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The Shuvaev Transform and the Skewness Effect at Small x

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I. Motivation: Generalised PDFs in (semi-) hard diffraction

II. The Shuvaev transform

III. Predictions for skewed partons at small x

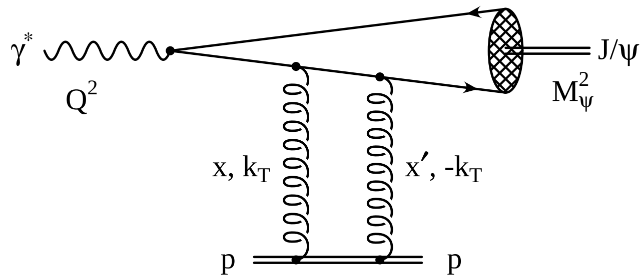
IV. Summary

Refs/more details: Martin+Nockles+Ryskin+Shuvaev+T, arXiv:0812.3558

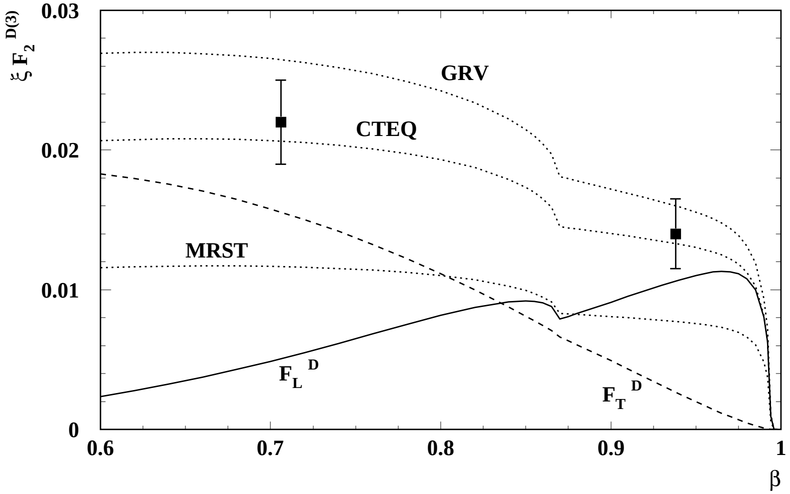
I. Motivation: Generalised PDFs in (semi-) hard diffraction

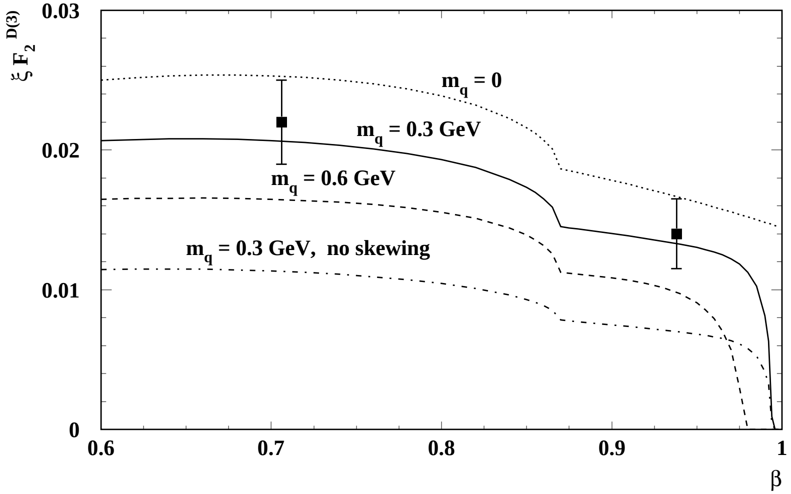
Why skewed PDFs at small x ?

- Description of diffractive processes involves generalised PDFs. Examples:
  - Diffractive vector meson production:
  - Elastic Higgs production at the LHC
  - Diffractive heavy quark or jet production



  - Inclusive diffraction in perturbative QCD regime, e.g. $F_2^{D(3)}$ for $\beta = Q^2/(Q^2+M^2) \to 1$ (within Parton-Hadron Duality):





A Hebecker+T, Phys.Lett.B498(2001)16

- Beyond the leading $\ln 1/x$ approximation cannot identify non-perturbative distribution with normal, diagonal PDF

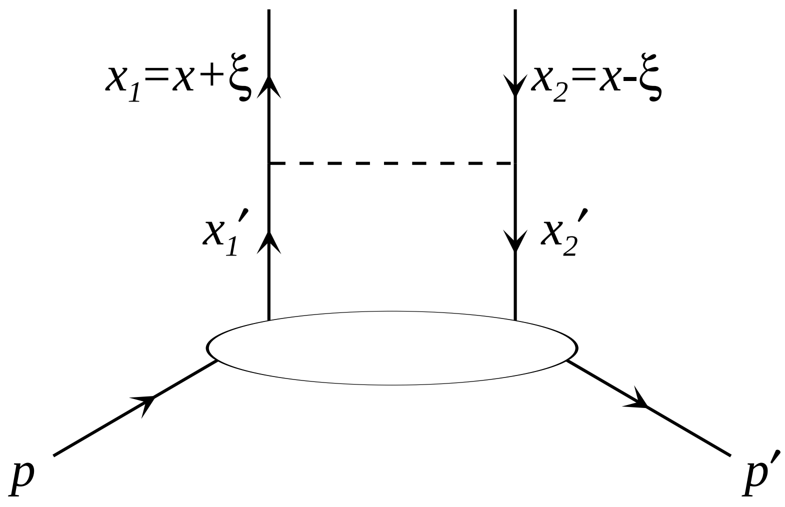
- $x_1 \neq x_2$, i.e. skewed PDF: $H(x_1, x_2; \mu^2; t)$ $\rightsquigarrow$ sizeable corrections w.r.t. diagonal limit (e.g. as large as ~ 2 for Υ production).

  Very active field of research to understand, measure, model Generalised PDFs.

- Theoretically ideal process to determine GPDFs: Deeply Virtual Compton Scattering.

- However, current data of limited accuracy and kinematic coverage.
  In addition: Connection with 'normal' (diagonal, global fit) PDFs?

- In the small x, ξ region, and under certain *physical small x assumptions* (see below), $H(x, \xi)$ can be determined from the diagonal $H(x, \xi = 0)$ distribution via the *Shuvaev transform*.
  [t dependence factored out; μ^2 dep. from diag. input and not discussed.]



II. Shuvaev Transform in a nutshell

- How can the non-diagonal PDF $H(x,\xi)$ at small x,ξ be reconstructed just from the diagonal $H(x,\xi=0)$?
  - Anomalous dimensions describing the evolution of Gegenbauer moments G_N of $H(x,\xi)$ are equal to the anomalous dimensions of the conventional Mellin moments, $M_N = \int_0^1 x^N H(x,0)\mathrm{d}x$; ξ is not changed during evolution.
    [This is a consequence of the conformal invariance of the evolution equations.]
  - *Polynomiality*, $G_N = \sum_{n=0}^{N} c_n^N \xi^{2n}$, allows to determine all Gegenbauer moments G_N up to accuracy $\mathcal{O}(\xi^2)$ from conventional PDFs, $c_0^N = M_N$.
  - The Shuvaev transform is an inverse (integral) transformation which determines the x dependence of $H(x,\xi)$ (for a given small ξ) from the diagonal $H(x,0)$.
- At NLO conformal invariance is broken and the parametric accuracy of the Shuvaev transform is only $\mathcal{O}(\xi)$. (Still sufficient for the small $\xi < |x|$ region under consideration.)

- *Caveat:* To get the x distribution from the G_N, an analytical continuation to complex N is needed $\rightsquigarrow$ there must be no NO singularities in the input distribution in the right-half plane (otherwise no unique solution).

- Hence we assume as an *additional condition* a Regge-based form of the diagonal low-x input distribution, with no singularities in the right-half plane $(j > 1)$ in the space-like $(\xi < |x|)$ domain.
  [There are higher-spin resonances, but in the time-like $(\xi > |x|)$ region.]

- The transform can be expressed by a double integral: Shuvaev+Golec-Biernat+Martin+Ryskin

$$\text{Quarks:} \quad H_q(x,\xi) = \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi}\,\mathrm{Im}\int_0^1 \frac{\mathrm{d}s}{y(s)\,\sqrt{1-y(s)x'}}\right] \frac{\mathrm{d}}{\mathrm{d}x'}\left(\frac{q(x')}{|x'|}\right)$$

$$\text{Gluons:} \quad H_g(x,\xi) = \int_{-1}^{1} \mathrm{d}x' \left[\frac{2}{\pi}\,\mathrm{Im}\int_0^1 \frac{\mathrm{d}s(x+\xi(1-2s))}{y(s)\,\sqrt{1-y(s)x'}}\right] \frac{\mathrm{d}}{\mathrm{d}x'}\left(\frac{g(x')}{|x'|}\right)$$

with $y(s) = \frac{4s(1-s)}{x+\xi(1-2s)}$.

These PV-$\int$s can be solved numerically by standard methods for given input distributions.

- But: Not practical to have this directly in codes for diffractive cross sections (let alone in MC codes...)

$\hookrightarrow$ We have calculated a package containing grid files for fast interpolation (in x, ξ, μ^2) with a simple Fortran routine.

- Files for MSTW2008, MRST2004, CTEQ6.6 including $q, \bar{q}, g$ are available from
  `http://www.maths.liv.ac.uk/TheorPhys/RESEARCH/pubcodes.html`
  Others can be produced easily on request.
- Easy application (file-size, speed; accuracy better than 0.2% at small $x < 10^{-2}$).

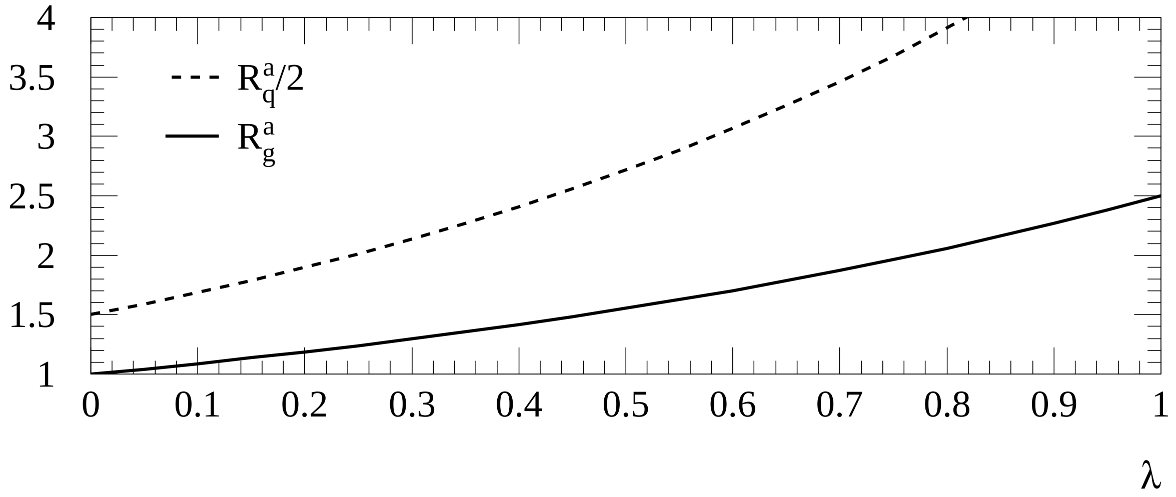
- Analytical result for $H(x, \xi = x)$ [In the LLA, $x_1 \gg x_2$, i.e. $\xi \to x$.]

In the case of a pure power form of the diagonal PDF, e.g. $xg \sim x^{-\lambda}$, the Shuvaev transform can be solved analytically for the limit $\xi = x$ ('maximal skewing'):

$$R^a = \frac{H(\xi,\xi)}{H(2\xi,0)} = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda+\frac{5}{2})}{\Gamma(\lambda+3+p)},$$

with $\lambda(\mu^2)$ the parton's *effective power* and $p = 0\ (1)$ for quarks (gluons).

Result for $R^a(\lambda)$ for quarks and gluons:



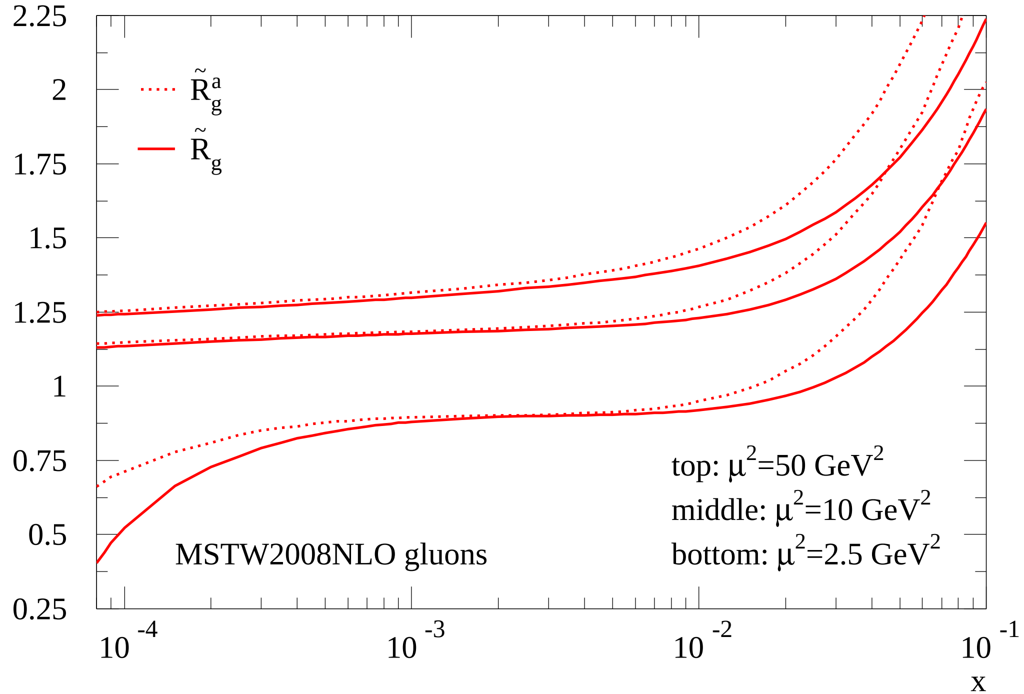
$\longrightarrow$ At given λ the skewing effect for quarks is larger than that for gluons.

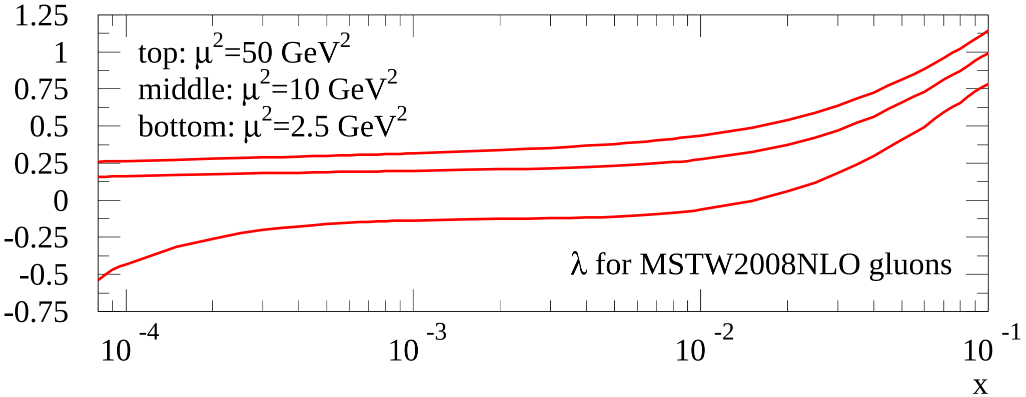
This can be understood from the strong ordering ($x_1' \gg x_1$, $x_2' \gg x_2$) in the double leading log approximation: For the gluon the ξ dependence mainly comes from the last P_{gg} splitting; (sea or singlet) quarks have to be generated from the last P_{qg} splitting with no $1/z = x_2'/x_2$ singularity, so $x_2' \sim x_2$ and both splittings $P_{qg}(x_2, x_2'; \xi)$ and $P_{gg}(x_2', x_2''; \xi)$ contribute to the skewing.

$\longrightarrow$ Many predictions of diffractive cross sections use this analytical approximation to account for the skewing effects.

We will demonstrate below how well this works for typical global fit PDFs.

III. Numerical results for $H(x,\xi)$ at small x





Full numerical Shuvaev transform of the MSTW2008 gluon at three scales:

$$\tilde{R} = \frac{H(x/2, x/2)}{H(x,0)},$$

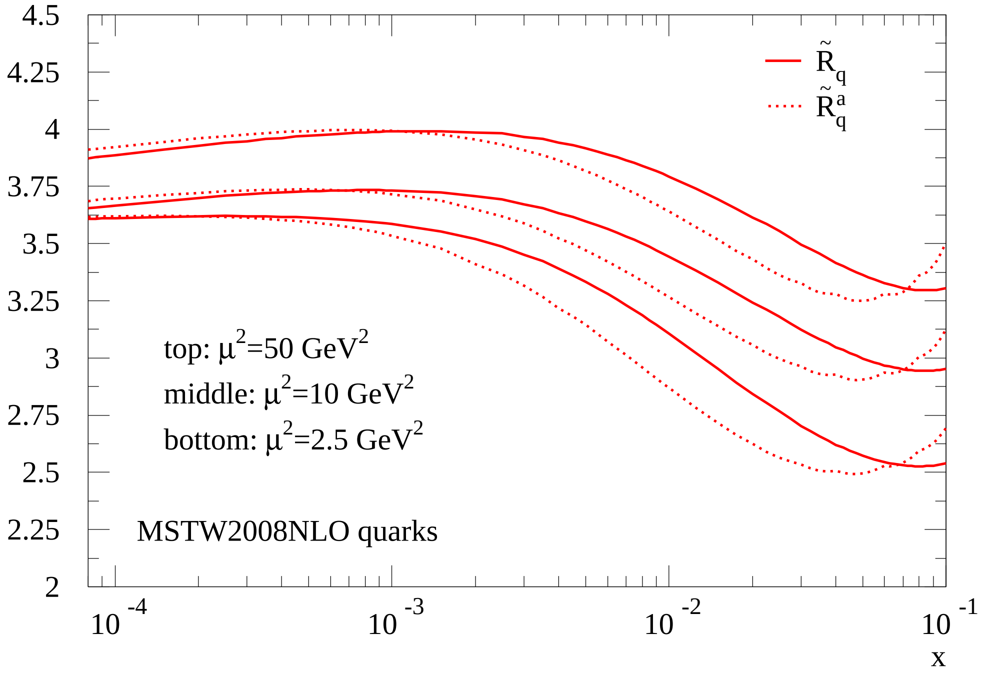
compared to analytical approximation

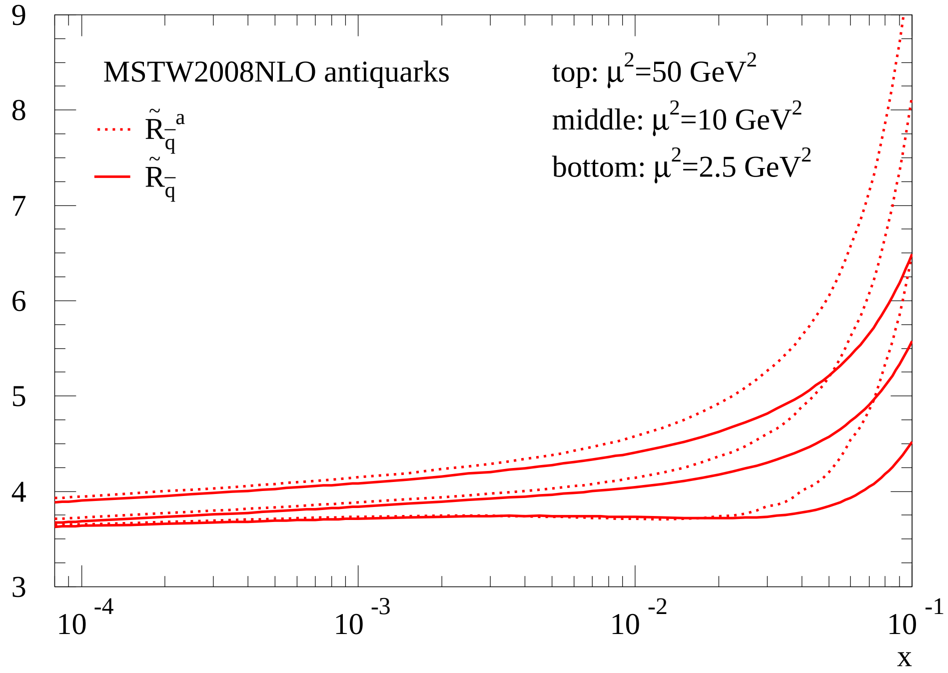
$$\tilde{R}^a = \frac{2^{2\lambda+3}}{\sqrt{\pi}} \frac{\Gamma(\lambda+\frac{5}{2})}{\Gamma(\lambda+3+p)},$$

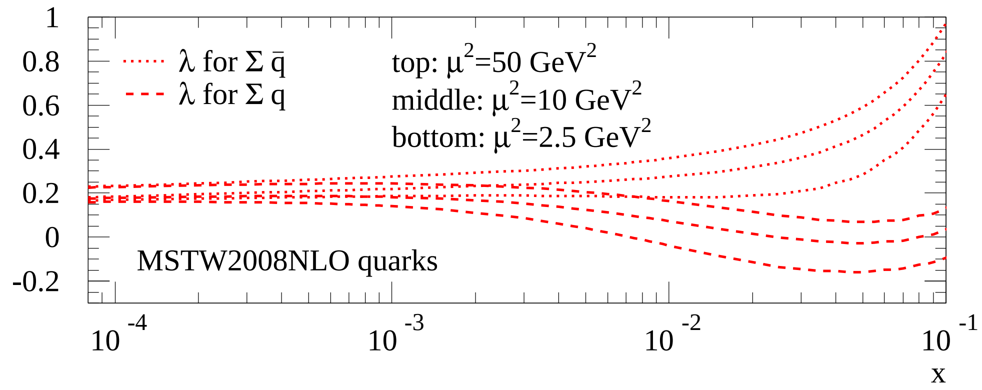
with λ evaluated at x.

- At small $x < 10^{-2}$ the MSTW2008 gluon has nearly perfect power form; larger deviations at larger x, and:
- Close to the input scale this gluon has neg. $\lambda \rightsquigarrow$ *skewing suppression* (and xg turns negative below $x \sim 10^{-5}$).

Skewing for MSTW2008 quarks and anti-quarks:





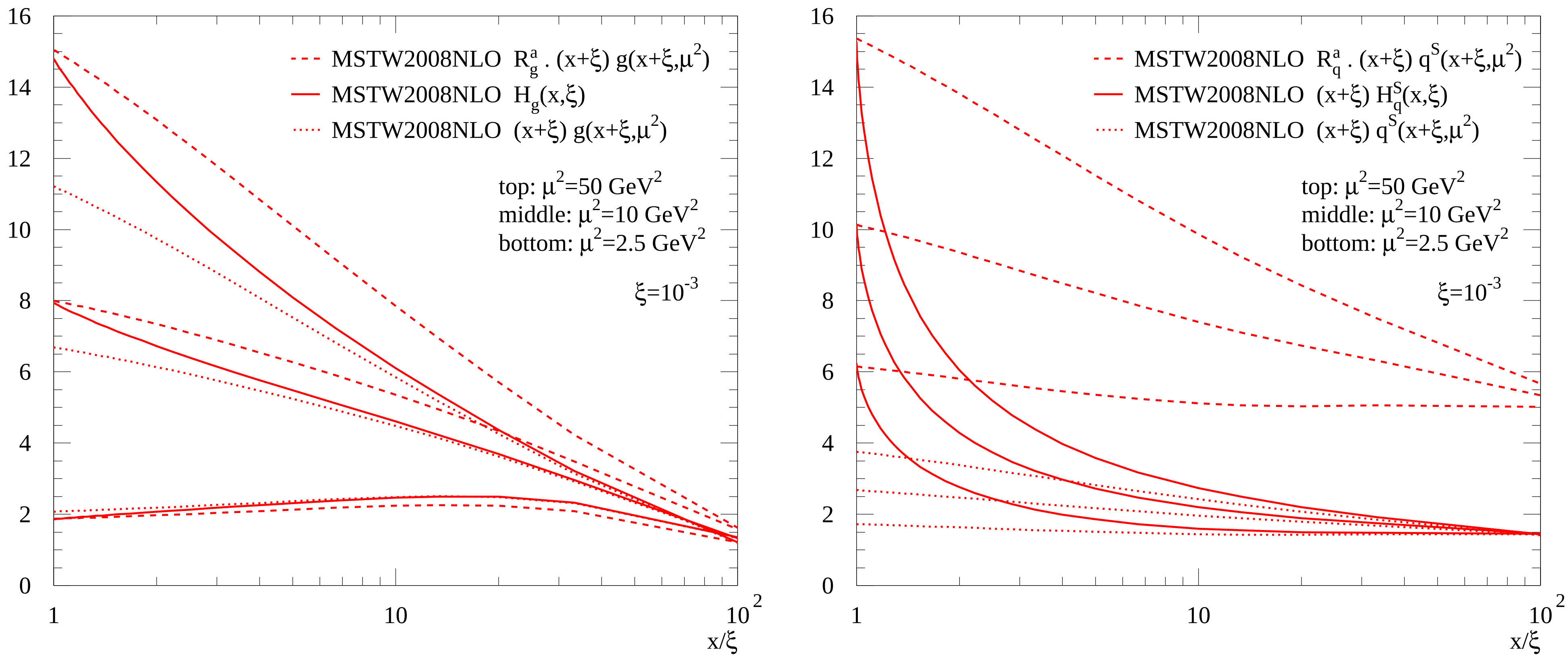


- As expected stronger skewing for quarks as compared to gluons.
- Also larger differences between approximation and full result.

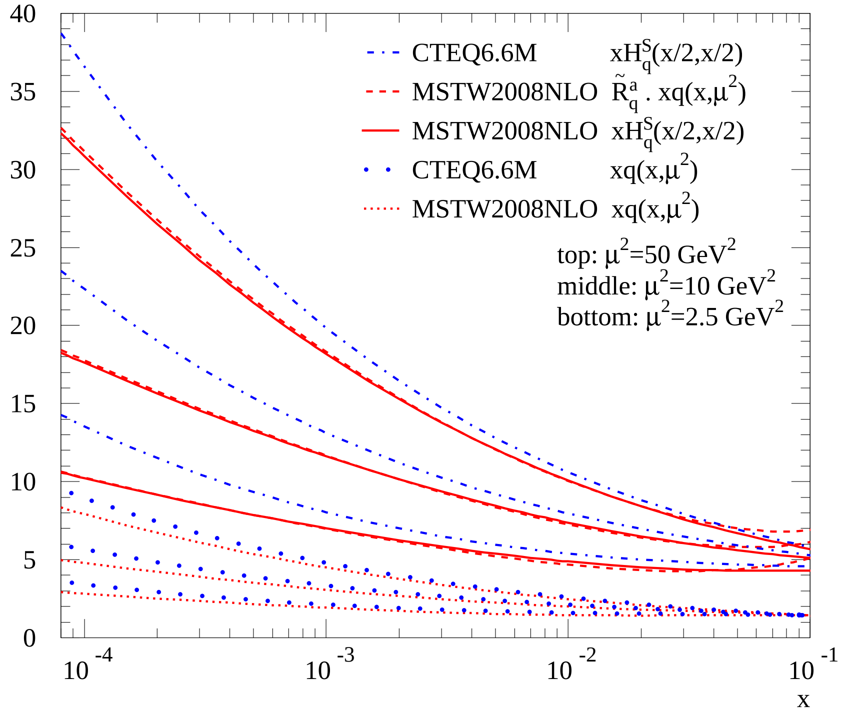
- Like for the gluon, λ grows with μ^2 and so does the skewing enhancement.

$H(x,\xi)$ plotted over x/ξ for MSTW2008 gluons and quarks (at three scales and $\xi = 10^{-3}$)

compared to the diagonal partons (dotted) and the diagonal PDFs multiplied with the analytical skewing factors R^a (dashed):



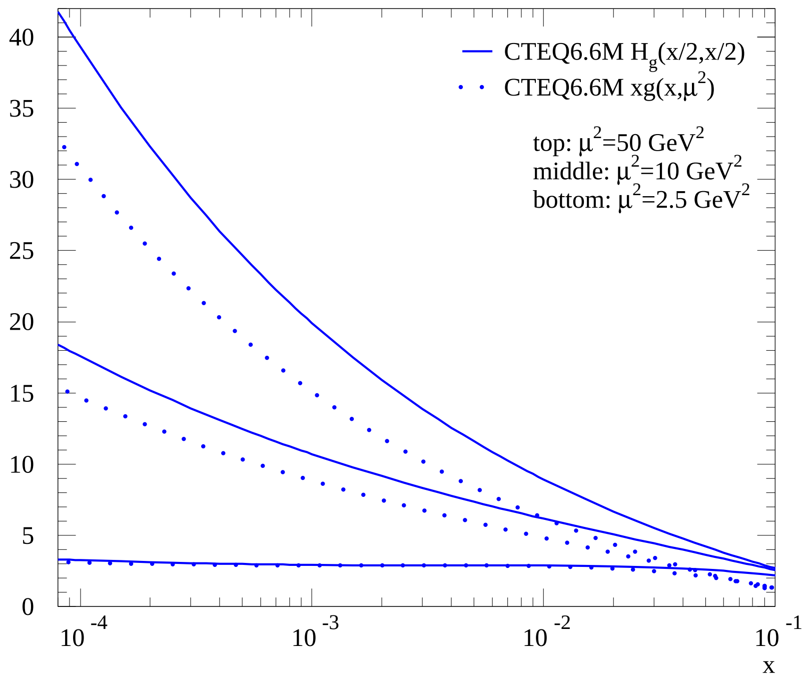
$\longrightarrow$ Simple application of (analytical) skewing correction in the limit $\xi \to x$ will typically over-estimate the skewing effect, especially for quarks.

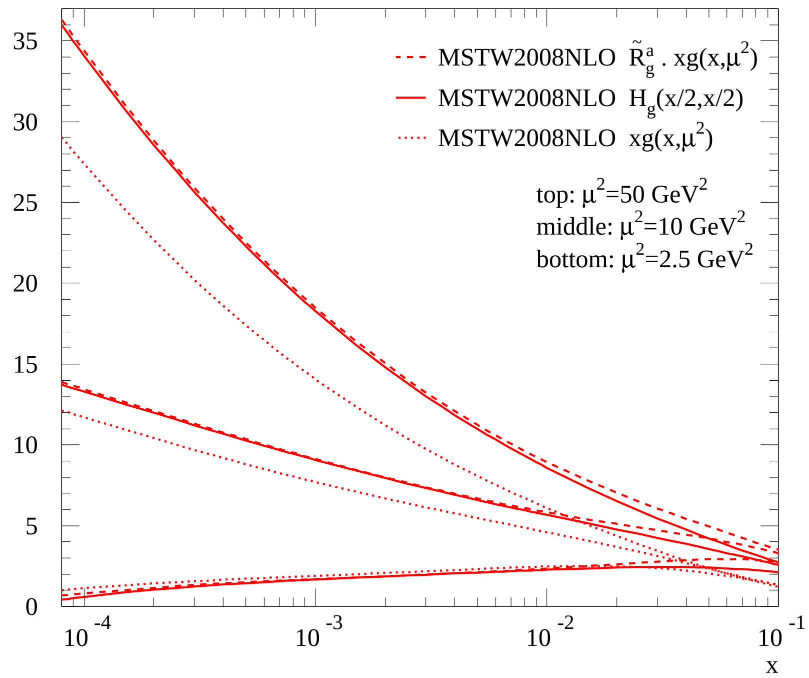


$H(x,\xi)$ as a function of x for CTEQ6.6M and MSTW2008 input

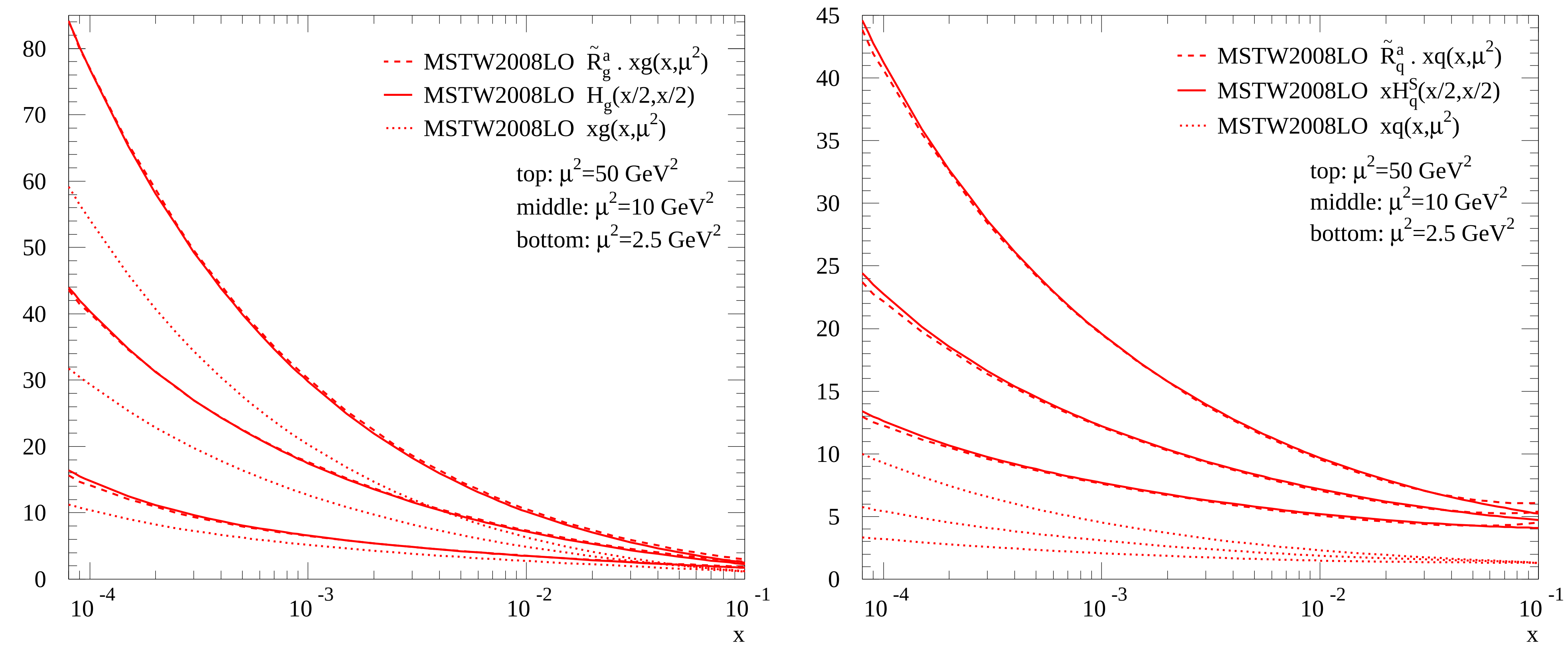
← Singlet quark distributions

- Similar enhancement for skewed (slightly steeper) CTEQ6.6M partons.
- Note the differences between the two global fits at smaller x.





Skewing for MSTW2008 **Leading Order** gluons and quarks:



- Qualitatively similar to NLO case;
- (artificially?) enhanced gluon in global fits to make up for missing LO $\gamma^* g$ splitting and no $1/z$ enhancement in LO P_{qq}. Better use NLO throughout for diffractive predictions?!

IV. Summary/Outline

- Generalised PDFs at small x crucial for the desciption of diffractive processes.
- Limited 'first principles' knowledge about about complicated GPDFs; not enough data to fully constrain them $\rightsquigarrow$ in general modelling required.
- In the small x regime one can predict the skewed PDFs purely from diagonal ones, using the *Shuvaev transform*.
- Prize to pay: Assumption of absence of additional singularities in the right-half plane.
  Note: This is a physically motivated conjecture based on the success of Regge theory for small x processes, but not following from first principles or symmetry arguments alone.
- The Shuvaev transform is solved in form of double integrals; results indicate sizeable corrections at small x, growing with μ^2, in general larger for q compared to g.
- We have provided an easy-to-use package for fast calculation (interpolation) of the skewed $H(x, \xi, \mu^2)$ for global fit partons from MSTW and CTEQ.
  No need to rely on analytical approximation used frequently which however turns out to be fairly accurate for global fit partons.

- Further studies and comparisons under way.
- Use of these results for $H(x, \xi, \mu^2)$ to better estimate skewing corrections when integrating over x to get e.g. quark contributions to diffractive VM production.

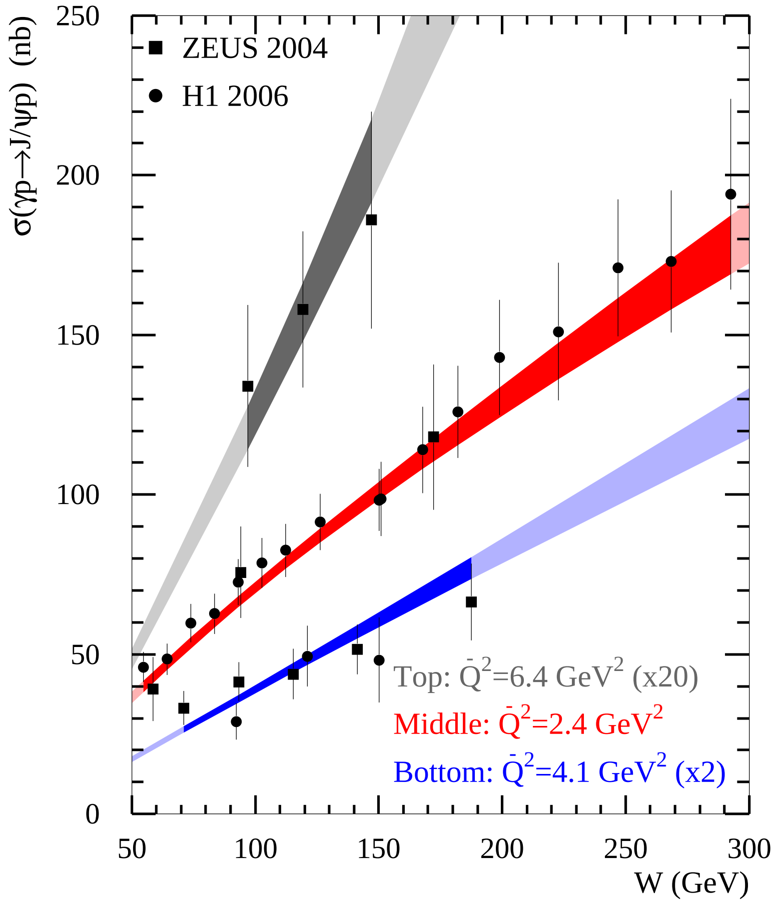
MNRT gluon fits

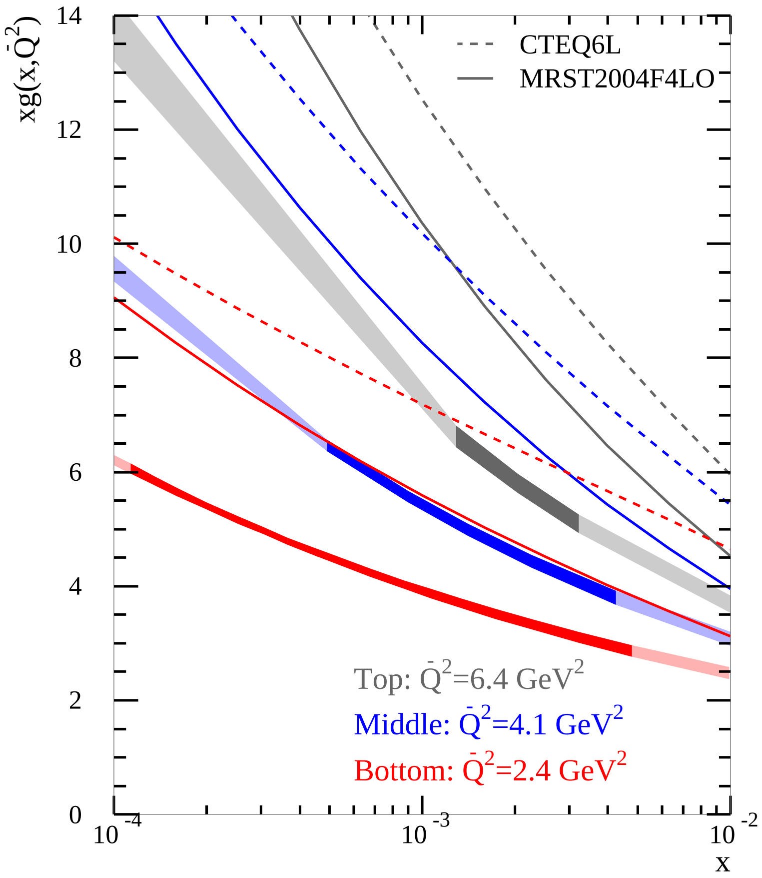
Martin+Nockles+Ryskin+T, Phys. Lett. B 662 (2008) 252

- Use of pQCD with Re part, skewing and a simple gluon ansatz with three free parameters:

$$xg(x,\mu^2) = N \cdot x^{-\lambda}\,, \quad \text{LO: } \lambda = a + b\ln\frac{\mu^2}{0.45\,\text{GeV}^2}, \quad \text{NLO: } \lambda = a + b\ln\ln\frac{\mu^2}{0.09\,\text{GeV}^2}$$

LO combined fit of H1 and ZEUS J/ψ data:





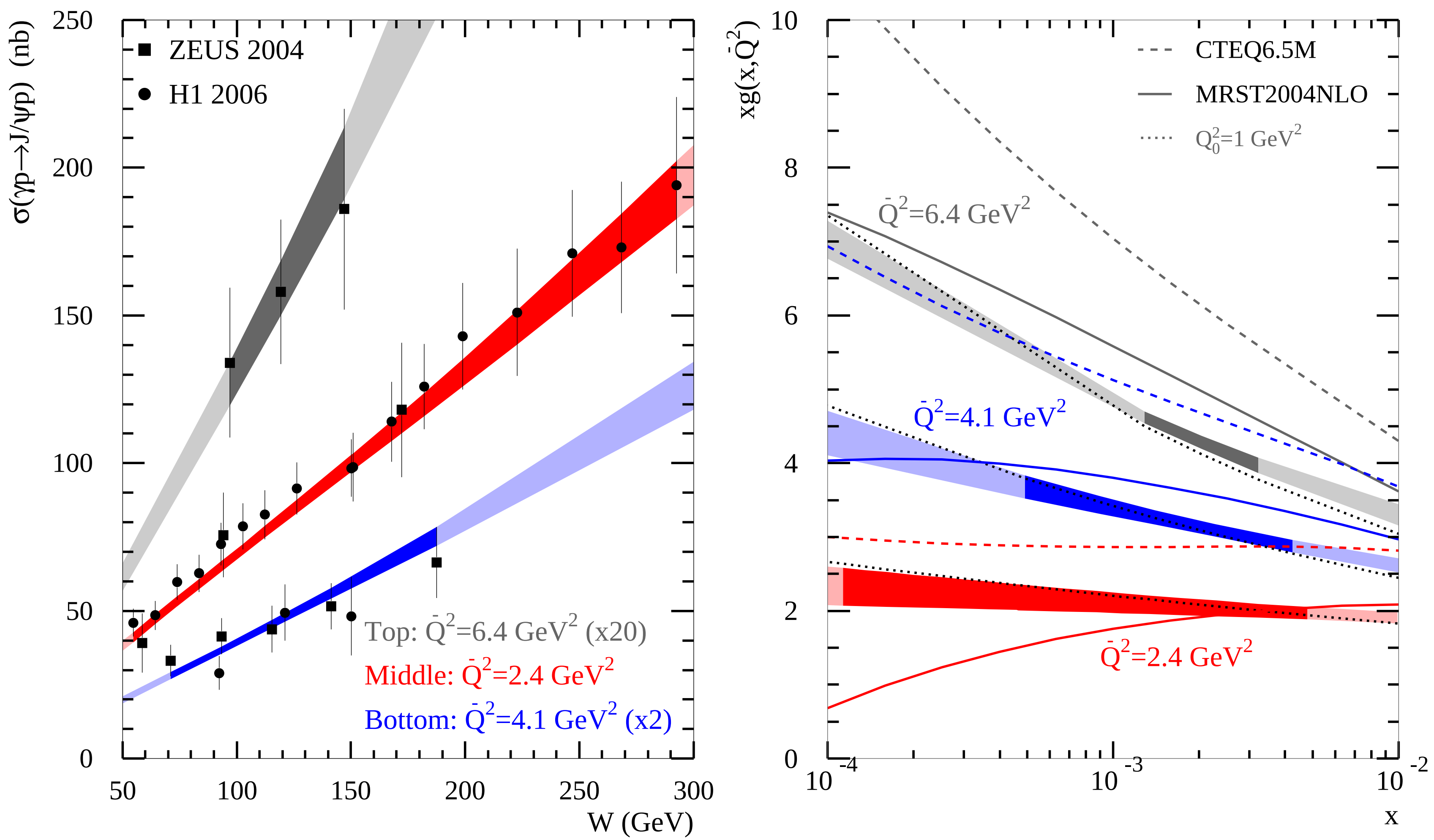
$$\frac{\chi^2_{\rm min}}{(d.o.f.=48)} = 0.9$$

$$N = 0.99 \pm 0.09,$$

$$a = 0.051 \pm 0.012,$$

$$b = 0.088 \pm 0.005.$$

NLO fit of H1 and ZEUS J/ψ data: (using k_T-fact. w. unint. gluon, Re part, skewing)

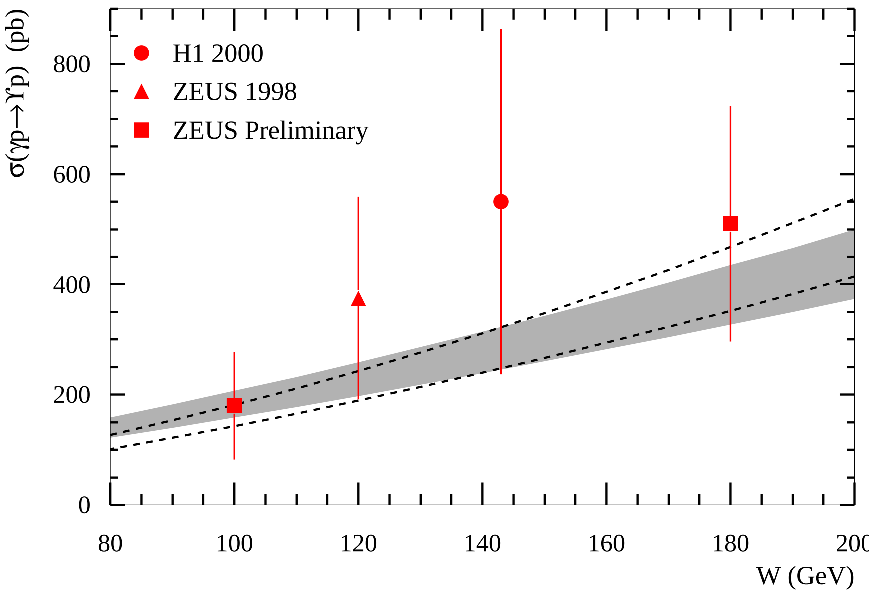


- Excellent overall fit to available J/ψ data: $\chi^2_{\rm min}/(d.o.f. = 48) \ = \ 0.8$
  $N = 1.55 \pm 0.18, \quad a = -0.50 \pm 0.06, \quad b = 0.46 \pm 0.03\,.$
- Tendency for slightly too steep higher-Q^2 cross sections, but statistically less significant.

Predictions for elastic Υ production

- Use of NLO gluon to predict Υ photoproduction in the same framework; applicability of pQCD and non-rel. approximation even better justified
- change of mass, electric charge, Γ_{ee}, but no other adjustments
- Comparison with HERA data: good description

MNHT plot



ZEUS `arXiv:0903.4205`