A Matrix Formulation for Small-x RG Improved Evolution

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- Some “historical” physical problems
  - Reliable description of rising “hard” cross sections and structure functions at high energies
  - Precise determination of parton splitting functions at small-x while keeping their well known behaviour at larger-x;
  - Providing a small-x resummation of parton ev. in matrix form: quarks and gluons are treated on the same ground and in a collinear factorization scheme as close as possible to $\overline{\mathrm{MS}}$

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- Outline
  - Generalizing BFKL and DGLAP evolutions
  - Criteria and mechanism of matrix kernel construction
  - Resummed results and partonic splitting function matrix
  - Conclusions

Generalizing BFKL and DGLAP eqs

- The BFKL equation (1976) predicts rising cross-sections but
  - Leading log predictions overestimate the hard Pomeron exponent, while NLL corrections are large, negative, and may make it ill-defined (Fadin, Lipatov; Camici, Ciafaloni: 1998)
  - Low order DGLAP evolution is consistent with rise of HERA SF, with marginal problems (hints of negative gluon density)
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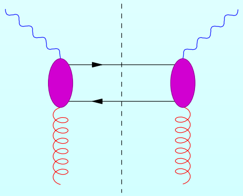
  - Need to reconcile BFKL and DGLAP approaches
- Collinear + small-x Resummations
  - In the last decade, various (doubly) resummed approaches (CCS + CCSS; Altarelli, Ball, Forte; Thorne, White ...)
  - Main idea: to incorporate RG constraints in the BFKL kernel
    Output: effective (resummed) BFKL eigenvalue $\chi_{\rm eff}(\gamma)$ or
    the "dual" DGLAP anomalous dimension $\Gamma_{\rm eff}(\omega)$ (+ running α_s)
  - So far, only the gluon channel is treated self-consistently;
    the quark channel is added by $\boldsymbol{k}$-factorization of the $q - \bar{q}$ dipole



The Matrix Approach

- Generalizes DGLAP self-consistent evolution for quarks and gluons in $\boldsymbol{k}$-factorized matrix form

$$F(x,Q^2) = \int_x^1 \frac{\mathrm{d}z}{z} \int \mathrm{d}^2\boldsymbol{k} \sum_{a=q,g} h_a\left(\frac{x}{z}, Q^2, \boldsymbol{k}\right) \mathcal{F}_a(z,\boldsymbol{k}) \ ,$$

so as to be consistent, at small x, with BFKL gluon evolution

- Defines, by construction, some unintegrated partonic densities $\mathcal{F}_a$ at any x; evolution of $\mathcal{F}_a$ governed by matrix kernel: $\mathcal{F}_a = \mathcal{F}_a^{(0)} + \mathcal{K}_{ab} \otimes \mathcal{F}_b$

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Main construction criteria for the matrix kernel

- Should incorporate exactly NLO DGLAP matrix evolution and the NLx BFKL kernel
- Should satisfy RG constraints in both ordered and antiordered collinear regions, and thus the $\gamma \leftrightarrow 1 - \gamma + \omega$ symmetry (see below)
- Is assumed to satisfy the Minimal-pole Assumption in the γ- and ω-expansions (see below)

BFKL vs DGLAP evolution

- Recall: DGLAP is an evolution equation for PDF $f_a(x, Q^2)$ in hard scale Q^2 and defines the anomalous dimension matrix $\Gamma(\omega)$, with the moment index $\omega = \partial/\partial(\log 1/x)$ conjugated to x

$$\frac{\partial}{\partial \log Q^2} f_a = [\Gamma(\omega)]_{ab} f_b$$

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$$\frac{\partial}{\partial \log \frac{1}{x}} \mathcal{F} = K(\gamma)\mathcal{F}$$

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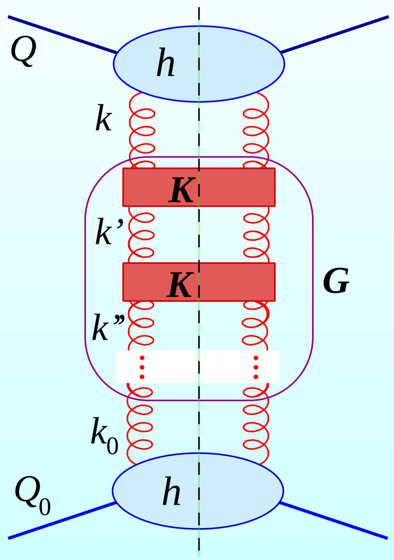
- Using $\boldsymbol{k}$-factorization, DGLAP evolution of the Green's function G corresponds to either ordered $\boldsymbol{k} \gg \boldsymbol{k}' \gg \ldots \boldsymbol{k}_0$ or antiordered $\boldsymbol{k} \ll \boldsymbol{k}' \ll \ldots \boldsymbol{k}_0$ momenta; BFKL incorporates all possible orderings (symm.)



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At fixed α_s, our RG-improved matrix kernel is perturbatively expanded $\mathcal{K}(\bar{\alpha}_s, \gamma, \omega) = \bar{\alpha}_s \mathcal{K}_0(\gamma, \omega) + \bar{\alpha}_s^2 \mathcal{K}_1(\gamma, \omega)$ and satisfies the minimal-pole assumption in the γ- and ω-expansions ($\gamma = 0 \leftrightarrow$ ordered $\boldsymbol{k}$'s)

$$\begin{aligned} \mathcal{K}(\bar{\alpha}_s, \gamma, \omega) &= (1/\gamma)\, \mathcal{K}^{(0)}(\bar{\alpha}_s, \omega) + \mathcal{K}^{(1)}(\bar{\alpha}_s, \omega) + O(\gamma) \\ &= (1/\omega)\, {}_0\mathcal{K}(\bar{\alpha}_s, \gamma) + {}_1\mathcal{K}(\bar{\alpha}_s, \gamma) + O(\omega) \end{aligned}$$

from which DGLAP anomalous dimension matrix Γ and BFKL kernel χ

$$\Gamma_0 = \mathcal{K}_0^{(0)}(\omega) \;, \qquad \Gamma_1 = \mathcal{K}_1^{(0)}(\omega) + \mathcal{K}_0^{(1)}(\omega)\Gamma_0(\omega) \;, \qquad \ldots$$

$$\chi_0 = [{}_0\mathcal{K}_0(\gamma)]_{gg} \;, \qquad \chi_1 = [{}_0\mathcal{K}_1(\gamma) + {}_0\mathcal{K}_0(\gamma)\; {}_1\mathcal{K}_0(\gamma)]_{gg} \;, \qquad \ldots$$

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- Such expressions are used to constrain $\mathcal{K}_0$ and $\mathcal{K}_1$ iteratively to yield the known NLO/NLx evolution, and approximate momentum conservation
- RG constraints in both ordered and antiordered collinear regions are met by the $\gamma \leftrightarrow 1 + \omega - \gamma$ symmetry of the kernel.

The Matrix Kernel

$$\mathcal{K}_0 = \begin{pmatrix} \Gamma^0_{qq}(\omega)\chi^\omega_c(\gamma) & \Gamma^0_{qg}(\omega)\chi^\omega_c(\gamma) \\ \Gamma^0_{gq}(\omega)\chi^\omega_c(\gamma) & \left[\Gamma^0_{gg}(\omega) - \frac{1}{\omega}\right]\chi^\omega_c(\gamma) + \frac{1}{\omega}\chi^\omega_0(\gamma) \end{pmatrix}$$

$$\begin{cases} \chi^\omega_c(\gamma) = \frac{1}{\gamma} + \frac{1}{1+\omega-\gamma} \\ \chi^\omega_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1+\omega-\gamma) \end{cases}$$

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- $\mathcal{K}_0$ has simple poles in γ (in χ^ω_c and χ^ω_0) and simple poles in ω in the gluon row
- No ω-poles are present in the quark row, consistently with LO DGLAP and reggeization of the quark at $\omega = -1$. We keep this structure also in $\mathcal{K}_1$

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$$\mathcal{K}_0 = \begin{pmatrix} \Gamma^0_{qq}(\omega)\chi^\omega_c(\gamma) & \Gamma^0_{qg}(\omega)\chi^\omega_c(\gamma) + \Delta_{qg}(\gamma,\omega) \\ \Gamma^0_{gq}(\omega)\chi^\omega_c(\gamma) & \left[\Gamma^0_{gg}(\omega) - \frac{1}{\omega}\right]\chi^\omega_c(\gamma) + \frac{1}{\omega}\chi^\omega_0(\gamma) \end{pmatrix} \qquad \begin{cases} \chi^\omega_c(\gamma) = \frac{1}{\gamma} + \frac{1}{1+\omega-\gamma} \\ \chi^\omega_0(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1+\omega-\gamma) \end{cases}$$

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- At NLO Γ^1_{qq} and Γ^1_{qg} contain $\frac{\bar{\alpha}_s^2}{\omega}$. Instead of adding such terms in $\mathcal{K}_1$ (see above) we add a proper non-singular $\Delta_{qg}(\gamma,\omega)$ term
- $\mathcal{K}_1$ is obtained by adding NLO DGLAP matrix Γ_1 and NLx BFKL kernel χ_1 (in $\mathcal{K}_{1,gg}$) with the subtractions due to the γ- and ω- expansions explained before

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- $\mathcal{K}_1$ is obtained by adding NLO DGLAP matrix Γ_1 and NLx BFKL kernel χ_1 (in $\mathcal{K}_{1,gg}$) with the subtractions due to the γ- and ω- expansions explained before
- In $(\boldsymbol{k},x)$ space one has the $\boldsymbol{k} \leftrightarrow \boldsymbol{k}'$ and $x \leftrightarrow xk^2/k'^2$ symmetry of the matrix elements and running coupling is introduced, as suggested by the RG and/or the NLx BFKL kernel

$$\mathcal{K}(\boldsymbol{k},\boldsymbol{k}';x) = \bar{\alpha}_s(\boldsymbol{k}^2_>)\mathcal{K}_0(\boldsymbol{k},\boldsymbol{k}';x) + \bar{\alpha}_s^2(\boldsymbol{k}^2_>)\mathcal{K}_1(\boldsymbol{k},\boldsymbol{k}';x)$$

(the scale $\boldsymbol{k}^2_> \equiv \max(\boldsymbol{k}^2,\boldsymbol{k}'^2)$ is replaced by $(\boldsymbol{k}-\boldsymbol{k}')^2$ in front of the BFKL kernel χ^ω_0)

Remarks

- Reproducing both low order DGLAP and BFKL evolutions provides novel Consistency Relations between the matrix $\boldsymbol{k}$-factorization scheme and $\overline{\mathrm{MS}}$, e.g. $[\Gamma_1]_{qq} = (C_F/C_A)[\Gamma_1]_{qg}$ at NLx order α_s^2/ω.
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- Note a source of ambiguity: integrated PDF are defined at $\gamma \sim 0$, all ω; but unintegrated ones are well defined by $\boldsymbol{k}$-factorization around different ω values: $\omega \sim 0$ (gluon) and $\omega \sim -1$ (quark)
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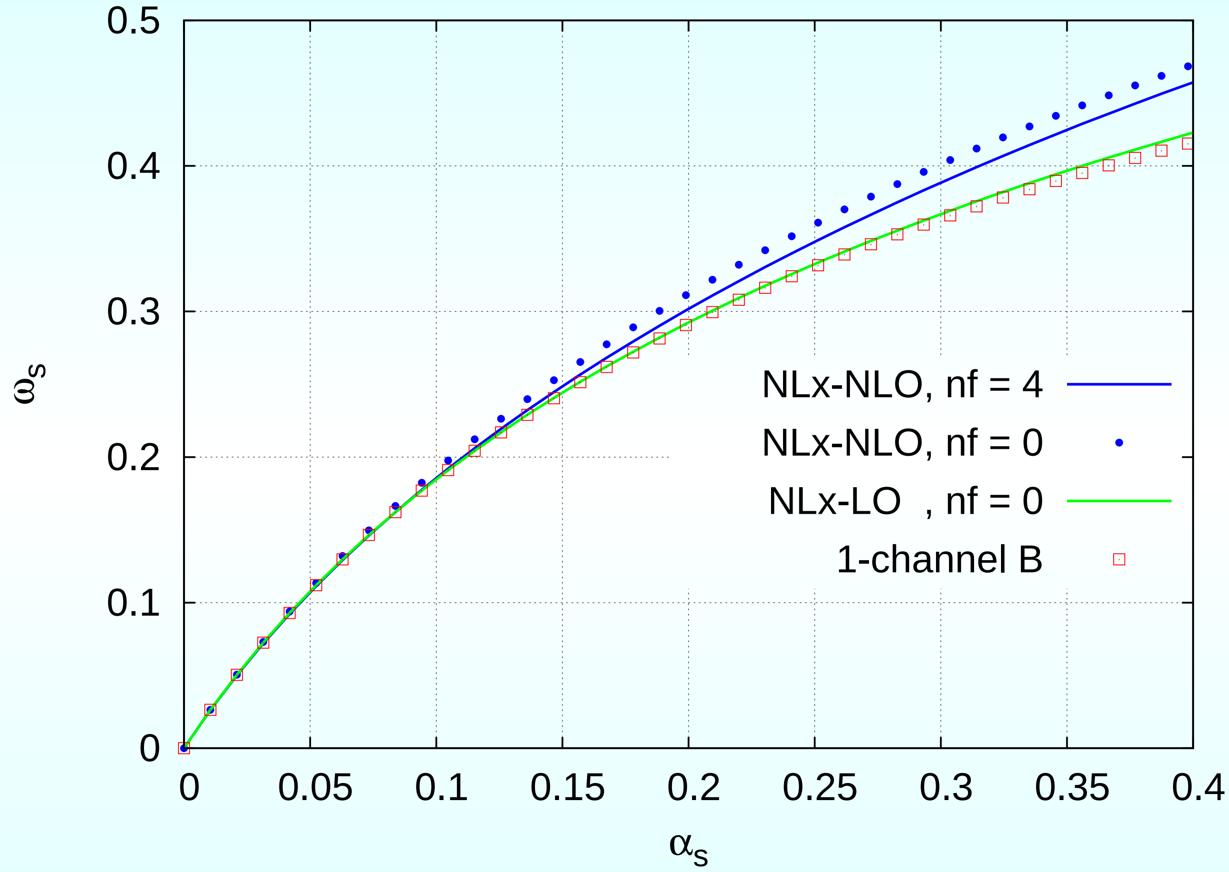
- We choose the NLO/NLx scheme: incorporates exact $\overline{\mathrm{MS}}$ anomalous dimension up to NLO and high-energy NLx BFKL kernel for the gluon channel
- Frozen coupling results are partly analytical, running coupling splitting functions obtained by a numerical deconvolution method.

Results: Hard Pomeron Exponent

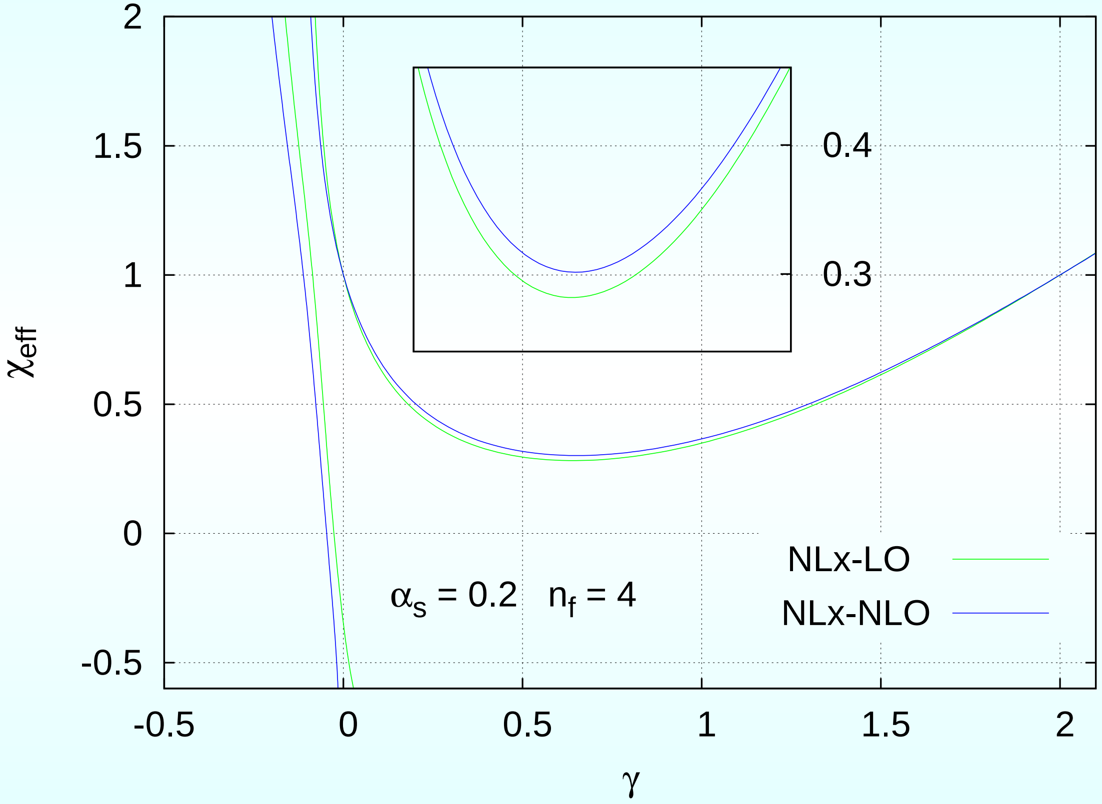
Frozen-α_s exponent $\omega_s(\alpha_s)$. LO/NLx scheme has only gg entry in $\mathcal{K}_1$



- Modest decrease from n_f-dependence (running α_s not included)
- LO/NLx scheme joins smoothly the gluon-channel limit at $n_f = 0$

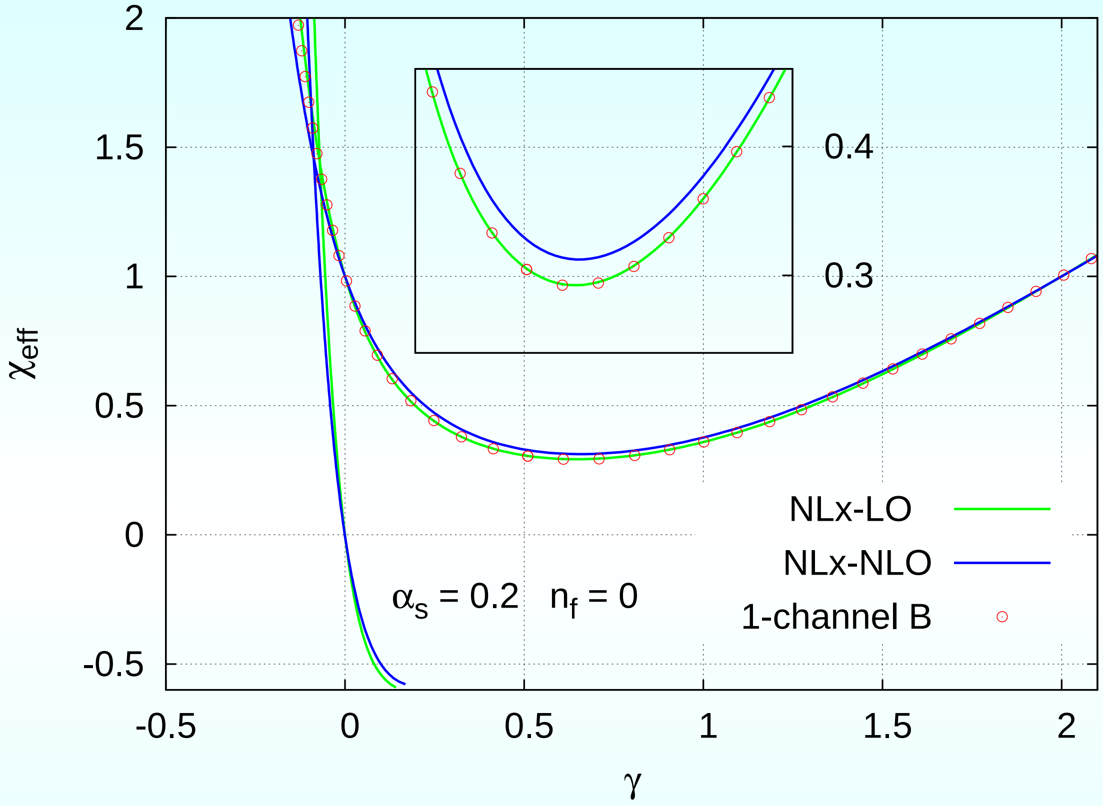
Effective Eigenvalue Functs ($n_f = 4$)

There are two, frozen α_s, resummed eigenvalue functions: $\omega = \chi_\pm(\alpha_s, \gamma)$ dual to the two anom. dim. eigenvalues $\gamma_\pm(\omega)$



- Fixed points at $\gamma = 0, 2$ and $\omega = 1 \Rightarrow$ momentum conservation in both collinear and anti-collinear limits.
- New subleading eigenvalue χ_-

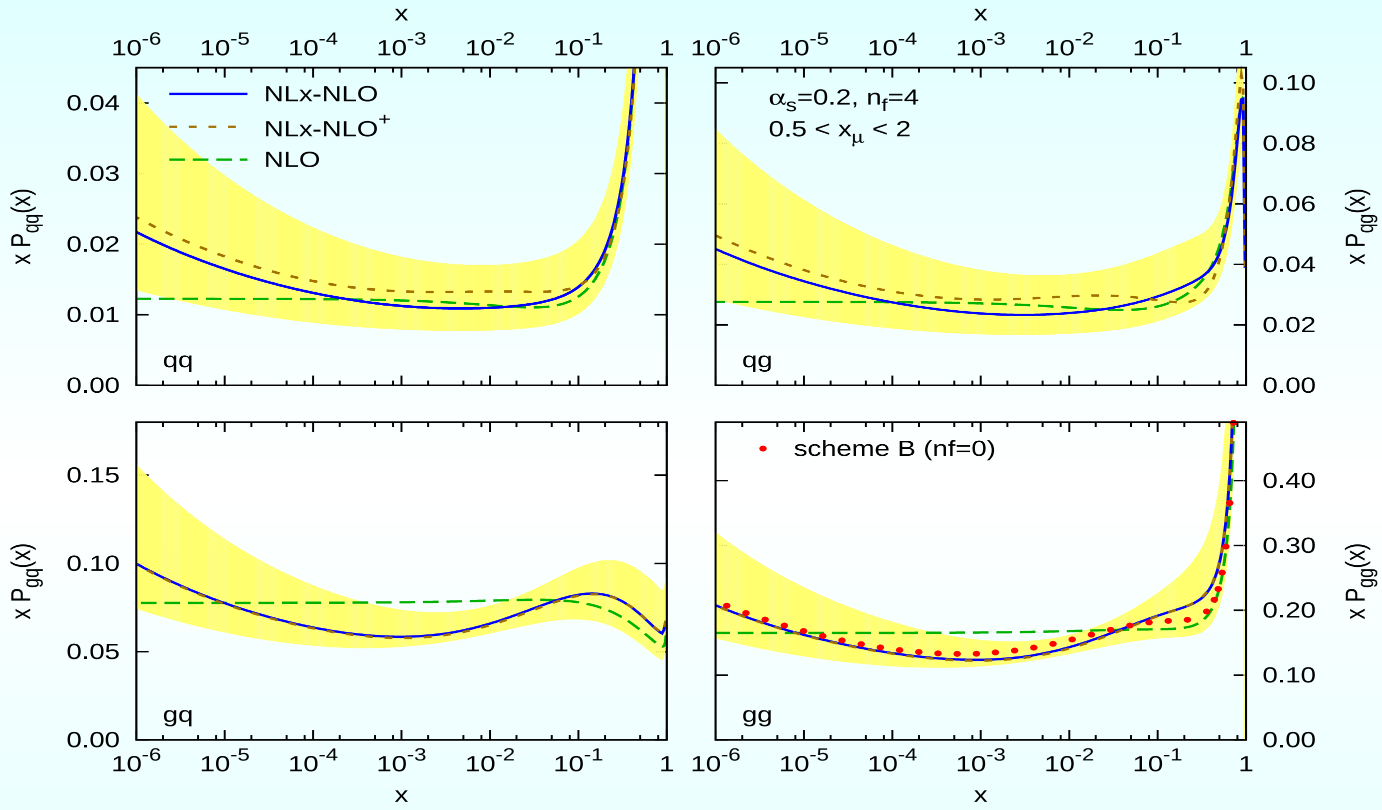
Effective Eigenvalue Functs ($n_f = 0$)



- Modest n_f-dependence of $\chi_+(\alpha_s, \gamma)$.
- NLx-LO scheme recovers the known gluon-channel result (in agreement with Altarelli Ball Forte) at $n_f = 0$.
- Level crossing of χ_- and χ_+ in the $n_f = 0$ limit.

Resummed Splitting Function Matrix

NLO$^+$ scheme includes, besides NLO, also NNLO terms $\sim \alpha_s^3/\omega^2$



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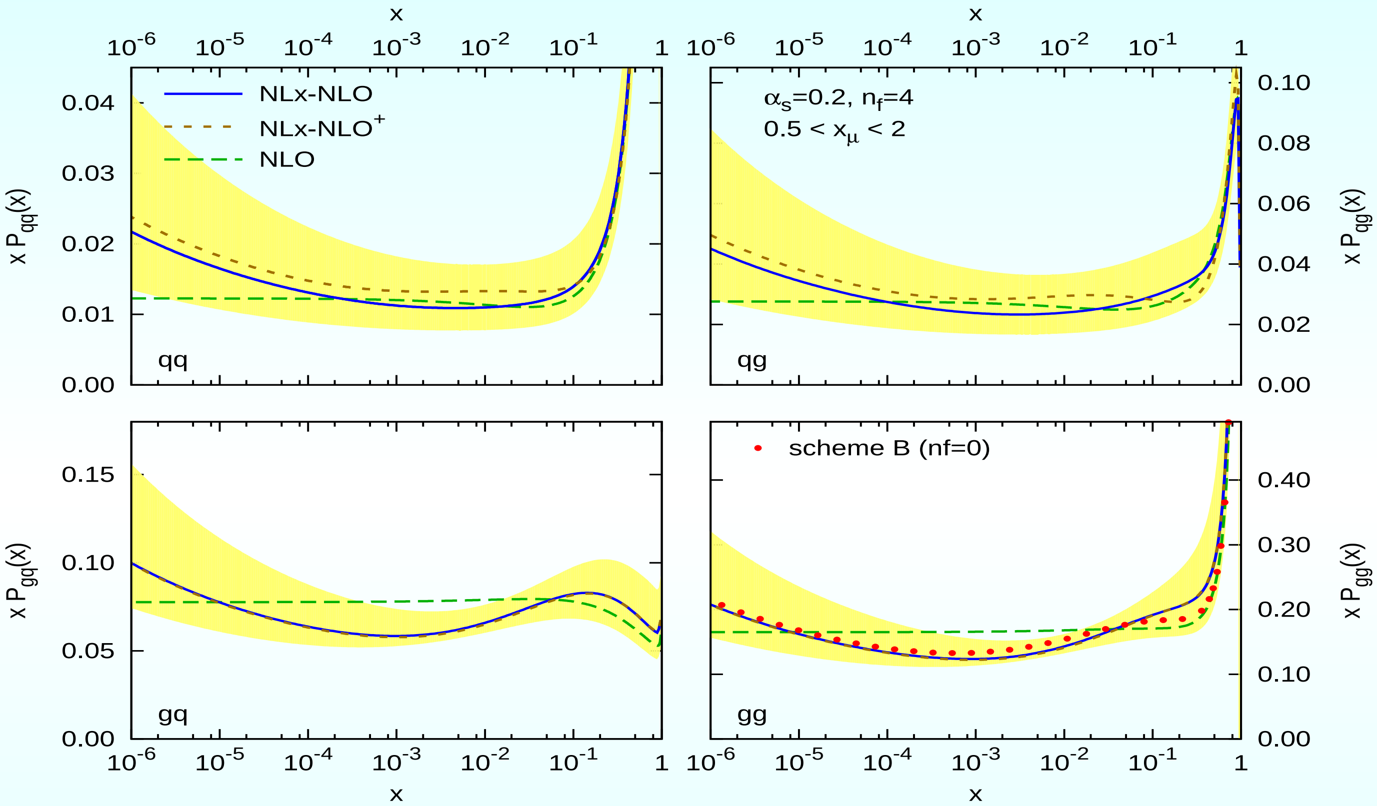

- Infrared cutoff independence insures (matrix) collinear factorization
- At intermediate $x \simeq 10^{-3}$ resummed P_{gg} and P_{gq} show a shallow dip
- Small-x rise of novel P_{qg} and P_{qq} delayed down to $x \simeq 10^{-4}$
- Scale uncertainty band $(0.25<x_\mu^2<4)$ larger for the (small) P_{qa} entries

Conclusions

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- Hard Pomeron and leading eigenvalue function are stable, with modest n_f-dependence.
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- Still need coefficient functions with comparable accuracy:
  take first LO impact factors with "exact kinematics"