The Color-Dipole Picture and F_L

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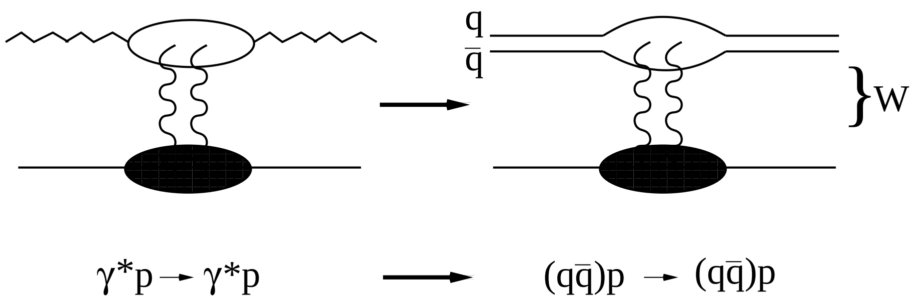
1) DIS at low $x_{bj} \lesssim 0.1$.



$$\frac{1}{\Delta E} = \frac{W^2}{Q^2 + M_{q\bar{q}}^2} \cdot \frac{1}{M_P} = \frac{1}{x_{bj} + \frac{M_{q\bar{q}}^2}{W^2}} \frac{1}{M_P} \gg \frac{1}{M_P};$$

Generalized Vector Dominance,
Sakurai, Schildknecht (1970ies)
Space-time interpretation
of generalized vector dominance
Gribov, Joffe (1970ies)

requires $x_{bj} \lesssim 0.1$,

$$M_{q\bar{q}}^2 \ll W^2.$$

Color-dipole interaction

$$m_{\rho,\omega}^2 \lesssim M_{q\bar{q}}^2, M_{q\bar{q}}'^2 \lesssim m_1^2(W^2);$$

For $W \simeq 225 GeV$, $M_{q\bar{q}} \lesssim 22.5 GeV$

$\approx$ upper bound of diffr. mass spectrum at HERA

Frequent approximation: $m_1^2 \to \infty$

$\gamma^* p$ **axis:**

$$M_{q\bar{q}}^2 = \frac{\vec{k}_\perp^{\,2}}{z(1-z)} = \frac{4\vec{k}_\perp^{\,2}}{\sin^2\vartheta} \qquad \text{where } 0 \le z \le 1$$

$$4z(1-z) = \sin^2\vartheta \quad \text{in } q\bar{q} \text{ rest frame}$$

$$z = \frac{1}{2} : \vartheta = 90^o.$$

$$M_{q\bar{q}}'^2 = \frac{(\vec{k}_\perp + \vec{l}_\perp)^2}{z(1-z)} = \frac{4(\vec{k}_\perp + \vec{l}_\perp)^2}{\sin^2\vartheta}$$

Difference $(M_{q\bar{q}}'^2 - M_{q\bar{q}}^2)$ **determined by magnitude of gluon transverse momentum,** $\vec{l}_\perp$.

2) Transverse size of $q\bar{q}$ fluctuation.

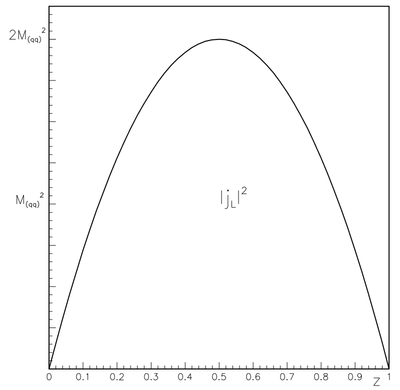
Electromagnetic coupling of γ^*(timelike) to $q\bar{q}$ of mass $M_{q\bar{q}}$

$$\gamma_L^* \quad : \sum_{\lambda=-\lambda'=\pm\frac{1}{2}} |j_L^{(\lambda,\lambda')}|^2 = 8M_{q\bar{q}}^2 z(1-z).$$

$$\gamma_T^* \quad : \sum_{\lambda=-\lambda'=\pm\frac{1}{2}} |j_T^{\lambda,\lambda'}(+)|^2 = \sum_{\lambda=-\lambda'=\pm\frac{1}{2}} |j_T^{\lambda,\lambda'}(-)|^2 = 2M_{q\bar{q}}^2 (1-2z(1-z)).$$

Angular distribution, i.e. dependence on $z(1-z) = \frac{1}{4}\sin^2\vartheta$,determines $\gamma_{L,T}^*$ coupling strengths at fixed $M_{q\bar{q}}^2$.

$q\bar{q}$ **pairs from longitudinal** $\gamma^* \equiv \gamma_L^*$ **are dominantly produced at** $\vartheta = 90^o$ $(z = \frac{1}{2})$ **and have "large" transverse momentum** $\vec{k}_\perp{}^2 = z(1-z)M_{q\bar{q}}^2$



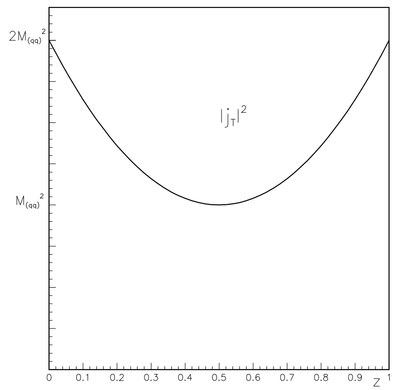
$$\langle \vec{k}_\perp^2 \rangle_L = M_{q\bar{q}}^2 \int_0^1 dz z(1-z) f_L(z) = \frac{4}{20} M_{q\bar{q}}^2$$

with

$$f_L(z) = \frac{z(1-z)}{\int dz z(1-z)} = 6z(1-z)$$

$|j_L|^2$**as function of** z

$q\bar{q}$ **pairs produced by transversely polarized** $\gamma^* \equiv \gamma_T^*$ **have "small"** $\vec{k}_\perp{}^2 = z(1-z)M_{q\bar{q}}^2$



$$\langle \vec{k}_\perp^2 \rangle_T = M_{q\bar{q}}^2 \int dz z(1-z) f_T(z) = \frac{3}{20} M_{q\bar{q}}^2$$

with

$$f_T(z) = \frac{1-2z(1-z)}{\int dz(1-2z(1-z))} = \frac{3}{2}(1-2z(1-z))$$

$|j_T|^2$ **as function of** z

So, with

$$\rho \equiv \frac{\langle \vec{k}_\perp^2 \rangle_L}{\langle \vec{k}_\perp^2 \rangle_T} = \frac{4}{3}$$

upon invoking the uncertainty relation,

$$\frac{\langle \vec{r}_\perp^{\,2} \rangle_T}{\langle \vec{r}_\perp^{\,2} \rangle_L} = \rho = \frac{4}{3}.$$

$$q\bar{q} \text{ from } \begin{cases} \gamma_L^* & \text{(i.e. } (q\bar{q})_L^{J=1}) : \quad \text{“small” (transverse) size,} \\ \gamma_T^* & \text{(i.e. } (q\bar{q})_T^{J=1}) : \quad \text{“large” (transverse) size.} \end{cases}$$

Cross section of transversely polarized $(q\bar{q})_T^{J=1}$ state enhanced relative to cross section of longitudinally polarized $(q\bar{q})_L^{J=1}$state:

$$\sigma_{(q\bar{q})_T^{J=1}p}(M_{q\bar{q}}^2, W^2) = \rho \; \sigma_{(q\bar{q})_L^{J=1}}(M_{q\bar{q}}^2, W^2).$$

3) Color Dipole Picture (CDP) I.

Nikolaev and Zakharov (1994)

Spacelike $\gamma^* : q^2 = -Q^2 < 0$;

A) Integration over massive $M_{q\bar{q}}$ continuum: $\int dz \int dM^2_{q\bar{q}} \int dM'^2_{q\bar{q}} \longrightarrow \int dz \int d^2 r_\perp$

$$\sigma_{\gamma^*_{L,T} p}(W^2, Q^2) =$$

$$= \int dz \int d^2 r_\perp |\psi_{L,T}(r_\perp, z(1-z), Q^2)|^2 \sigma_{(q\bar{q})p}(r_\perp, z(1-z), W^2)$$

$$= \frac{\alpha}{\pi} R_{e^+e^-} Q^2 \begin{cases} \int 4 d^2 r'_\perp K_0^2(r'_\perp Q) \int dz z(1-z) \sigma_{(q\bar{q})p}\left(\frac{r'_\perp}{\sqrt{z(1-z)}}, z(1-z), W^2\right) \\ \int d^2 r'_\perp K_1^2(r'_\perp Q) \int dz (1-2z(1-z)) \sigma_{(q\bar{q})p}\left(\frac{r'_\perp}{\sqrt{z(1-z)}}, z(1-z), W^2\right) \end{cases}$$

$$= \frac{2\alpha R_{e^+e^-}}{3\pi^2} Q^2 \int d^2 r'_\perp K^2_{0,1}(r'_\perp Q) \sigma_{(q\bar{q})^{J=1}_{L,T} p}(r'_\perp, W^2);$$

$Q \equiv \sqrt{Q^2}$;

$R_{e^+e^-} \equiv 3 \sum_f Q_f^2$;

$r'_\perp \equiv \sqrt{z(1-z)} r_\perp$:

$$\left. \begin{array}{l} K_0(y) \sim -\ln y \\ K_1(y) \sim \frac{1}{y}, \end{array} \right\} \text{for } y \to 0$$

- **Factorization between Q^2 dependence (photon wave function) and W^2 dependence ($(q\bar{q})p$ interaction):**

$$\sigma_{\gamma^*_{L,T}p}(W^2, Q^2) \sim \int d^2r'_\perp K^2_{0,1}(r'_\perp Q)\sigma_{(q\bar{q})^{J=1}_{L,T}p}(r'_\perp, W^2)$$

No change of dipole cross section under change of Q^2 at fixed W

- **Frequent assumption:** $\sigma_{(q\bar{q})p} = \sigma_{(q\bar{q})p}(r_\perp, x_{bj} = \frac{Q^2}{W^2})$

Change of dipole cross section under change of Q^2.

Strictly speaking inconsistent with CDP, justified at most as an approximation.

e.g. Ewerz and Nachtmann (2006)

B) Color-dipole interaction.

$$\sigma_{(q\bar{q})^{J=1}_{L,T}p}(r'_\perp, W^2) = \int d^2l'_\perp \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}'^2_\perp, W^2)\left(1 - e^{-i\vec{l}_\perp{}'\cdot\vec{r}_\perp{}'}\right)$$

$$= \begin{cases} \int d\vec{l}'^2_\perp \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}\left(\vec{l}'^2_\perp, W^2\right), & \text{for } r'_\perp \to \infty, \\ r_\perp'^2 \frac{\pi}{4} \int d\vec{l}'^2_\perp \vec{l}'^2_\perp \bar{\sigma}_{(q\bar{q})^{J=1}_{L,T}p}(\vec{l}'^2_\perp, W^2), & \text{for } r'_\perp \to 0. \end{cases}$$

"color transparency"

Transverse-size enhancement:

$$\sigma_{(q\bar{q})^{J=1}_{T}p}(r'_\perp, W^2) = \rho\sigma_{(q\bar{q})^{J=1}_{L}p}(r'_\perp, W^2)$$

4) $R(W^2,Q^2) = \frac{\sigma_{\gamma^*_L p}(W^2,Q^2)}{\sigma_{\gamma^*_T p}(W^2,Q^2)}$

Kuroda, Schildknecht (2008)

For Q^2 sufficiently large:

$$R(W^2,Q^2) = \underbrace{\frac{\int d^2r'_\perp r'^2_\perp K_0^2(r'_\perp Q)}{\int d^2r'_\perp r'^2_\perp K_1^2(r'_\perp Q)}}_{\frac{1}{2}} \cdot \underbrace{\frac{\int d\vec{l}'^2_\perp \vec{l}'^2_\perp \bar{\sigma}_{(q\bar{q})^{J=1}_L}(\vec{l}'^2_\perp, W^2)}{\int d\vec{l}'^2_\perp \vec{l}'^2_\perp \bar{\sigma}_{(q\bar{q})^{J=1}_T}(\vec{l}'^2_\perp, W^2)}}_{\frac{1}{\rho}} = \frac{1}{2\rho}$$

$$R(W^2,Q^2) = \begin{cases} \frac{1}{2} = 0.5 & \text{for } \rho = 1, \text{ helicity independence} \\ \frac{1}{2\cdot\frac{4}{3}} = \frac{3}{8} = 0.375 & \text{for } \rho = \frac{4}{3}, \text{ transverse size enhancement} \end{cases}$$

Based on

i) photon wave function

ii) color transparency

iii) transverse size enhancement

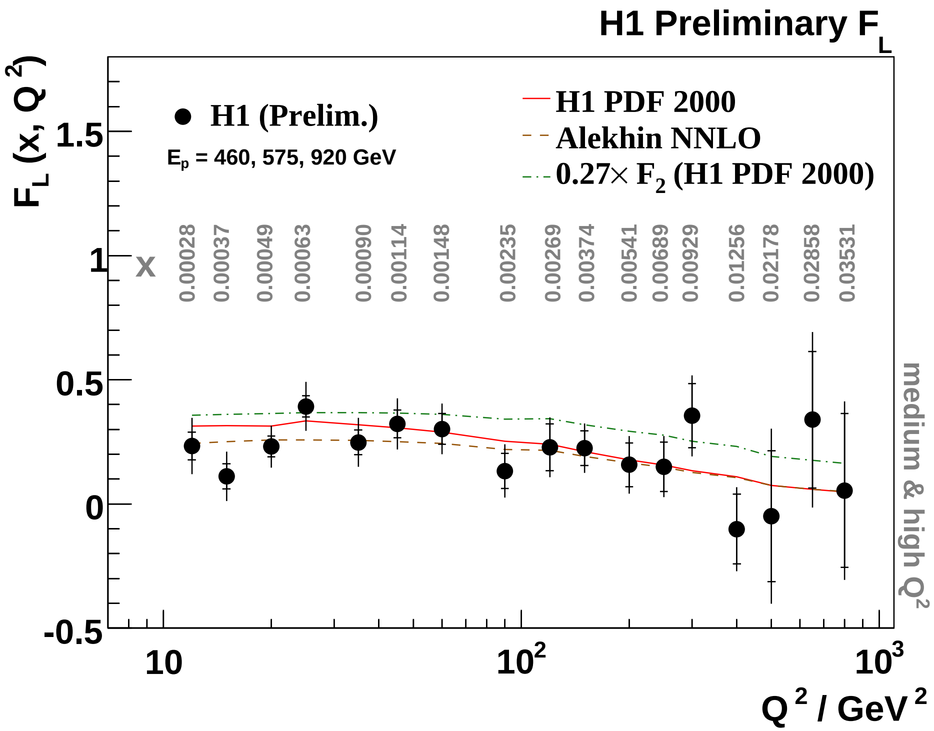
Measurement of $\rho = 2R(W^2,Q^2)$ provides direct test of the CDP (independent of specific ansatz for dipole cross section).

Measurement

$$\sigma_r(x,y,Q^2) = F_2(x,Q^2)\left(1 - \frac{y^2}{1+(1-y)^2}\frac{1}{1+2\rho}\right), \qquad y \equiv \frac{Q^2}{xs}; \quad 0 \le y \le 1$$

In terms of $F_2(x,Q^2)$ we have

$$F_L(x,Q^2) = \frac{1}{1+2\rho}F_2(x,Q^2) = \\ = \begin{cases} \frac{1}{3}F_2(x,Q^2) = 0.33F_2(x,Q^2), & \text{for} \rho = 1 (\text{helicity independence}) \\ \frac{1}{1+\frac{8}{3}}F_2(x,Q^2) = 0.27F_2(x,Q^2), & \text{for } \rho = \frac{4}{3} \ (\text{transverse} - \text{size enhancement}) \end{cases}$$



V. Chekelian, private communication

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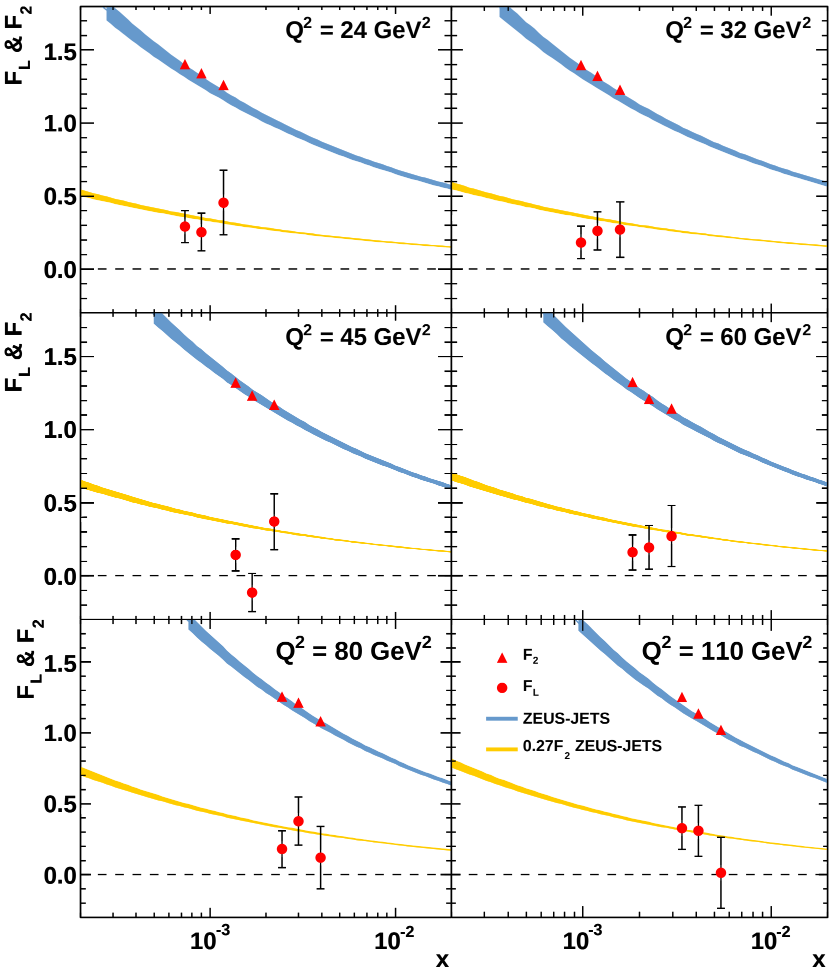


$$F_L = 0.27 \cdot F_2$$

B. Reisert, private communication

5) Color Dipole Picture II

Model-independent analysis "Low x-scaling".

$$\sigma_{\gamma^* p}(W^2, Q^2) = \sigma_{\gamma^* p}\left(\eta(W^2, Q^2)\right);$$

$$\eta(W^2, Q^2) \equiv \frac{Q^2 + m_0^2}{\Lambda^2_{sat}(W^2)};$$

Cvetic, Schildknecht, Surrow, Tentyukov (2000/2001)

$$\Lambda^2_{sat}(W^2) = B\left(\frac{W^2}{1GeV^2}\right)^{C_2}; \quad \eta(W^2, Q^2) \sim \frac{Q^2}{(W^2)^{C_2}}; \quad \sigma_{(q\bar{q})p} = \sigma_{(q\bar{q})p}\left(r_\perp \cdot \Lambda_{sat}(W^2)\right)$$

At variance with: "Geometric scaling"

$$\sigma_{\gamma^* p}(W^2, Q^2) = \sigma_{\gamma^* p}\left(\tau(x, Q^2)\right);$$

$$\tau(x, Q^2) \equiv Q^2 \quad R_0^2(x);$$

Stasto, Golec–Biernat, Kwiecinski (2000/2001)

$$R_0^2(x) \sim x^\lambda = \frac{(Q^2)^\lambda}{(W^2)^\lambda} \qquad \tau(x, Q^2) \sim Q^2 x^\lambda = \frac{Q^2}{(W^2)^\lambda}(Q^2)^\lambda; \qquad \sigma_{(q\bar{q})p} = \sigma_{(q\bar{q})p}\left(r_\perp \frac{1}{x^\lambda}\right)$$

In the CDP, from a principal point of view, $\eta(W^2, Q^2)$ preferable to $\tau(x, Q^2)$.

3984 A. BIALAS

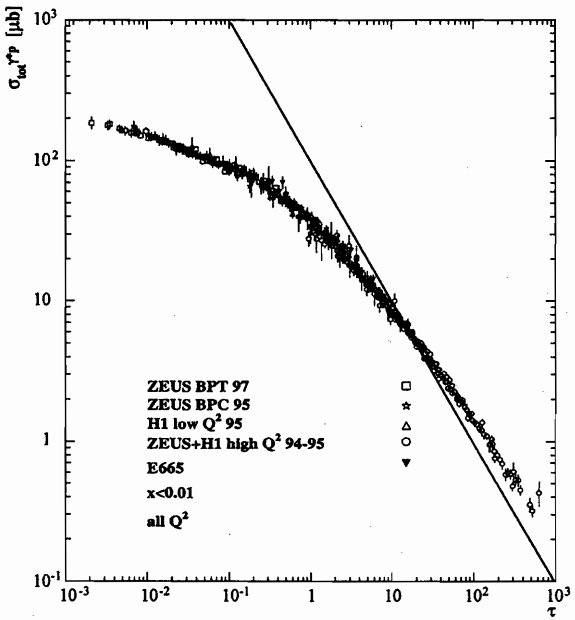


Fig. 1. Scaling from [1].

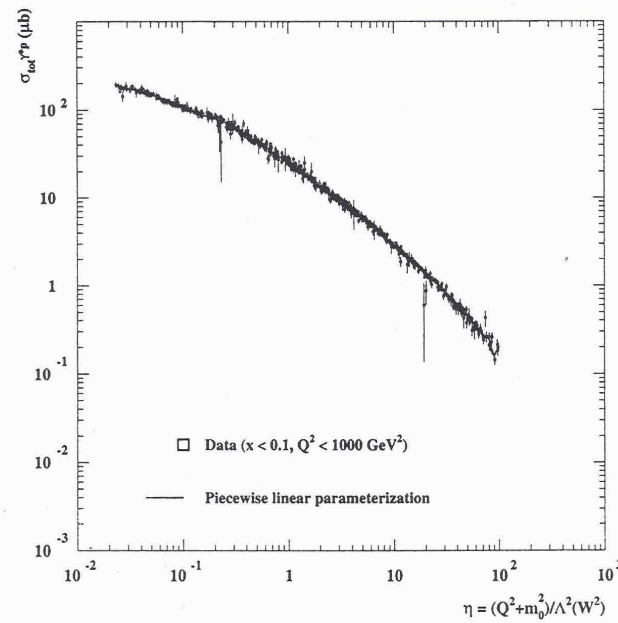


Fig. 2. Geometrical scaling from [4].

$$\sigma_{\gamma^* p} = \sigma_{\gamma^* p}(\tau)$$

$$\sigma_{\gamma^* p} = \sigma_{\gamma^* p}(\eta)$$

A. Bialas, Summary Talk, DIS 2002

Why low-x scaling?

Dipole interaction: Two scales:

$$\sigma_{(q\bar{q})^{J=1}}(r'_\perp, W^2) \sim \sigma^{(\infty)}(W^2) \begin{cases} 1, & r'_\perp \to \infty, \\ r'^2_\perp \Lambda^2_{sat}(W^2), & r'_\perp \to 0, \end{cases}$$

$$\Lambda^2_{sat}(W^2) \sim \int d\vec{l}'^2_\perp \vec{l}'^2_\perp \bar{\sigma}_{(q\bar{q})^{J=1}_L}(\vec{l}'^2_\perp, W^2)$$

Dimensional analysis:

$$\begin{aligned} \sigma_{\gamma^* p}(W^2, Q^2) &\sim \int d^2 r'_\perp \left(K_0^2(r'_\perp Q) + K_1^2(r'_\perp Q)\right) \sigma_{(q\bar{q})^{J=1}}(r'_\perp, W^2) \\ &\sim \sigma^{(\infty)}(W^2) f\left(\frac{Q^2 + m_0^2}{\Lambda^2_{sat}(W^2)}, \underbrace{\frac{m_0^2}{\Lambda^2_{sat}(W^2)}}_{\approx 0}, \underbrace{\frac{\Lambda^2_{sat}(W^2)}{m_1^2(W^2)}}_{\approx 0}\right) \\ &\sim \sigma^{(\infty)} f\left(\eta(W^2, Q^2)\right). \end{aligned}$$

QCD-Dipole interaction with dimensional analysis implies low-x scaling.

Why different W dependence for Q^2 small and Q^2 large?

Dipole cross section interpolating $r'_\perp \to \infty$ and $r'_\perp \to 0$ dependence of QCD dipole interaction, generically

$$\begin{aligned}\sigma_{(q\bar{q})^{J=1}_{L,T}p}(r'_\perp, W^2) &= \sigma^{(\infty)}(W^2)\left(1 - J_0(r'_\perp \Lambda_{sat}(W^2))\right)\\ &= \sigma^{(\infty)}(W^2)\begin{cases} 1, & \text{for } r'^2_\perp \to \infty, \\ \frac{1}{4}\vec{r}'^2_\perp \Lambda^2_{sat}(W^2), & \text{for } r'^2_\perp \to 0. \end{cases}\end{aligned}$$

yields

$$\sigma_{\gamma^* p}(W^2, Q^2) = \frac{\alpha R_{e^+e^-}}{3\pi}\sigma^{(\infty)}(W^2)\frac{1}{1+4\eta}\ln\frac{\sqrt{1+4\eta}+1}{\sqrt{1+4\eta}-1}$$

$$\cong \frac{\alpha R_{e^+e^-}}{3\pi}\sigma^{(\infty)}(W^2)\begin{cases} \ln\frac{1}{\eta} = \ln\frac{\Lambda^2_{sat}(W^2)}{Q^2+m^2_0}, & \eta \ll 1, \quad \text{or } Q^2 \ll \Lambda^2_{sat}(W^2) \\ \frac{1}{2\eta} = \frac{1}{2}\frac{\Lambda^2_{sat}(W^2)}{Q^2}, & \eta \gg 1, \quad \text{or } Q^2 \gg \Lambda^2_{sat}(W^2) \end{cases}$$

Since η-dependence different for small and large η , also W-dependence different

$$\sigma_{\gamma^* p}(W^2, Q^2) \sim \begin{cases} \ln \Lambda^2_{sat}(W^2) & , \quad \text{for} \quad Q^2 \ll \Lambda^2_{sat}(W^2), \\ \Lambda^2_{sat}(W^2) \sim (W^2)^{c_2 = const} & , \quad \text{for} \quad Q^2 \gg \Lambda^2_{sat}(W^2). \end{cases}$$

Scale for Q^2 dependence is $\Lambda^2_{sat}(W^2)$, where at HERA $2GeV^2 \lesssim \Lambda^2_{sat}(W^2) \lesssim 7GeV^2$

Since $\Lambda^2_{sat}\left(W^2 = \frac{Q^2}{x}\right) \sim \alpha_s(Q^2) xg(x, Q^2)$, soft energy dependence (as $\ln \Lambda^2_{sat}(W^2)$) not due to change in functional form of gluon density when decreasing Q^2 from $Q^2 \gg \Lambda^2_{sat}(W^2)$ to $Q^2 \ll \Lambda^2_{sat}(W^2)$.

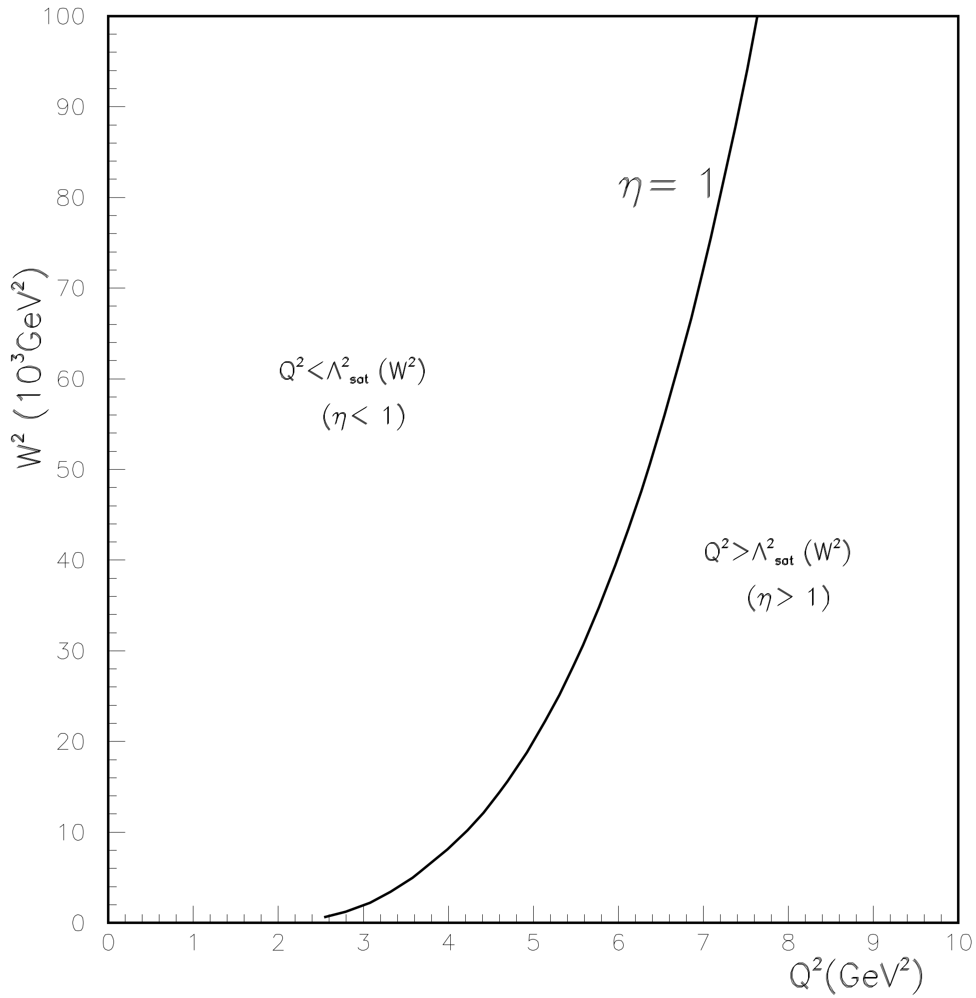
Note also:

$$\lim_{\substack{W^2 \to \infty \\ Q^2 \text{fixed}}} \frac{\sigma_{\gamma^* p}(\eta(W^2, Q^2))}{\sigma_{\gamma p}(\eta(W^2, Q^2 = 0))} = \lim_{\substack{W^2 \to \infty \\ Q^2 \text{fixed}}} \frac{\ln \frac{\Lambda^2_{sat}(W^2)}{Q^2 + m_0^2}}{\ln \frac{\Lambda^2_{sat}(W^2)}{m_0^2}} =$$

$$\lim_{\substack{W^2 \to \infty \\ Q^2 \text{fixed}}} \frac{\ln r(Q^2) \frac{\Lambda^2_{sat}(W^2)}{m_0^2}}{\ln \frac{\Lambda^2_{sat}(W^2)}{m_0^2}} = 1$$

"saturation"

$$\eta(W^2, Q^2) = 1$$



100
90
80
70
60
50
40
30
20
10
0
W² (10³GeV²)
η= 1
Q²<Λ²sat (W²)
(η< 1)
Q²>Λ²sat (W²)
(η> 1)
0
1
2
3
4
5
6
7
8
9
10
Q²(GeV²)

Why strong rise with W^2 as $\Lambda^2_{sat}(W^2) \sim (W^2)^{C_2 \simeq 0.28}$?

a)

$$\gamma^*(q\bar{q}) \text{ fluctuation : } \begin{cases} (q\bar{q})_{M_{q\bar{q}}} & \longrightarrow \quad (q\bar{q})_{M_{q\bar{q}}}, \\ (q\bar{q})_{M_{q\bar{q}}} & \longrightarrow \quad (q\bar{q})_{M'_{qq'} \neq M_{q\bar{q}}}, \end{cases}$$

$M^2_{q\bar{q}}, M'^2_{q\bar{q}} \leq m^2_1(W^2)$

$\Delta M^2_{q\bar{q}} = (M'^2_{q\bar{q}} - M^2_{q\bar{q}}) \sim \Lambda^2_{sat}(W^2)$

expect increase of $\Lambda^2_{sat}(W^2)$with W^2.

b) DGLAP evolution at low x

$$\frac{\partial F_2\left(\frac{x}{2}, Q^2\right)}{\partial \ln Q^2} = \frac{R_{e^+e^-}}{9\pi}\alpha_s(Q^2)xg(x, Q^2)$$

CDP for $\eta \gg 1 \quad (Q^2 \gg \Lambda^2_{sat}(W^2))$, at HERA: $2GeV^2 \lesssim \Lambda^2_{sat}(W^2) \lesssim 7GeV^2$

$$\left.\begin{array}{l} F_2(x, Q^2) \\ \alpha_s(Q^2)xg(x, Q^2) \end{array}\right\} \sim \Lambda^2_{sat}(W^2)$$

One finds

$$\frac{\partial}{\partial \ln W^2}\Lambda^2_{sat}(2W^2) = \frac{1}{2\rho + 1}\Lambda^2_{sat}(W^2);$$

$(2\rho + 1)c_2 2^{c_2} = 1;$ Kuroda, Schildknecht (2005)

and

$$\rho = \begin{cases} 1, & , \quad c_2^{theor.} = 0.276 \qquad c_2^{exp}|_{Model-indep.} = 0.28 \pm 0.06 \\ \frac{4}{3} & , \quad c_2^{theor.} = 0.23 \end{cases}$$

Low-x scaling consistent with evolution and $c_2^{theor.} \cong c_2^{exp}$

Treating ρ as free parameter (violating $\rho = \frac{4}{3}$)

ρ	c_2^{theor}	$\frac{\sigma_{\gamma_L^*}}{\sigma_{\gamma_T^*}}$	$F_2\left(W^2 = \frac{Q^2}{x}\right)$
$\to \infty$	0	0	$\left(\frac{Q^2}{x}\right)^0 = \text{const}$
0	0.65	∞	$\left(\frac{Q^2}{x}\right)^{0.65}$

Related theoretical argument:

Double asymptotic scaling solution of DGLAP evolution yields geometric scaling and $\lambda \simeq 0.3$ where $\tau = \frac{Q^2}{Q_0^2} x^\lambda$. Caola and Forte (2008)

6. Conclusions:

- Color transparency plus transverse-size enhancement of $(q\bar{q})_T^{J=1}$ relative to $(q\bar{q})_L^{J=1}$ implies

  $F_L = 0.27 \cdot F_2$, consistent with experiment.

- Low-x scaling: $\sigma_{\gamma^* p}(W^2, Q^2) = \sigma_{\gamma^* p}\left(\eta(W^2, Q^2) = \frac{Q^2 + m_0^2}{\Lambda^2_{sat}(W^2)}\right)$, $\Lambda^2_{sat} \sim (W^2)^{C_2 \simeq 0.28}$.

  Why low-x scaling?

  Consequence of color-dipole interaction plus dimensional analysis

  Why different W-dependence for small and large Q^2 ?

  Color-dipole interaction (generically) implies different η dependence

  Different η-dependence implies different W-dependence.

$$\sigma_{\gamma^* p}(\eta) \sim \begin{cases} \ln \frac{1}{\eta} \sim \ln \Lambda^2_{sat}(W^2) & , \quad \text{for } Q^2 \ll \Lambda^2_{sat}(W^2), \\ \frac{1}{\eta} \sim \Lambda^2_{sat}(W^2) & , \quad \text{for } Q^2 \gg \Lambda^2_{sat}(W^2). \end{cases}$$

$$\lim_{\substack{W^2 \to \infty \\ Q^2 \text{fixed}}} \frac{\sigma_{\gamma^* p}(W^2, Q^2)}{\sigma_{\gamma p}(W^2)} = 1. \qquad \text{“Saturation”}$$

Why $c_2^{exp} \simeq 0.28 \neq 0$, where $\Lambda^2_{sat}(W^2) \sim (W^2)^{C_2}$?

Consistency of CDP with DGLAP evolution implies $c_2^{theor.} \simeq c_2^{exp} \simeq 0.28$

- The same functional form of the gluon density

$$\Lambda^2_{sat}\left(W^2 = \frac{Q^2}{x}\right) \sim \alpha_s(Q^2) x\, g(x, Q^2) \sim (W^2)^{C_2 \simeq 0.28}$$

leads to "soft" $(Q^2 \ll \Lambda^2_{sat}(W^2))$ and "hard" $(Q^2 \gg \Lambda^2_{sat}(W^2))$ energy dependence of $\sigma_{\gamma^* p}(\eta(W^2, Q^2))$.

- Interpretation in the CDP of the experimentally observed low-x scaling of $\sigma_{\gamma^* p} = \sigma_{\gamma^* p}(\eta(W^2, Q^2))$ does not require non-linear evolution.