Fast NLO calculation of jet and charm production cross sections by the pseudo-moment method

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Parton Distribution Function at LHC era

LHC : “QCD machine”

- Precise knowledge of parton distribution is vital for signal/background evaluation.
- NLO (NNLO) PDF from global fits:

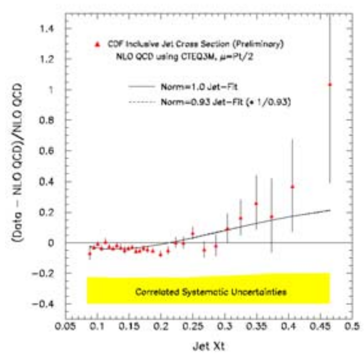
  CTEQ, MSTW, H1, ZEUS, Alekhin, NNPDF, BBG, etc.

- Plentiful jet(s) data from LHC

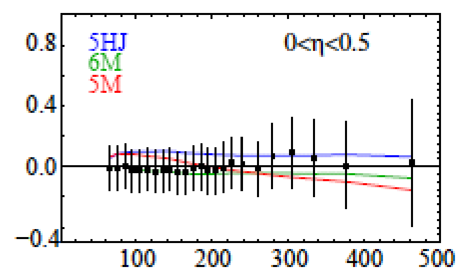
  → “Feedback” from LHC jet data (+ Tevatron . HERA)

  ex. “anomaly” in jet data ↔ new physics or new pdf ?

Inclusive jet data at Tevatron



CTEQ3M



CTEQ5HJ,5M, 6M

K-factor method

NLO calculation of jet cross section

$\rightarrow$ Monte Carlo integration

Jet definition, experimental cuts

$$\sigma = \sum_n \alpha_s^n(\mu_r) \sum_{a,b} \int dx_1 \int dx_2 \boxed{\hat{\sigma}_{ab}^{(n)}(x_1, x_2)} f_a(x_1, \mu_f) f_b(x_2, \mu_f)$$

— For PDF fit, repeated calculations take too much time.

- K-factor method

Calculate $K = \dfrac{d\sigma^{NLO}}{d\sigma^{LO}}$ for a given PDF

$\rightarrow$ Assume $\sigma^{NLO} \approx K \sigma^{LO}$

— Actually, K-factor does depend on PDF.

— To go beyond the K-factor method, we need to separate PDF from MC integration !

- Interpolation method (x-space)

fastNLO (Kluge,Rabbertz,Wobisch)
ApplGrid (Carli et al.)

$$f_a(x) = \sum_i f_a(x^{(i)}) E^{(i)}(x)$$

$E^{(i)}(x)$: interpolation functions

Interpolate PDF with 10s of grid points

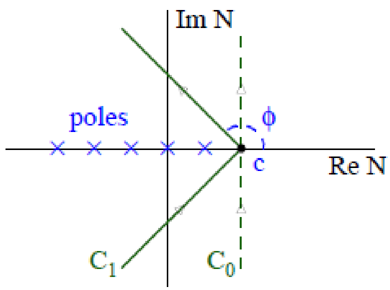
$$\sigma = \sum_{a,b} \sum_{i,j} f_a(x_1^i) f_b(x_2^j) \times \sum_n \alpha_s^n(\mu_r) \boxed{\int dx_1 \int dx_2 \hat{\sigma}_{a,b}^{(n)}(x_1, x_2) E_a^{(i)}(x_1, \mu_f) E_b^{(j)}(x_2, \mu_f)}$$

MC Integration → sum over the grid pts with pre-calculated coefficients

- "Pseudo moment" method (N-space)

Stratmann, Vogelsang; Berger, Graudenz, Hampel, Vogt; Kosower

$$\sigma = \sum_k \alpha_s^k(\mu_r) \sum_{a,b} \int dx_1 \int dx_2 \sigma_{ab}^{(k)}(x_1, x_2, \mu_r, \mu_f) f_a(x_1, \mu_f) f_b(x_2, \mu_f)$$



Inverse Mellin transform

$$f_a(x, \mu_f) = \frac{1}{2\pi i} \int_{\mathcal{C}} dN x^{-N} f_a^N(\mu_f)$$

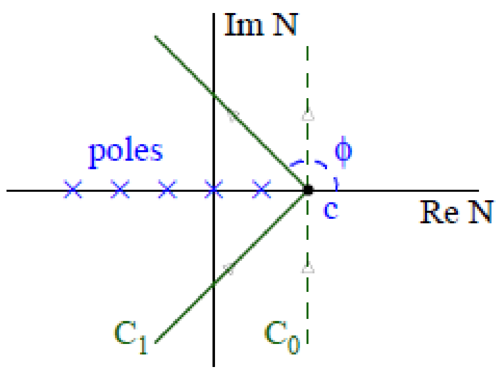
$$\sigma = \frac{1}{(2\pi i)^2} \int_{\mathcal{C}_{N_1}} dN_1 \int_{\mathcal{C}_{N_2}} dN_2 \sum_{a,b} f_a^{N_1}(\mu_f) f_b^{N_2}(\mu_f)$$
$$\times \sum_k \alpha_s^k(\mu_r) \boxed{\int dx_1 \int dx_2 x_1^{-N_1} x_2^{-N_2} \hat{\sigma}_{a,b}^{(k)}(x_1, x_2)}$$

Pseudo moments: $\left[\hat{\sigma}_{a,b}^{(k)}\right]_{N_1, N_2}$

Pseudo moment method

$$\sigma = \frac{1}{(2\pi i)^2}\int_{\mathcal{C}_{N_1}} dN_1 \int_{\mathcal{C}_{N_2}} dN_2 \sum_a H_a^{N_1,N_2}(\mu_f) \sum_i \alpha_s^i(\mu_r)\left[\hat{\sigma}_a^{(k)}\right]_{N_1,N_2}$$

$$= \frac{1}{(2\pi i)^2}\sum_{i,j}\Delta u_i \Delta v_j \mathrm{Re}\left\{ e^{2i\phi} H_a^{N_{1,i},N_{2,j}}(\mu_f)\left[\hat{\sigma}_a^{(k)}\right]_{N_{1,i},N_{2,j}} + H_a^{N_{1,i},N_{2,j}^*}(\mu_f)\left[\hat{\sigma}_a^{(k)}\right]_{N_{1,i},N_{2,j}^*}\right\}$$



sub-process $H_{a=1}^{m,n}(\mu_f) = G^m(\mu_f)\cdot G^n(\mu_f)$

Discritize for $(N_1,\ N_2)$ integration:

$$N_{1,i} = u_i e^{i\phi},\ \ N_{2,j} = v_j e^{i\phi}$$

\# pseudo moment

= # sub-process x 2 (NLO) x (# points in N-space)^2 x # 2 (branches)

~ O(10^5) for each bin

— Pre-calculate them only once before fitting and restore as grid data.

— Moreover, we can make use of fastNLO to calculate grid data for jet (dijet) production.

Towards a Global Fit

- Initial distributions

$$\boxed{x f_i(x, Q_0) = N_i p_{i,1} x^{p_{i,2}} (1-x)^{p_{i,3}} \left[1 + p_{i,5} x^{p_{i,4}} + p_{i,6} x\right] \quad + \quad \alpha_s(Q_0)}$$

- Evolution

QCD-PEGASUS (Vogt)

$$\boxed{\mathrm{P}_{\mathrm{NS}}^{(i)}(N), \;\; \mathbf{P}_S^i(N), \;\; A_{qq,h}^{ns,(2)}(N), \cdots}$$

- Coeff. functions for DIS, DY, …

$$\boxed{C_{2,q}^{(i)}(N), \overline{C}_{2,q}^{(i)}(N), C_{2,g}^{(i)}(N), \overline{C}_{2,g}(N) \cdots}$$

$$\boxed{q_i^N(\mu_f), \;\; g^N(\mu_f)}$$

- Pseudo moment method for jet(s), heavy quark production

fastNLO (Kluge et al.)
HVQDIS (Harris et al.) etc.

- N-space parameterizaton for massive coeff. functions

(Alekhin, Blümlein)

$$\boxed{\left[\hat{\sigma}_a^{(k)}\right](N), \;\; \left[\hat{\sigma}_a^{(k)}\right](N_1, N_2)} \longrightarrow \sigma^{\text{th.}} \longleftarrow \boxed{C_{2,hq}^{(i)}(N), \overline{C}_{2,hq}^{(i)}(N), C_{2,hg}^{(i)}(N), \overline{C}_{2,hg}(N) \cdots}$$

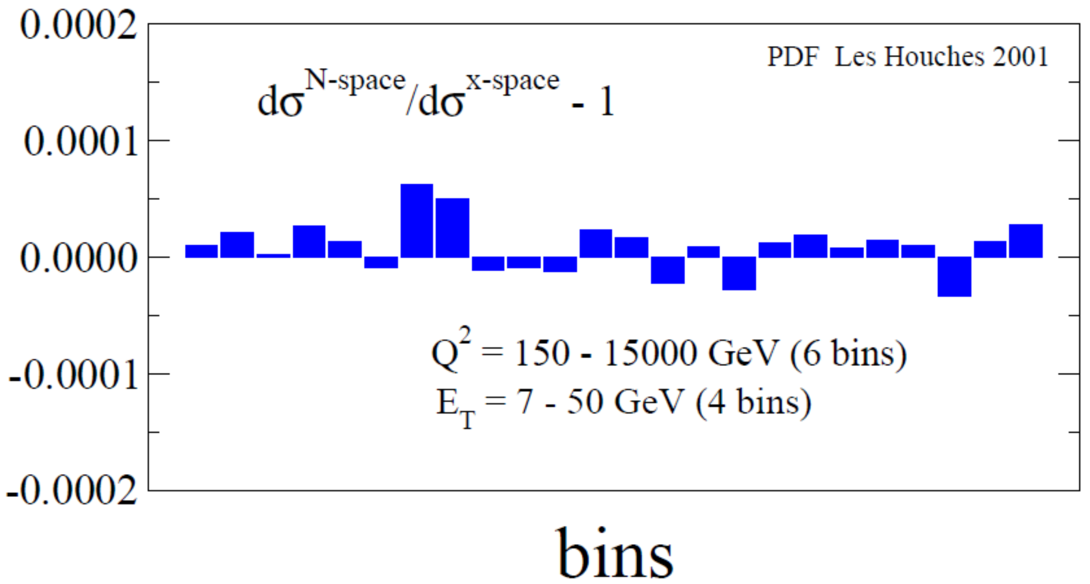
- Experimental data $\sigma^{\text{exp.}}$

$$\longrightarrow \chi^2 = \sum_i \frac{\sigma_i^{\text{th.}} - \sigma_i^{\text{exp.}}}{\delta \sigma_i^{\text{exp.}}}$$

Numerical accuracy

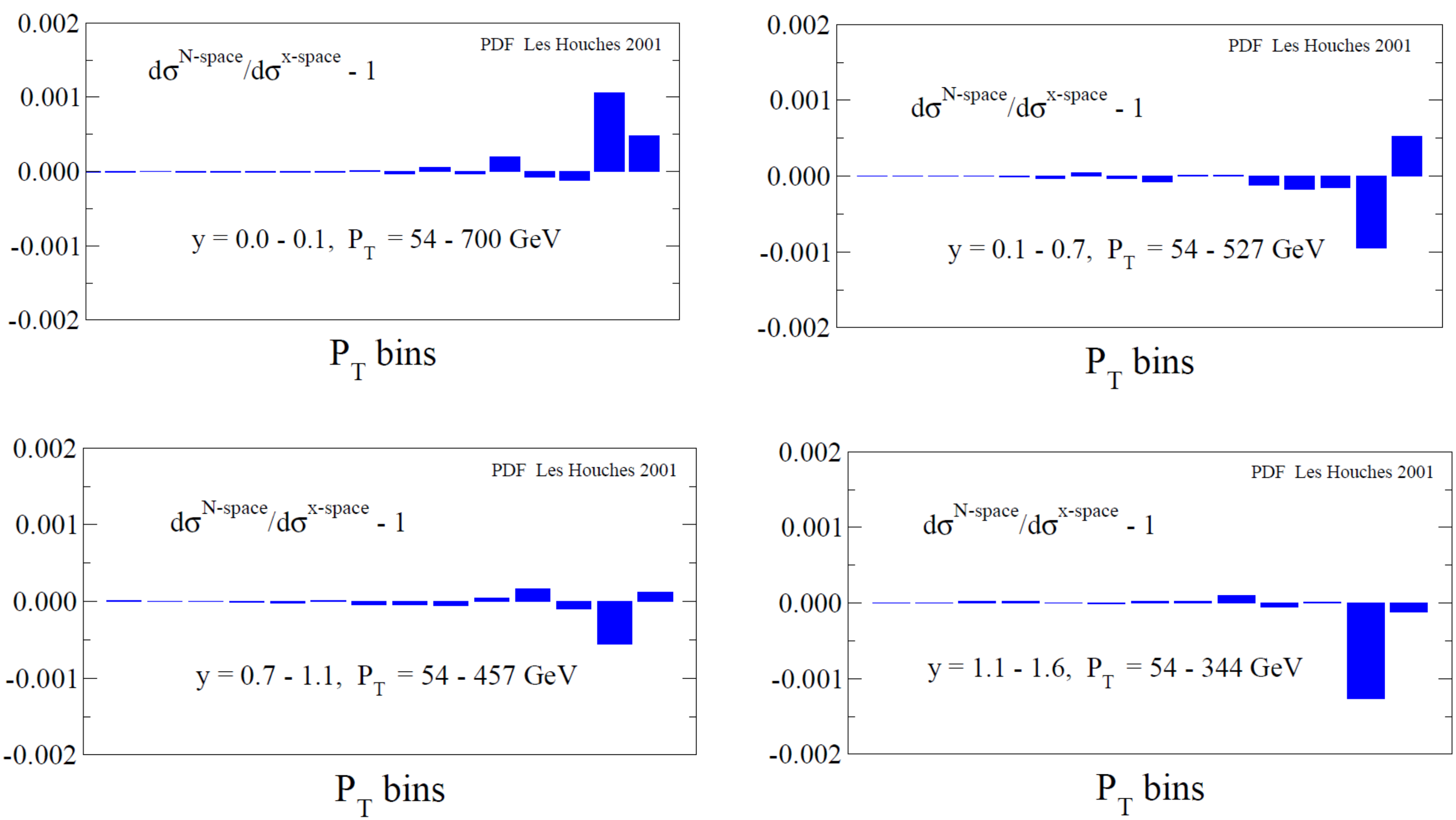
Comparison between results by direct x-space calculation and by N-space calculation with Pseudo moment method

- Jet production at eP collider (HERA kinematics)

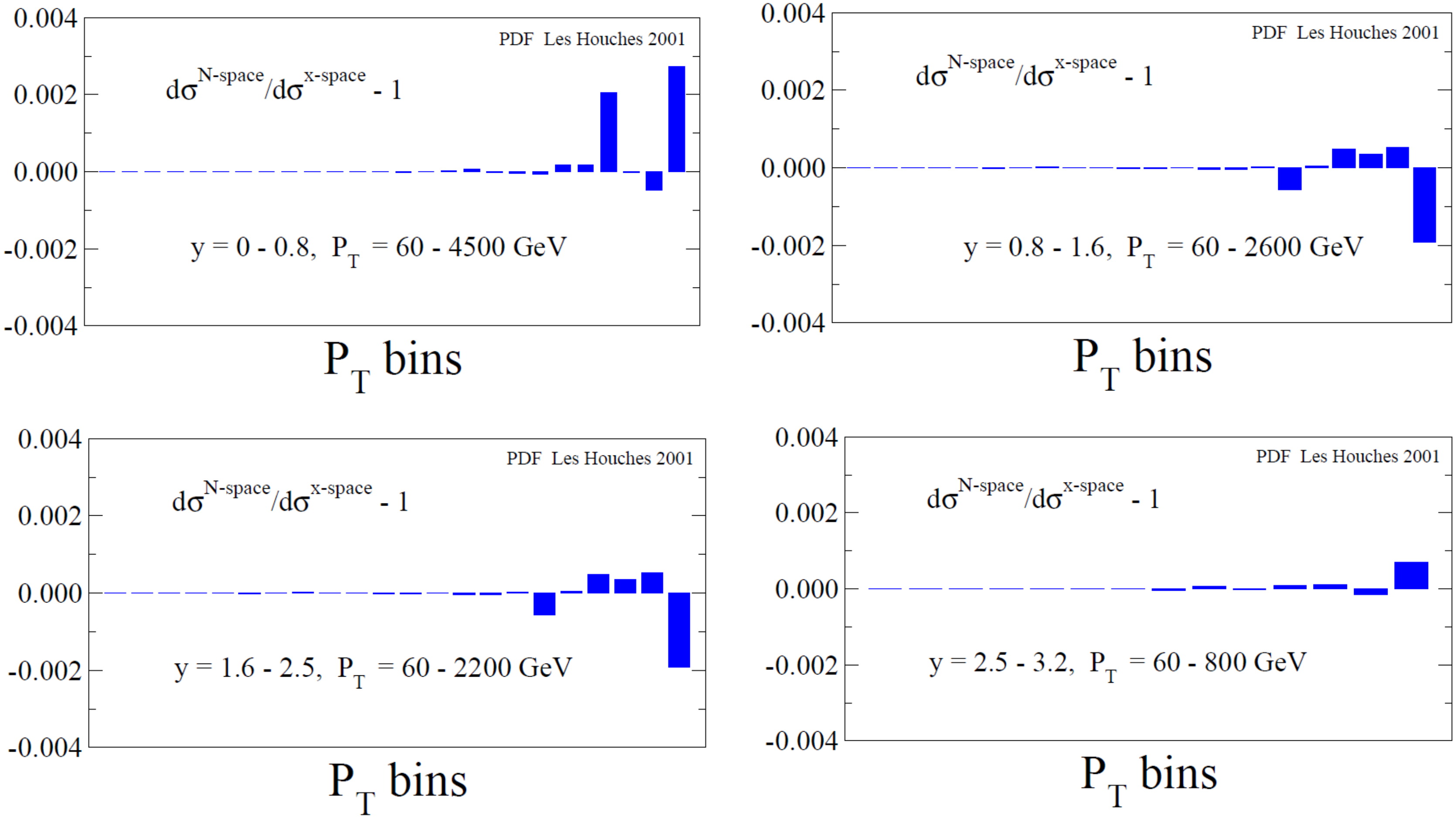


The relative error is less than 0.01% in the whole region.

- Jet production in PPbar collider (Tevatron Run II kinematics)



● Jet production at PP collider (LHC kinematics)



- ~ 0.01% , except very high P_T region with 4-digits grid data.
  ↔ enough for practical use.
- More accuracy possible by taking more precise grid data.

Heavy quark structure functions

$$F_2^{Q\bar{Q}}(x, Q^2, m^2) = \sum_i \int_x^{\mathrm{Zmax}} \frac{dz}{z} C_{2,i}(\xi, \eta) f_i\left(\frac{x}{z}, \mu^2\right)$$

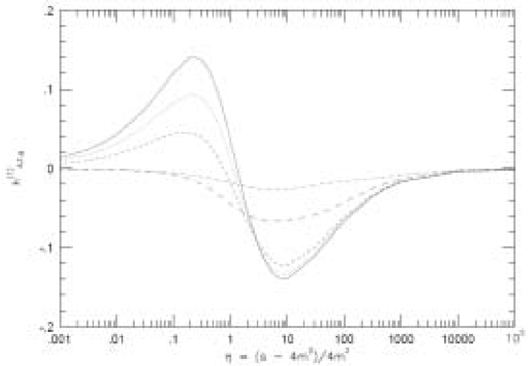
$$\xi = \frac{Q^2}{m^2}, \quad \eta = \frac{s}{4m^2} - 1$$

- NLO massive coeff. functions does not have a simple analytic form.
- For fast NLO calculation, we need parameterization for them.

(1) Riemersma, Simth, van Neerven ('95)

$$h_{2,i}^{(1)}(\xi, \eta) = C_{2,i}^{(1)}(\xi, \eta) - C_{2,i}^{(1),\eta\to 0}(\xi, \eta) - C_{2,i}^{(1),\eta\to\infty}(\xi, \eta)$$

threshold asymptotic form



parameterize in (ξ, η)

→ x-space calculation

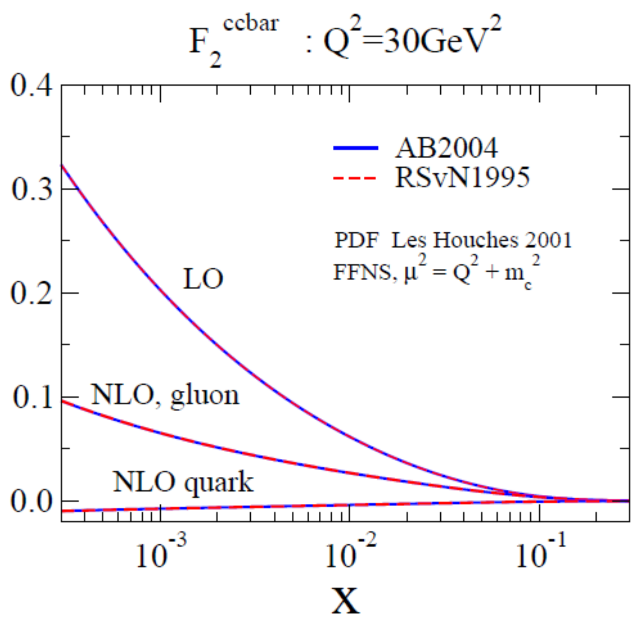
Heavy quark structure functions

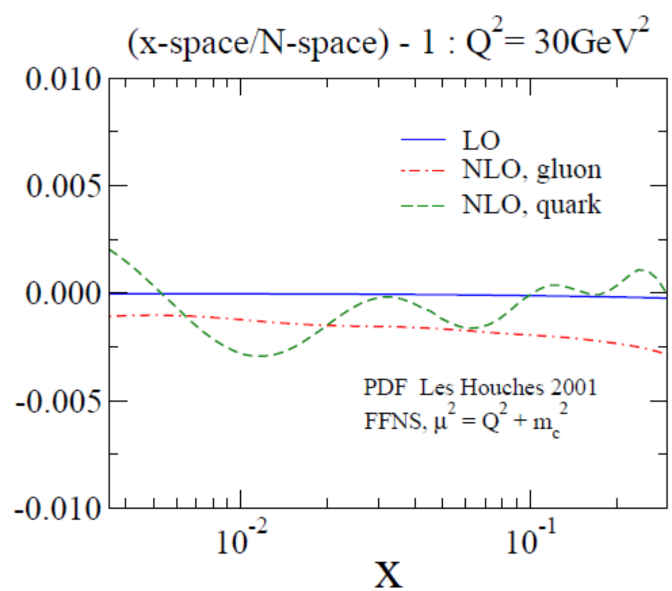
(2) Alekhin, Blümlein ('04)

parametrize by MINIMAX-method

$$C^{\text{MINIMAX}}(z,\xi)(\rho - z)^{\kappa} = \sum_{k=0}^{K} a_k(\rho) z^k$$

$$\to \mathbf{M}[C^{\text{MINIMAX}}(z,\xi)](N) = \sum_{k=0}^{K} a_k(\rho)\rho^{N+k-\kappa} B(N+k, 1-\kappa)$$





~ 0.1% accuracy for the sum of 3 contributions

Summery

- Fast NLO calculation is necessary for PDF fits with QCD final state data beyond K-factor approximation.

- Pseudo moment method is one of the solutions.
  - — Pre-calculate the pseudo moments for partonic cross sections combine them with each PDF.
  - — ~ 0.1% accuracy for jet cross sections .

- Heavy-quark structure functions
  - — MINIMAX-method is useful for N-space calculation

- A global fit with full NLO calculation for jet, dijet, c,b productions, etc.

cf. Polarized PDF by De Florian, Sassot, Stratmann,Vogelsang