{a_2} \ldots t^{a_n})\, m(p_1,\epsilon_1;p_2,\epsilon_2;\ldots;p_n,\epsilon_n)\ ,$$

Color part

Kinematical part

Maximally Helicity Violating amplitude for gluons: 2 to n

Here: all gluons are outgoing



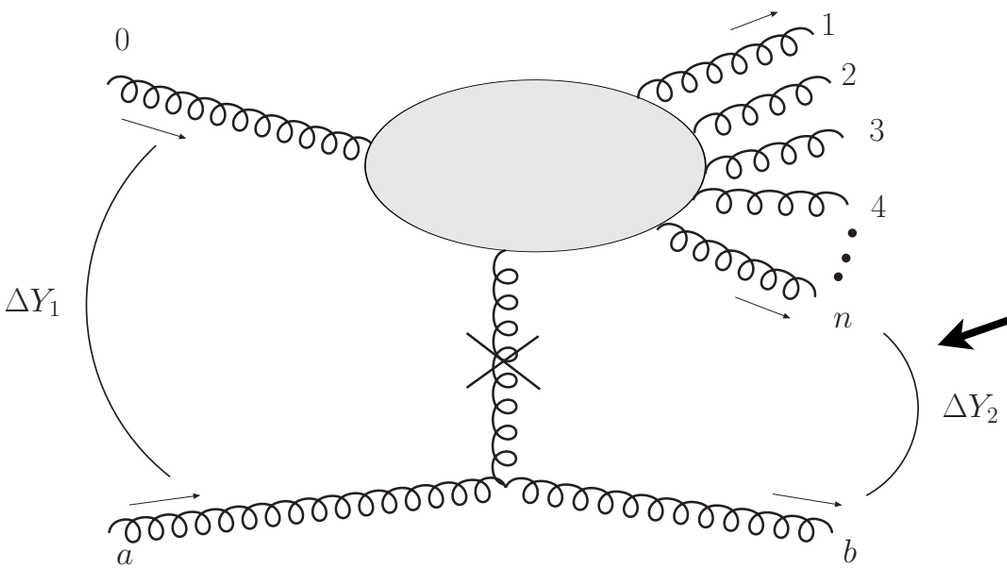
$$\langle ij \rangle = \sqrt{z_i z_j}\ \underline{\epsilon}^{(+)} \cdot \left(\frac{\underline{k}_i}{z_i} - \frac{\underline{k}_j}{z_j} \right)$$

$$\langle\, ij \,\rangle = \sqrt{z_i z_j}\ \underline{\epsilon}^{(+)} \cdot \underline{v}_{ij},$$

Tree level, Parke-Taylor formula

$$m(1^-,2^-,3^+,\ldots,n^+) \;=\; i g^{n-2}\, \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle \ldots \langle n{-}2\ n{-}1\rangle\langle n{-}1\ n\rangle\langle n1\rangle}\ ,$$

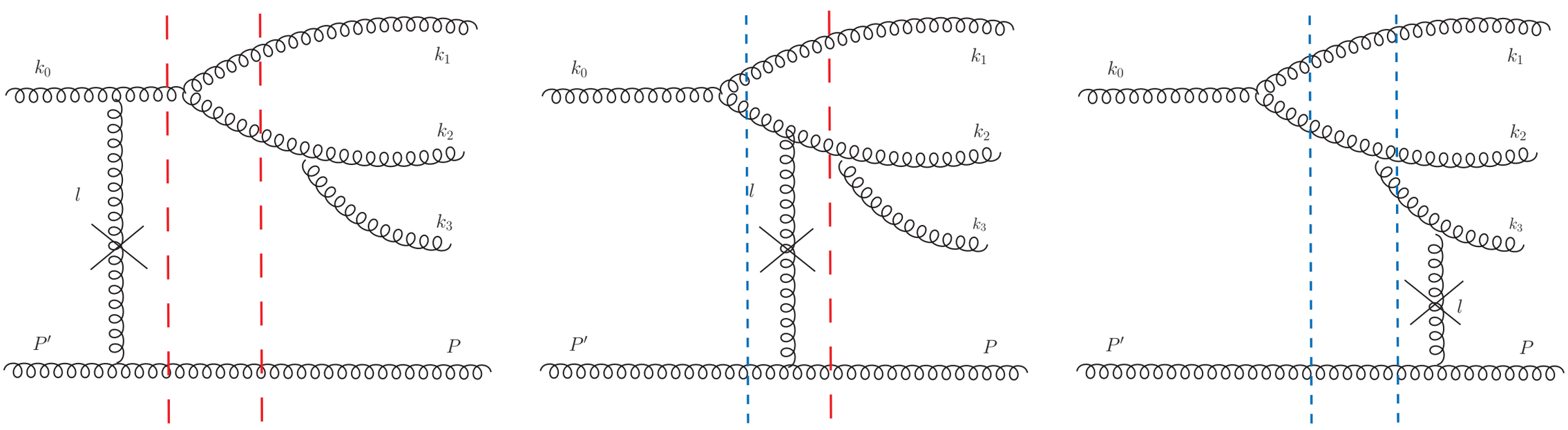
Scattering from light -cone wave functions



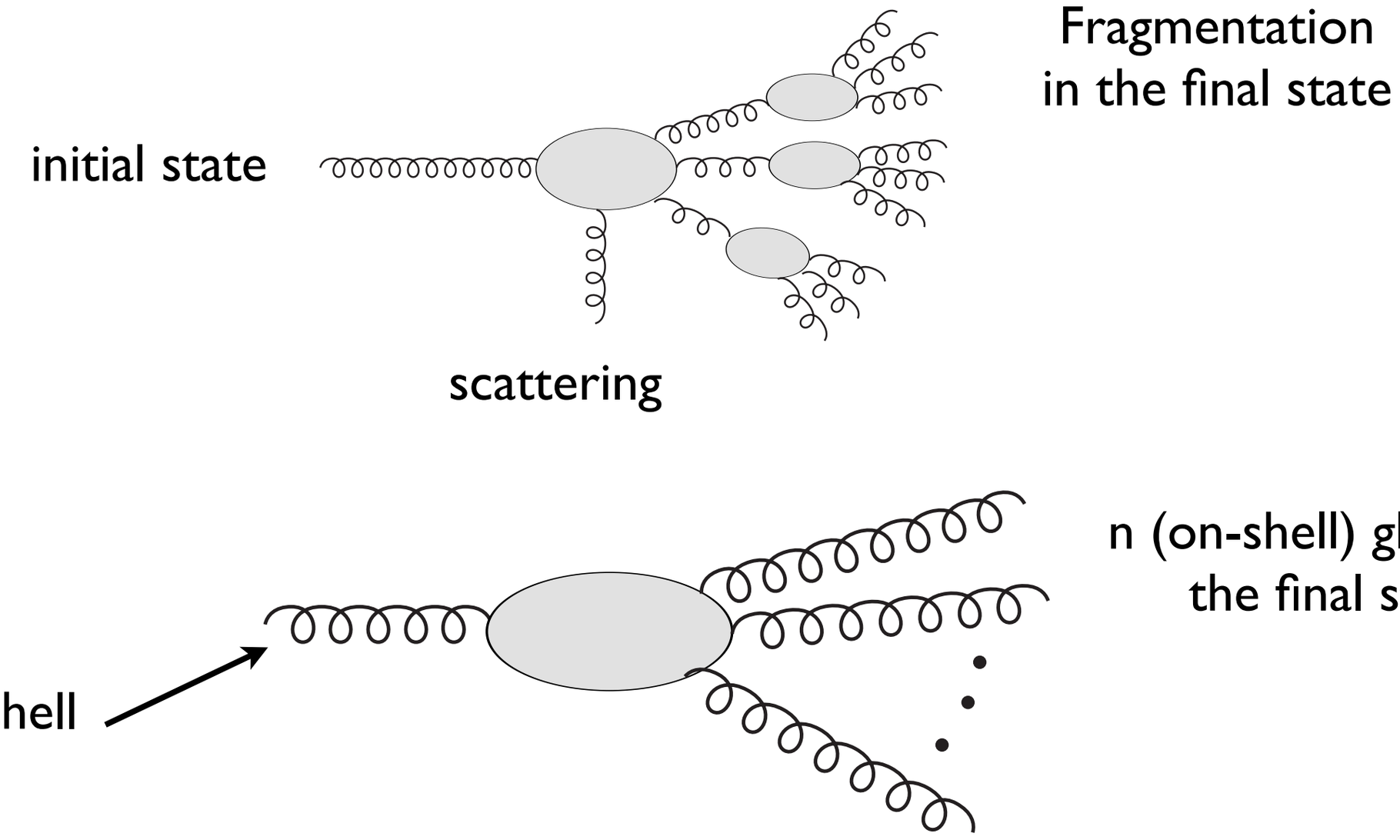
Rapidity gap

Helicity conserving.
High energy approximation: instantaneous gluon in the light-cone gauge

Sum over initial and final state emissions



Final state emissions: gluon fragmentation



Amplitude for fragmentation

$$T[(12\ldots n)\to 1,2,\ldots,n] \;=\; g^{n-1}\left(\frac{z_{(12\ldots n)}}{z_1 z_2\ldots z_n}\right)^{3/2}\frac{1}{v_{12}v_{23}\ldots v_{n-1\,n}}$$

Duality: wave function vs fragmentation

Wave function
initial state

$\longleftrightarrow$

Fragmentation
final state

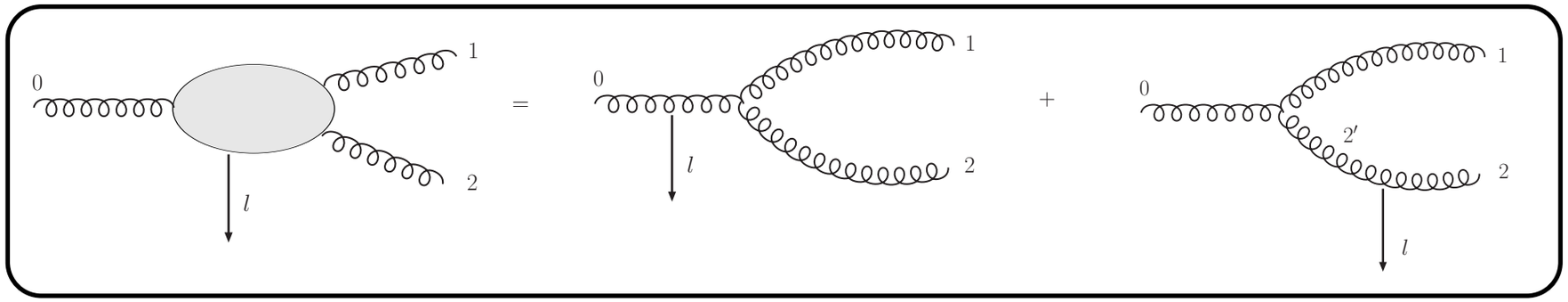
$$\Psi_n \sim \frac{1}{v_{(12\ldots n-1)n}\, v_{(12\ldots n-2)(n-1\,n)} \cdots v_{1(2\ldots n)}}$$

$$T_n \sim \frac{1}{v_{12} v_{23} \cdots v_{n-1\,n}}$$

Nearly identical expressions (the same topology of graphs): different combinations of momenta

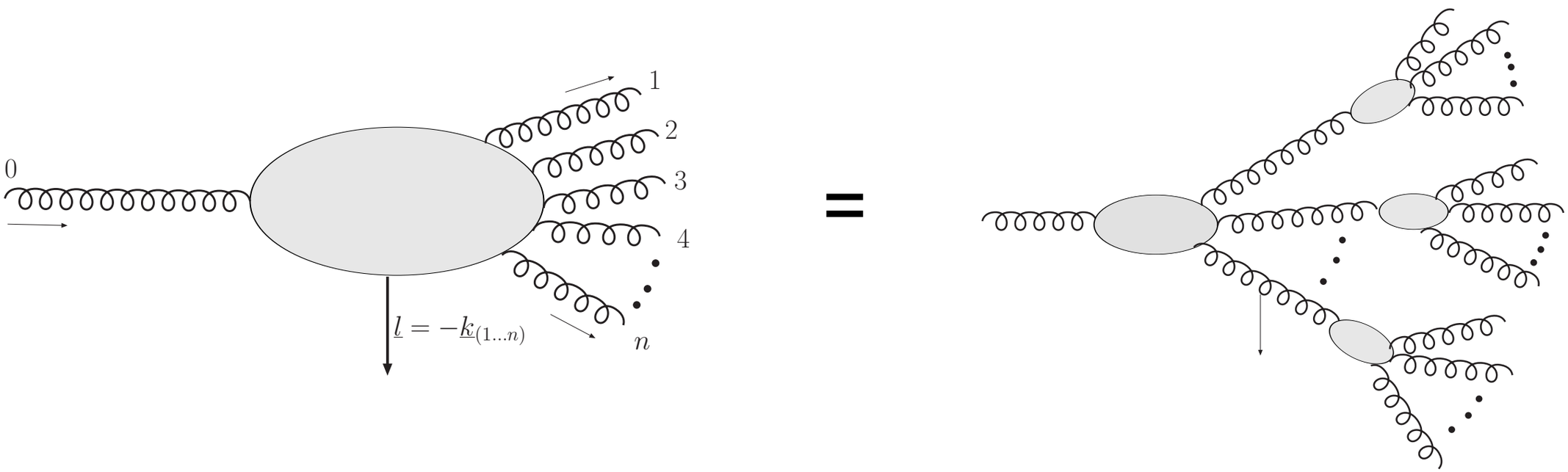
$$v_{(i_1 i_2 \ldots i_p)(j_1 j_2 \ldots j_q)} = \frac{k_{i_1} + k_{i_2} + \ldots + k_{i_p}}{z_{i_1} + z_{i_2} + \ldots + z_{i_p}} - \frac{k_{j_1} + k_{j_2} + \ldots + k_{j_q}}{z_{j_1} + z_{j_2} + \ldots + z_{j_q}},$$

Relation with MHV

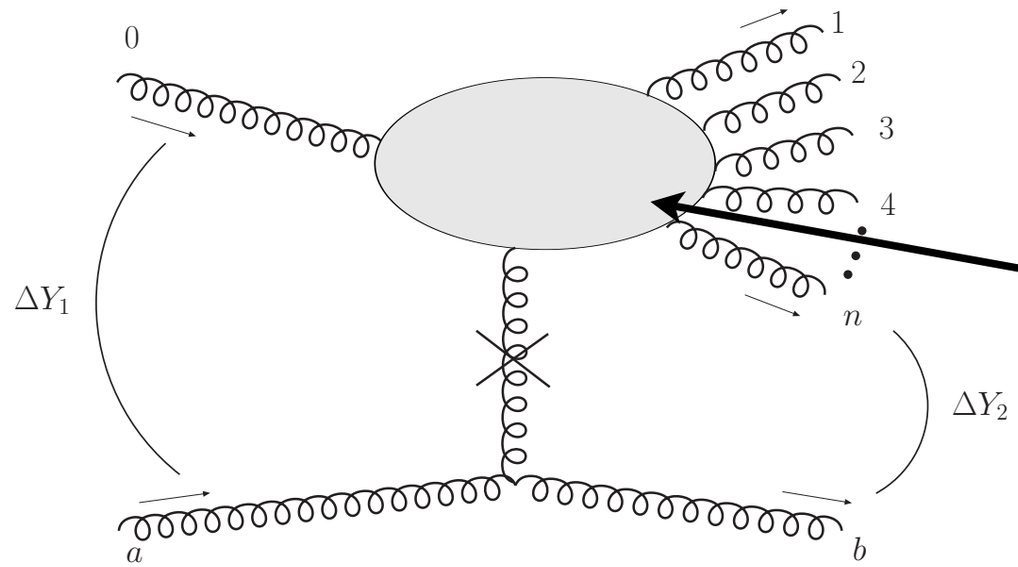


$$\tilde{\Psi}_2(1,2) \;=\; T[(12)\to 1,2] \;+\; \Psi_2(1,2')\,,$$

Master formula for arbitrary number of gluons



$$\tilde{\Psi}_n(1,2,\ldots,n) \;=\; \sum_{m=1}^{n} \sum_{(1\le n_1<n_2<\ldots<n_{m-1}\le n)} \Psi_m((1\ldots n_1)(n_1+1\ldots n_2)\ldots(n_{m-1}+1\ldots n))$$
$$\times\; T[(1\ldots n_1)\to 1,\ldots,n_1]\, T[(n_1+1\ldots n_2)\to n_1+1,\ldots,n_2]\,\ldots\, T[(n_{m-1}+1\ldots n)\to n_{m-1}+1,\ldots,n]\,.$$



Upper part: 1 to n with momentum transfer. Obtained by summing all possible attachments.

$$M(0; a \to 1, \ldots, n; b) \simeq \frac{s}{t} \times \tilde{\Psi}_n$$

2 to 2 amplitude

Spinor products:

$$\langle i i+1 \rangle = \sqrt{z_i z_{i+1}} v_{i i+1}$$

Recover MHV amplitude in the light cone formalism

$$M(0; a \to 1, \ldots, n; b) \simeq g^{n+1} \frac{\langle a0 \rangle^4}{\langle a0 \rangle \langle 01 \rangle \langle 12 \rangle \langle n-1\, n \rangle \langle nb \rangle \langle ba \rangle},$$

Summary

Reformulation of the dipole kernel to include kinematical effects.

Impact parameter dependence significantly modified: exponential tails with the energy-dependent cutoff.

Resummation of the light cone wave function with exact kinematics.

Resummation of the fragmentation amplitudes. Duality between the fragmentation and the wave functions.

Derivation of the scattering amplitudes in the light cone formalism.
Consistency check with (new derivation of) the MHV amplitudes.