High energy resummation of Drell-Yan processes

Simone Marzani
University of Manchester

XVII International Workshop on
Deep-Inelastic Scattering and Related Subjects
Madrid 26th – 30th April 2009

In collaboration with Richard Ball,
arXiv0812.3602 [hep-ph]
Nucl.Phys.B814:246-264,2009.

Outline

- DY and W/Z cross sections at the LHC
- High energy (small-x) logarithms:
  - Perturbative evolution of parton densities
  - Coefficient functions
- Resummation of DY and W/Z cross sections
  - Analytic results
  - Some phenomenology
- Conclusions and Outlook

QCD corrections to DY and W/Z cross-sections

- Outstanding precision in the computation of QCD corrections to Drell-Yan and W/Z cross sections
- Fixed order results:
  - NNLO predictions for the inclusive cross-section known for a long time Hamberg, van Neervan and Matsuura, Nucl.Phys.B359:343-405,1991
  - More recently NNLO rapidity distributions as well Anastasiou *et al* ., Phys.Rev.D69:094008,2004
- Resummation: threshold logarithms up to N^3LL

  Moch and Vogt, Phys.Lett.B631:48-57,2005

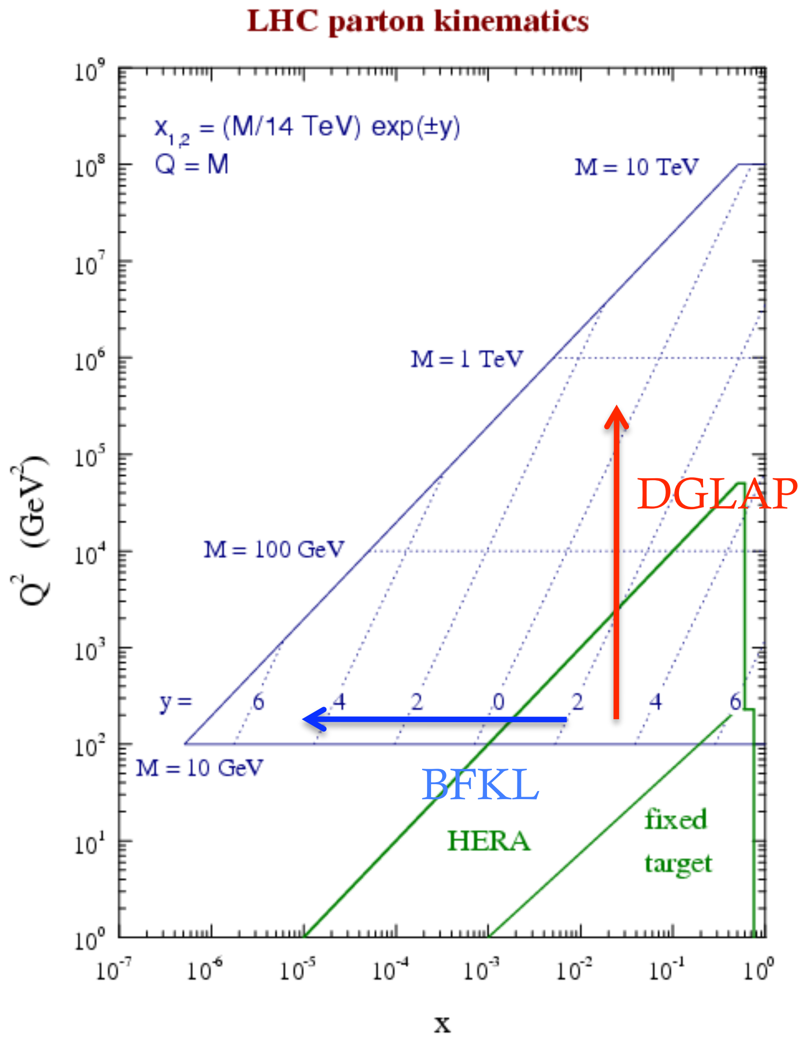
  Laenen and Magnea, Phys.Lett.B632:270-276,2006

- Use these cross-section as standard candles at the LHC (~ 5 % accuracy)

LHC kinematics

- LHC kinematic coverage: larger than any other collider
- Evolution in Q^2: the well established DGLAP equation resums collinear logs
- Small-x evolution given by the BFKL equation

BFKL:
- Imporant contributions at large rapidities
- Inclusive observables less sensitive
- Small-x effects to be taken into account if the we want a few percent precision



QCD evolution equations

DGLAP: Q^2 evolution for N moments of the parton density

$$\frac{d}{d\ln(Q^2/\mu^2)}G(N,Q^2) = \gamma(N,\alpha_s)G(N,Q^2)$$

BFKL: small-x evolution for M moments of the parton density

$$\frac{d}{d\ln(1/x)}G(x,M) = \chi(M,\alpha_s)G(x,M)$$

Mellin moments:

logs $\leftrightarrow$ poles

$$\ln^k\frac{Q^2}{\mu^2} \leftrightarrow \frac{1}{M^{k+1}}$$

$$\ln^k\frac{1}{x} \leftrightarrow \frac{1}{N^{k+1}}$$

Can we write an anomalous dimension which resums both logs of Q^2 and x?

Parton Evolution at small-*x*

- Problem now solved by different groups
  - Altarelli, Ball, Forte (ABF)
  - Ciafaloni, Colferai, Salam, Stasto (CCSS)
  - Thorne, White (TW)

    for a comparative review see HERA-LHC Proc. arXiv:0903.3861
- Stable solution of the running coupling BFKL equation
- Match to standard DGLAP at moderate *x*

The ABF approach

- Resummation performed in double Mellin space (N,M)
- Main ingredients:
  - Duality relations (which lead to double leading expansion)

$$\chi(\gamma(N,\alpha_s),\alpha_s) = N$$

  - Exchange symmetry of the BFKL kernel

$$M \leftrightarrow 1 - M$$

  - Resummation of running coupling corrections
  - Consistent treatment of the factorization scheme

Altarelli, Ball, Forte
Nucl.Phys.B742:1-40,2006
Nucl.Phys.B799:199-240,2008

- Studies of the phenomenological impact of small-*x* resummation are now possible see also Juan Rojo's talk

But we need resummed coefficient functions as well

Hadronic cross section

$$\sigma = \left(C^{FO} + C^{Lx} - d.c.\right) \otimes f_1 f_2$$

Fixed order coefficient function

Double counting

Tower of small-x logs

Parton densities
(resummed evolution)

- Consistent choice of factorization scheme
- We currently know how to compute resummed coefficient functions at the leading (non trivial) logarithmic accuracy, for the inclusive cross-section

High energy factorization (I)

- In the high energy limit the following k_t-dependent factorization formula holds:

$$\sigma = \int \frac{d^2k_1}{\pi k_1^2} \int \frac{d^2k_2}{\pi k_2^2} \hat{\sigma}^{\rm off}(k_1, k_2) \otimes \mathcal{F}_1(k_1^2, \mu^2)\mathcal{F}_2(k_2^2, \mu^2)$$

- The high energy singularities of the collinear factorized coefficient function are obtained by computing Mellin moments of the off-shell cross section:

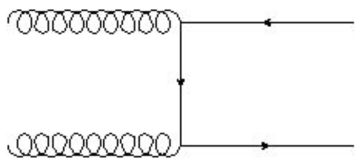
$$h(M_1, M_2) = M_1 M_2 \int_0^\infty (k_1^2)^{M_1-1} \int_0^\infty (k_2^2)^{M_2-1} \hat{\sigma}^{\rm off}$$

- and then by solving for M_i using the evolution equations

$$h(\gamma_s(N), \gamma_s(N)) \quad \text{with} \quad \gamma_s = \sum_k a_k \left(\frac{\alpha_s}{N}\right)^k$$

High energy factorization (II)

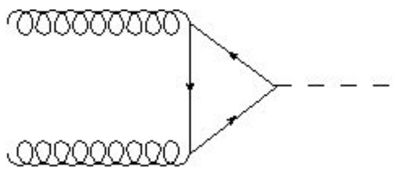
- Originally used for heavy flavour production



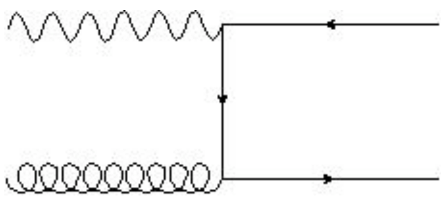
Catani, Ciafaloni, Hautmann Nucl.Phys.B366:135-188,1991.
Ball, Ellis JHEP 0105:053,2001.

- and more recently for Higgs in gluon gluon fusion

SM, Ball, Del Duca, Forte, Vicini
Nucl.Phys.B800:127-145,2008.



- DIS is more delicate because collinear singularities (due to massless quarks) must be consistently factorized



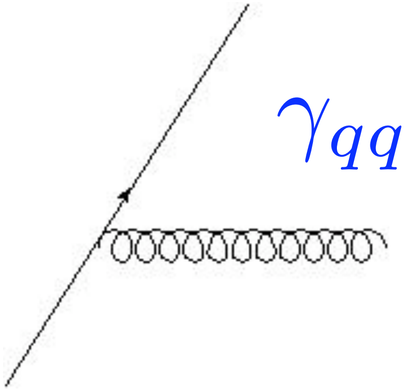
Catani, Hautmann
Nucl.Phys.B427:475-524,1994.

- DY same problem as DIS and also more complicated flavour structure

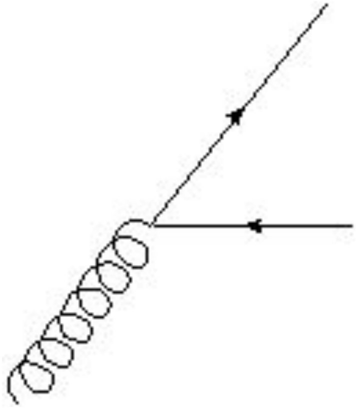
DY sub-processes

- DY processes have a rather complicated flavour structure
  - LO: $q\bar{q} \to \gamma^*$
  - NLO: $q\bar{q} \to \gamma^* \quad qg \to q\gamma^*$
  - NNLO: $q\bar{q} \to \gamma^* \quad qg \to q\gamma^* \quad gg \to q\bar{q}\gamma^* \quad qq \to qq\gamma^*$
- We need to understand which diagrams have high energy logarithms
- Similar structure for W/Z production

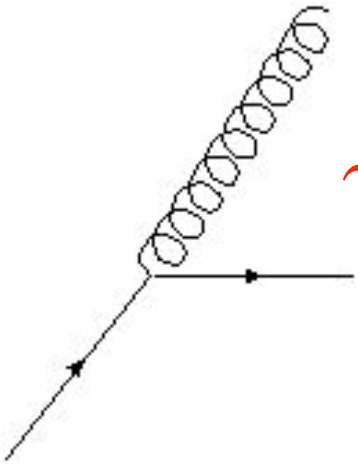
What is singular at high energy ?



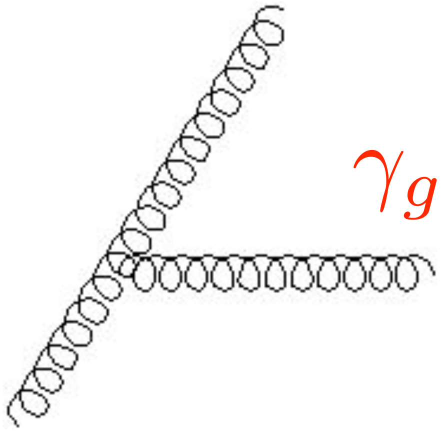
$$\gamma_{qq} \sim \alpha_s C_F$$



$$\gamma_{qg} \sim \alpha_s T_r$$

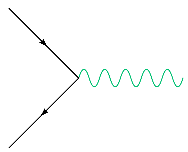


$$\gamma_{gq} \sim C_F \frac{\alpha_s}{N}$$

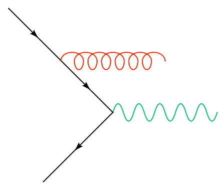


$$\gamma_{gg} \sim C_A \frac{\alpha_s}{N}$$

DY at high energy (I)

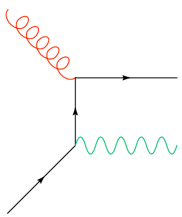


LO

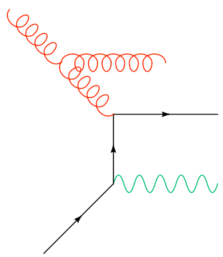


NLO

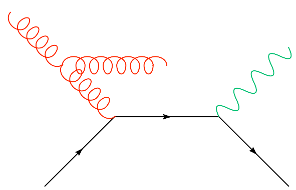
- The emission of a gluon along a quark line is regular at high energy



NLO

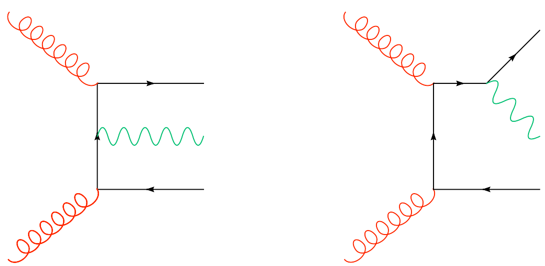


NNLO



NNLO

- The emission of a gluon along a gluon line gives a contribution



NNLO

- Subsequent gluon emissions are singular but all the contributions suppressed by α_s

DY at high energy (II)

$$D_{q\bar{q}} = 1 + \mathcal{O}(\alpha_s^k)$$

$$D_{qg} = \mathcal{O}(\alpha_s) + \mathcal{O}\left(\alpha_s \left(\frac{\alpha_s}{N}\right)^k\right)$$

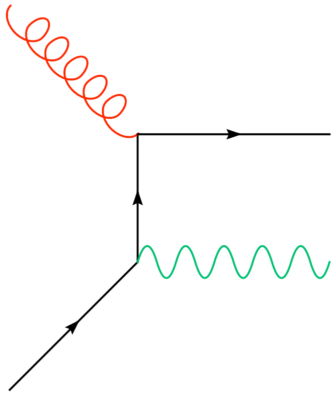
$$D_{gg} = \mathcal{O}(\alpha_s^2) + \mathcal{O}\left(\alpha_s^2 \left(\frac{\alpha_s}{N}\right)^k\right)$$

$$D_{qq} = \frac{C_F}{C_A}\mathcal{O}\left(\alpha_s \left(\frac{\alpha_s}{N}\right)^k\right)$$

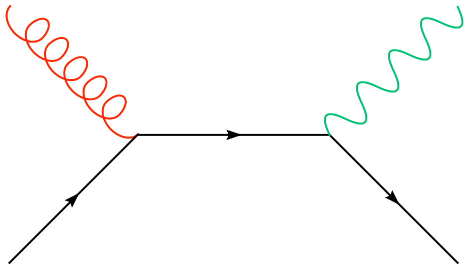
Leading (non-trivial)
small-x logs

- We calculate the high energy singularities in the qg channel using k_t-factorization
- We then compute the ones in the qq channel via colour-charge relations
- Anything else is sub-leading

Calculation of the impact factor



(a)



(b)

$$\tau = Q^2/\hat{s}$$

$$\xi = k_t^2/Q^2$$

$$h_{qg}(N,M) = M^2 \int_0^\infty d\xi \xi^{M-1} \int_0^1 d\tau \tau^{N-1} \sigma^{\text{off}}(\tau,\xi)$$

The full result rather complicated sum of generalized hypergeometric functions

The high energy limit is rather simple

$$h_{qg}(0,M) = \frac{\alpha_s}{2\pi} T_R \frac{4\Gamma(1-M)^2\Gamma(1+M)^2}{(1-M)(2-M)(3-M)}$$

Collinear singularities

- Collinear singularities because of the massless lines

- The gluon k_t provides a regulator: no singularities but this is not consistent with collinear factorization in $\overline{\mathrm{MS}}$

- We have to write high energy factorization in *D*- dimensions
- This leads to a relation between the impact factor and the anomalous dimensions

$$\gamma_{qg} + \gamma_s D_{qg} = h_{qg}(\gamma_s) R(\gamma_s)$$

- We are not able to provide a result in a closed form, but an expansion to any desired order

Analytic results

- The analytic result for the coefficient function is

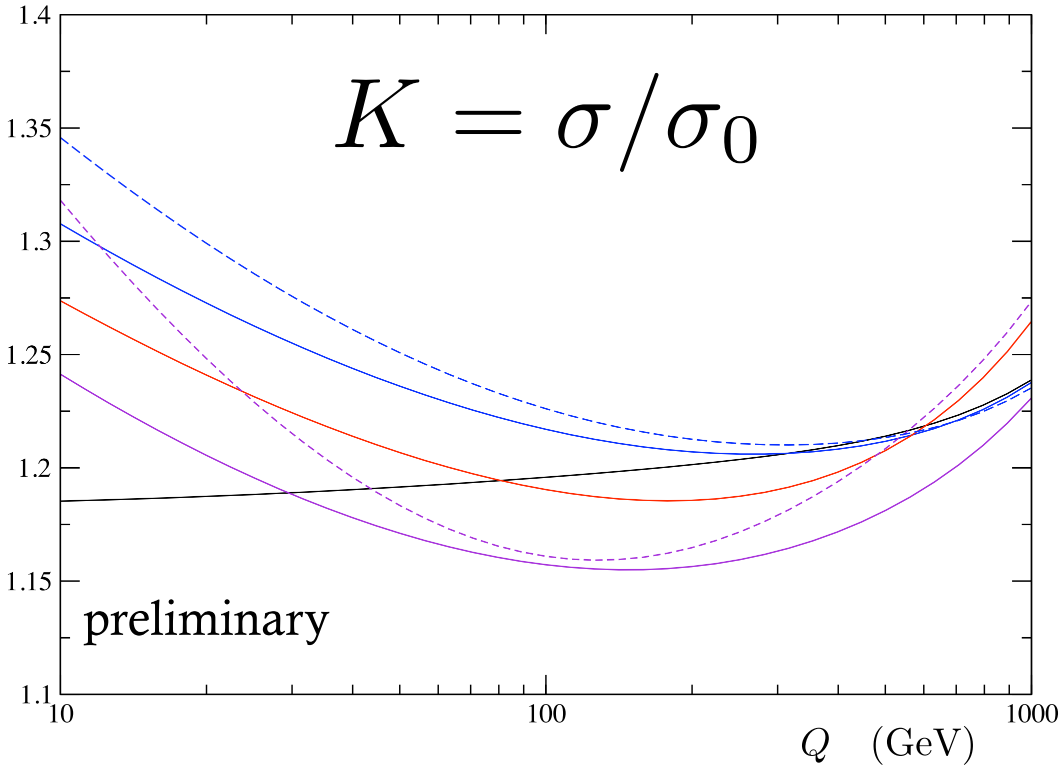
$$D_{qg}(N,\alpha_s) = \frac{\alpha_s}{18\pi}T_R\Big[1+\left(\frac{29}{6}+2\pi^2\right)\frac{C_A}{\pi}\frac{\alpha_s}{N}+\left(\frac{1069}{108}+\frac{11}{3}\pi^2+4\zeta_3\right)\left(\frac{C_A}{\pi}\frac{\alpha_s}{N}\right)^2+$$
$$\left(\frac{9031}{648}+\frac{85}{18}\pi^2+\frac{7}{20}\pi^4+\frac{73}{3}\zeta_3\right)\left(\frac{C_A}{\pi}\frac{\alpha_s}{N}\right)^3+\ldots\Big]$$

- The first two coefficients are in agreement with fixed-order results

- Using high energy colour-charge relations, we have:

$$D_{qq}(N,\alpha_s) = \frac{C_F}{C_A}\left[D_{qg}-\frac{\alpha_s}{18\pi}T_R\right]$$

Phenomenology: LHC at 14 TeV



- Computed with NNPDF1.0
- Fixed order in Mellin space from Blumlein, Ravindran Nucl.Phys.B716:128-172,2005.
- Running coupling resummation Ball, Nucl.Phys.B796:137-183,2008.

- solid NLO with NLO PDFs
- dashed NLO*res* with NLO PDFs
- solid NLO*res* with NLO*res* PDFs

- NNLO with NNLO PDFs
- dashed NNLO*res* with NNLO PDFs
- solid NNLO*res* with NLO*res* PDFs

- NLO: small-*x* resummation corrects the fixed-order result by 5 % at Q = 100 GeV and 10 % at Q = 10 GeV
- NNLO:
  - NNLO coefficient functions seem to get most of the low-*x* shape
  - We don't have yet NNLO*res* PDFs
  - A few percent uncertainty due to small-*x* effects for Q < 100 GeV

Conclusions

- We have computed the resummed DY coefficient function at high energy
- Small-x singularities are the same for W/Z production (although flavour structure is different)
- We have computed DY K-factors with resummed partons
- A few percent correction to NNLO is found
- Calculation for W/Z production at the LHC is work in progress

Outlook

- Extend this formalism to rapidity distributions
  - they are experimentally more relevant
  - they are more sensitive to small-x effects
- Compute more the resummed coefficient function for other processes (inclusive jets)
- Combine different small-x and large-x resummations