Higher-order soft corrections to $t\bar{t}$-production

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The Top Quark

- Mass of the top quark:

$$m_t = 173.1 \pm 0.6 \ (\text{stat.}) \pm 1.1 \ (\text{syst.}) \, \text{GeV}$$

[Tevatron Electroweak Working Group and CDF Collaboration and D0 Collab], arXiv:0903.2503 [hep-ex].

- Production mechanisms on the partonic level:
$gg \to t\bar{t}$, $q\bar{q} \to t\bar{t}$ @ all orders, $gq \to t\bar{t}$ @ N^kLO, $k \geq 1$

- Measured pair production cross section at the Tevatron:

$$\sigma(p\bar{p} \to t\bar{t}) = 7.0 \pm 0.3 \ (\text{stat.}) \pm 0.4 \ (\text{syst.}) \pm 0.4 \ (\text{lumi.}) \, \text{pb}$$

A. Lister [CDF and D0 Collaborations], arXiv:0810.3350 [hep-ex].

Theoretical Ingredients

- The hadronic cross section:

$$\sigma_{pp\to t\bar{t}X}(S, m_t, \mu) = \sum_{i,j=q,\bar{q},g} \int_{4m_t^2}^{S} ds\, L_{ij}(s, S, \mu_f^2)\, \hat{\sigma}_{ij}(s, m_t, \mu_r, \mu_f)$$

  where S is the hadronic cms energy and L the parton luminosity

- Theory results on $\hat{\sigma}$:
  - LO and NLO scaling functions well known since long, see f. e.
    P. Nason, S. Dawson and R. K. Ellis, Nucl. Phys. B **303** (1988) 607.
    W. Beenakker, H. Kuijf, W. L. van Neerven and J. Smith, Phys. Rev. D **40** (1989) 54.
    W. Bernreuther, A. Brandenburg, Z. G. Si and P. Uwer, Nucl. Phys. B **690** (2004) 81 [arXiv:hep-ph/0403035].
  - new since DIS 2008: NNLO approx. cross section
    S. Moch and P. Uwer, Phys. Rev. D **78** (2008) 034003 [arXiv:0804.1476 [hep-ph]].
    N. Kidonakis and R. Vogt, Phys. Rev. D **78** (2008) 074005 [arXiv:0805.3844 [hep-ph]].
    NLO + NLL cross section
    M. Cacciari, S. Frixione, M. L. Mangano, P. Nason and G. Ridolfi, JHEP **0809** (2008) 127 [arXiv:0804.2800 [hep-ph]].

Theoretical Ingredients: Scale dependence

- μ_r, μ_f: renormalisation and factorisation scale: arbitrary parameters
- μ_f: separates long and short distant interactions
  short distant interactions can be described bei pQCD, because α_s is small at high energies,
  long distant interactions are absorbed into the parton distribution functions (PDF)
- μ_r: “Expansion parameter” of the perturbation series
- use scale dependence of the cross section to estimate the theoretical uncertainty
  common choice: $\frac{1}{2} m_q \leq \mu_r = \mu_f \leq 2 m_q$

Theoretical Ingredients: Scale dependence

- at infinite order, scale dependence vanishes: $\longrightarrow$ renormalizsation group equation (RGE)
- use RGE to calculate the scale dependent terms
- Example: For the $gg/q\bar{q}$ channel at NLO, these terms are given by

$$\sigma^{\mathrm{NLO}}_{gg/q\bar{q}} = P^{\mathrm{LO}}_{gg/q\bar{q}} \otimes \sigma^{\mathrm{LO}}_{gg/q\bar{q}} - \beta_0 \sigma^{\mathrm{LO}}_{gg/q\bar{q}}$$

- $P^{\mathrm{LO}}_{gg/q\bar{q}}$: splitting functions, govern the scale evolution of the PDFs and are calculable functions
- NLO scale dependent terms are expressed in terms of LO functions

Theoretical Ingredients NNLO cross section

- The partonic, NNLO approx cross section:

$$\hat{\sigma}_{ij}(s, m_t, \mu) = \frac{{\alpha_s}^2}{{m_t}^2}\left[f_{ij}^{(00)} + 4\pi\alpha_s\left(f_{ij}^{(10)} + f_{ij}^{(11)}\log\left(\frac{\mu^2}{{m_t}^2}\right)\right) + (4\pi\alpha_s)^2\left(f_{ij}^{(20)} + f_{ij}^{(21)}\log\left(\frac{\mu^2}{{m_t}^2}\right) + f_{ij}^{(22)}\log^2\left(\frac{\mu^2}{{m_t}^2}\right)\right)\right]$$

  with $ij = q\bar{q}, gg, gq$ and s the partonic center of mass (cms) energy

- Renormalization Group Equation: $\longrightarrow$ scale dependent scaling functions

- Fits on $f_{gg/qq}^{(10)}$ from the (new since DIS 2008) analytic NLO result and on the numerically calculable scaling functions $f^{(21)}$,$f^{(22)}$

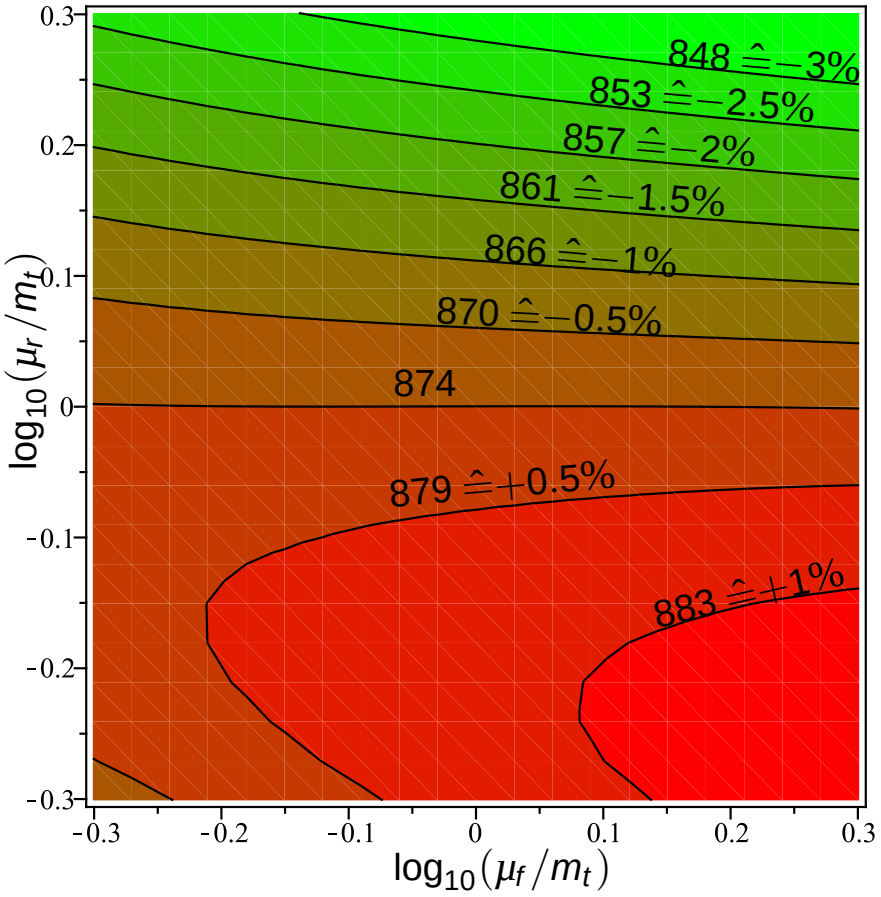
  M. Czakon and A. Mitov, arXiv:0811.4119 [hep-ph]

  $\longrightarrow$ Easier handling of the scaling functions, faster computations. Provide `Fortran` code.

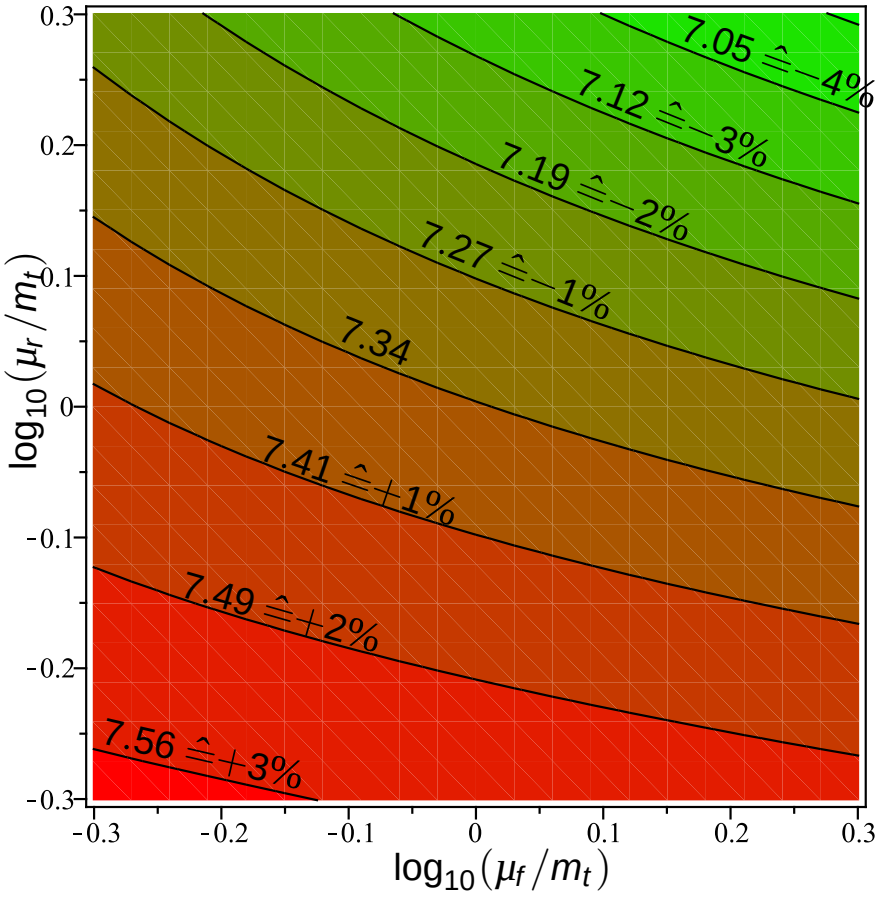
- Resummation: $\longrightarrow$ Estimate of the NNLO contribution at the production threshold $\longrightarrow f^{(20)}$

Scale dependence of the $t\bar{t}$ production cross section

Contour lines of the total hadronic cross section for top pair production in the μ_f - μ_r - plane in pb ($\log_{10}$ - scale)



LHC



Tevatron

Scale dependence of the $t\bar{t}$ production cross section, cont'd

- Maximal deviation from the cros section at $\mu_f = \mu_r = m_t$ and for $1/2m_t \leq \mu_{f/r} \leq 2m_t$:

  -5% at the Tevatron, -3.5% at the LHC

  $\Rightarrow$ Very mild scale dependence at NNLO

- At the Tevatron:
  cross section maximal at $\mu_f = \mu_r = 1/2m_t$ and minimal at $\mu_f = \mu_r = 2m_t$

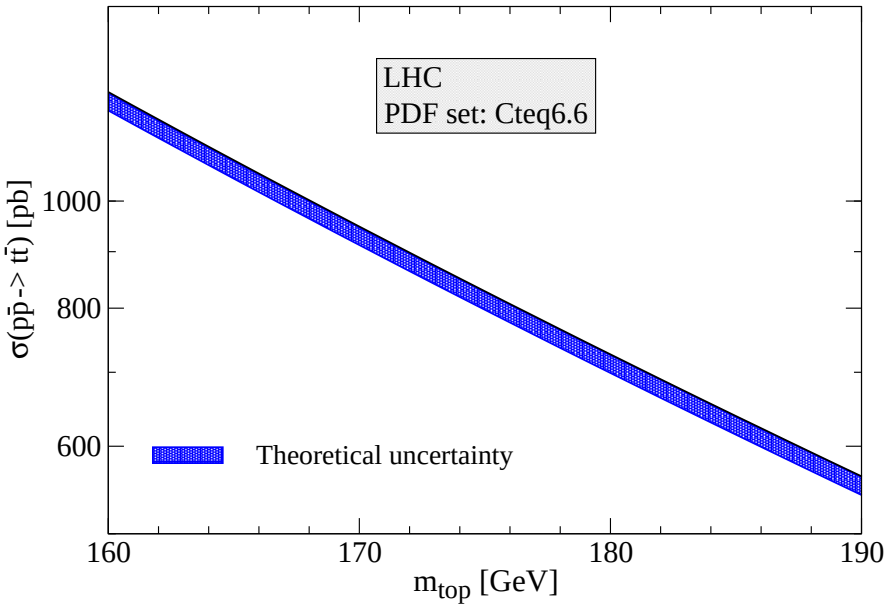
- At the LHC:
  cross section minimal at $\mu_f = \mu_r = 2m_t$

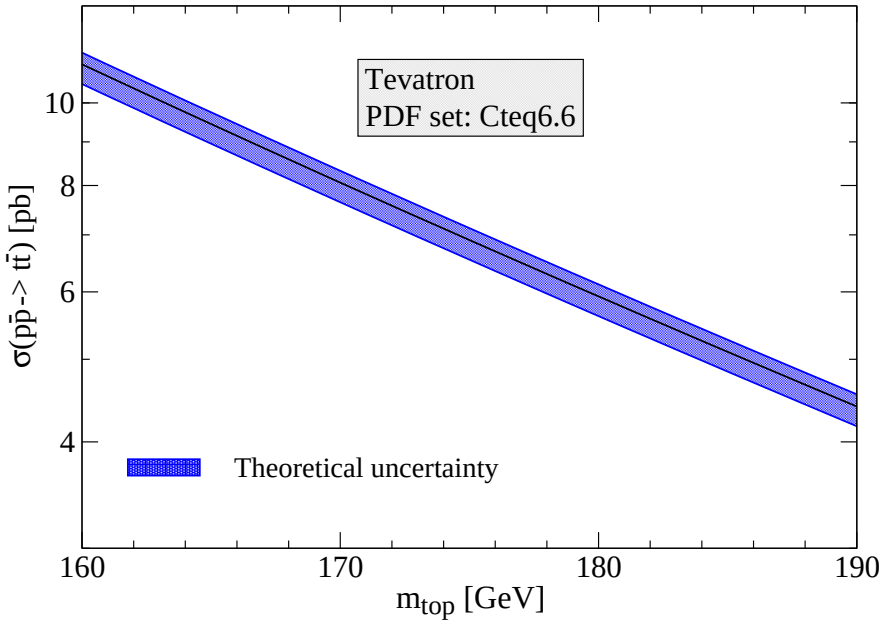
- Total cross section prediction for $m_t = 173\,\text{GeV}$ and PDF set Cteq6.6:

$$\sigma(p\bar{p} \to t\bar{t}) = 7.34^{+0.23}_{-0.38}\,\text{pb @ Tevatron}$$

$$\sigma(pp \to t\bar{t}) = 874^{+14}_{-33}\,\text{pb @ LHC 14 TeV}$$

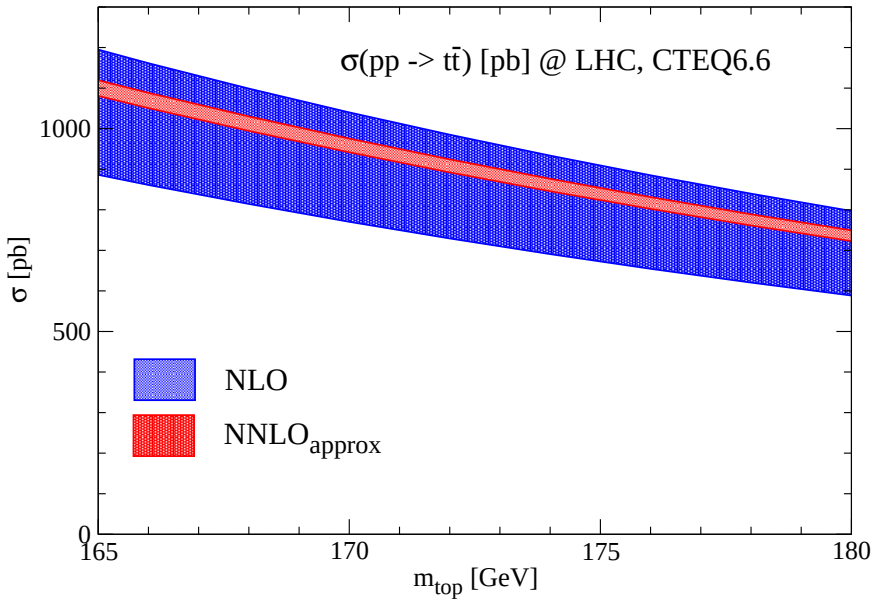
Mass dependence

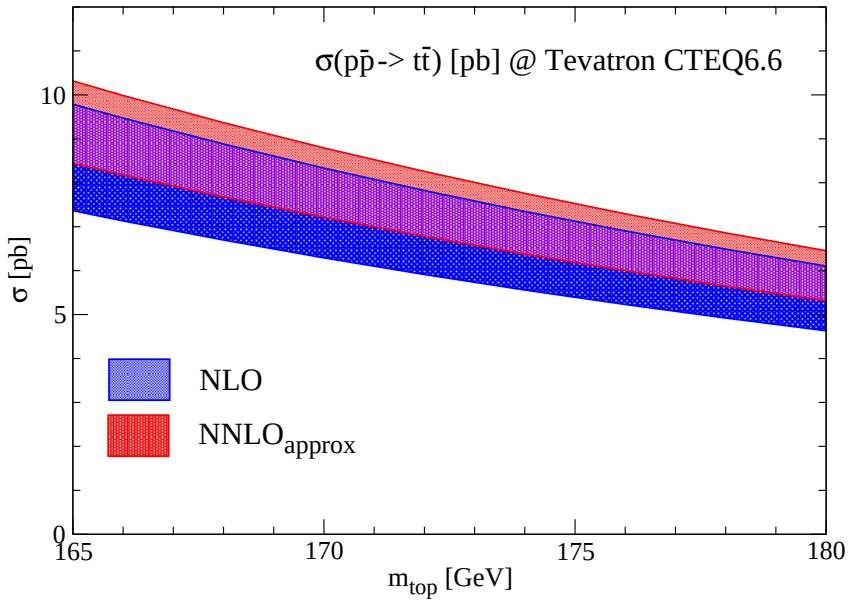




- Fairly strong mass dependence
- Fit functions for different scenarios:
  $\sigma(m_t) = a + bx + cx^2 + dx^3 + ex^4 + fx^5 + gx^6, x = (m_t/\text{GeV} - 173).$

PDF error





- Define PDF error $\Delta O = \frac{1}{2}\left(\sum_{i=1}^{n_{\mathrm{PDF}}}(O_{i+} - O_{i-})^2\right)^{1/2}$.
- upper and lower total error bound: $\sigma(\mu = 1/2 m_t) + \Delta O$, $\sigma(\mu = 2m_t) - \Delta O$
- Total cross section with combined theoretical and PDF error:

$$\sigma(p\bar{p} \to t\bar{t}) = 7.34^{+0.66}_{-0.76}\,\mathrm{pb}\ @\ \text{Tevatron}$$

$$\sigma(pp \to t\bar{t}) = 874^{+26}_{-50}\,\mathrm{pb}\ @\ \text{LHC 14 TeV}$$

Summary

We calculated

- the top pair production cross section at the LHC and the Tevatron with the full factorisation and renormalisation scale dependence.
- the total cross section as a function of the top mass and produced fit functions for different scales and PDF sets for both colliders
- the PDF uncertainty and presented a combined (theoretical and PDF) total error on the top pair production cross section.