High energy resummation of Drell-Yan processes

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Outline

• DY and W/Z cross sections at the LHC
• High energy (small-\( x \)) logarithms:
  – Perturbative evolution of parton densities
  – Coefficient functions
• Resummation of DY and W/Z cross sections
  – Analytic results
  – Some phenomenology
• Conclusions and Outlook
QCD corrections to DY and W/Z cross-sections

• Outstanding precision in the computation of QCD corrections to Drell-Yan and W/Z cross sections

• Fixed order results:
  – NNLO predictions for the inclusive cross-section known for a long time
  – More recently NNLO rapidity distributions as well

• Resummation: threshold logarithms up to N^3LL

• Use these cross-section as standard candles at the LHC
  (~ 5 % accuracy)
LHC kinematics

- LHC kinematic coverage: larger than any other collider
- Evolution in $Q^2$: the well established DGLAP equation resums collinear logs
- Small-$x$ evolution given by the BFKL equation

BFKL:
- Important contributions at large rapidities
- Inclusive observables less sensitive
- Small-$x$ effects to be taken into account if we want a few percent precision
QCD evolution equations

DGLAP: $Q^2$ evolution for $N$ moments of the parton density

\[
\frac{d}{d \ln(Q^2/\mu^2)} G(N, Q^2) = \gamma(N, \alpha_s) G(N, Q^2)
\]

BFKL: small-$x$ evolution for $M$ moments of the parton density

\[
\frac{d}{d \ln(1/x)} G(x, M) = \chi(M, \alpha_s) G(x, M)
\]

Mellin moments:

\[
\ln^k \frac{Q^2}{\mu^2} \leftrightarrow \frac{1}{M^{k+1}}
\]

\[
\ln^k \frac{1}{x} \leftrightarrow \frac{1}{N^{k+1}}
\]

Can we write an anomalous dimension which resums both logs of $Q^2$ and $x$?
Parton Evolution at small-\(x\)

- Problem now solved by different groups
  - Altarelli, Ball, Forte (ABF)
  - Ciafaloni, Colferai, Salam, Stasto (CCSS)
  - Thorne, White (TW)

- Stable solution of the running coupling BFKL equation

- Match to standard DGLAP at moderate \(x\)
The ABF approach

- Resummation performed in double Mellin space \((N,M)\)
- Main ingredients:
  - Duality relations (which lead to double leading expansion)
    \[ \chi(\gamma(N, \alpha_s), \alpha_s) = N \]
  - Exchange symmetry of the BFKL kernel
    \[ M \leftrightarrow 1 - M \]
  - Resummation of running coupling corrections
  - Consistent treatment of the factorization scheme

- Studies of the phenomenological impact of small-\(x\) resummation are now possible

But we need resummed coefficient functions as well

see also Juan Rojo’s talk

Altarelli, Ball, Forte

Hadronic cross section

\[ \sigma = (C^{FO} + C^{Lx} - d.c.) \otimes f_1 f_2 \]

- Consistent choice of factorization scheme
- We currently know how to compute resummed coefficient functions at the leading (non trivial) logarithmic accuracy, for the inclusive cross-section

Fixed order coefficient function

Tower of small-\(x\) logs

Double counting

Parton densities (resummed evolution)
High energy factorization (I)

- In the high energy limit the following $k_t$-dependent factorization formula holds:

$$\sigma = \int \frac{d^2 k_1}{\pi k_1^2} \int \frac{d^2 k_2}{\pi k_2^2} \hat{\sigma}^{\text{off}}(k_1, k_2) \otimes \mathcal{F}_1(k_1^2, \mu^2) \mathcal{F}_2(k_2^2, \mu^2)$$

- The high energy singularities of the collinear factorized coefficient function are obtained by computing Mellin moments of the off-shell cross section:

$$h(M_1, M_2) = M_1 M_2 \int_0^\infty (k_1^2)^{M_1-1} \int_0^\infty (k_2^2)^{M_2-1} \hat{\sigma}^{\text{off}}$$

- and then by solving for $M_i$ using the evolution equations

$$h(\gamma_s(N), \gamma_s(N)) \quad \text{with} \quad \gamma_s = \sum_k a_k \left( \frac{\alpha_s}{N} \right)^k$$
High energy factorization (II)

- Originally used for heavy flavour production

\[ \text{Ball, Ellis JHEP 0105:053,2001.} \]

- and more recently for Higgs in gluon gluon fusion

\[ \text{SM, Ball, Del Duca, Forte, Vicini Nucl.Phys.B800:127-145,2008.} \]

- DIS is more delicate because collinear singularities (due to massless quarks) must be consistently factorized


- DY same problem as DIS and also more complicated flavour structure
DY sub-processes

• DY processes have a rather complicated flavour structure
  – LO: \( q\bar{q} \to \gamma^* \)
  – NLO: \( q\bar{q} \to \gamma^* \) \( qg \to q\gamma^* \)
  – NNLO: \( q\bar{q} \to \gamma^* \) \( qg \to q\gamma^* \) \( gg \to q\bar{q}\gamma^* \) \( qq \to qq\gamma^* \)

• We need to understand which diagrams have high energy logarithms

• Similar structure for W/Z production
What is singular at high energy?

\[ \gamma_{qq} \sim \alpha_s C_F \]

\[ \gamma_{qg} \sim \alpha_s T_r \]

\[ \gamma_{gq} \sim C_F \frac{\alpha_s}{N} \]

\[ \gamma_{gg} \sim C_A \frac{\alpha_s}{N} \]
DY at high energy (I)

- The emission of a gluon along a quark line is regular at high energy.

- The emission of a gluon along a gluon line gives a contribution.

- Subsequent gluon emissions are singular but all the contributions suppressed by $\alpha_s$. 
DY at high energy (II)

\[ D_{q\bar{q}} = 1 + \mathcal{O}(\alpha_s^k) \]

\[ D_{qg} = \mathcal{O}(\alpha_s) + \mathcal{O}\left(\alpha_s \left(\frac{\alpha_s}{N}\right)^k\right) \]

\[ D_{gg} = \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\alpha_s^2 \left(\frac{\alpha_s}{N}\right)^k\right) \]

\[ D_{qq} = \frac{C_F}{C_A} \mathcal{O}\left(\alpha_s \left(\frac{\alpha_s}{N}\right)^k\right) \]

- We calculate the high energy singularities in the qg channel using \( k_t \)-factorization
- We then compute the ones in the qq channel via colour-charge relations
- Anything else is sub-leading
Calculation of the impact factor

\[ \tau = \frac{Q^2}{\hat{s}} \]

\[ \xi = \frac{k_t^2}{Q^2} \]

\[ h_{qg}(N, M) = M^2 \int_0^\infty d\xi \xi^{M-1} \int_0^1 d\tau \tau^{N-1} \sigma^{\text{off}}(\tau, \xi) \]

The full result rather complicated sum of generalized hypergeometric functions

The high energy limit is rather simple

\[ h_{qg}(0, M) = \frac{\alpha_s}{2\pi} T_R \frac{4\Gamma(1-M)^2\Gamma(1+M)^2}{(1-M)(2-M)(3-M)} \]
Collinear singularities

• Collinear singularities because of the massless lines

• The gluon $k_t$ provides a regulator: no singularities but this is not consistent with collinear factorization in $\overline{\text{MS}}$

• We have to write high energy factorization in $D$-dimensions
• This leads to a relation between the impact factor and the anomalous dimensions

$$\gamma_{qg} + \gamma_s D_{qg} = h_{qg}(\gamma_s) R(\gamma_s)$$

• We are not able to provide a result in a closed form, but an expansion to any desired order
Analytic results

• The analytic result for the coefficient function is

\[ D_{qg}(N, \alpha_s) = \frac{\alpha_s}{18\pi} T_R \left[ 1 + \left( \frac{29}{6} + 2\pi^2 \right) \frac{C_A}{\pi} \frac{\alpha_s}{N} + \left( \frac{1069}{108} + \frac{111}{3} \pi^2 + 4\zeta_3 \right) \left( \frac{C_A}{\pi} \frac{\alpha_s}{N} \right)^2 + \left( \frac{9031}{648} + \frac{85}{18} \pi^2 + \frac{7}{20} \pi^4 + \frac{73}{3} \zeta_3 \right) \left( \frac{C_A}{\pi} \frac{\alpha_s}{N} \right)^3 + \ldots \right] \]

• The first two coefficients are in agreement with fixed-order results

• Using high energy colour-charge relations, we have:

\[ D_{qq}(N, \alpha_s) = \frac{C_F}{C_A} \left[ D_{qg} - \frac{\alpha_s}{18\pi} T_R \right] \]
Phenomenology: LHC at 14 TeV

\[ K = \frac{\sigma}{\sigma_0} \]

- Computed with NNPDF1.0
- Fixed order in Mellin space from Blumlein, Ravindran
  \( \text{Nucl.Phys.B716:128-172,2005.} \)
- Running coupling resummation Ball, \( \text{Nucl.Phys.B796:137-183,2008.} \)

- solid NLO with NLO PDFs
- dashed NLO\text{res} with NLO PDFs
- solid NLO\text{res} with NLO\text{res} PDFs

- NNLO with NNLO PDFs
- dashed NNLO\text{res} with NNLO PDFs
- solid NNLO\text{res} with NNLO\text{res} PDFs

- NLO: small-\( x \) resummation corrects the fixed-order result by
  5 \% at \( Q = 100 \) GeV and 10 \% at \( Q = 10 \) GeV
- NNLO:
  - NNLO coefficient functions seem to get most of the low-\( x \) shape
  - We don’t have yet NNLO\text{res} PDFs
  - A few percent uncertainty due to small-\( x \) effects for \( Q < 100 \) GeV
Conclusions

• We have computed the resummed DY coefficient function at high energy
• Small-\(x\) singularities are the same for W/Z production (although flavour structure is different)
• We have computed DY K-factors with resummed partons
• A few percent correction to NNLO is found
• Calculation for W/Z production at the LHC is work in progress
Outlook

• Extend this formalism to rapidity distributions
  – they are experimentally more relevant
  – they are more sensitive to small-$x$ effects

• Compute more the resummed coefficient function for other processes (inclusive jets)

• Combine different small-$x$ and large-$x$ resummations