e+A physics at a future Electron-Ion Collider

Outline

- What is the nature of gluonic fields?
- Non-linear QCD and Saturation physics
  - Nuclear Enhancement Factor
- The 4 key gluon measurements in e+A
  - Momentum distributions
  - Space-time distributions
  - The role of the Pomeron
  - Energy loss in cold nuclear matter
- Universality at small-x

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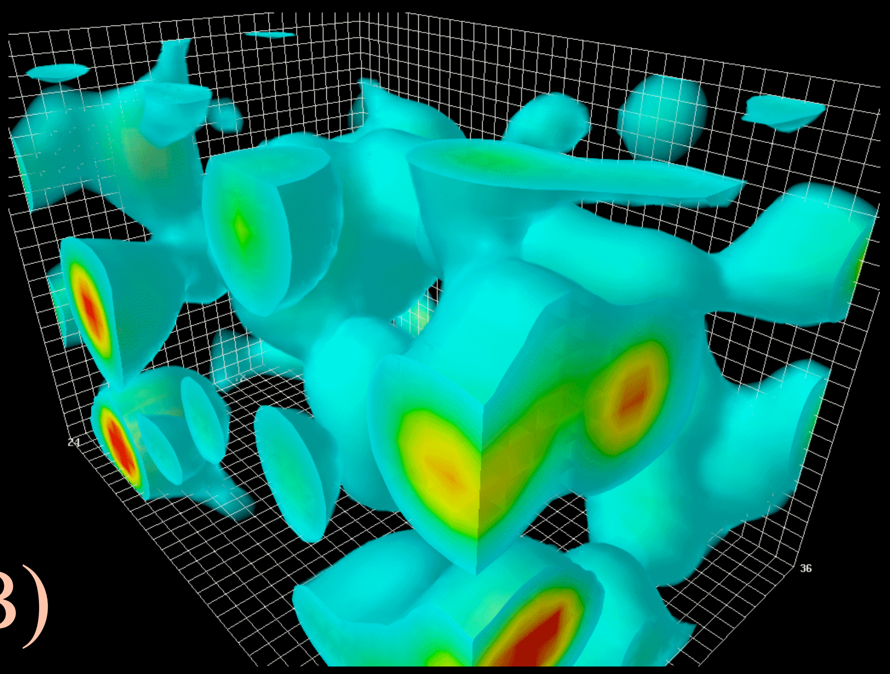
What do we know about gluons?

Gluons and the QCD Lagrangian:

$$L_{QCD} = \bar{q}(i\gamma^{\mu}\partial_{\mu} - m)q - g(\bar{q}\gamma^{\mu}T_a q)A^a_{\mu} - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a$$

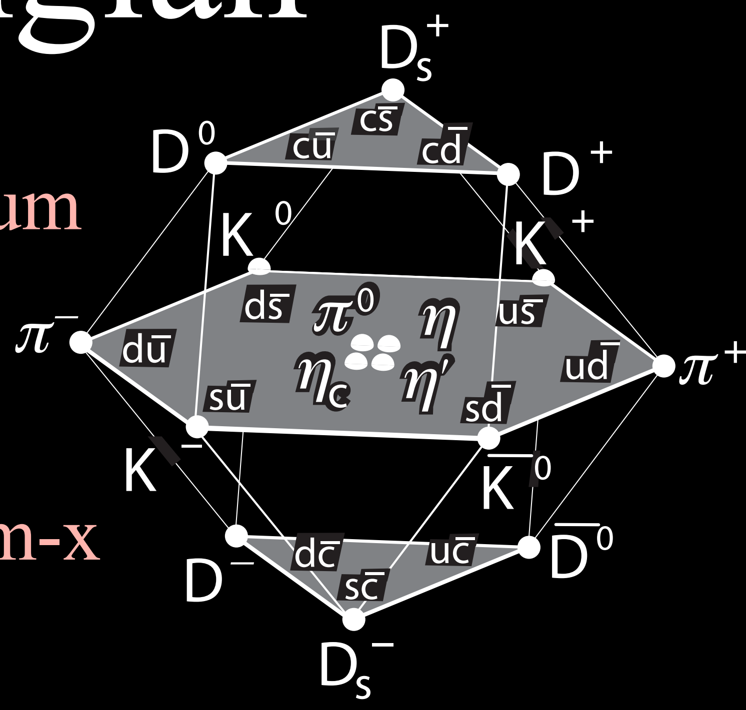
- **Despite the success of QCD, our knowledge of glue is very limited. We know:**



Action (~energy) density fluctuations of gluon-fields in QCD vacuum (2.4 ×2.4× 3.6 fm) (Derek Leinweber)

- Mediators of the strong interaction
- Determine essential features of QCD
  - Asymptotic freedom from gluon loops
- Dominate structure of QCD vacuum (χSB)
  - L_{QCD} gets hadron masses correct to ~ 10%

Gluons and the Lagrangian



- **Hard to "see" gluons in the low-energy world**
  - Gluon degrees of freedom "missing" in hadronic spectrum
    - Constituent Quark Picture?
- From DIS:
  - Drive the structure of baryonic matter already at medium-x
- Crucial players at RHIC and the LHC
  - Drive the entropy

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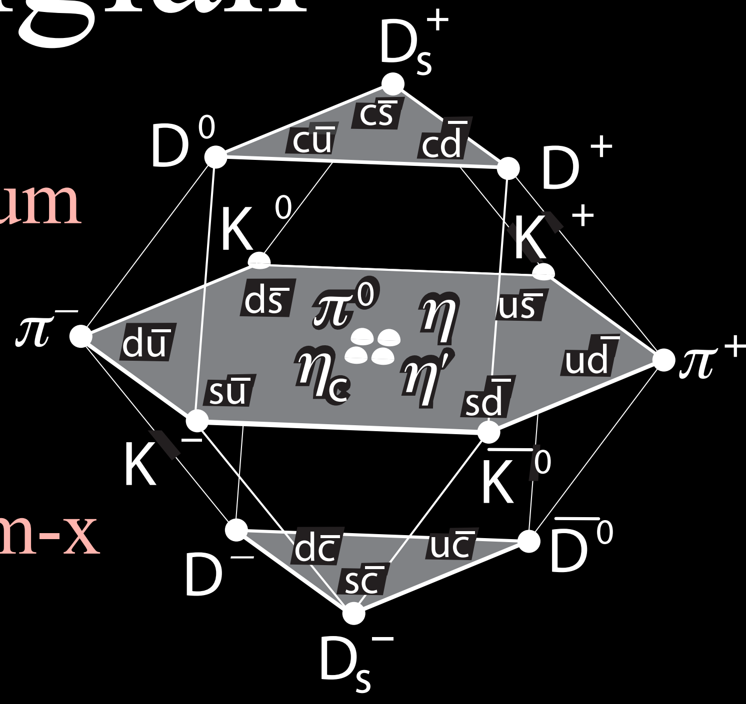

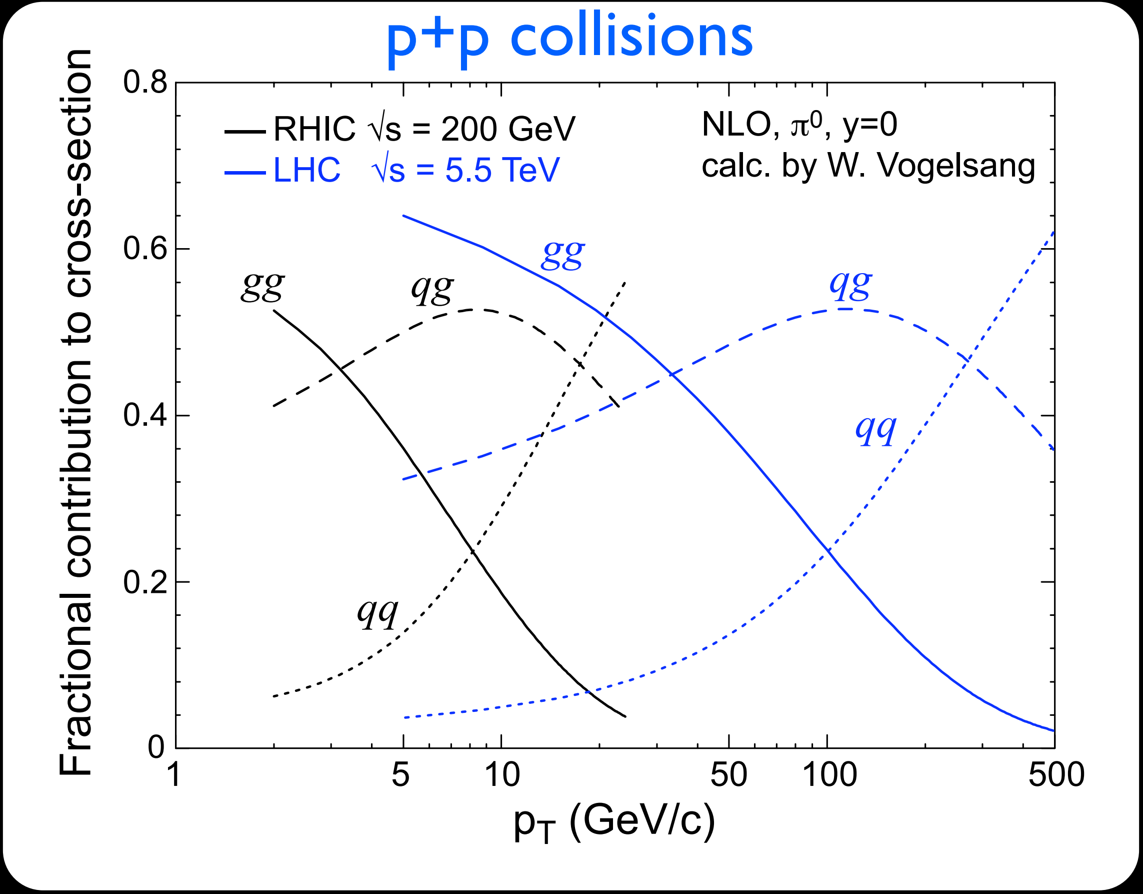

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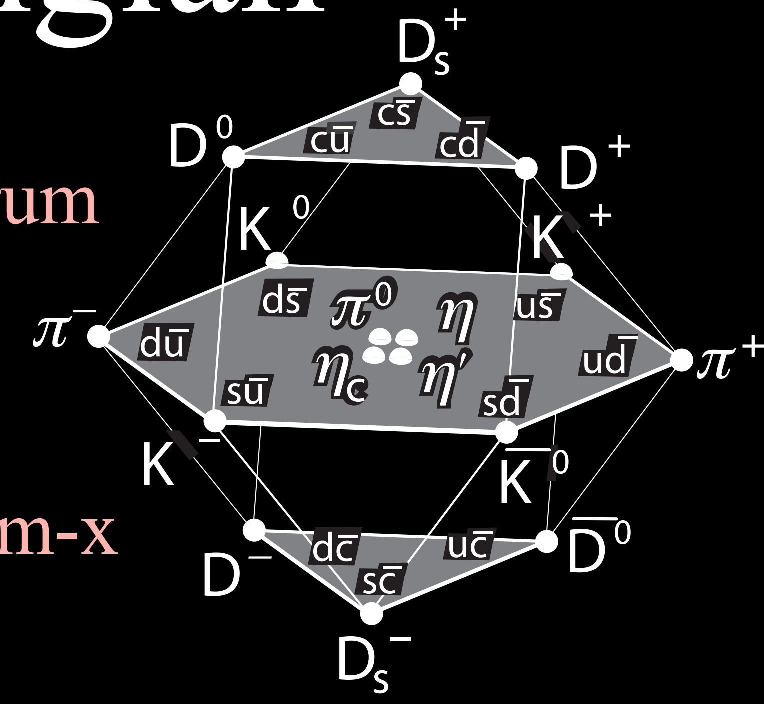

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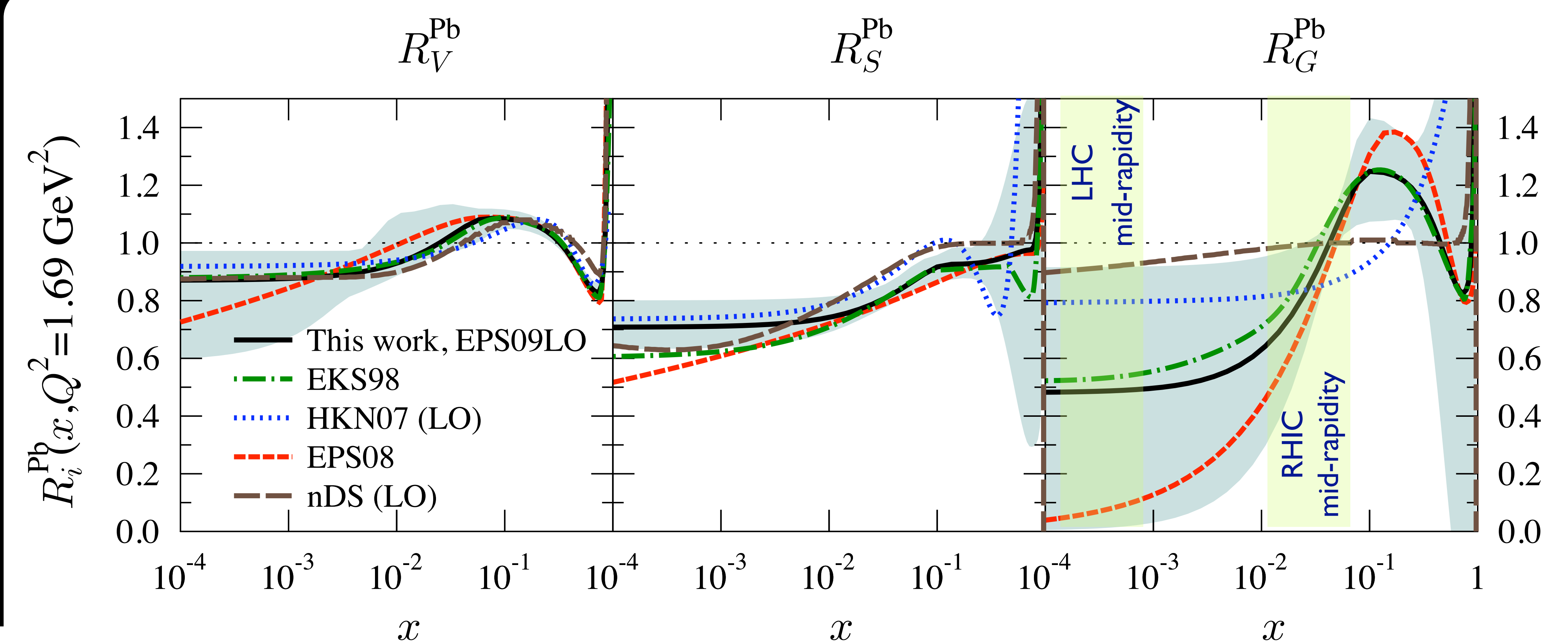
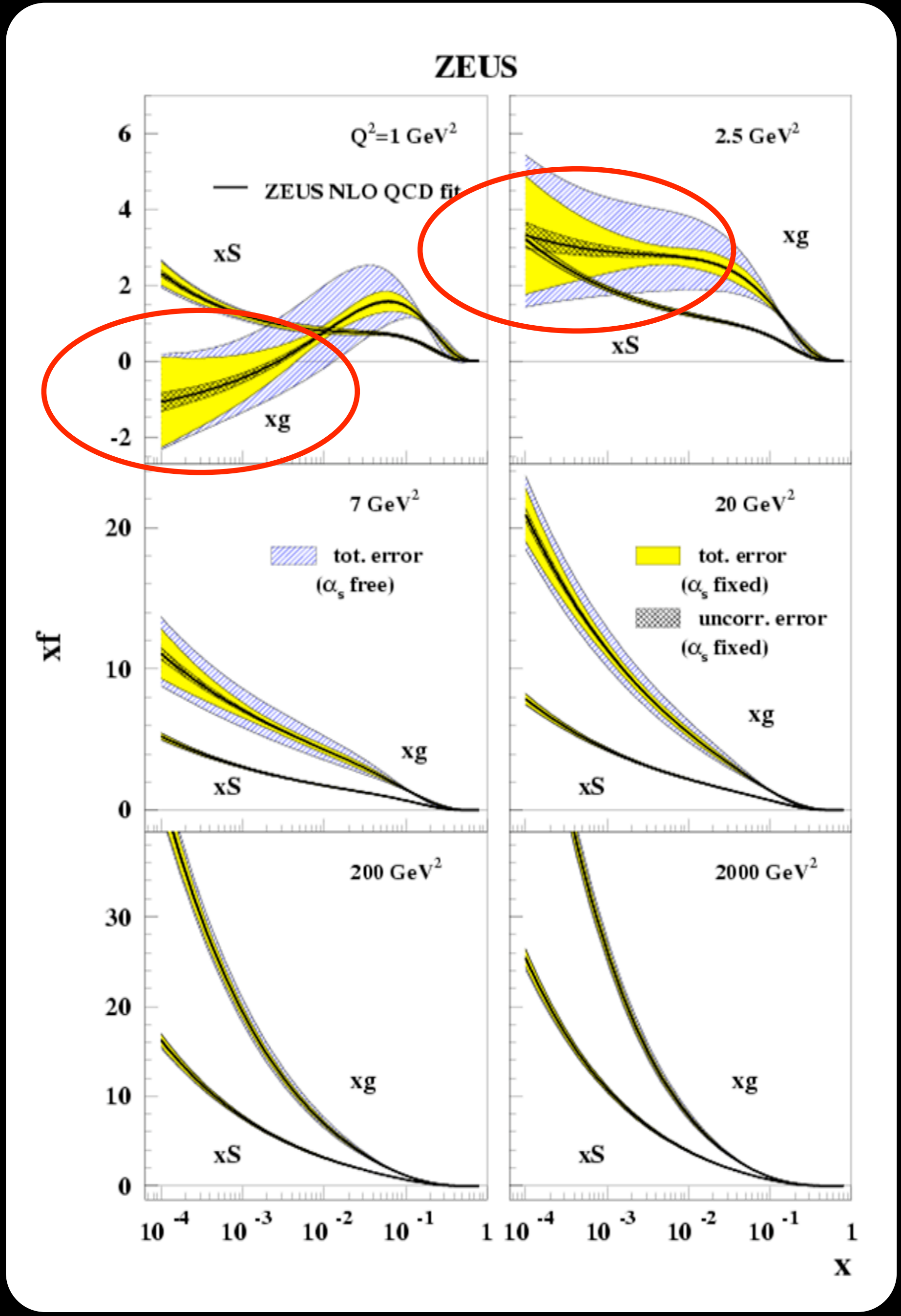
Eskola, Paukkonen, Salgado: arXiv0902.4154



The problem with our current understanding

- Using the Linear DGLAP evolution model:
  - Weird behaviour of xG at low-x and low-Q^2 in HERA data
    - xG goes negative
    - xG < xS (even though sea quarks come from gluon splitting)



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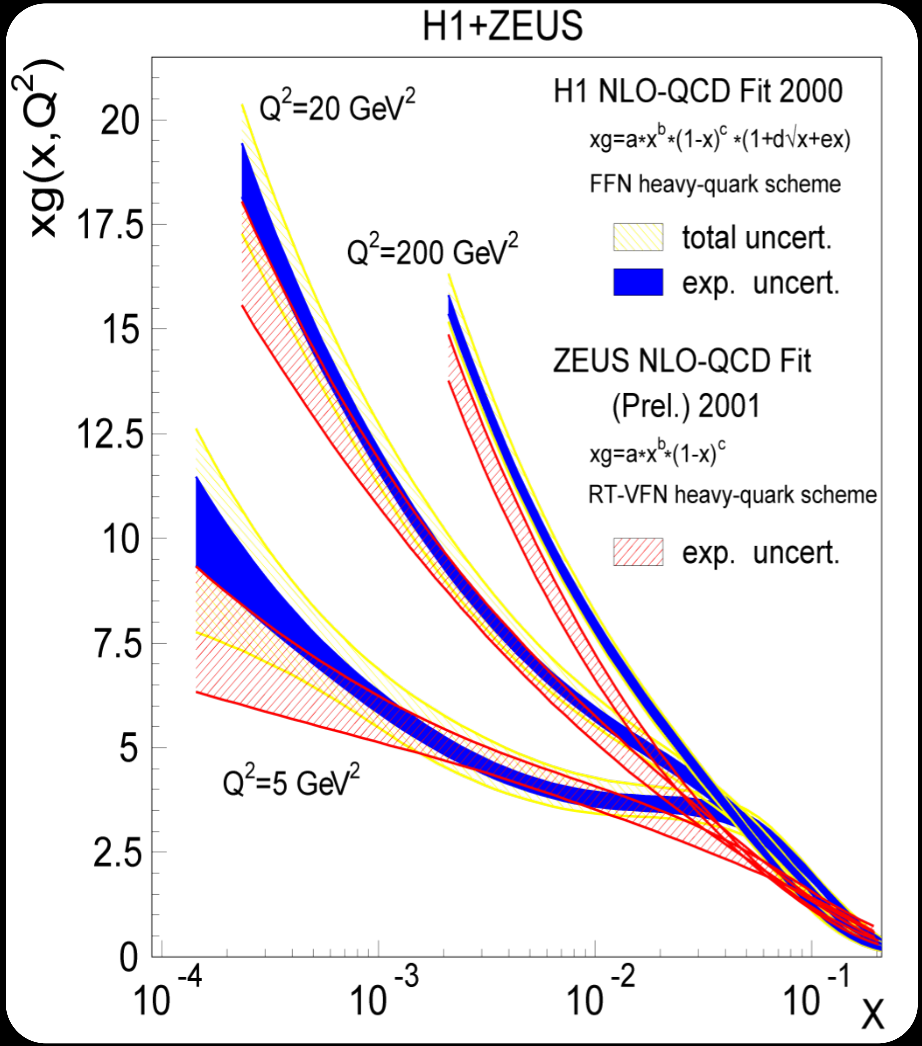
    - Must have saturation !!



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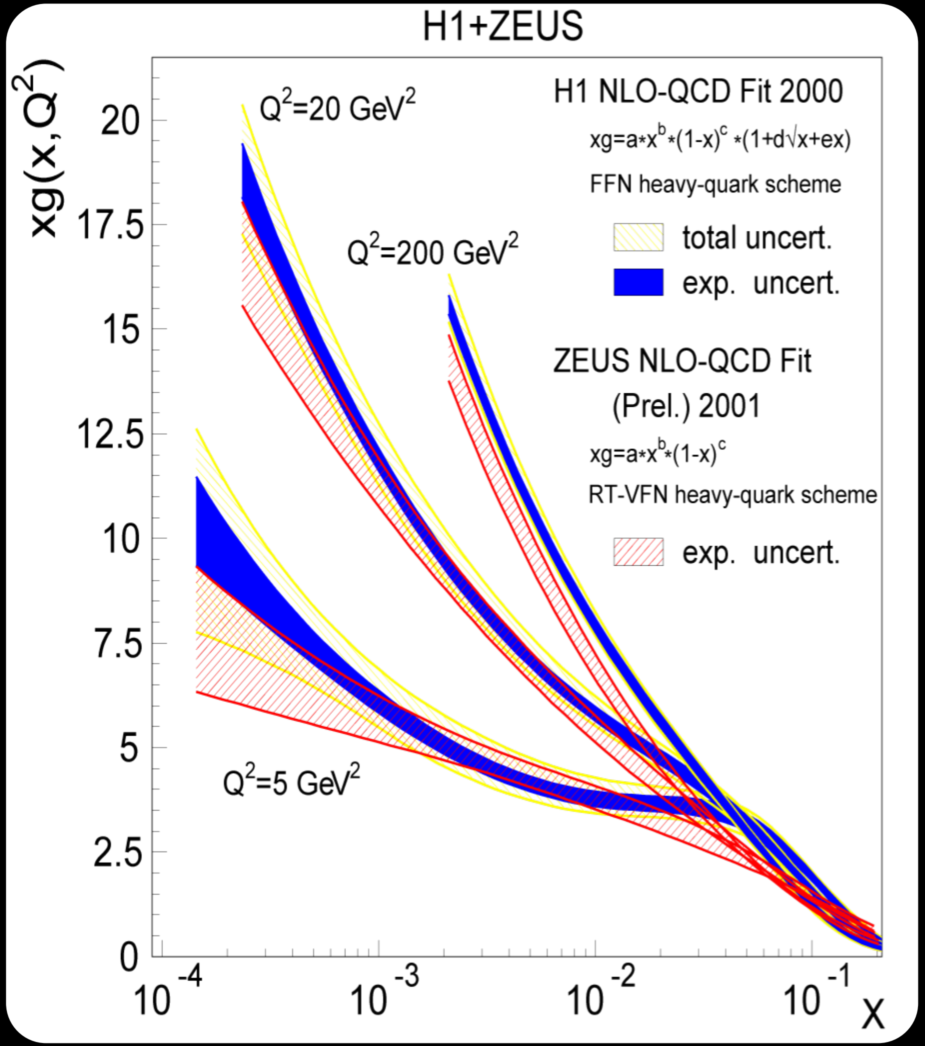
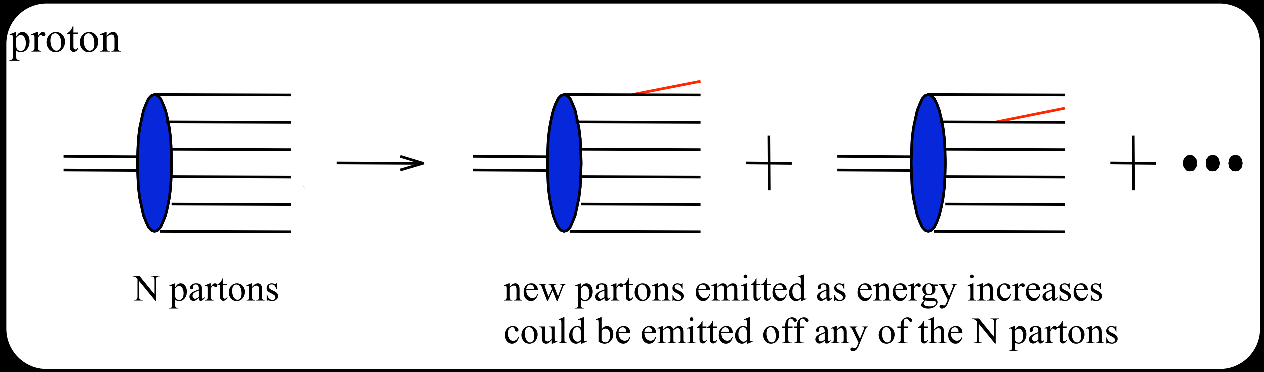
- What is the underlying dynamics?



Non-linear QCD and Saturation



- BFKL evolution in x:
  - linear
    - explosion in colour field at small-x

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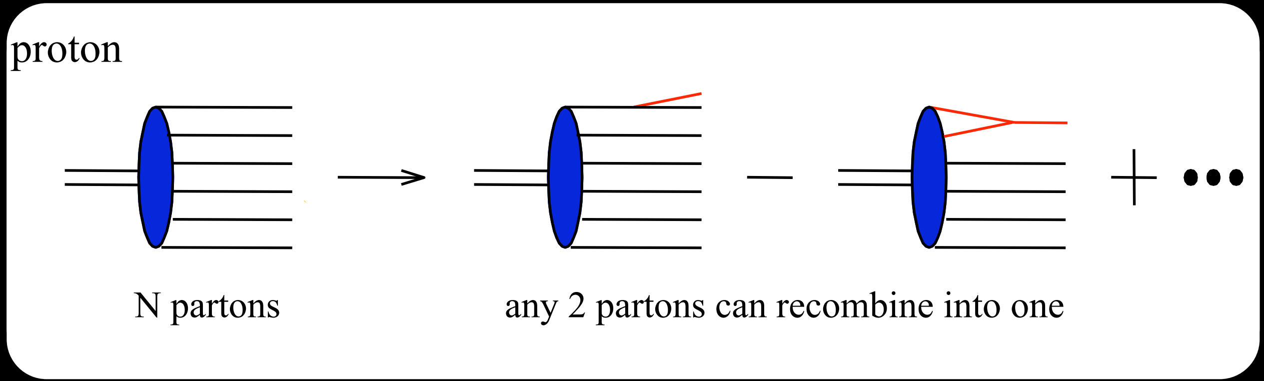
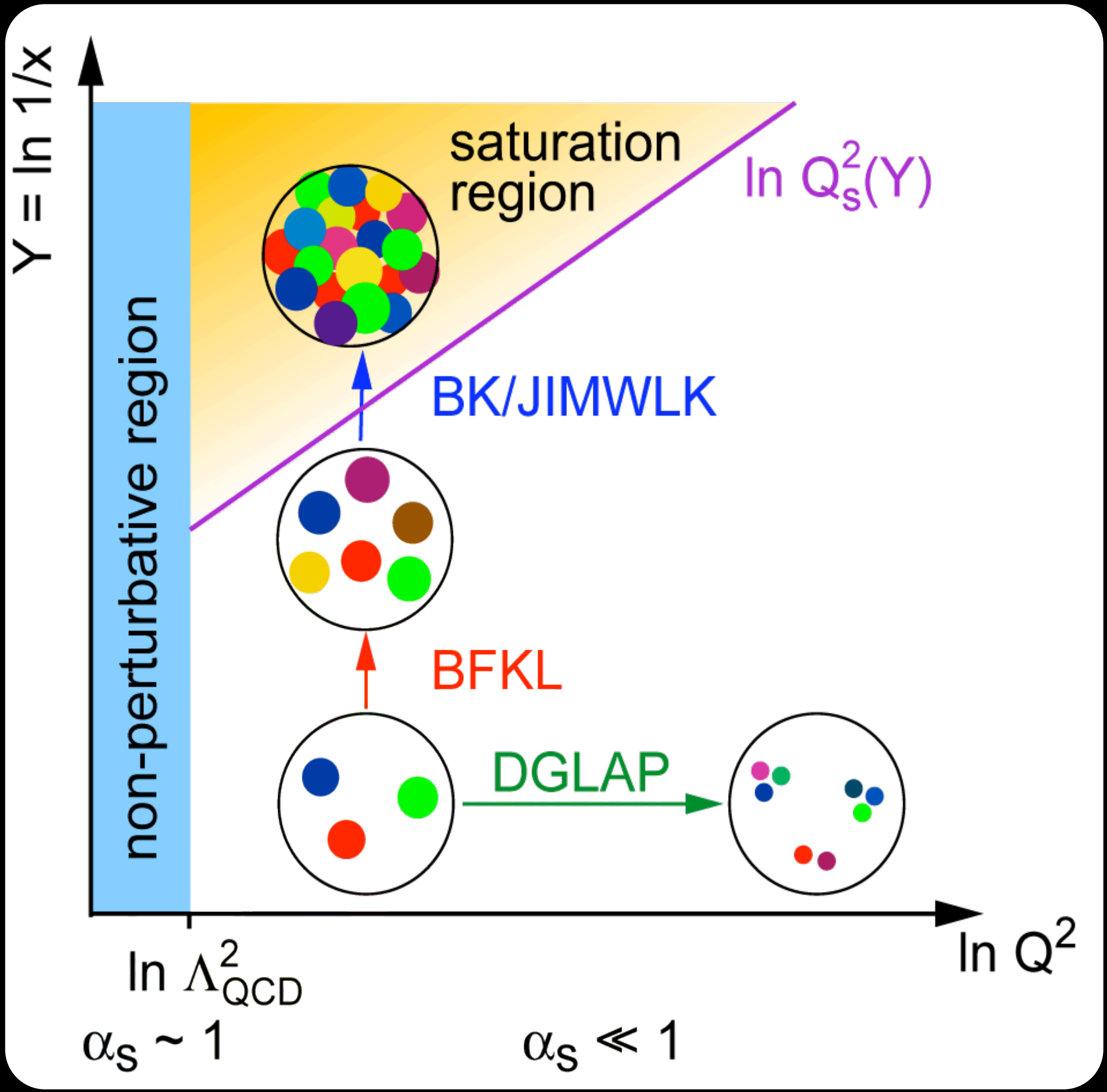
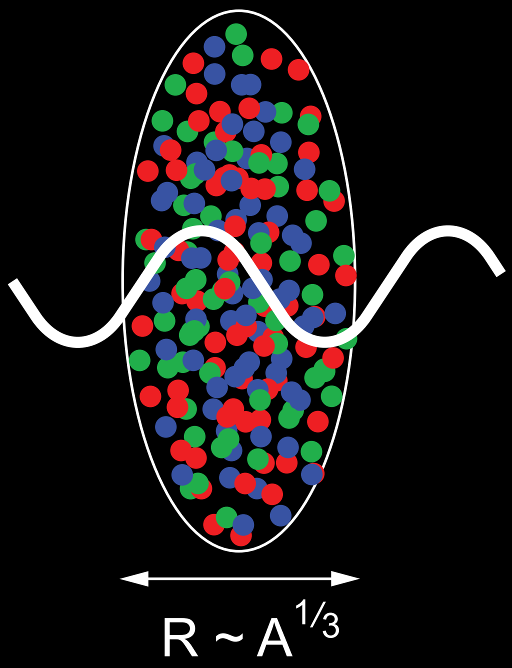


- BFKL evolution in x:
  - linear
    - explosion in colour field at small-x
- Non-linear JIMWLK/BK equations:
  - non-linearity ⇒ saturation
  - characterised by the saturation scale, $Q_S(x,A)$
  - arises naturally in the Colour-Glass Condensate EFT



The Nuclear Enhancement Factor

- Enhancing Saturation effects:
  - Probes interact over distances $L \sim (2m_n x)^{-1}$
  - For probes where $L > 2R_A$ ($\sim A^{1/3}$), cannot distinguish between nucleons in front or back of the nucleus.
    - Probe interacts coherently with all nucleons.
  - Probes with transverse resolution $1/Q^2$ ($\ll \Lambda^2_{QCD}$) $\sim$ 1 fm^2 will see large colour charge fluctuations.
    - This kick experienced in a random walk is the resolution scale.



The Nuclear Enhancement Factor

Simple geometric considerations lead to:

$Q_S^2 \propto \dfrac{\alpha_s xG(x, Q_S^2)}{\pi R_A^2}$ HERA: $xG \propto \dfrac{1}{x^{1/3}}$ A dependence: $xG_A \propto A$

Nuclear Enhancement Factor: $(Q_S^A)^2 \approx cQ_0^2(\dfrac{A}{x})^{1/3}$

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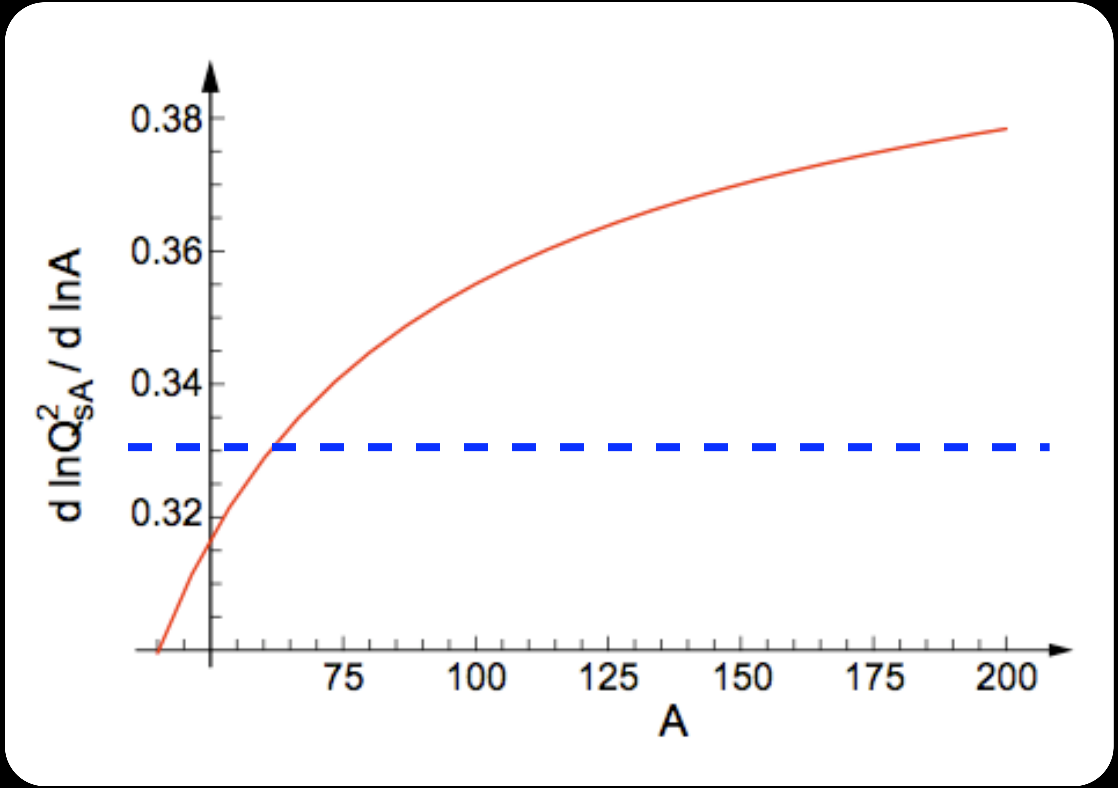
Enhancement of Qs with A: ⇒ non-linear QCD regime reached at significantly lower energy in e+*A* than in *e*+*p*

The Nuclear Enhancement Factor

More sophisticated analyses
⇒ confirm (exceed) pocket formula for high A

e.g. Kowalski, Lappi and Venugopalan, PRL 100, 022303 (2008); Armesto et al., PRL 94:022002; Kowalski, Teaney, PRD 68:114005



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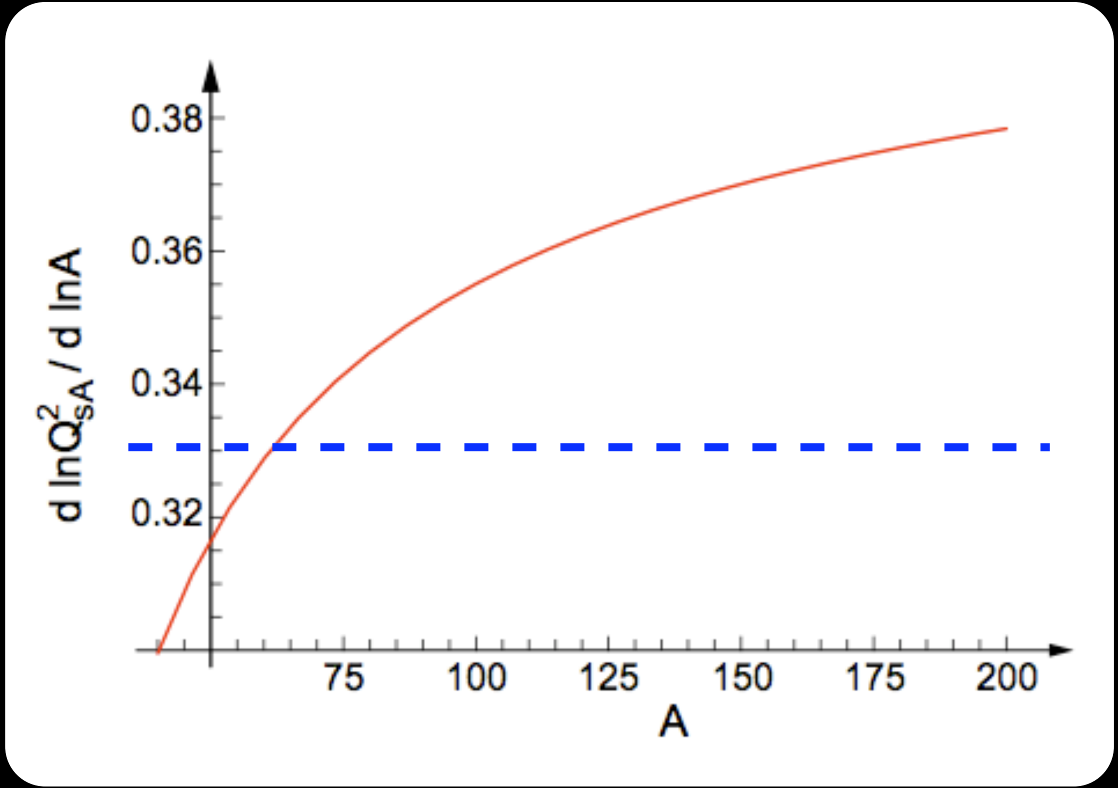
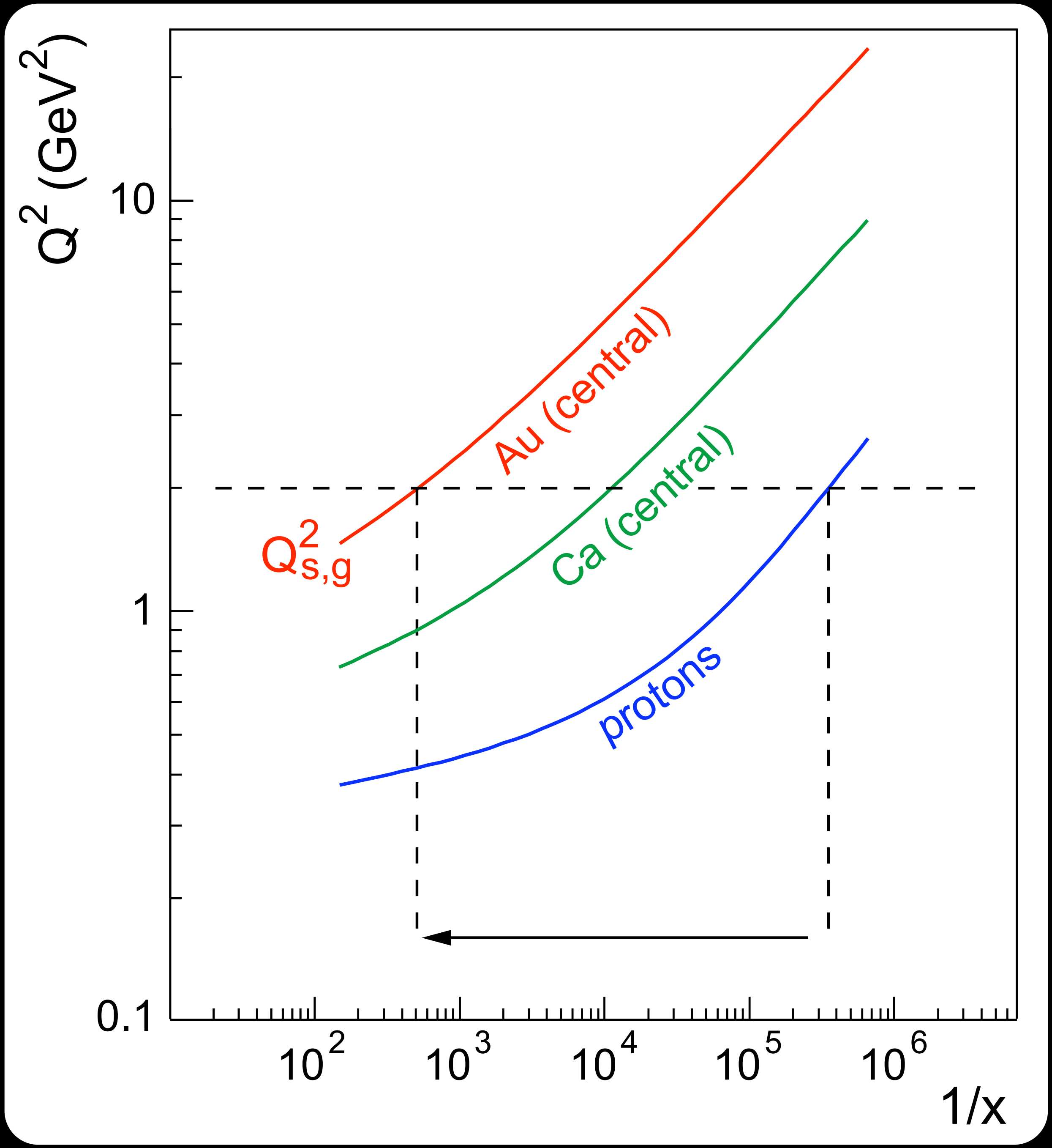




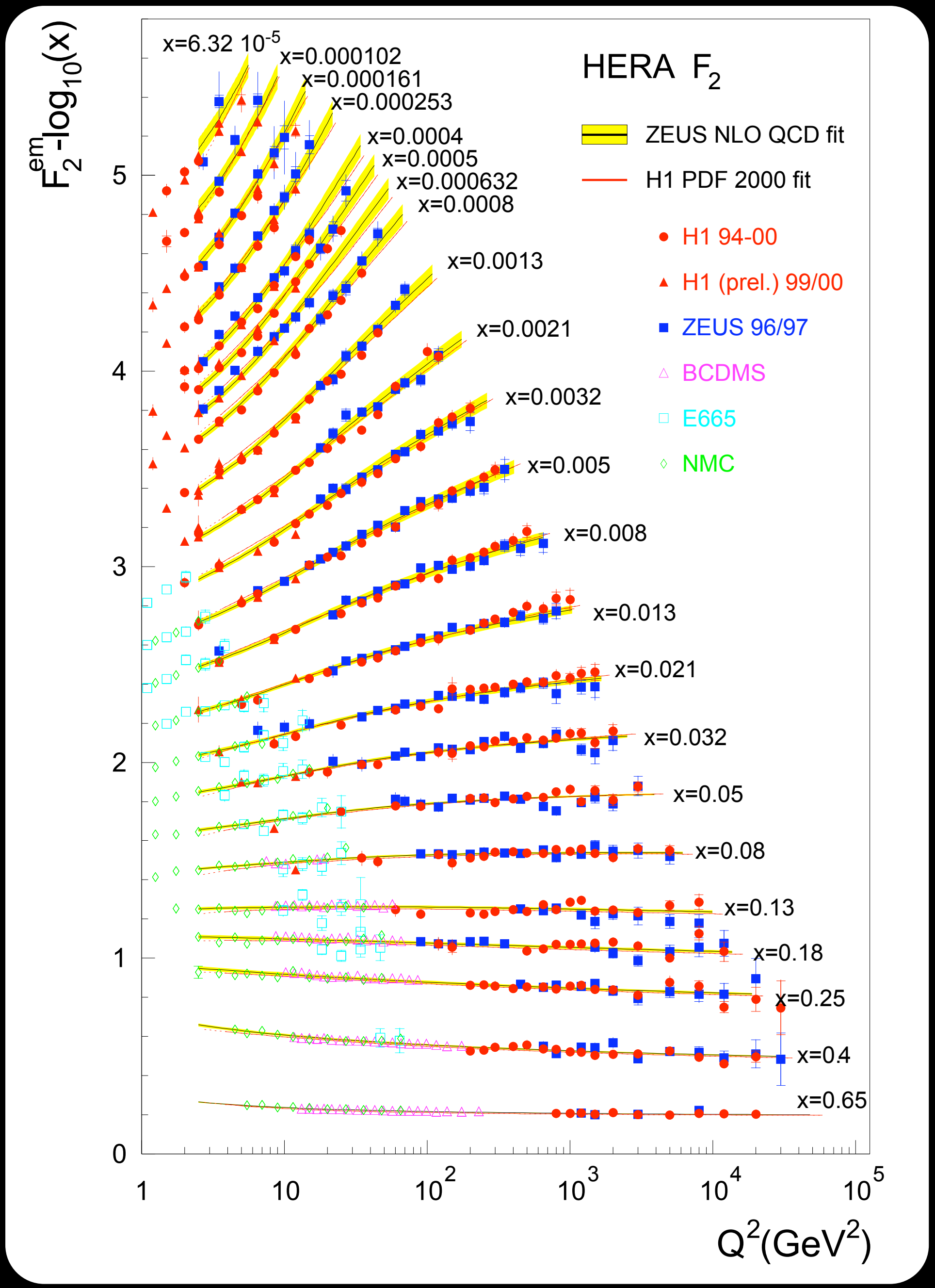
One would require an energy in e+p ~ 200-300 x e+A to get to same $Q^2{}_s$

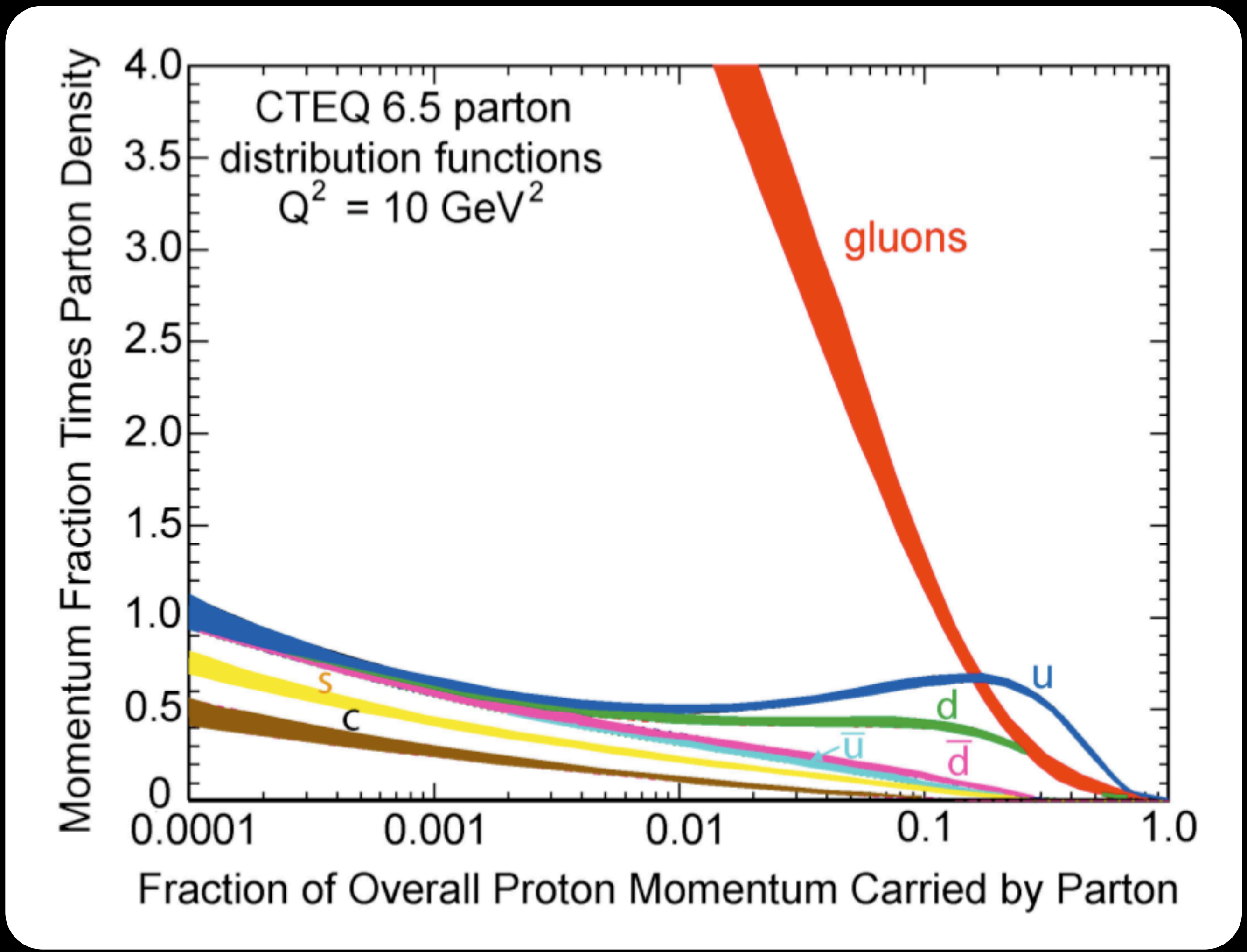
4 Key Measurements in e+A Physics

- Momentum distribution of gluons in nuclei?
  - Extract via scaling violation in F_2: $\partial F_2/\partial \ln Q^2$
  - Direct Measurement: $F_L \sim xG(x,Q^2)$ - requires $\sqrt{s}$ scan
  - Inelastic vector meson production (e.g. J/Ψ, ρ)
  - Diffractive vector meson production ($\sim [xG(x,Q^2)]^2$)
  - 2+1 jet rates

Gluon momemtum distributions: i) F_2 scaling violation

$$\frac{d^2\sigma^{ep\to eX}}{dxdQ^2} = \frac{4\pi\alpha^2_{e.m.}}{xQ^4}\left[\left(1-y+\frac{y^2}{2}\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right]$$





Scaling violation of $dF_2/d\ln Q^2$ and linear DGLAP evolution $\Rightarrow$ $xG(x,Q^2)$

Gluon momemtum distributions: ii) F_L measured directly

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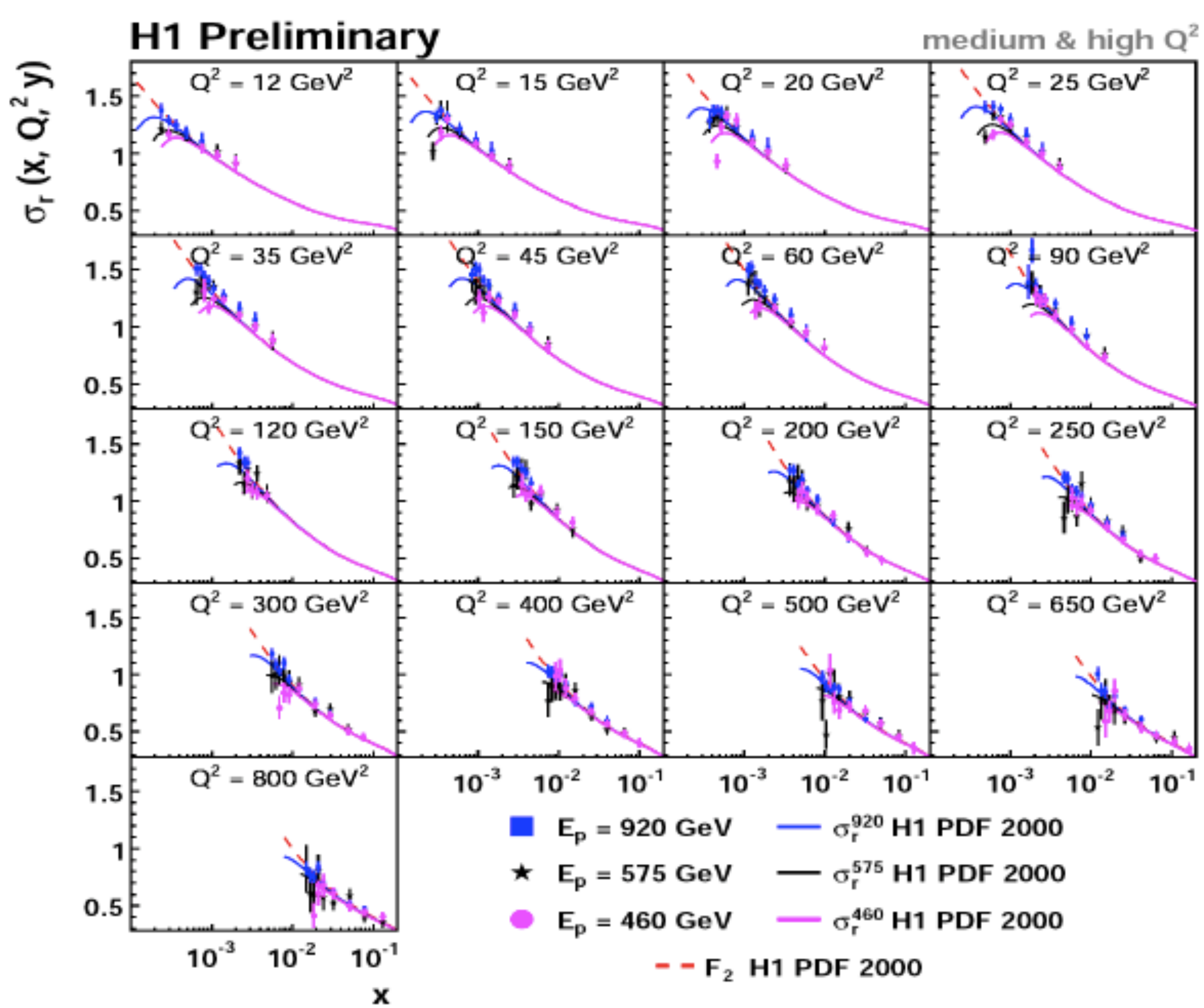
requires $\sqrt{s}$ scan, $Q^2/xs = y$



Preliminary F_L measurements

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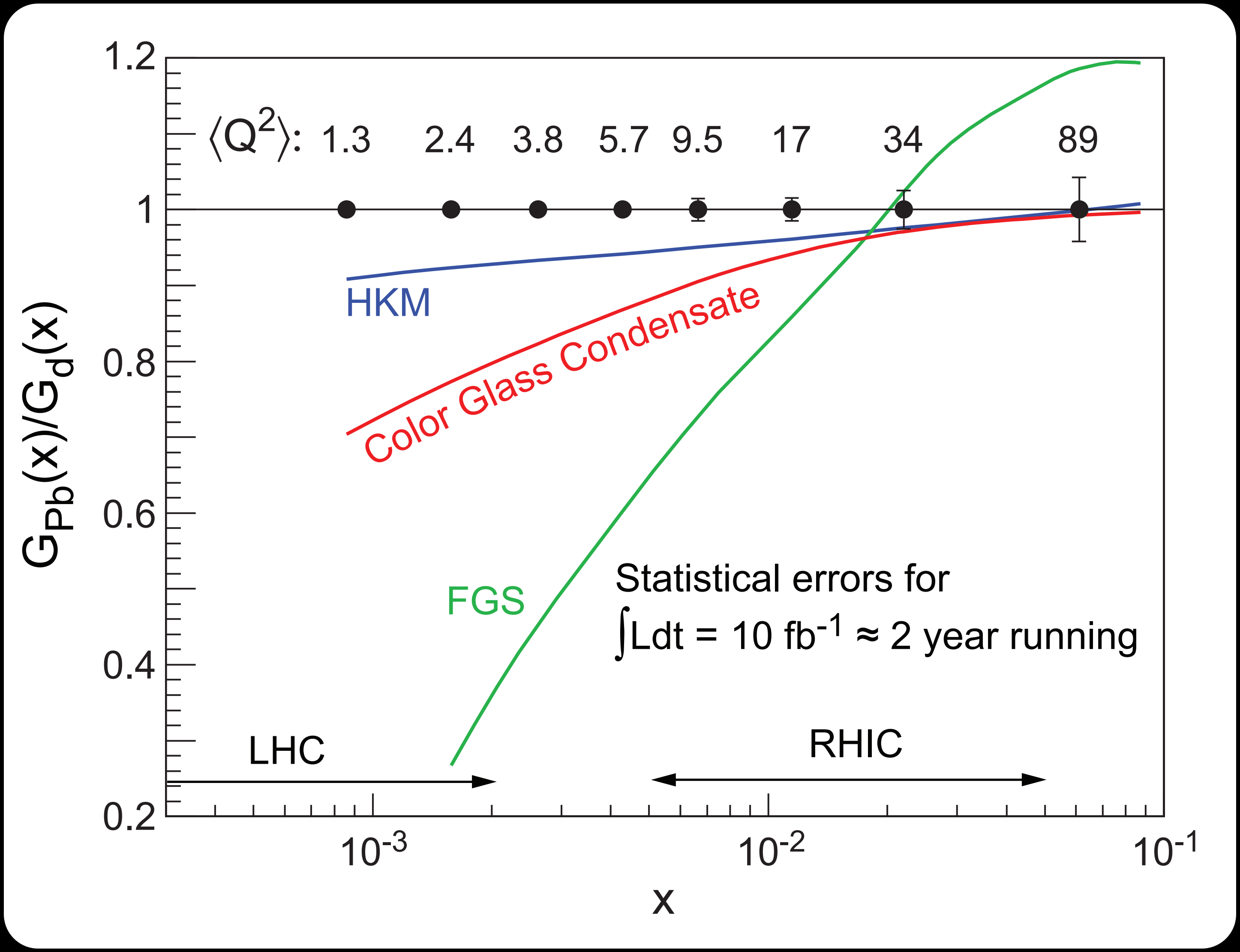
Here:
$\int L dt$ = 4/A fb^{-1} (10+100) GeV
= 4/A fb^{-1} (10+50) GeV
= 2/A fb^{-1} (5+50) GeV

statistical error only



Syst. studies of $F_L(A,x,Q^2)$:

- $xG(x,Q^2)$ with great precision
- Distinguish between models

HKM and FGS are "standard" shadowing parameterizations that are evolved with DGLAP

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  - Exclusive final states (e.g. ρ, J,Ψ)
  - Deep Virtual Compton Scattering (DVCS) - $\sigma \sim A^{4/3}$
  - F_2, F_L for various impact parameters

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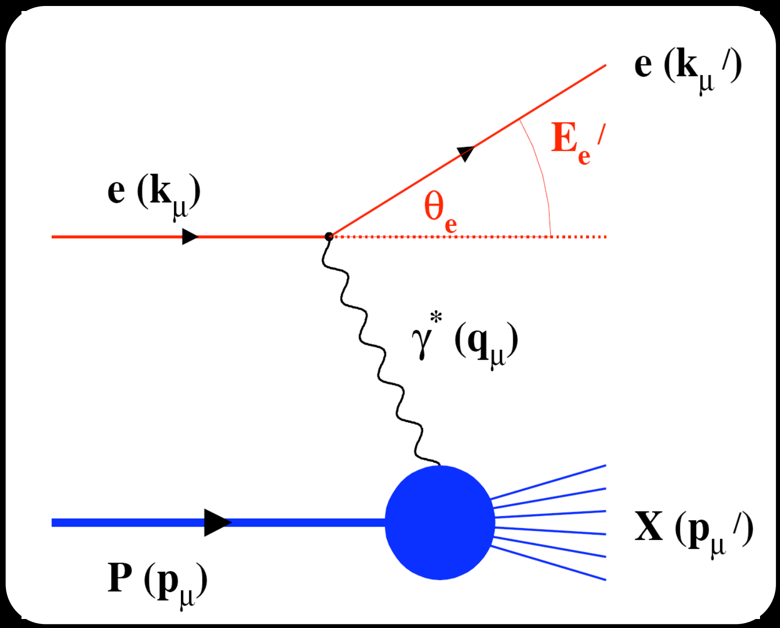
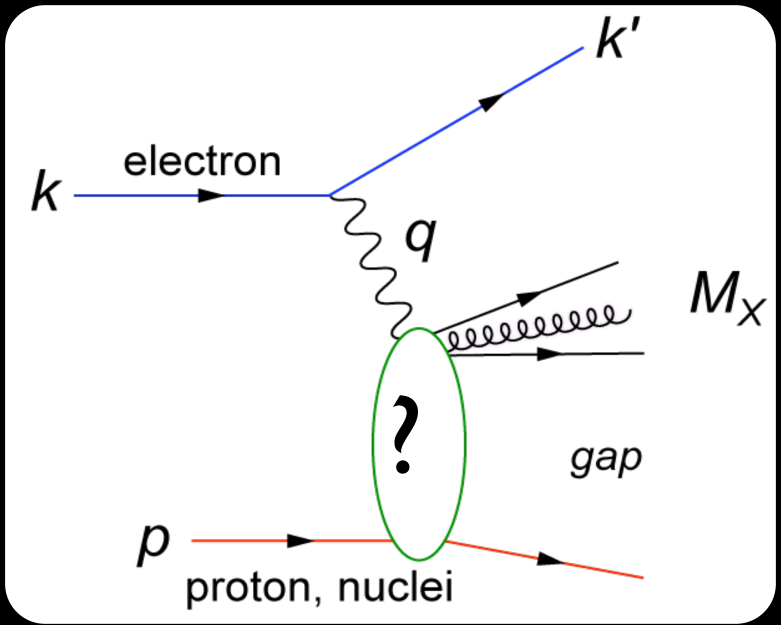
- Role of colour-neutral (Pomeron) excitations?
  - Diffractive vector meson production ($\sim [xG(x,Q^2)]^2$)
  - Diffractive cross-section: $\sigma_{diff}/\sigma_{tot}$ (~ 10%: HERA e+p; 30%? EIC e+A?)

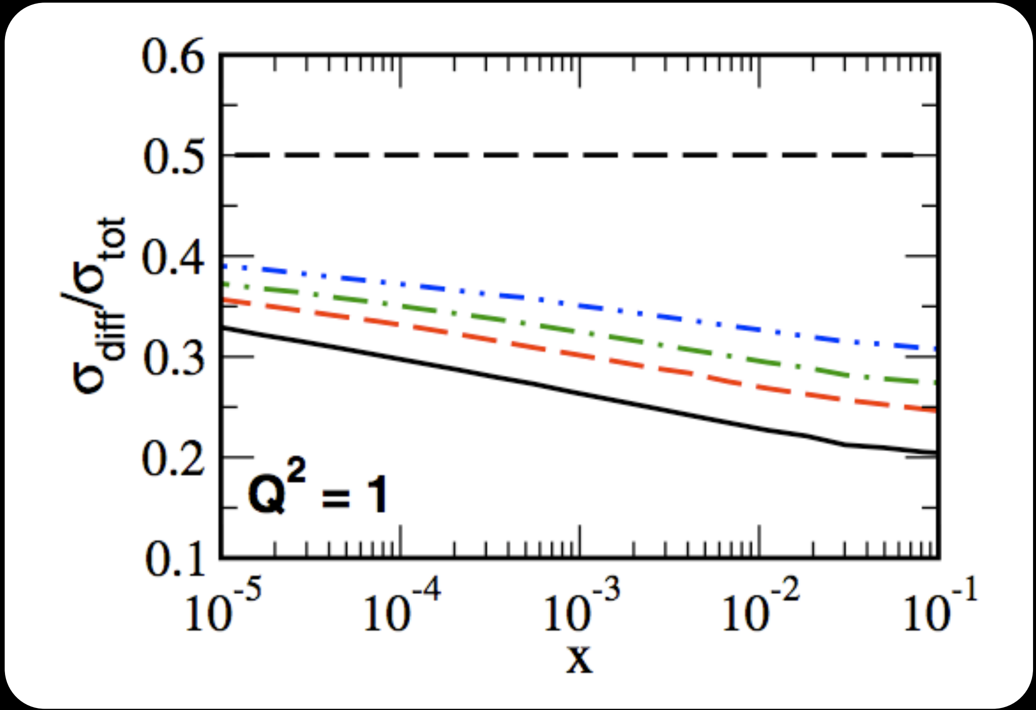
Role of colour-neutral (Pomeron) excitations

$$\frac{d\sigma}{dt}|_{t=0}(\gamma^* A \to M_X A) \propto \alpha^2[G_A(x,Q^2)]^2$$

Curves: Kugeratski, Goncalves, Navarra, EPJ C46, 413







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  - Predictions: ~25-40%?

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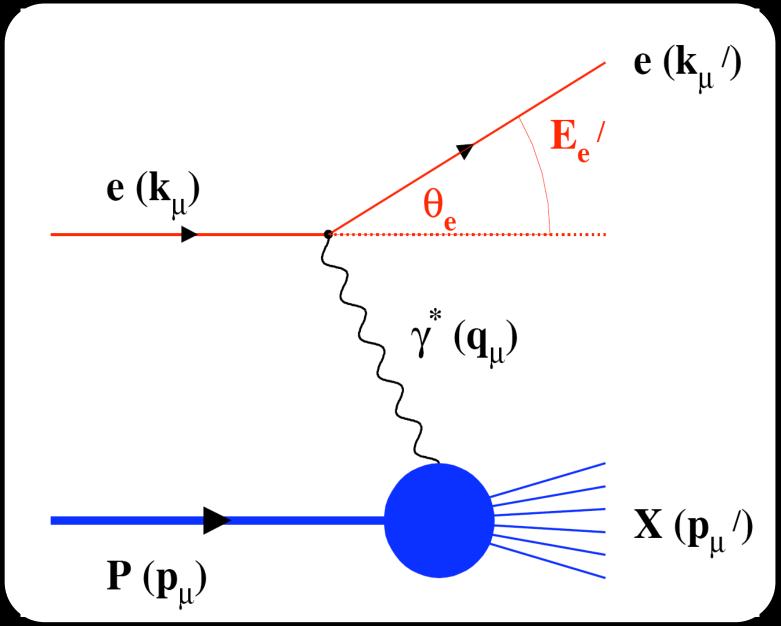

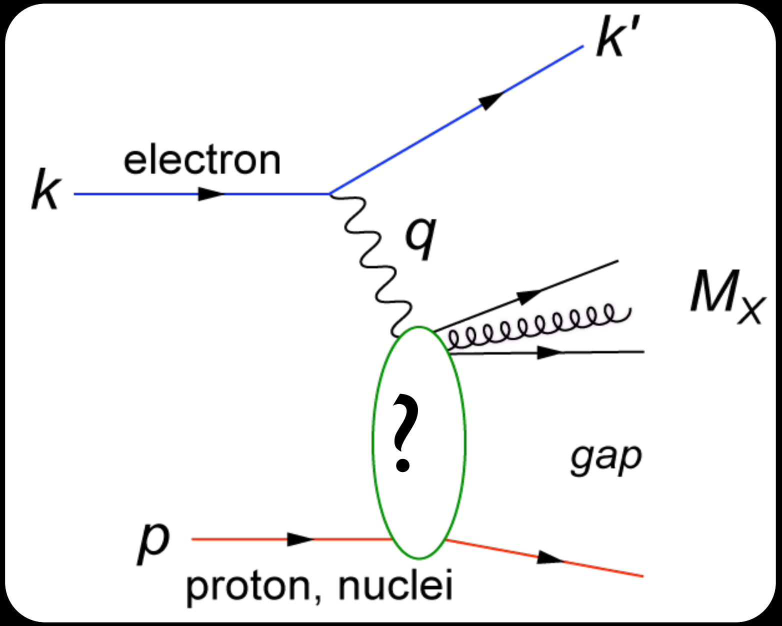

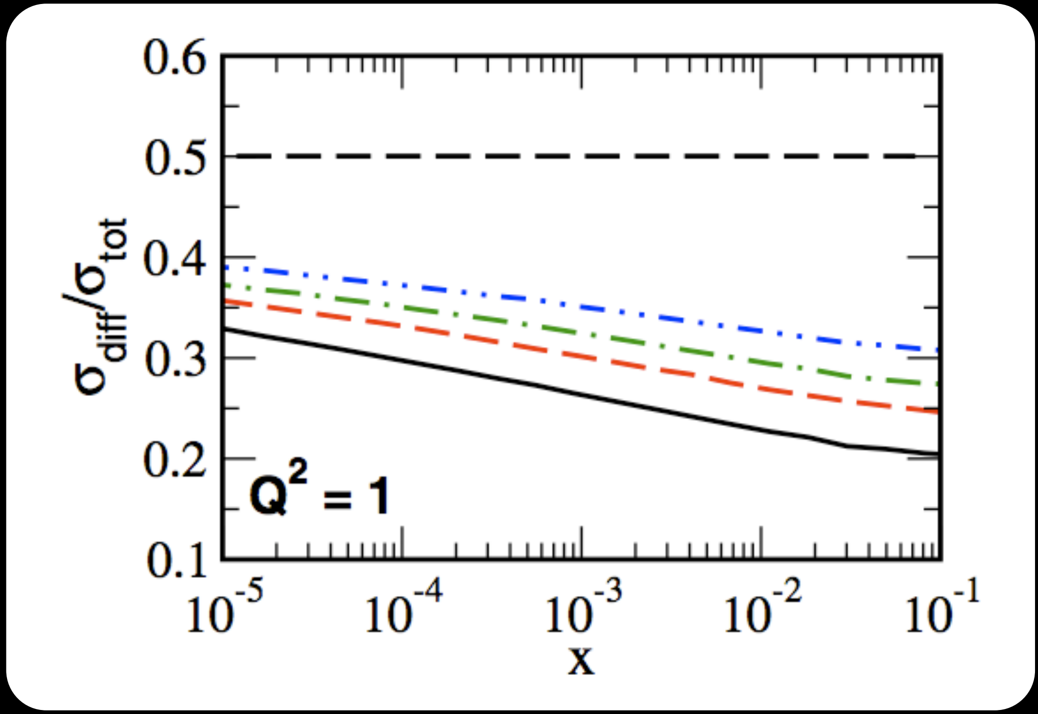

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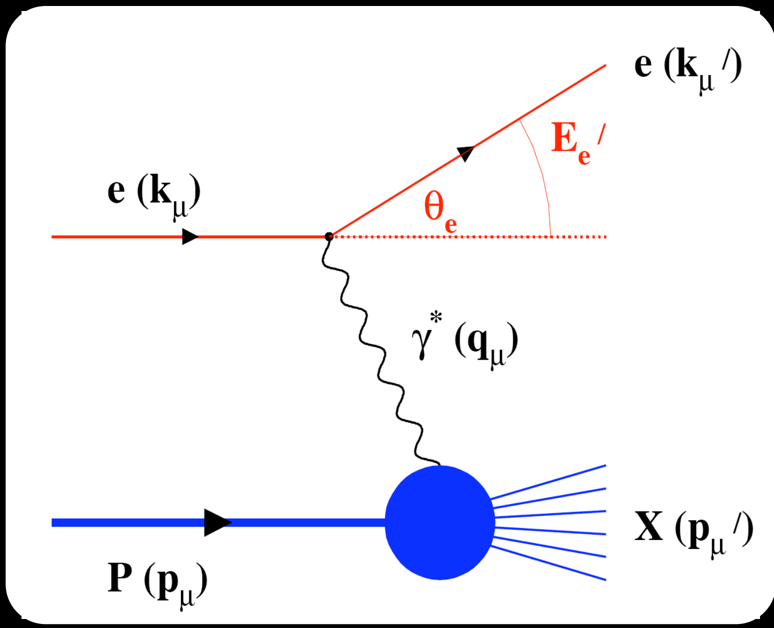

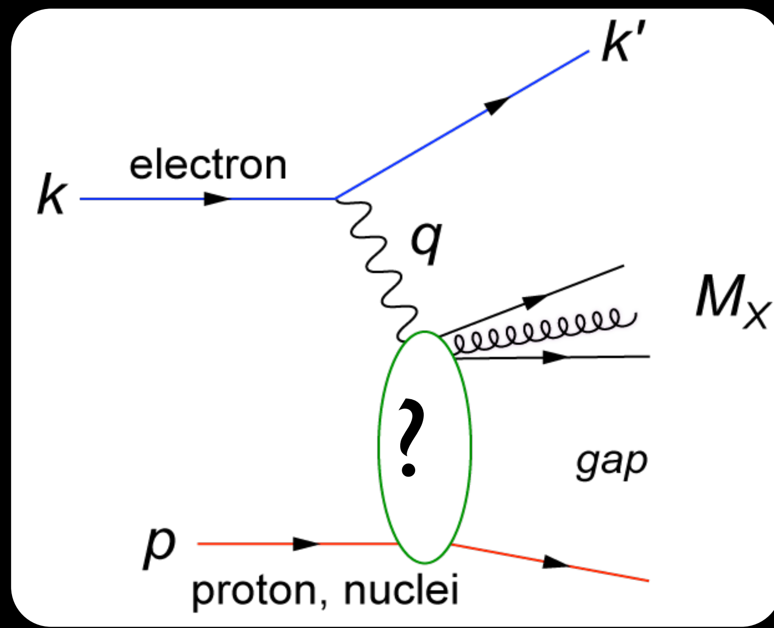

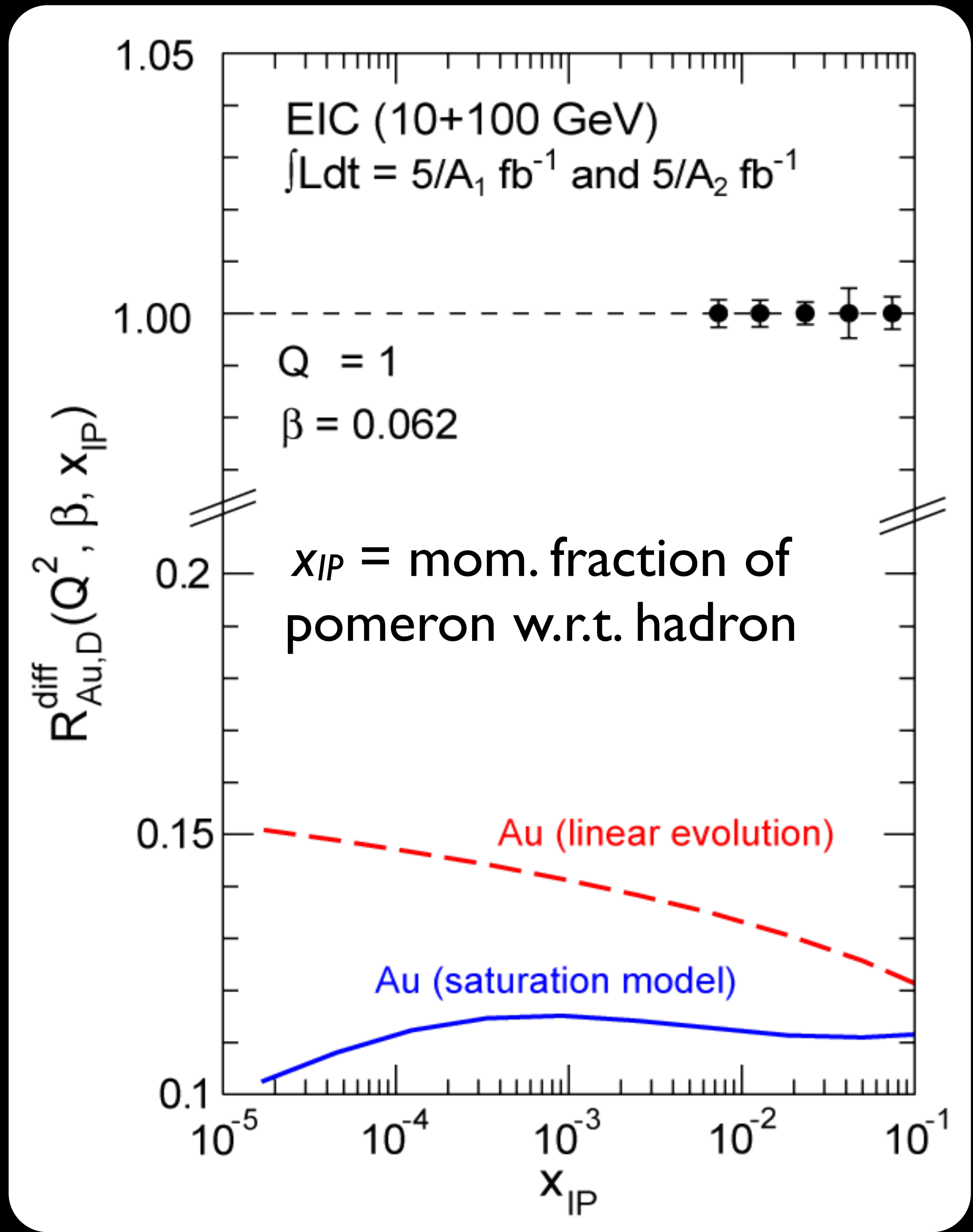

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- Look inside the "Pomeron"
  - Diffractive structure functions
- Distinguish between linear evolution and saturation models

How to measure coherent diffraction in e+A ?

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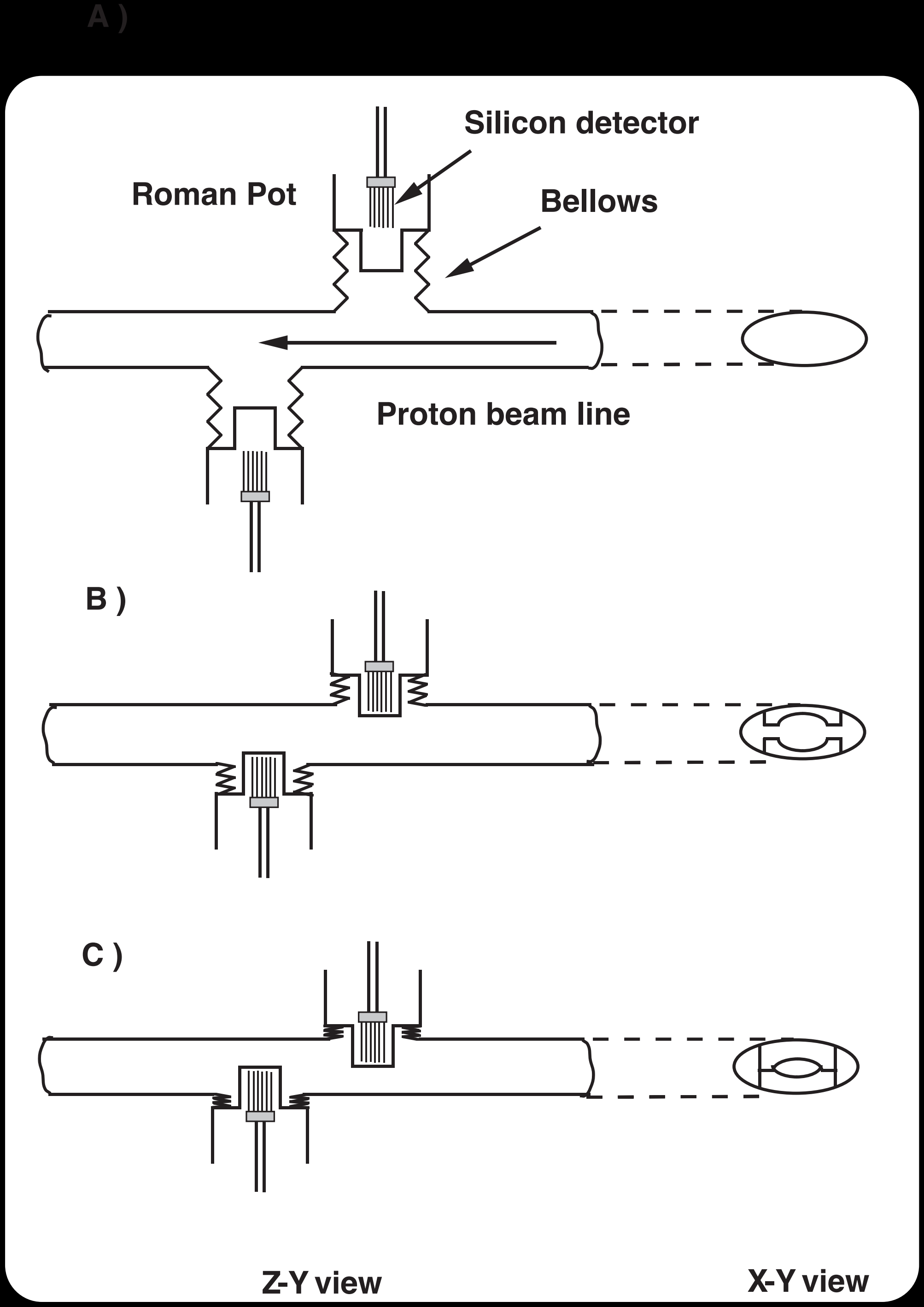

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- Coherent diffraction == low t
- Can measure the nucleus if it is separated from the beam in Si (Roman Pot) “beamline” detectors
  - $p_T^{min} \sim pA\theta_{min}$
    - For beam energies = 100 GeV/n and θ_{min} = 0.08 mrad:
- These are large momentum kicks, >> the binding energy (~ 8 MeV)

species (A)	p_T^{min} (GeV/c)
d (2)	0.02
Si (28)	0.22
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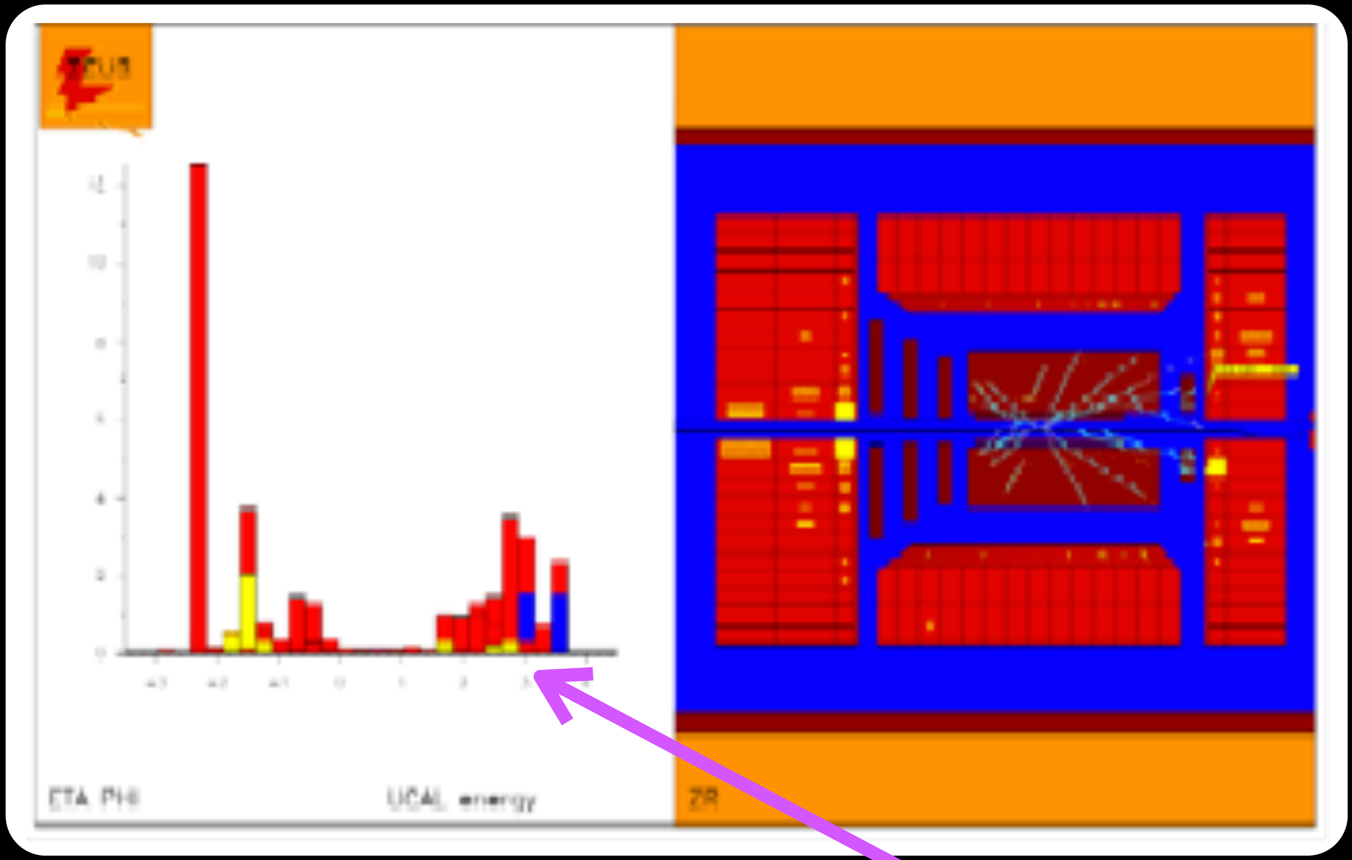
For large A, nucleus cannot be separated from beam without breaking up

How else to measure diffraction in e+A?

Method used at HERA:

Large Rapidity Gap Method:

In diffractive events, a large gap in rapidity occurs between outgoing p and final state particles



activity in the proton direction

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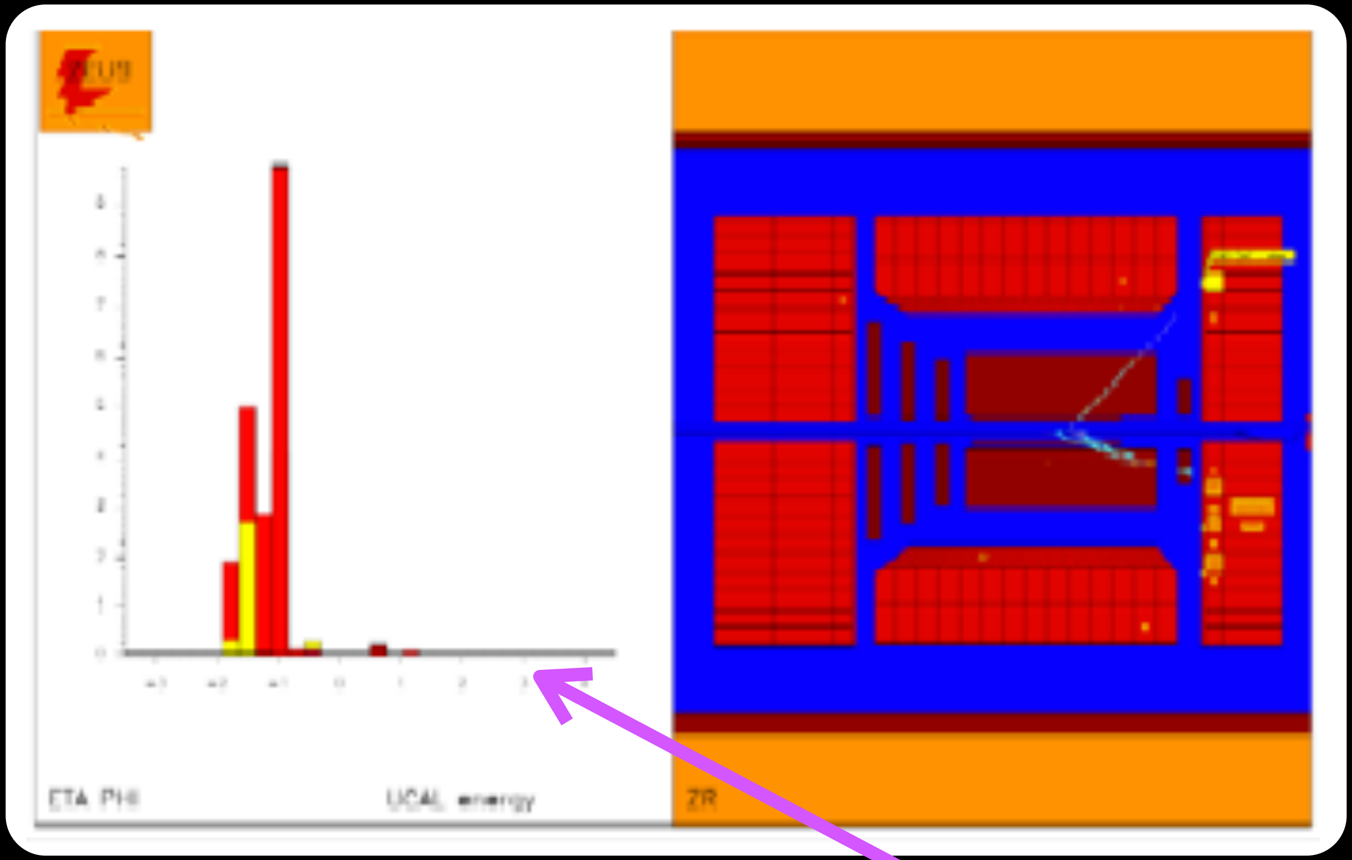

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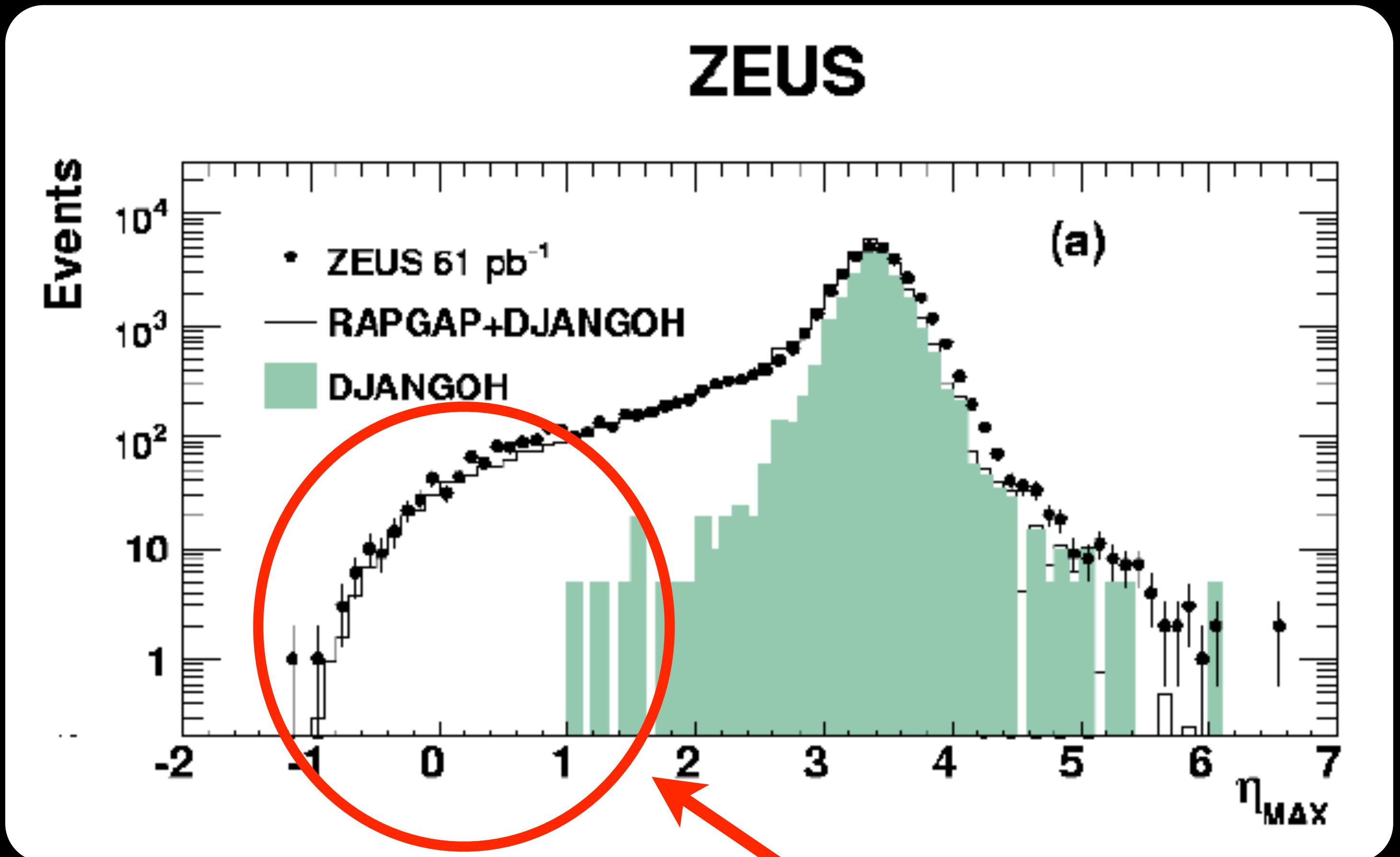

Diffractive events

- At HERA: $\Delta\eta \sim 7 \Rightarrow$ hadronization reduces this to ~2.5
- Pros
  - Lots of statistics
- Cons
  - Sensitive to hadronization models
  - No information on t

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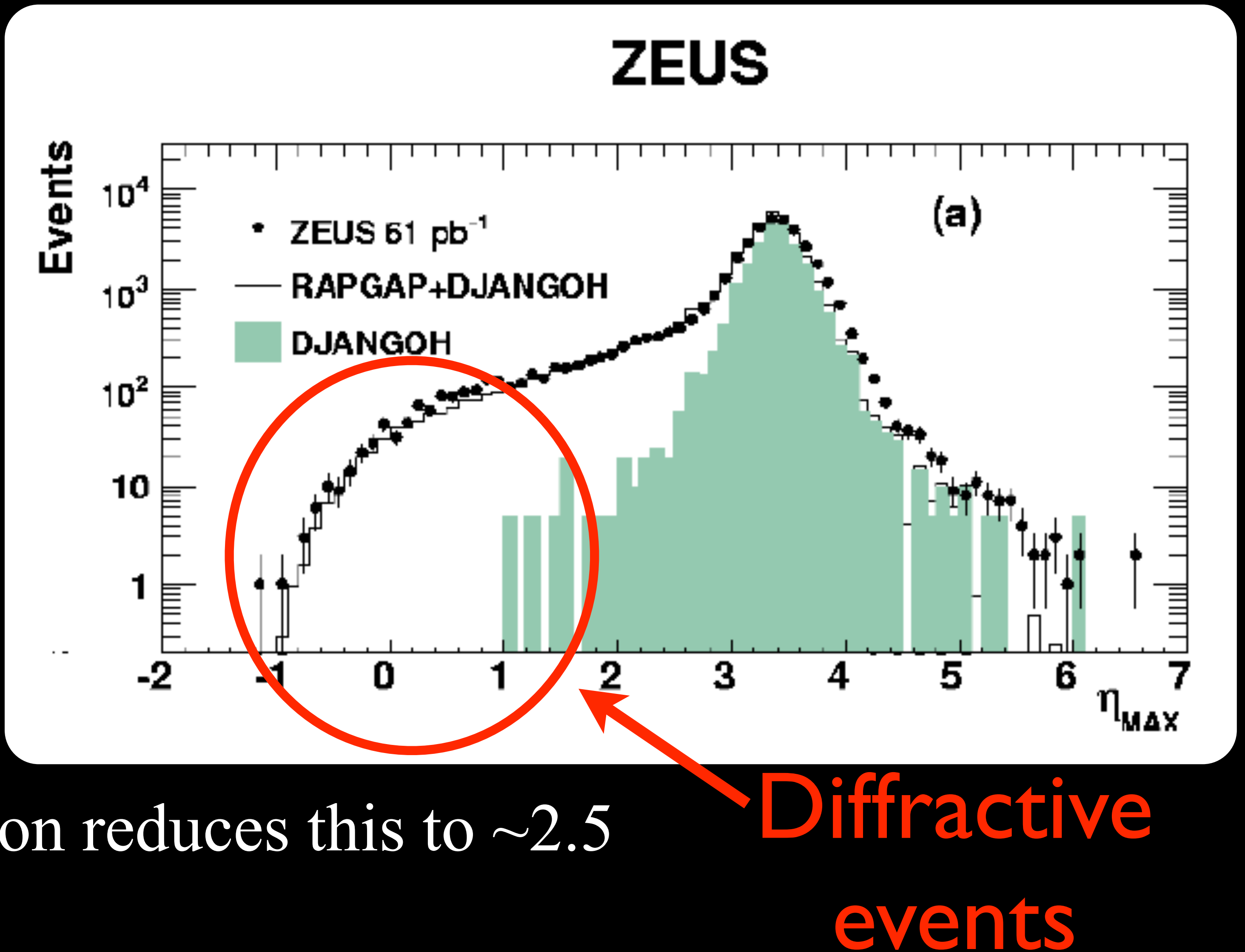

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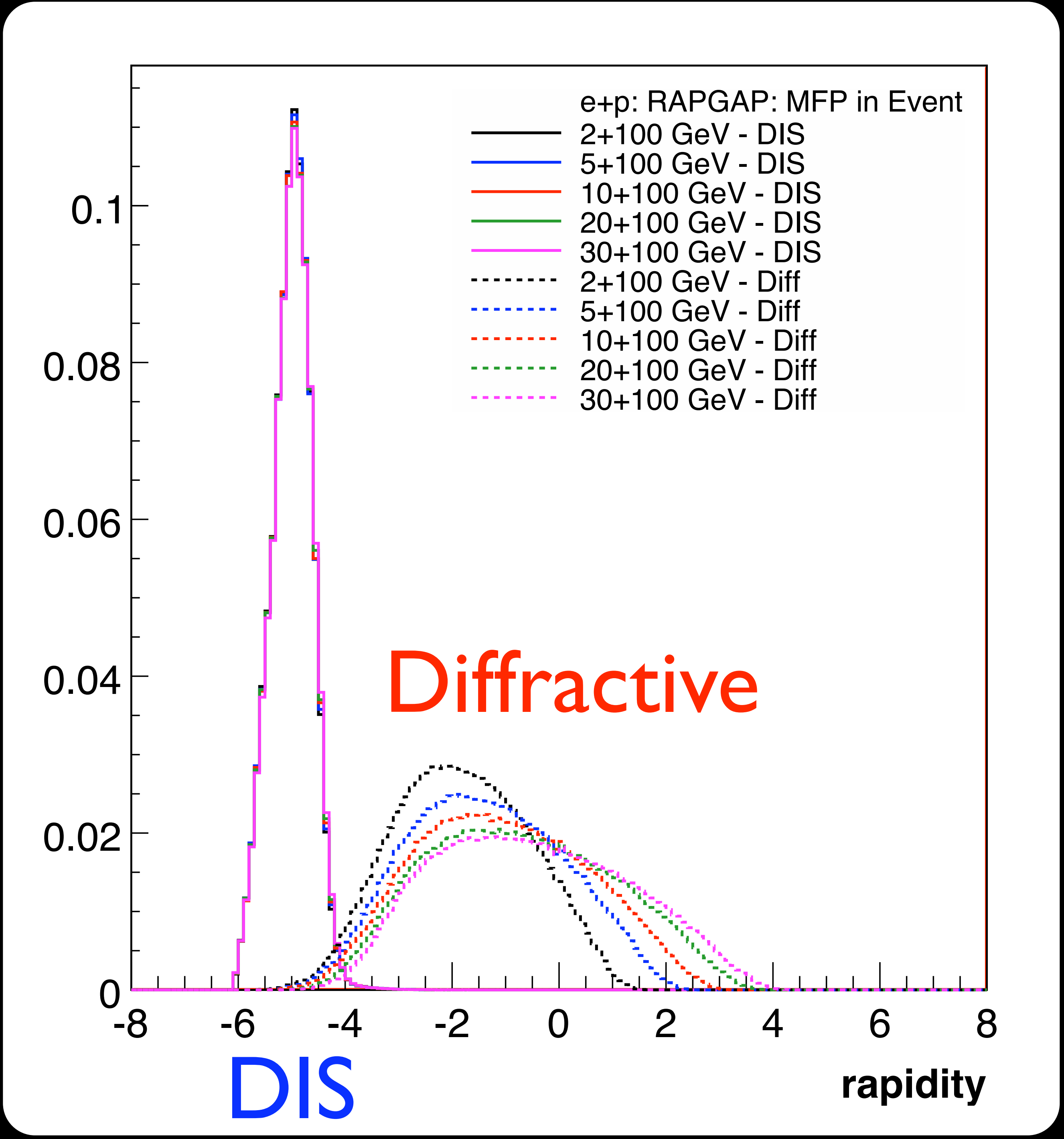
Can this method be used at an EIC?

Large rapidity gaps at an EIC

- Method:
  - Use RAPGAP in diffractive and DIS modes to simulate e+p collisions at EIC energies
  - Clear difference between DIS and Diffractive modes in "most forward particle in event" distributions
    - Little change in distributions with increasing energy



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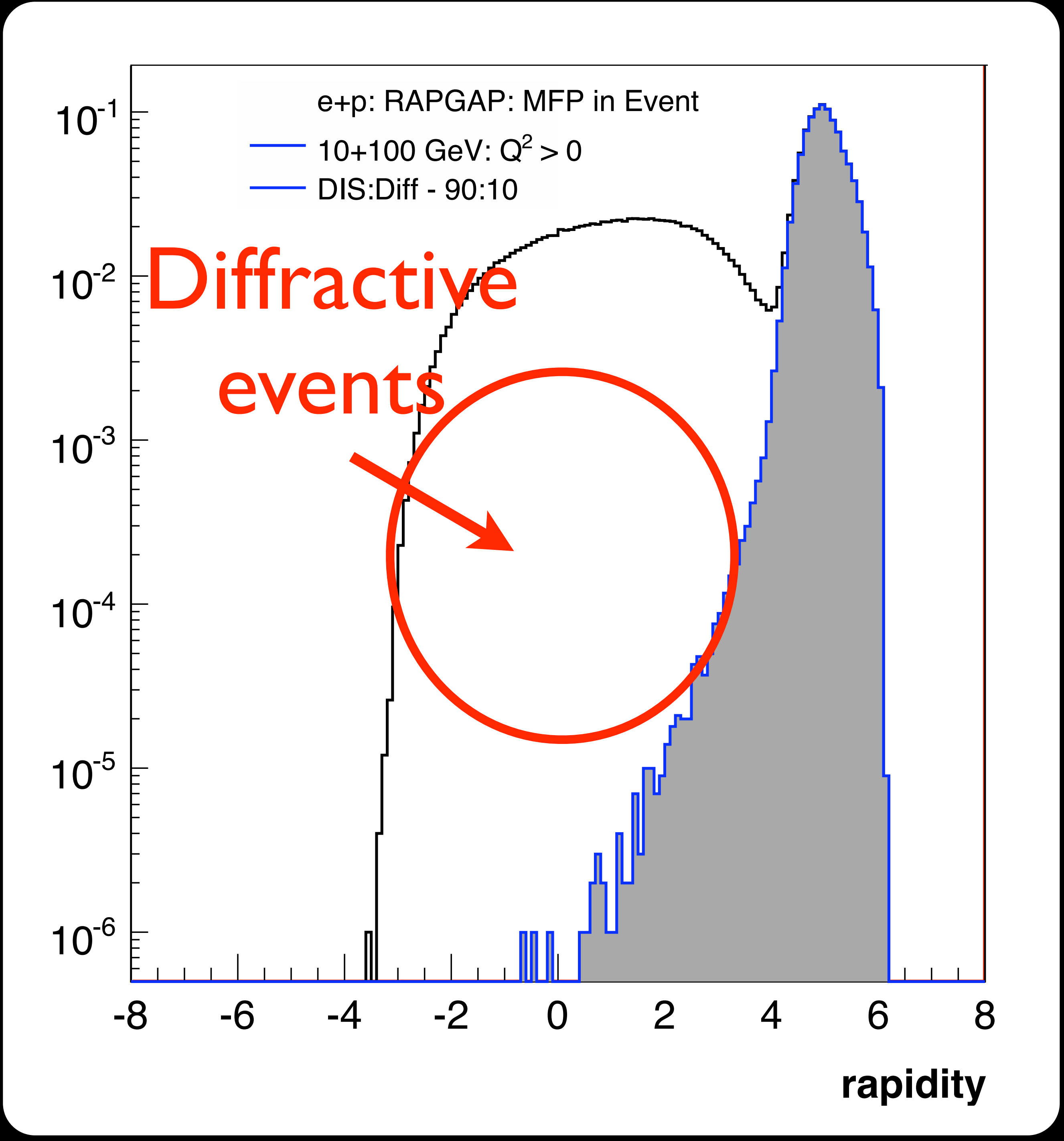
    - Little change in distributions with increasing energy
  - Can reproduce “ZEUS-like” plots



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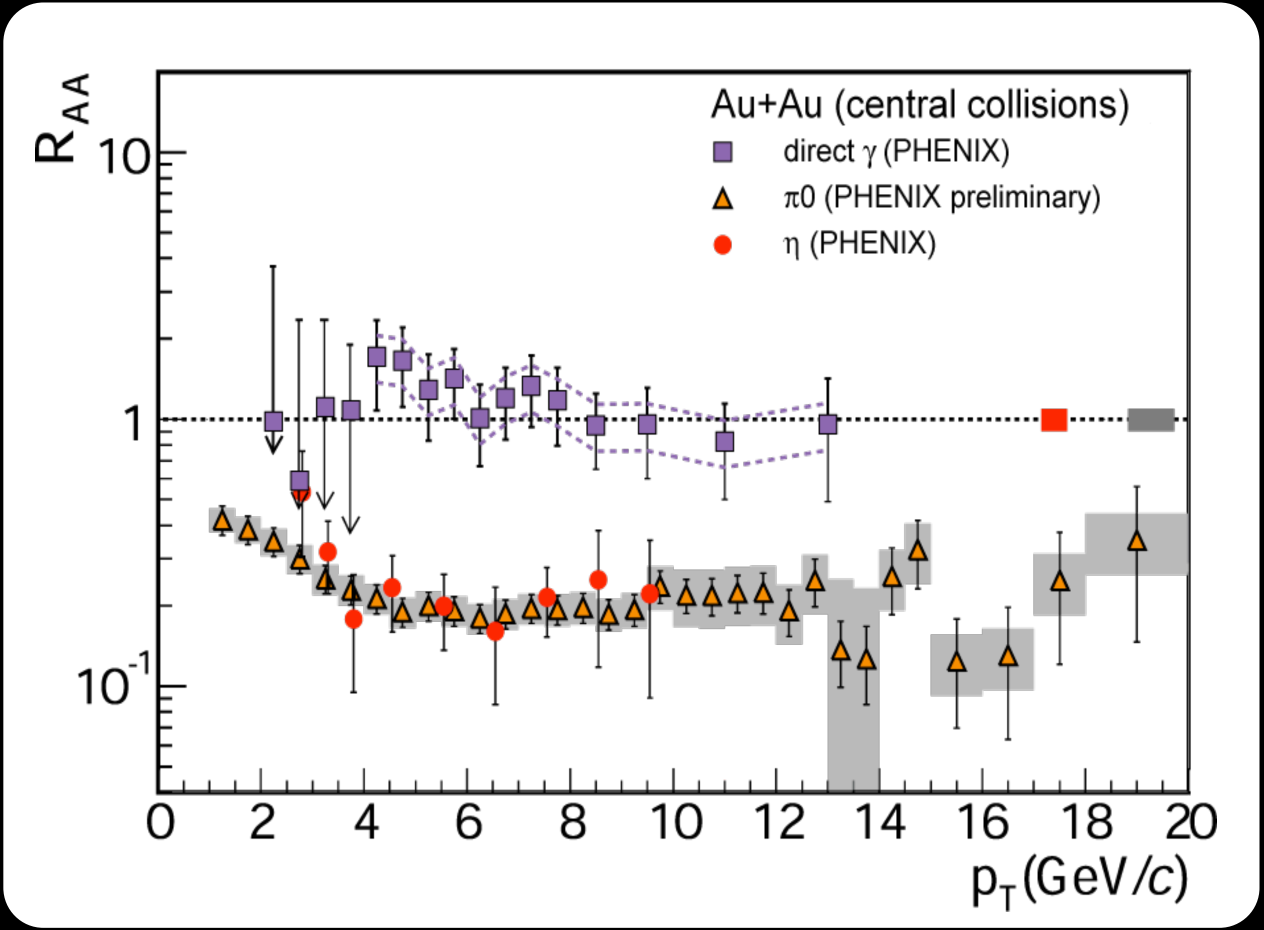
4 Key Measurements in e+A Physics

- Momentum distribution of gluons in nuclei?
  - Extract via scaling violation in F_2: $\partial F_2/\partial \mathit{ln} Q^2$
  - Direct Measurement: $F_L \sim xG(x,Q^2)$ - requires $\sqrt{s}$ scan
  - Inelastic vector meson production (e.g. J/Ψ, ρ)
  - Diffractive vector meson production ($\sim [xG(x,Q^2)]^2$)
  - 2+1 jet rates
- Space-time distribution of gluons in nuclei?
  - Exclusive final states (e.g. ρ, J,Ψ)
  - Deep Virtual Compton Scattering (DVCS) - $\sigma \sim A^{4/3}$
  - F_2, F_L for various impact parameters
- Role of colour-neutral (Pomeron) excitations?
  - Diffractive vector meson production ($\sim [xG(x,Q^2)]^2$)
  - Diffractive cross-section: $\sigma_{diff}/\sigma_{tot}$ (~ 10%: HERA e+p; 30%? EIC e+A?)
- Interaction of fast probes with gluonic medium?
  - Hadronization, Fragmentation
  - Energy loss (charm!!)

Interaction of fast probes with gluonic medium

RHIC Au+Au @ 200 GeV/n



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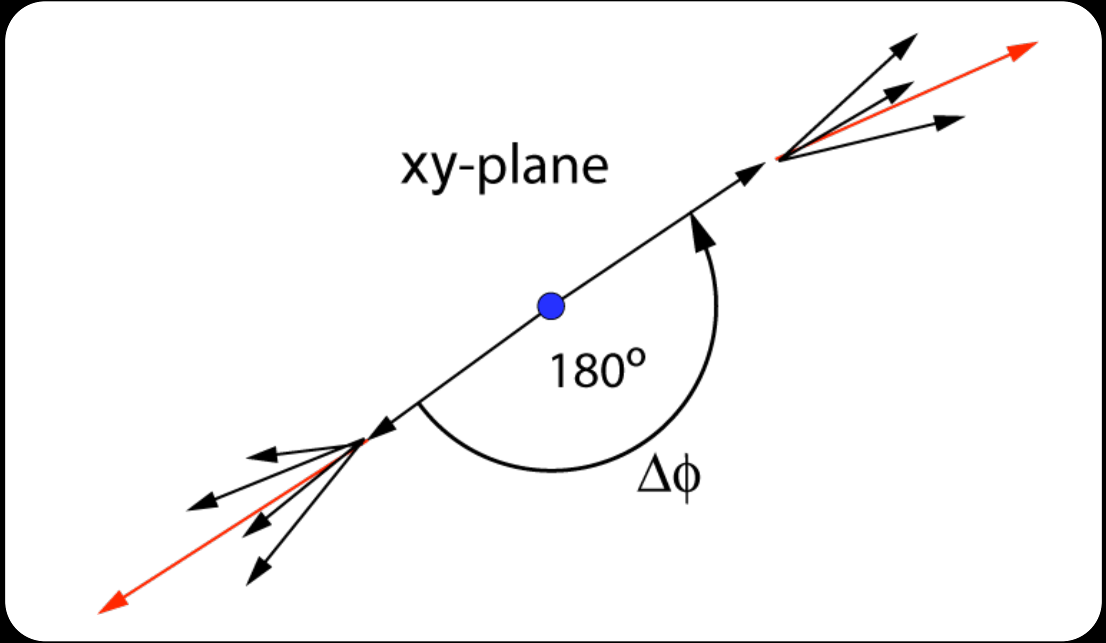

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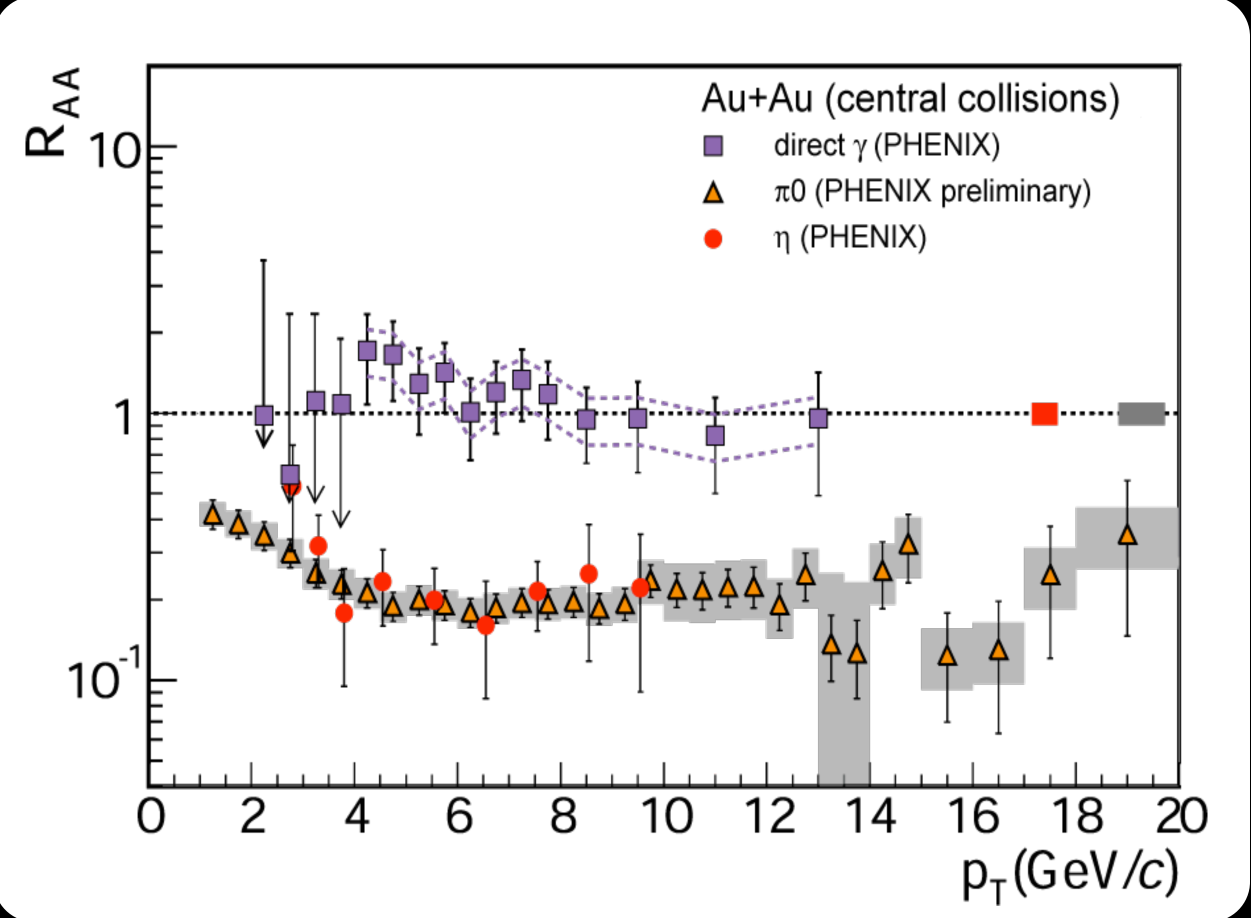

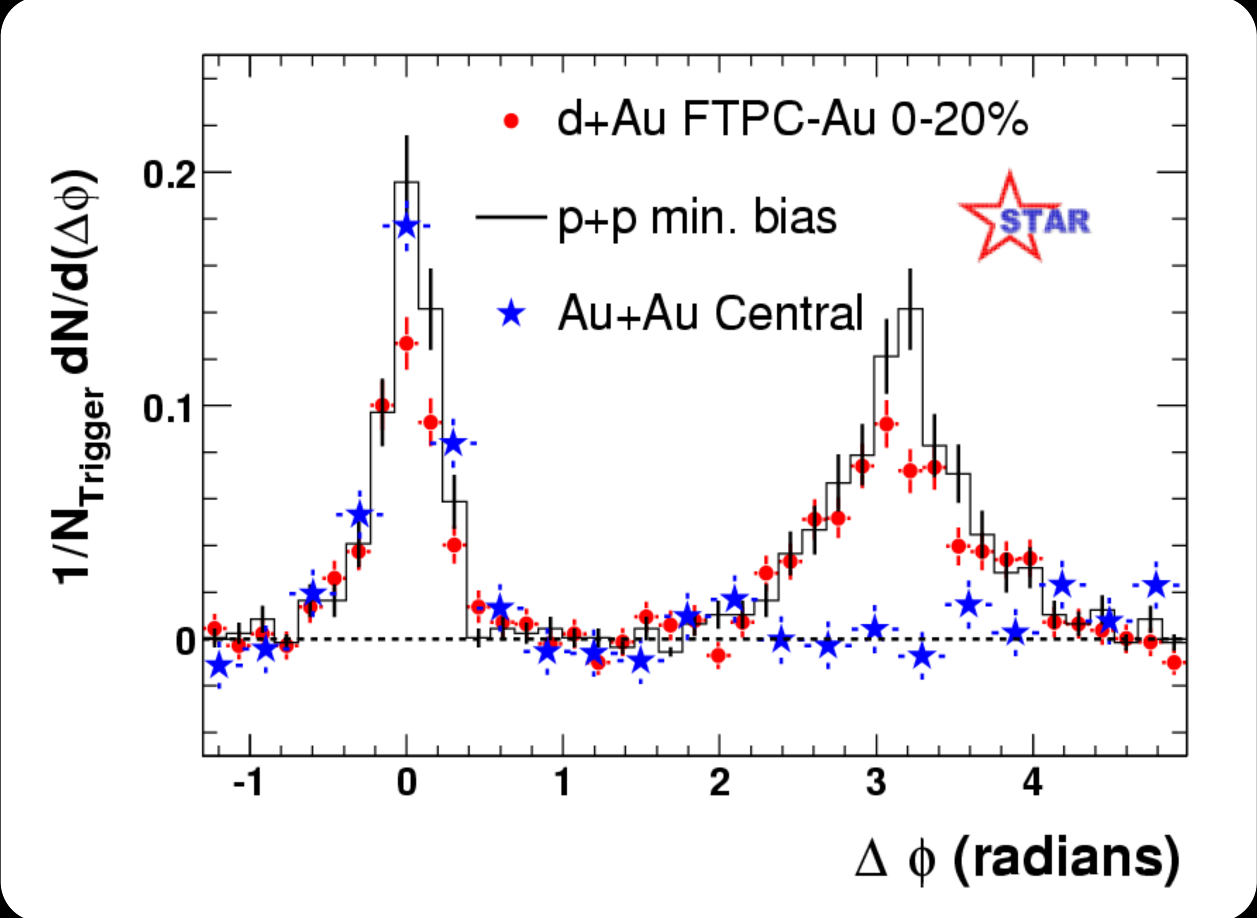

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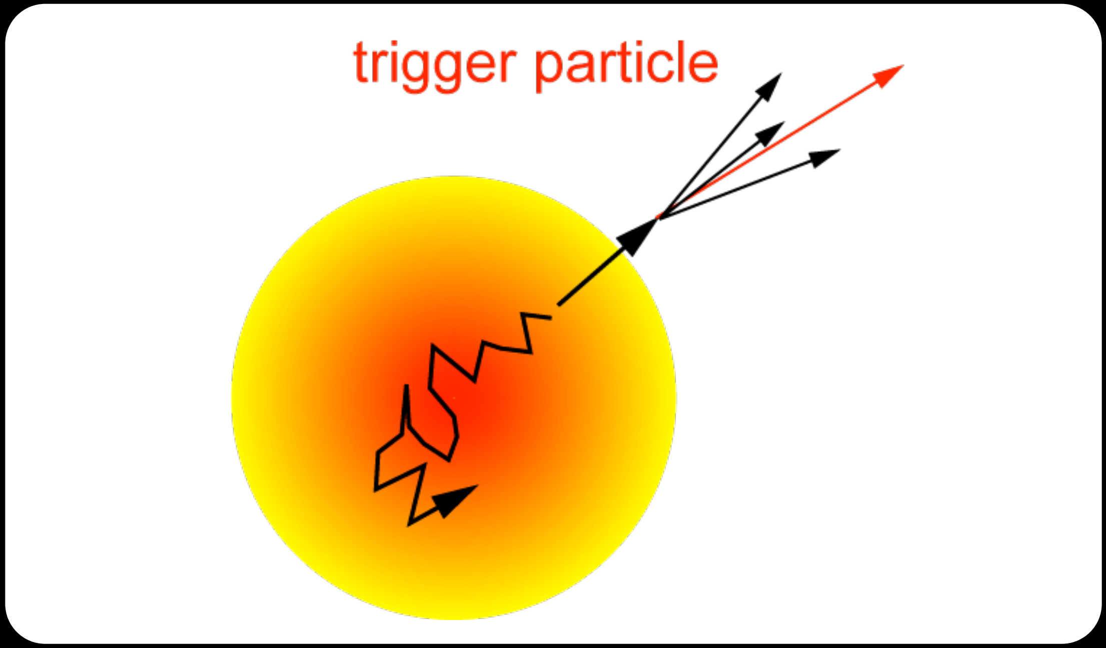

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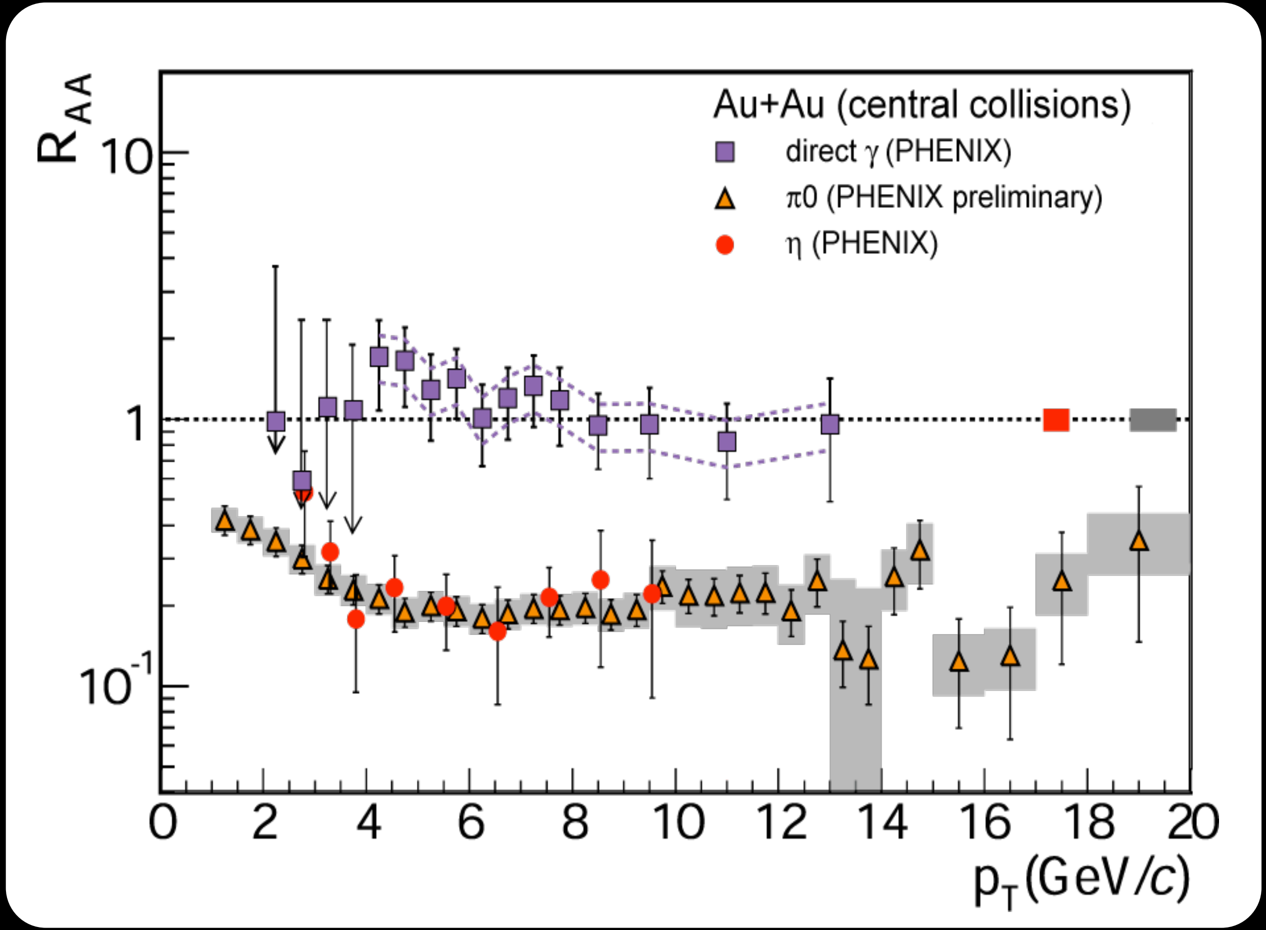
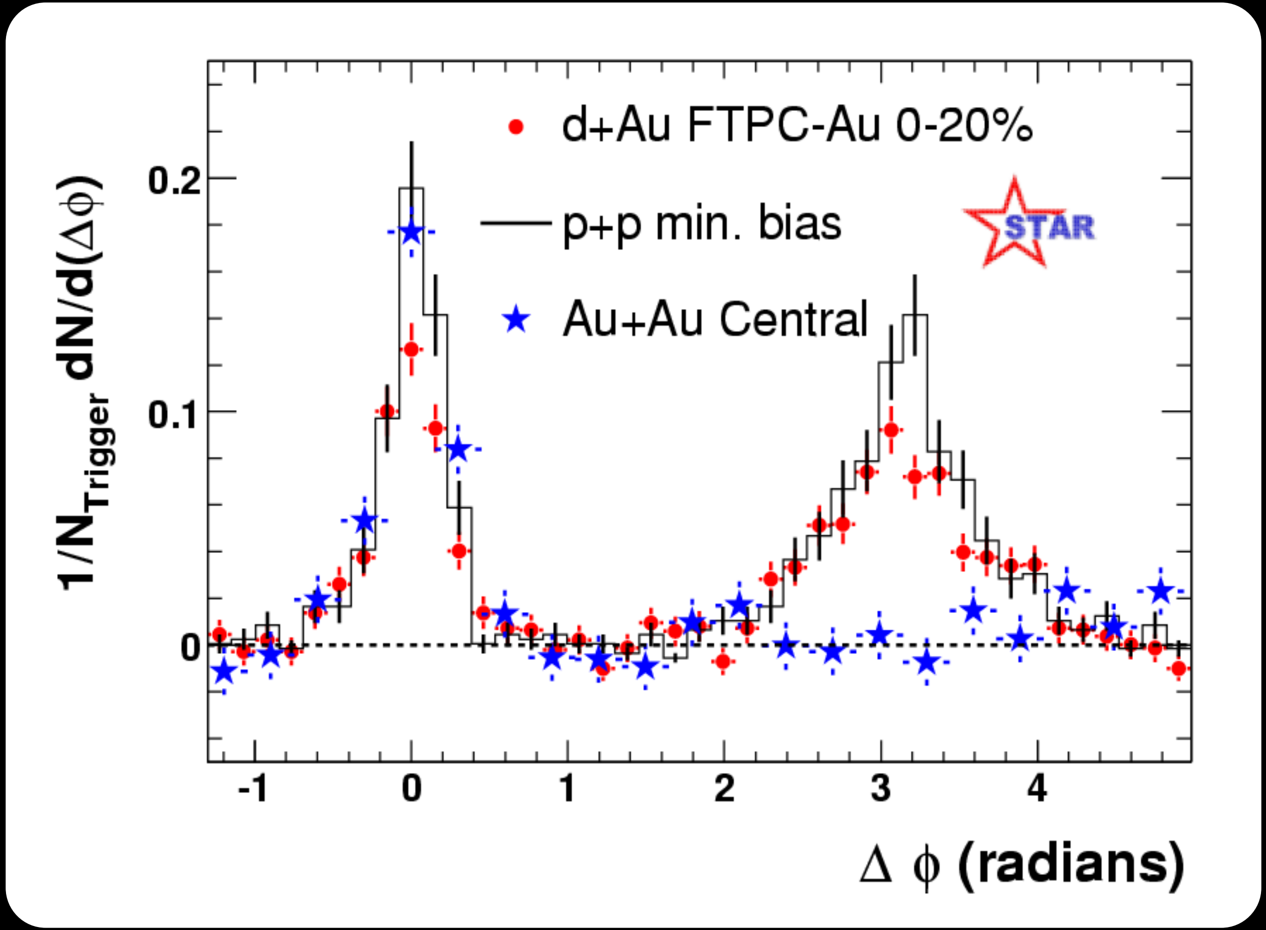
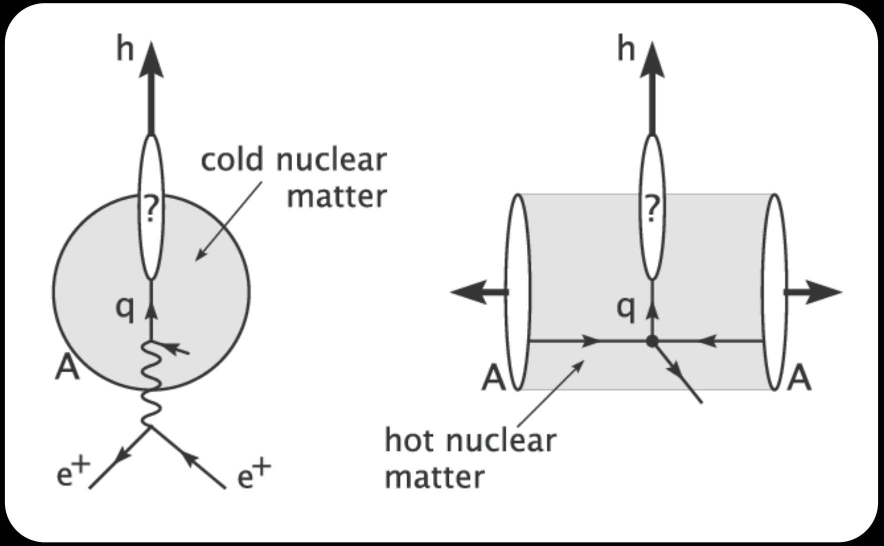




Interaction of fast probes with gluonic medium

- nDIS:
  - Clean measurement in 'cold' nuclear matter
  - Suppression of high-p_T hadrons analogous to, but weaker than at RHIC



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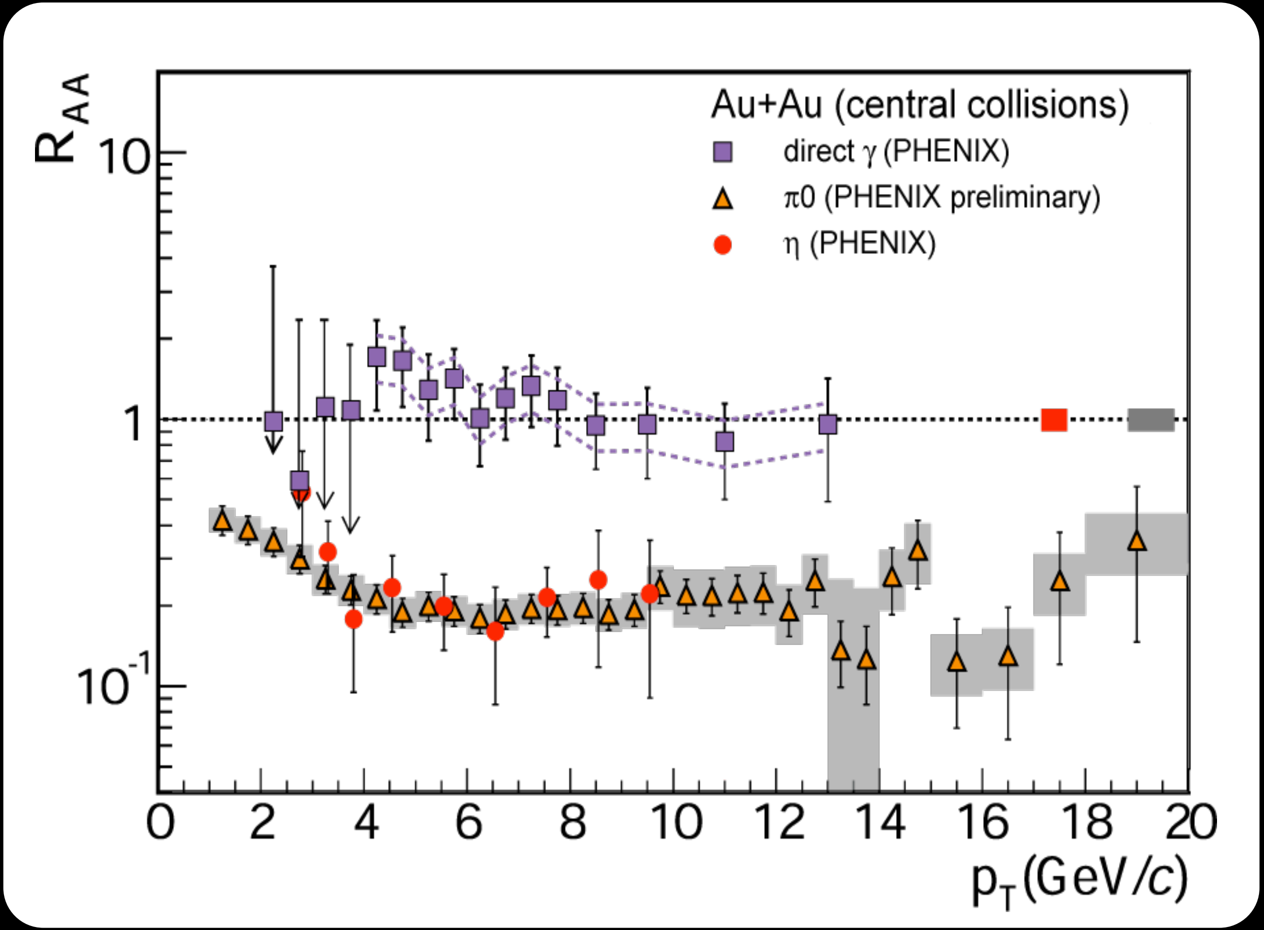

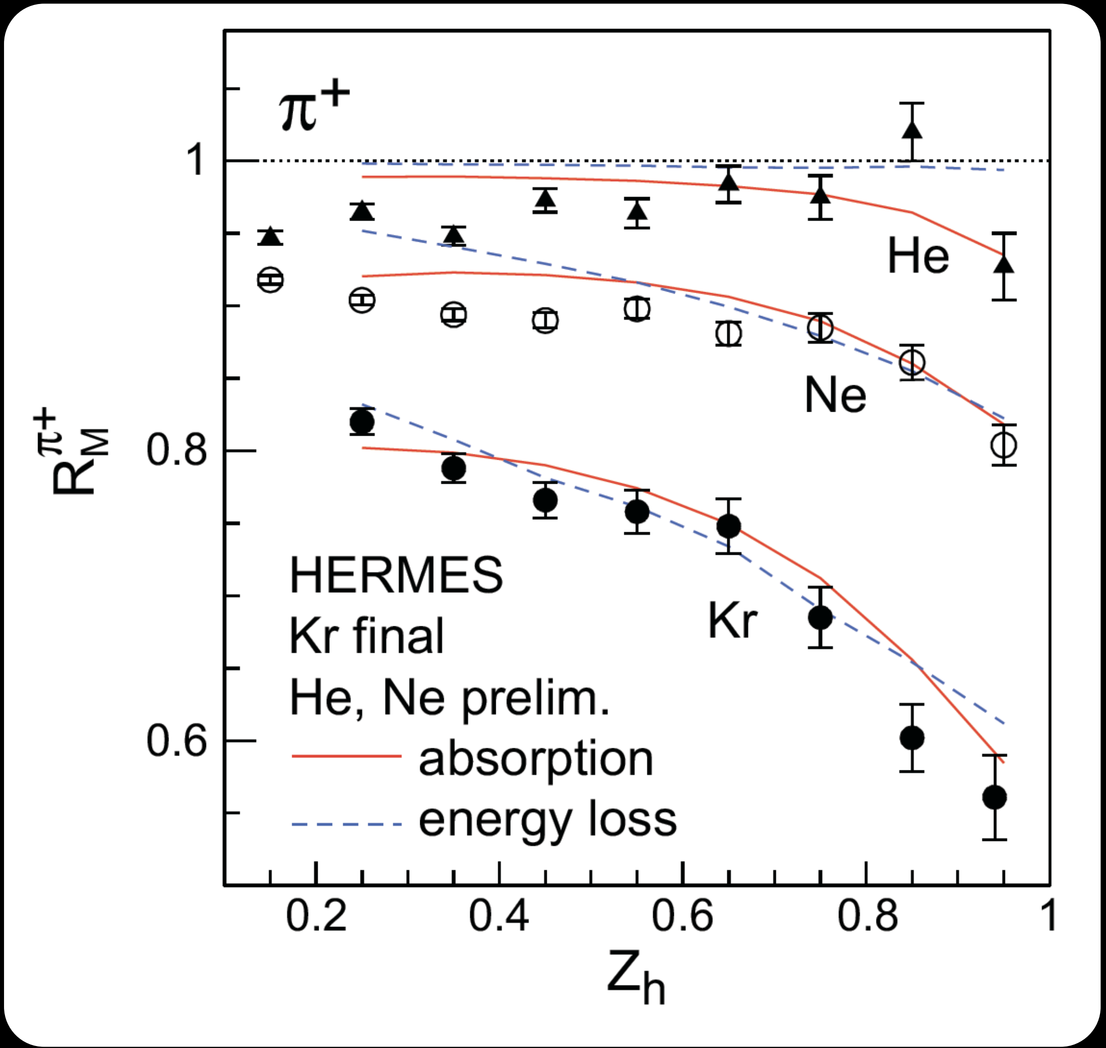

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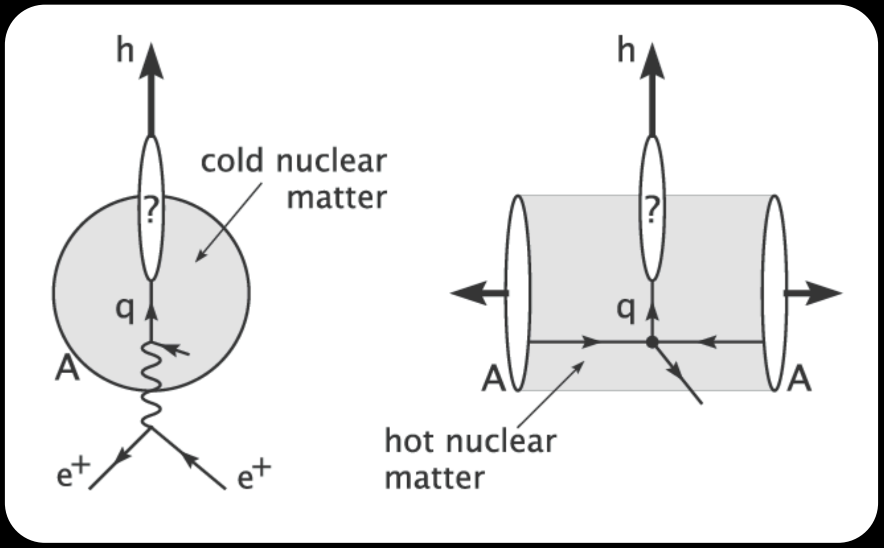
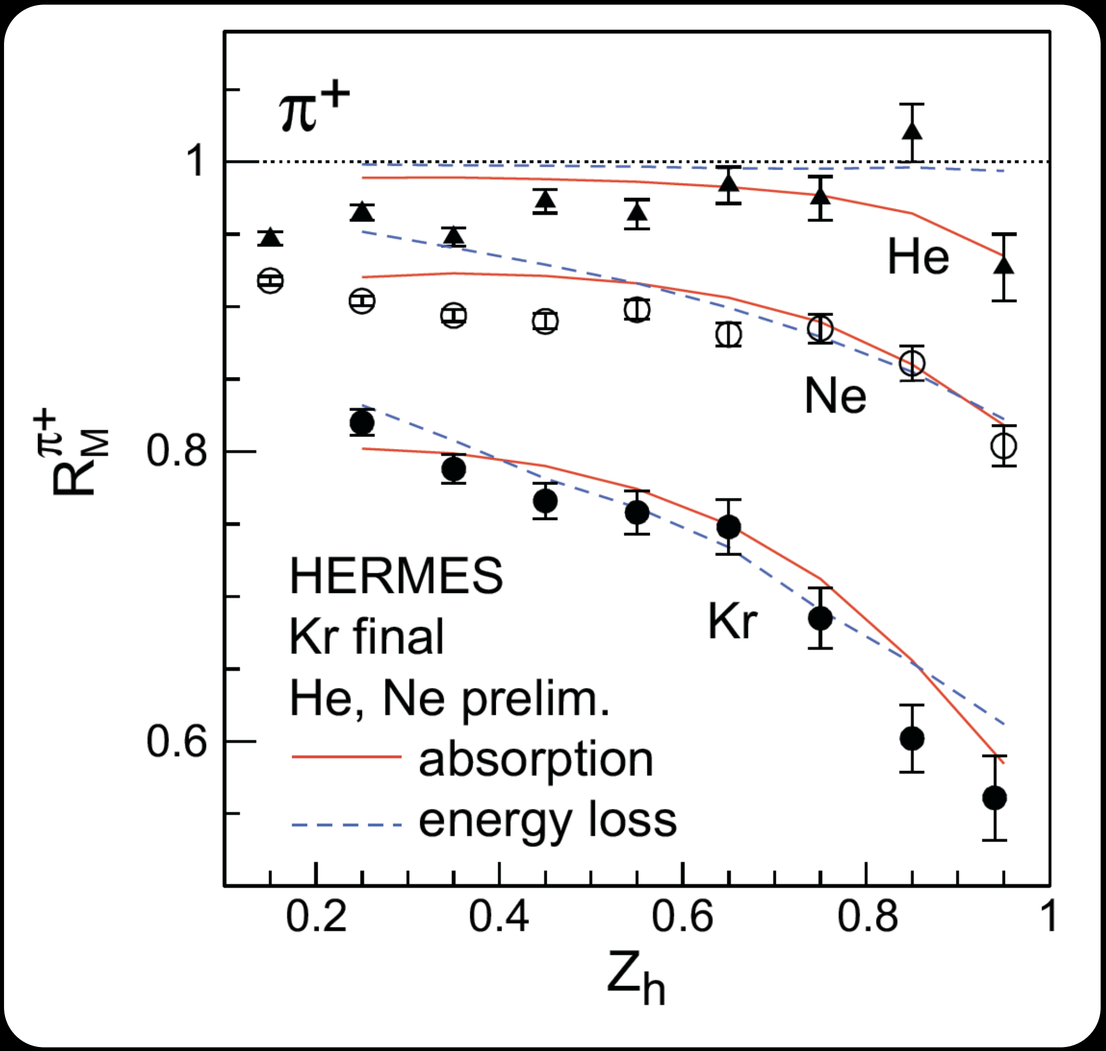


- Fundamental question:
  - When do partons get colour neutralized?

Parton energy loss vs. (pre)hadron absorption



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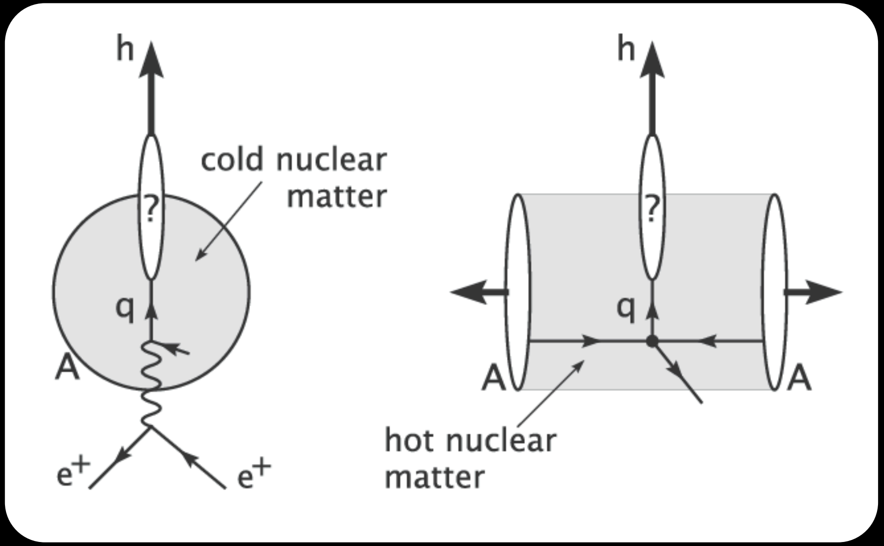

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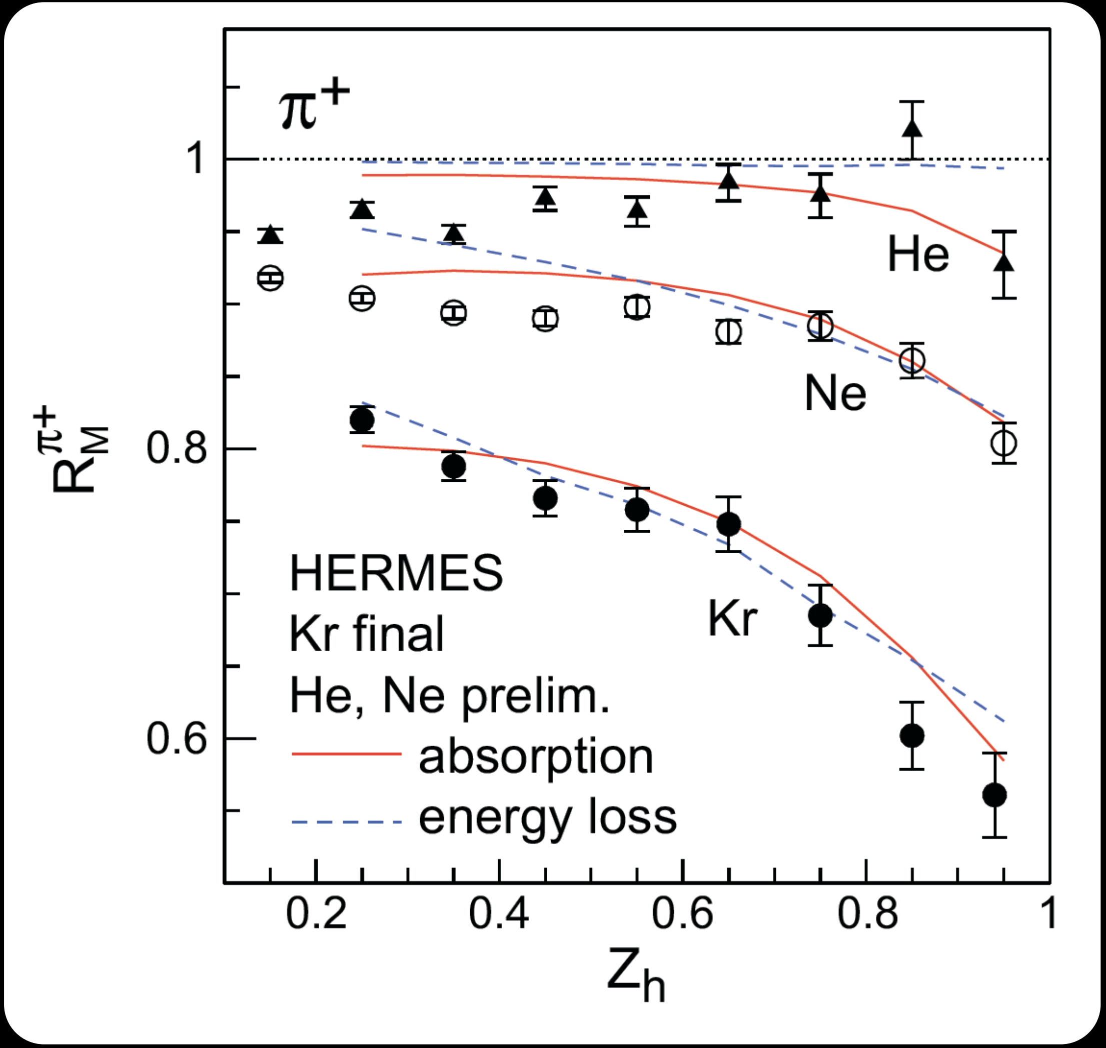
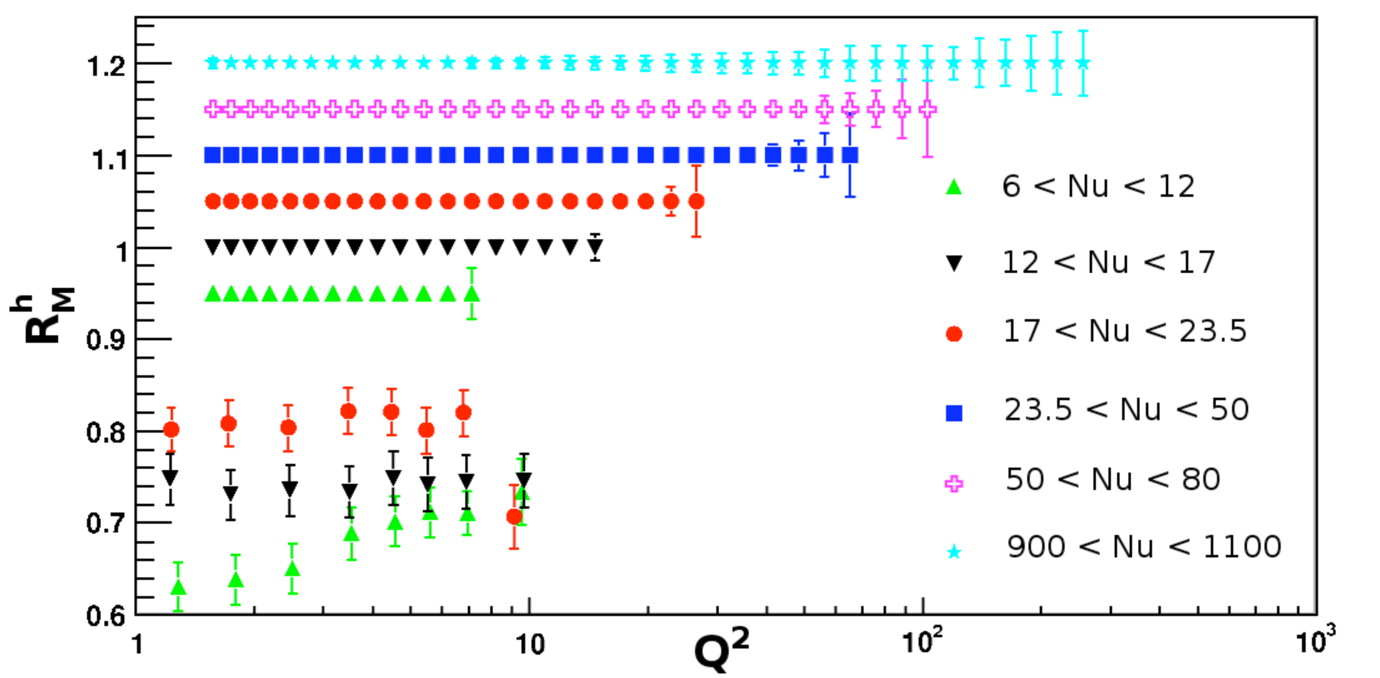
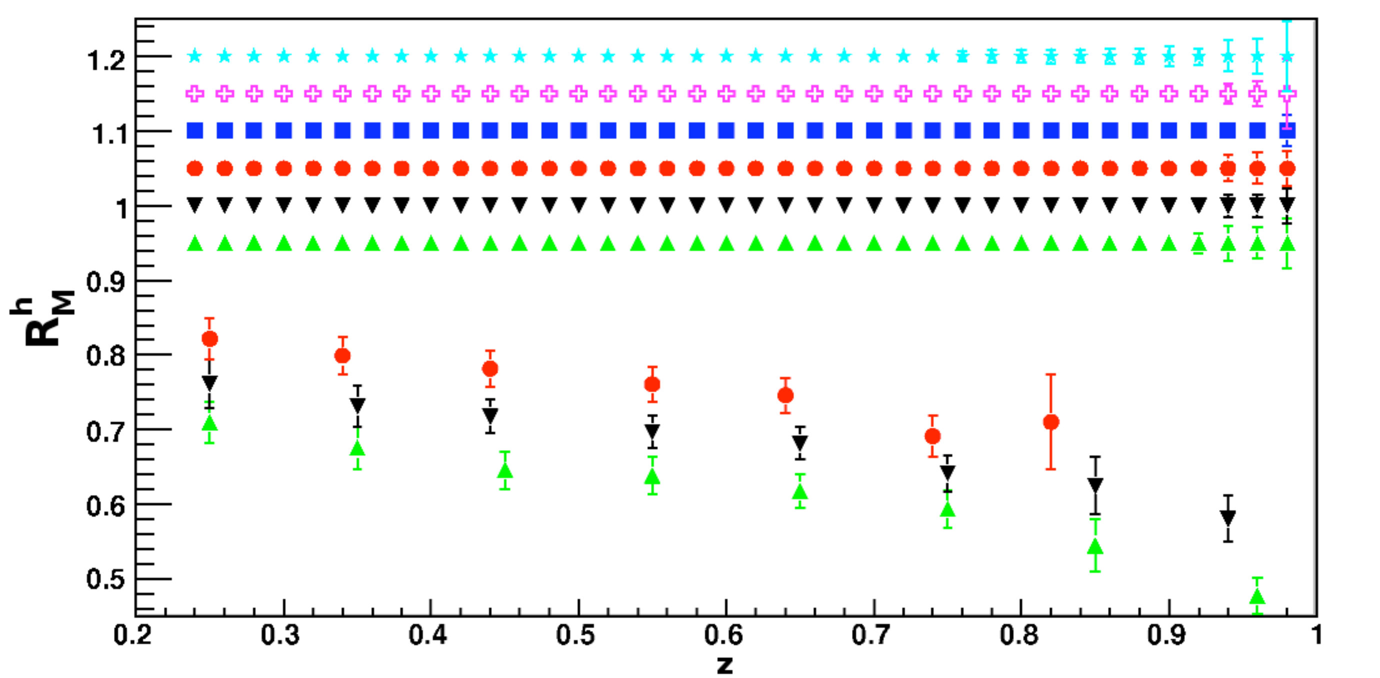
Parton energy loss vs. (pre)hadron absorption

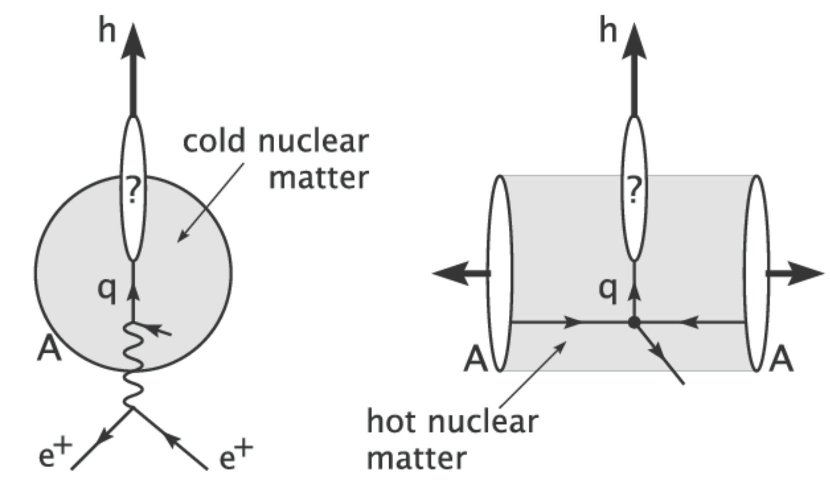
Energy transfer in lab rest frame:
EIC: $10 < \nu < 1600$ GeV
HERMES: 2-25 GeV
EIC: can measure *heavy flavour* energy loss

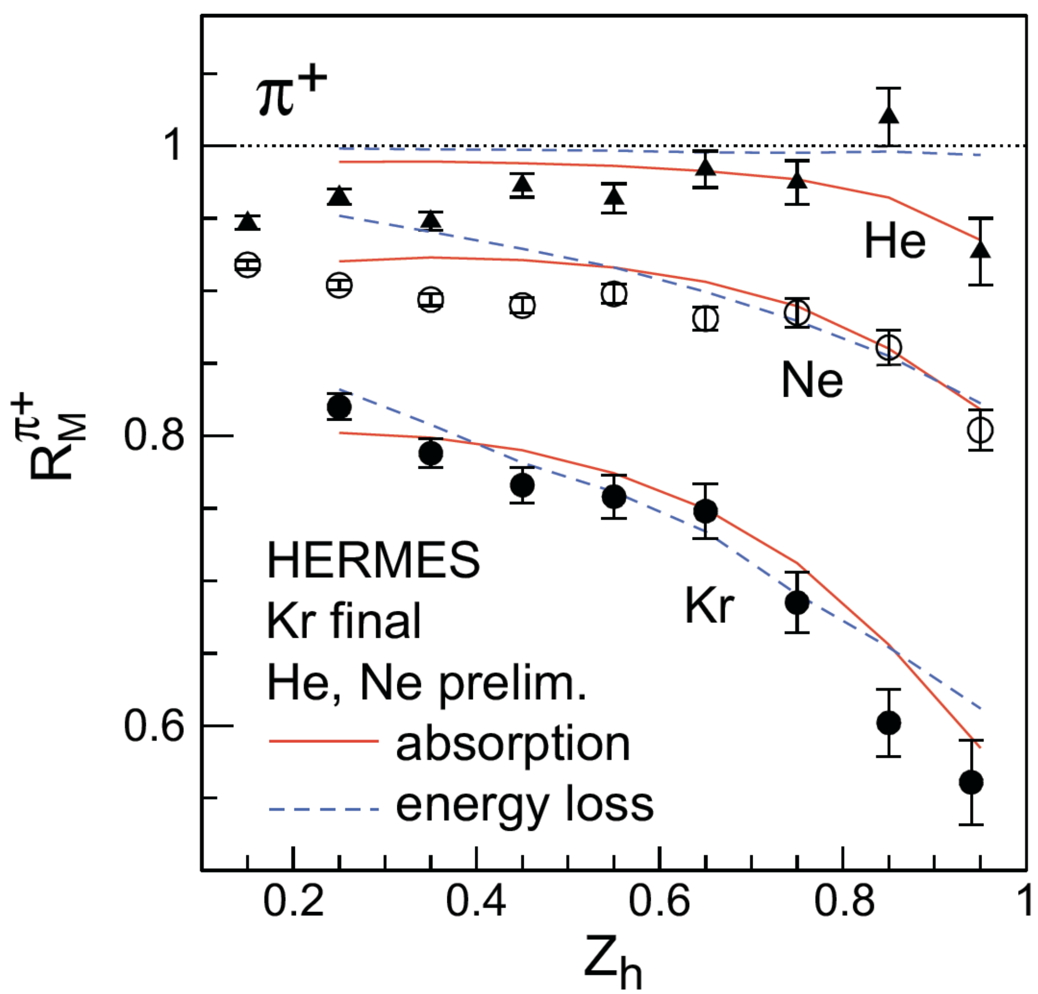


Interaction of fast probes with gluonic medium



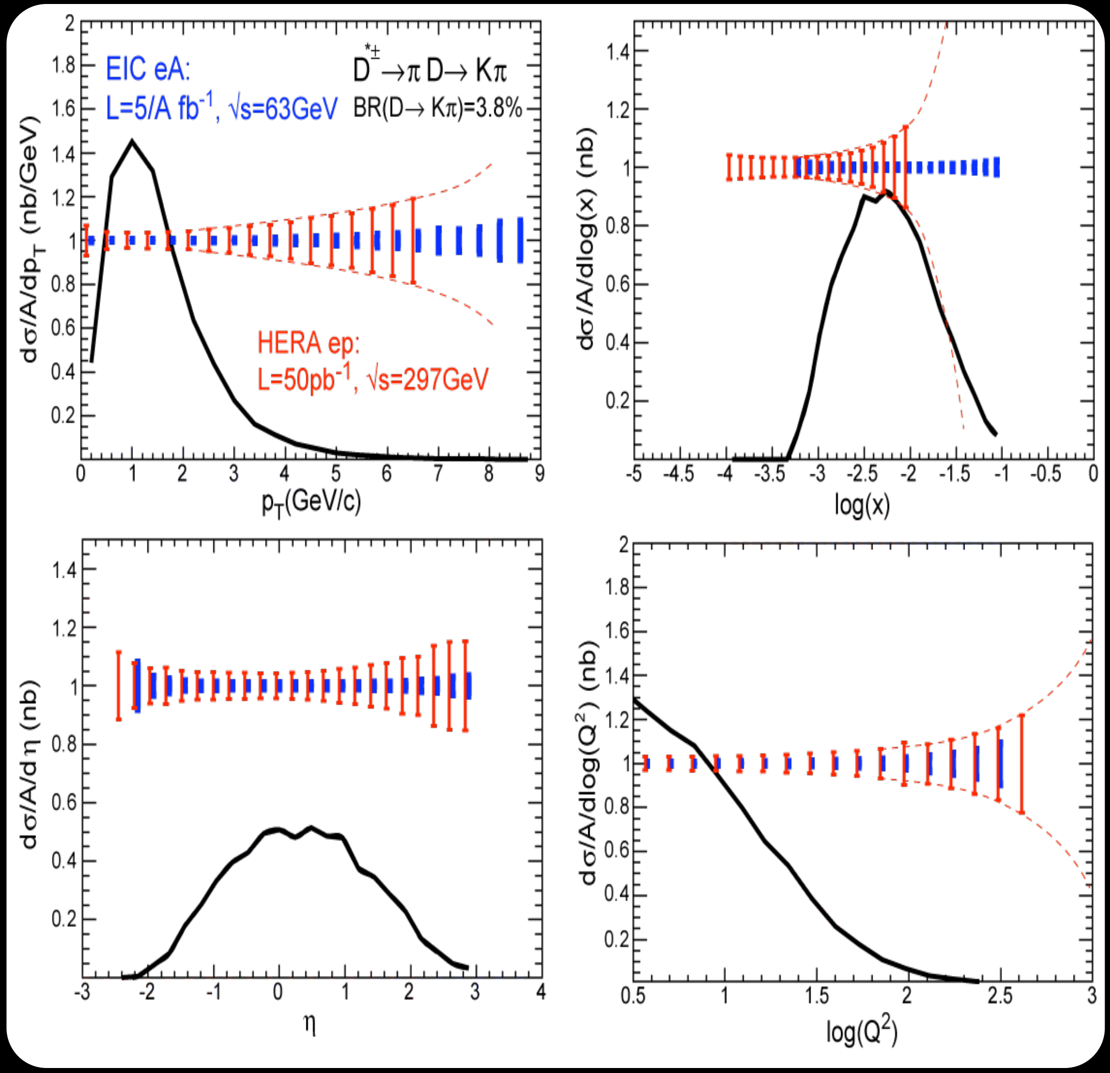


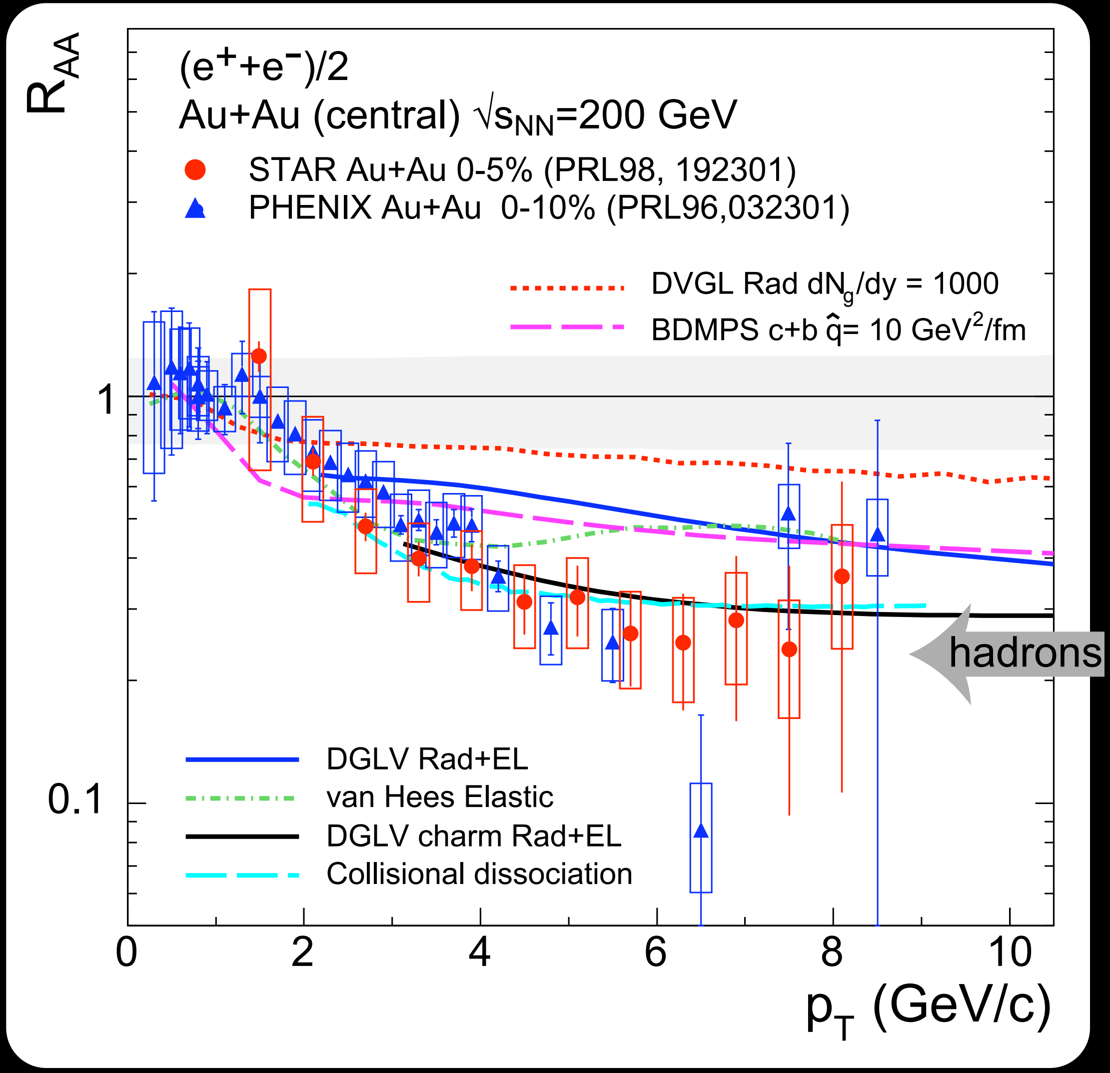




Charm measurements at an EIC

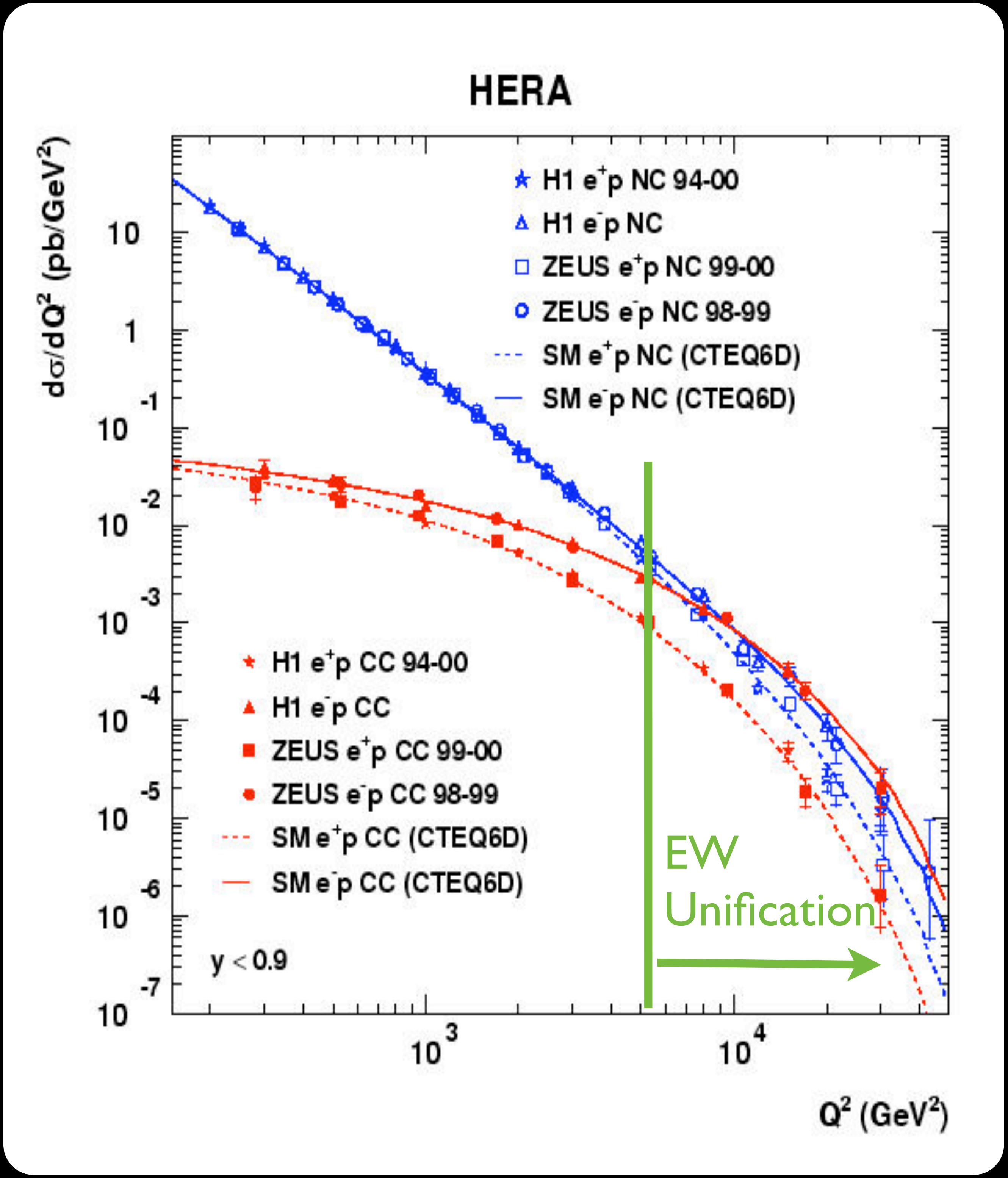
Charm also suppressed at RHIC - above and beyond model predictions





- EIC: allows multi-differential measurements of heavy flavour
- Covers and extends energy range of SLAC, EMC, HERA, and JLAB allowing for the study of wide range of formation lengths

EW Unification at HERA



- From DIS at HERA:
  - At small-medium Q^2, $\sigma(NC) >> \sigma(CC)$
  - For $Q^2 > M_Z^2$ and M_W^2, $\sigma(NC) \sim \sigma(CC)$
    - EW Unification
- Already a textbook figure ...

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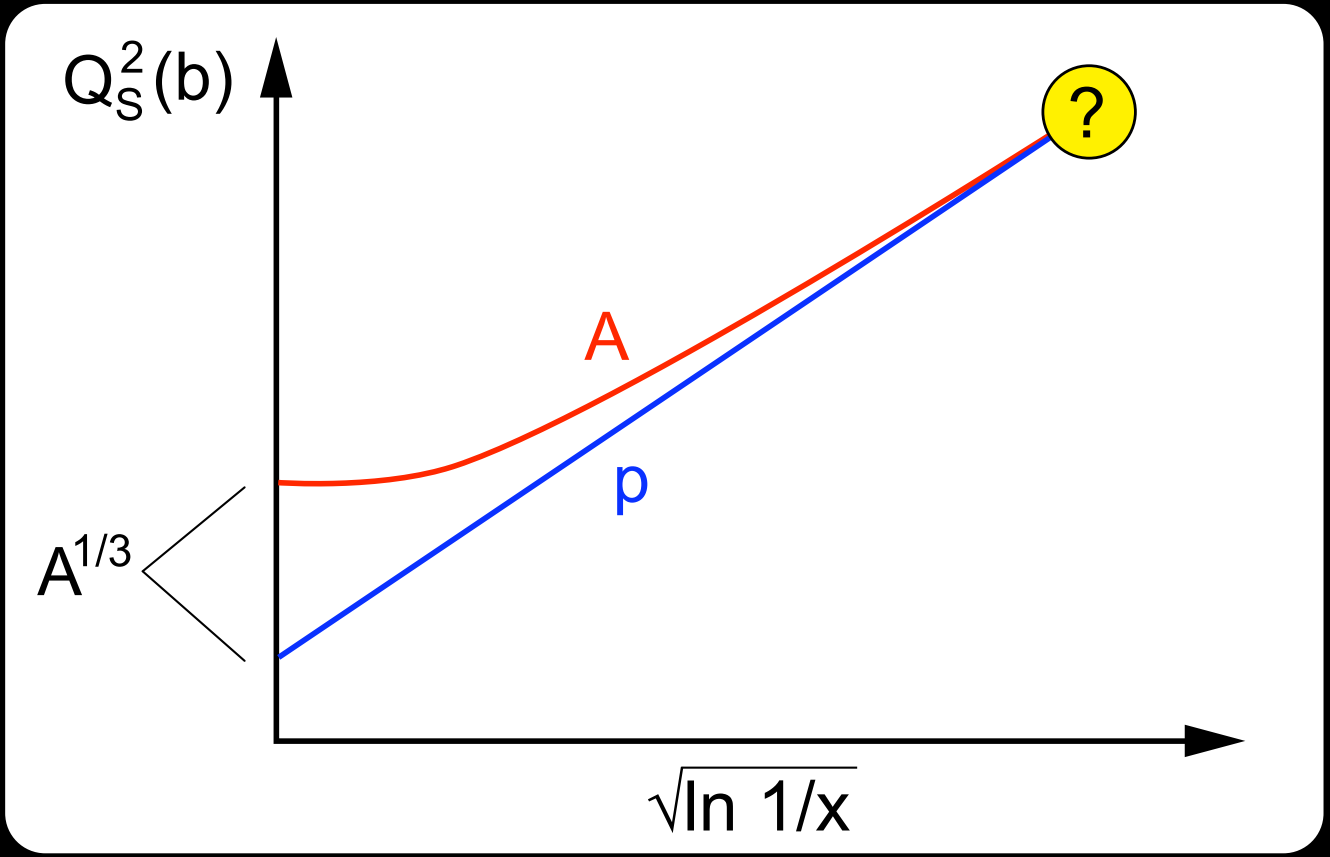
Matter at low-x: A truly universal regime?

What about on the parton scale?



A.H. Mueller, hep-ph/0301109

- Small-x running-coupling BFKL QCD evolution predicts:
  - Q_S approaches universal behaviour for all hadrons and nuclei
  - No dependence on A!!
  - Not only functional form $f(Q_S)$ universal, but even Q_S itself becomes universal

- Nuclei and all hadrons have a component of their wave function with the *same* behaviour
- This is a conjecture! Needs to be tested

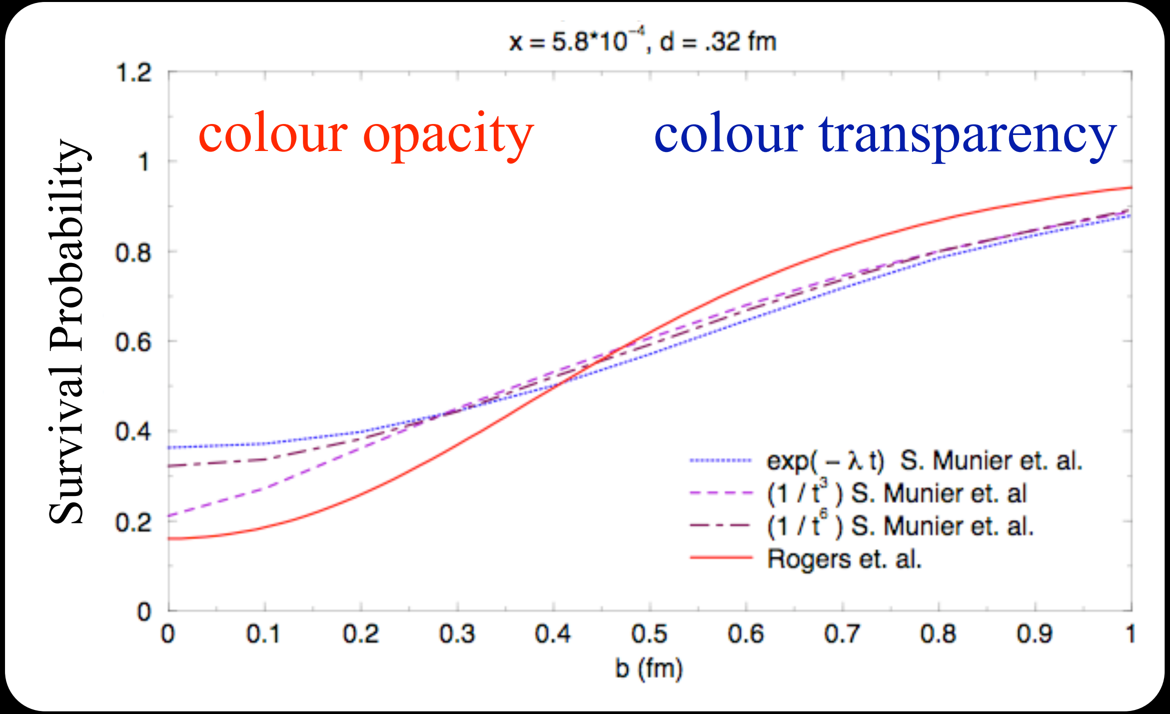
Summary

- The study of e+A collisions at an EIC allow us to explore the physics of Strong Colour Fields and the nature of non-linear QCD and saturation. We can address the questions:
  - What are the momentum distributions of gluons in nuclei?
    - Measure F_2, F_L distributions
  - What are the space-time distributions of gluons in nuclei?
    - Vector meson survival probability
  - What is the role of colour-neutral excitations (Pomerons)?
    - Diffractive physics
  - How do fast partons interact with cold nuclear matter?
    - Measure energy loss of fast-moving hadrons
  - Is the saturation scale also universal at low-x?

BACKUP

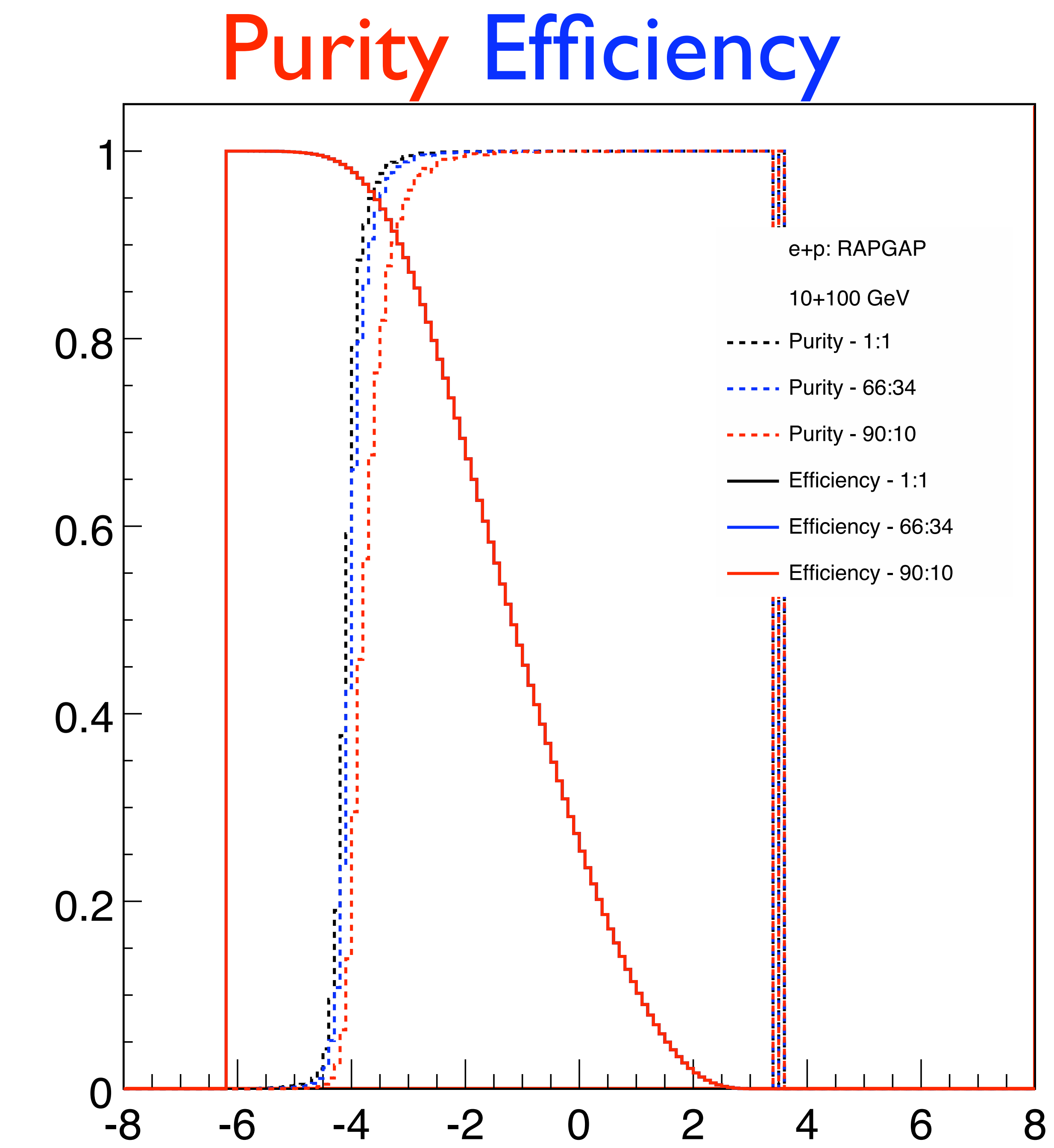
Gluon space-time distributions

- In the "colour dipole" picture:
  - virtual photon fluctuates into a qq-bar dipole and scatters coherently on the nucleus
  - calculate the survival probability of qq-bar pair to propogate through the target without interacting
    - Calculate by measuring the vector meson cross-section
  - pQCD ⇒ survival ~ 1
  - dipole models ⇒ ~ x5 smaller
  - HERA data limited on this
  - b profile of nuclei more uniform



Large rapidity gaps at an EIC

- Efficiency vs Purity:
  - Efficiency = fraction of diffractive events out of all diffractive events in sample
  - Purity = fraction of diffractive events out of all events in sample
  - Possible to place a cut to have both high efficiency and high purity
    - However, reduce the acceptance by 1 or 2 units of rapidity and these values drop significantly
  - Need hermetic detector coverage!!



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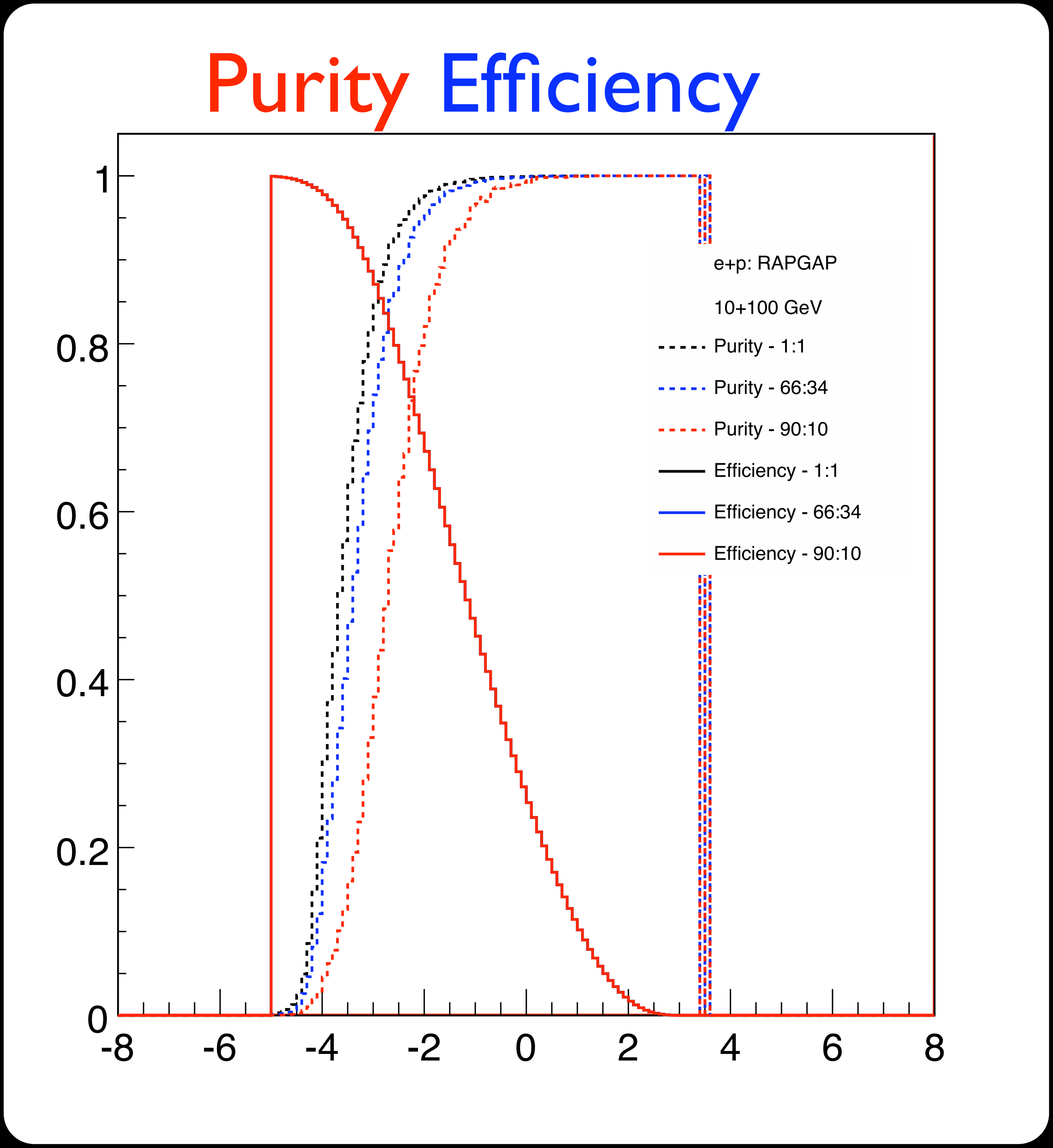

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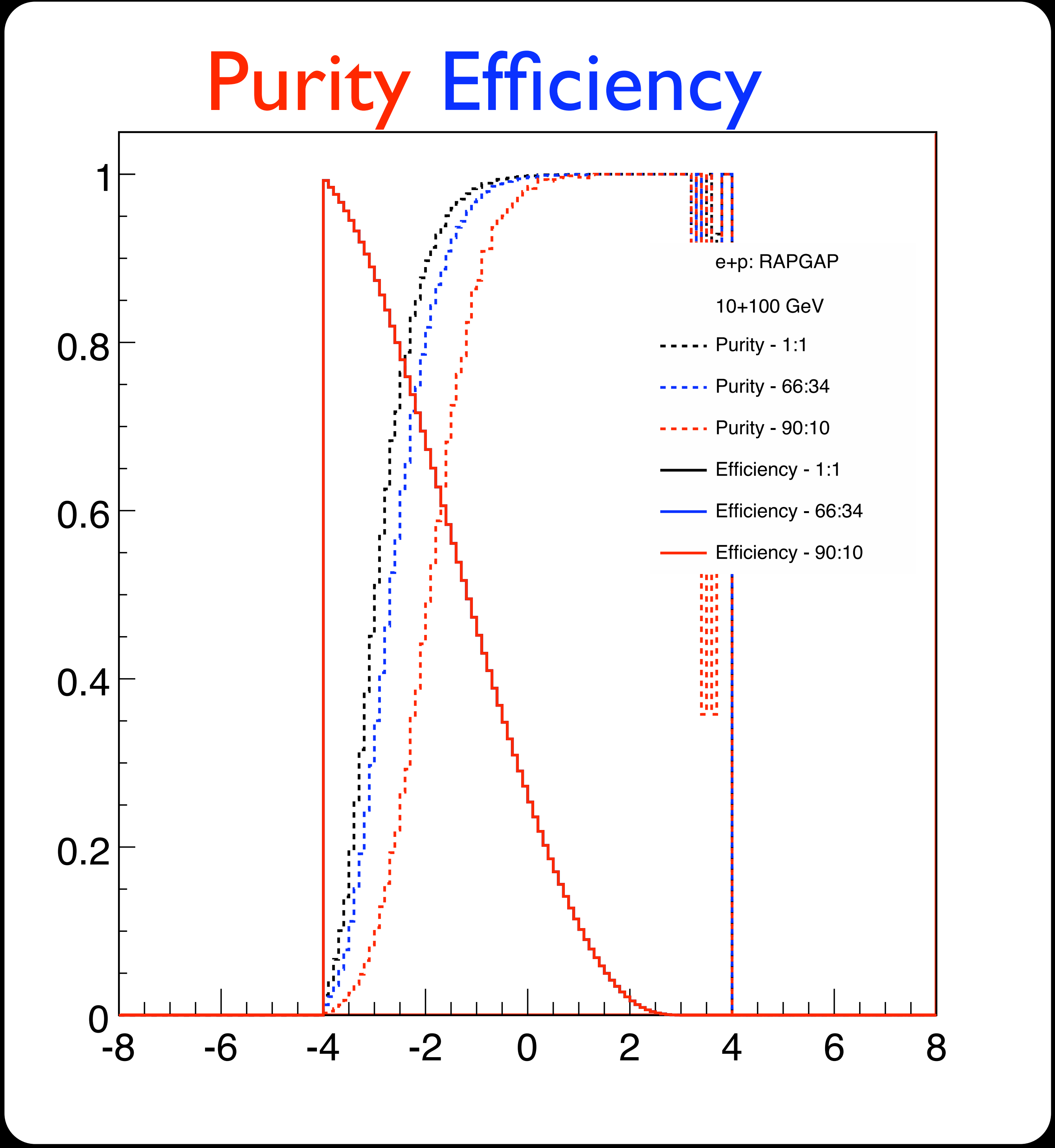
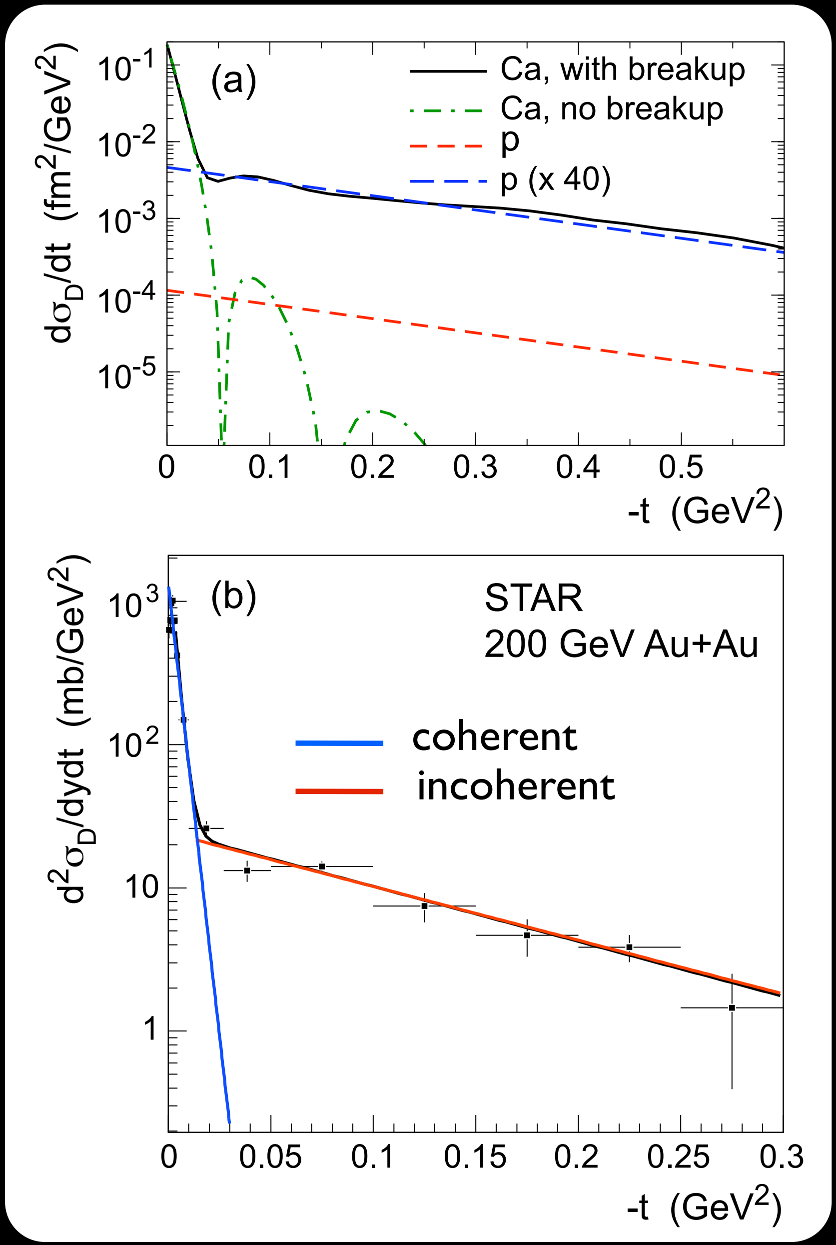


Role of colour-neutral (Pomeron) excitations

$$\frac{d\sigma}{dt}|_{t=0}(\gamma^* A \to VA) \propto \alpha_s^2 [G_A(x, Q^2)]^2$$

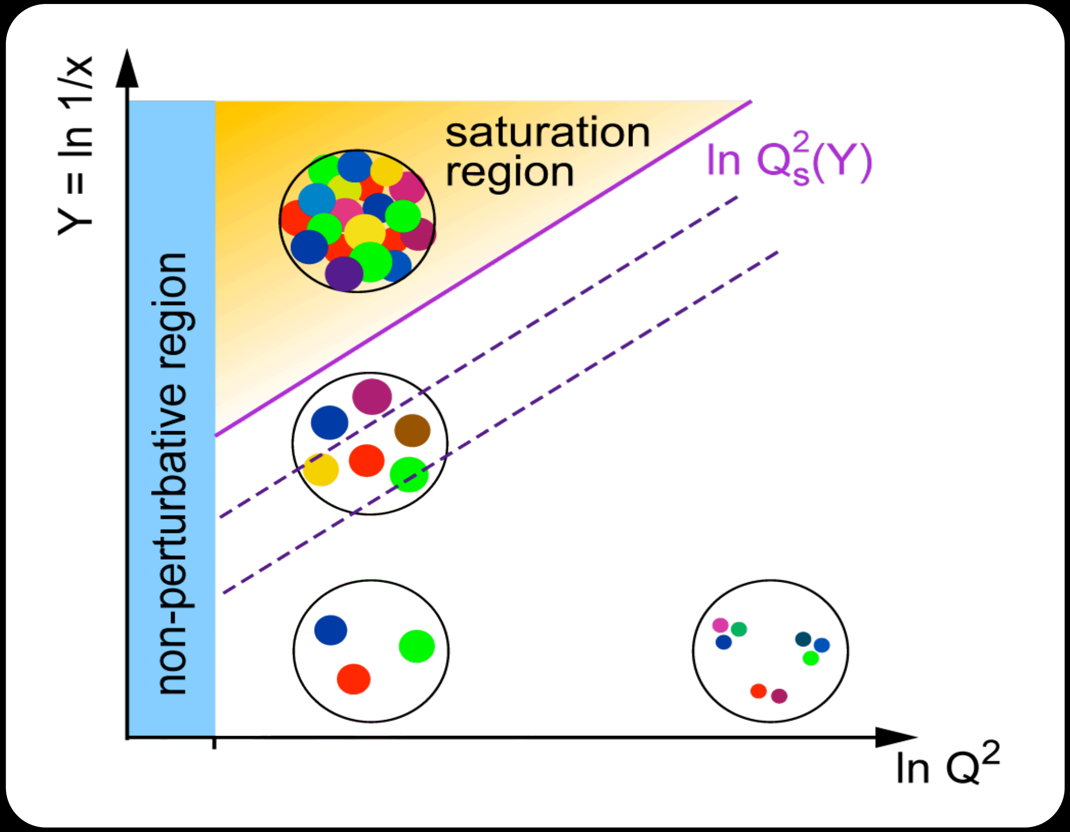
- Knowledge of t is important
  - small t ⇒ coherent diffraction
  - large t ⇒ incoherent diffraction
    - Results from STAR UPC Au+Au collisions
      - coherent diffraction ⇒ t < 0.03 GeV^2
      - incoherent diffraction ⇒ t > 0.03 GeV^2



STAR Ultra-Peripheral Au+Au Collisions:
Phys. Rev. C **77** (2008) 34910

On the parton scale: Geometric scaling of $\sigma_{DIS}(x, Q^2)$

- Although saturation is reached only when $Q_S \sim Q$, observables are sensitive to the saturation scale as they approach saturation.
- Crucial consequence of non-linear evolution towards saturation:
  - Physics invariant along lines of constant gluon density (parallel to saturation regime)



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- Crucial consequence of non-linear evolution towards saturation:
  - Physics invariant along lines of constant gluon density (parallel to saturation regime)
- For inclusive events in DIS, this manifests itself in "Geometric Scaling"
  - Total cross-section scales with $Q^2/Q^2_S(x)$ instead of x and Q^2/Q^2_0 separately