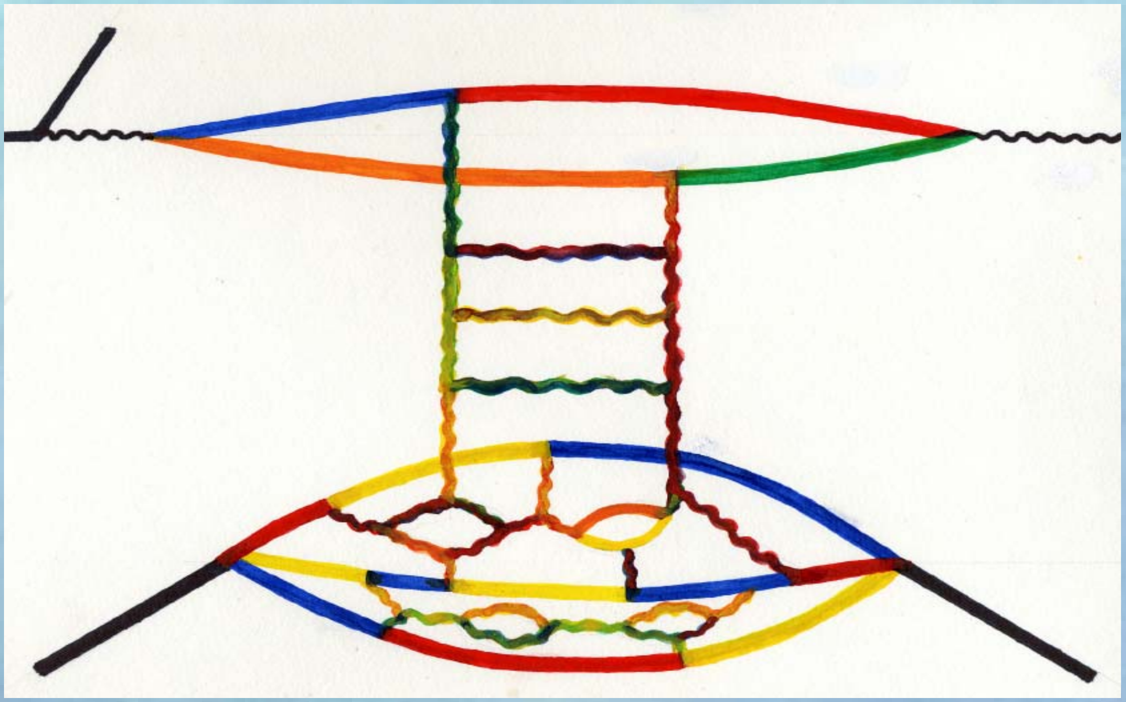
Gluon density from Discrete-BFKL Pomeron and HERA data

Henri Kowalski

& J. Ellis, L. Lipatov, D. Ross, G. Watt



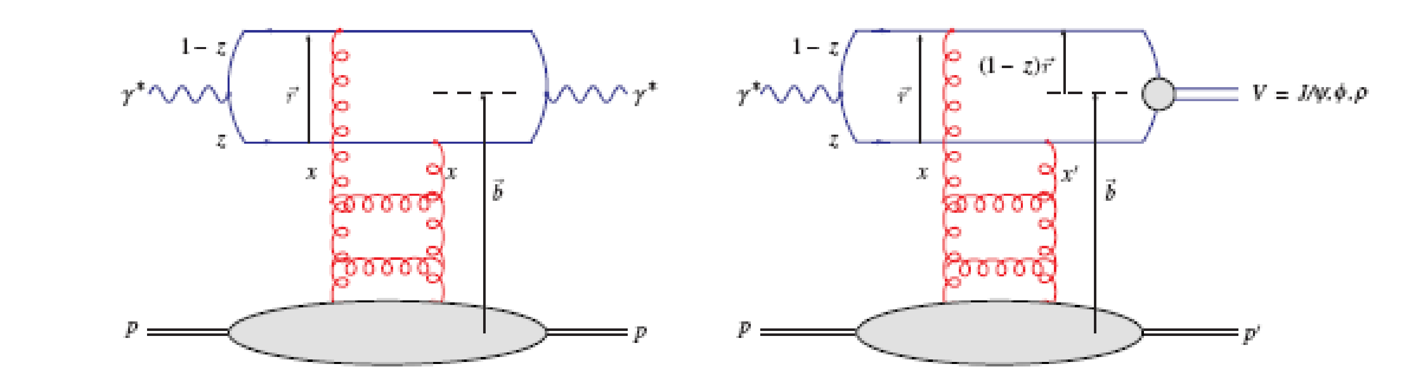
DIS 2009

Madrid, 27th of April 2009

Why Pomeron at HERA?

Because the same, universal, gluon density describes the properties of many reactions measured at HERA:

F_2 , inclusive diffraction,
exclusive J/Psi, Phi and Rho production
DVCS, diffractive jets



Successful description of gluon density evolulution by DGLAP but

DGLAP fits to HERA data produce gluon densities which tend to become negative at low-x and low Q^2

negative – MRST, MSTW
valence like – CTEQ, dipole model

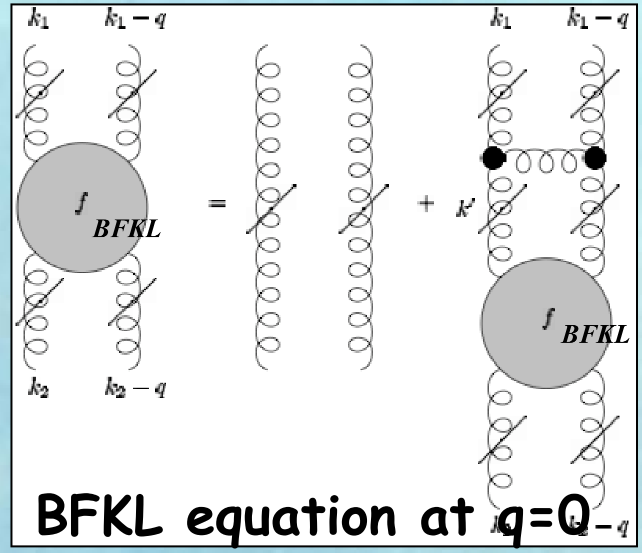
Why the Discrete-BFKL Pomeron?

BFKL gluon density could provide an alternative

Discrete-BFKL Pomeron

similarity with the non-perturbative Pomeron picture,
correspondence with the graviton in AdS/CFT
precision of HERA data

Basics of BFKL



BFKL equation at q=0

Conformal invariance

solved by finding a complete set of eigenfunctions

$$\omega \tilde{f}(\omega, \mathbf{k_1}, \mathbf{k_2}) = \delta^2(\mathbf{k_1} - \mathbf{k2}) + \frac{\alpha C_A}{\pi^2} \int \frac{d^2\mathbf{k}'}{(\mathbf{k_1} - \mathbf{k}')^2} \left[\tilde{f}(\omega, \mathbf{k}', \mathbf{k_2}) - \frac{\mathbf{k_1}^2}{\mathbf{k}'^2 + (\mathbf{k}' - \mathbf{k_1})^2} \tilde{f}(\omega, \mathbf{k_1}, \mathbf{k_2}) \right]$$

Eigen-functions

$$f_\omega(k^2) = \frac{(k^2)^{i\nu}}{\sqrt{k^2}}$$

$$\omega = \bar{\alpha}_s \chi(\nu)$$

Characteristic function

$$\chi(\nu) = -2\gamma_E - \psi(1/2 + i\nu) - \psi(1/2 - i\nu)$$

ψ is the Digamma function

Green function

$$f_{BFKL}(\omega, k_1, k_2) = \int_{-\infty}^{\infty} d\nu \left(\frac{k_1^2}{k_2^2}\right)^{i\nu} \frac{1}{2\pi^2 k_1 k_2} \frac{1}{(\omega - \bar{\alpha}_s \chi(\nu))}$$

Green function

$$f_{BFKL}(\omega, k_1, k_2) = \int_{-\infty}^{\infty} d\nu \left(\frac{k_1^2}{k_2^2}\right)^{i\nu} \frac{1}{2\pi^2 k_1 k_2} \frac{1}{(\omega - \bar{\alpha_s}\chi(\nu))}$$

usually approximated by $\chi(\nu) = 4\ln 2 - 14\zeta(3)\nu^2 + \ldots$ at small ν:

$$f(\sqrt{s}, \mathbf{k_1}, \mathbf{k_2}) \sim \frac{1}{\mathbf{k_1 k_2}} s^{4\overline{\alpha_s}\ln(2)} \frac{1}{\sqrt{\ln(s)}} \exp\left\{\frac{-\ln^2(\mathbf{k_1}/\mathbf{k_2})}{14\zeta(3)\overline{\alpha_s}\ln(s)}\right\}$$

dominated by the leading singularity ⇔ $\omega_0 = \alpha_S 4\ln(2)$

No direct generalization to running α_S, the function

$$f_\omega(k^2) = \frac{(k^2)^{i\nu}}{\sqrt{k^2}}$$

has varying ω because $\omega = \bar{\alpha}_s \chi(\nu)$

Constructing eigenfunc with fixed ω when α_s is running

Lipatov 86

NLO BFKL
$$\omega \equiv \chi(\alpha_s,\nu) = \bar{\alpha}_s\,(1 - A\bar{\alpha}_s)\,\chi_0\left(\frac{1}{2} + \bar{\alpha}_s B + i\nu + \frac{\omega}{2}\right) + \bar{\alpha}_s^2\chi_1(\nu).$$

Fadin, Lipatov
G. Salam
resummation, 98

with $\alpha_s(k)$ running ➔ ν becomes $\nu(k)$

BFKL equation simplifies around k_{crit}, definition: $\nu(k_{crit})=0$

BFKL ➔ Airy eq. around $k\sim k_{crit}$, solutions – Airy functions

$$\left[\frac{d^2}{d[\ln(k^2/k^2_{\mathrm{crit}})]^2} + \frac{\beta_0}{2\pi}\,\frac{\dot\chi\,(\alpha_s(k_{\mathrm{crit}}),0)}{\chi''(\alpha_s(k_{\mathrm{crit}}),0)}\ln\left(\frac{k^2}{k^2_{\mathrm{crit}}}\right)\right]\bar{f}_\omega(k) = 0,$$

away of k_{crit}

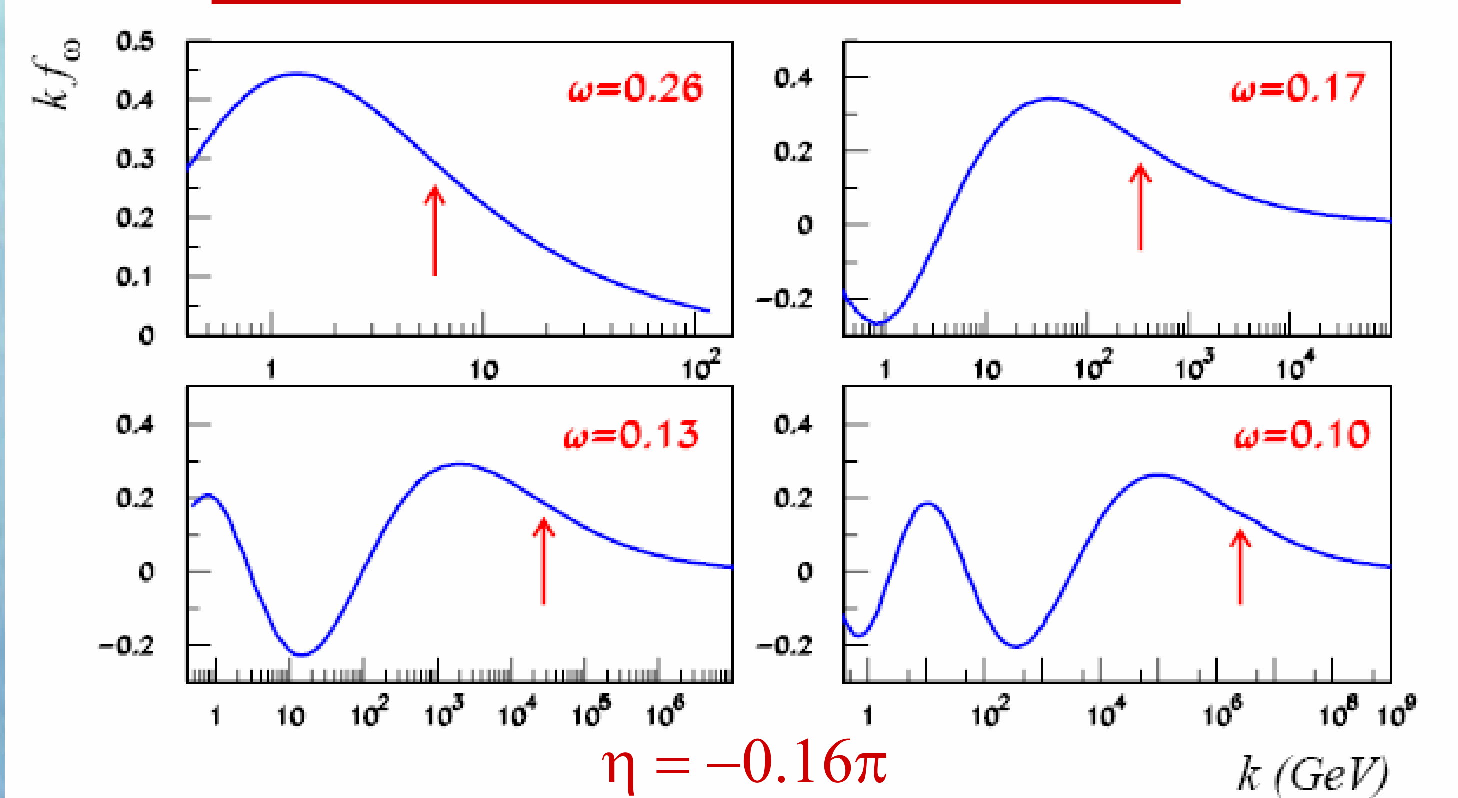
$$\boxed{\overline{f_\omega}(k) \;=\; e^{\pm i\varphi_\omega(k)},}$$

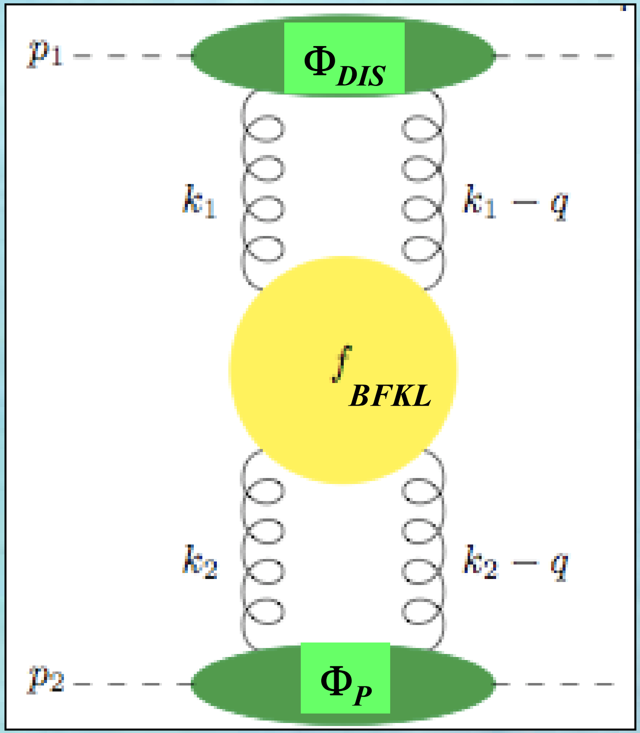
$$\varphi_\omega(k) = 2\int_k^{k_{\rm crit}} \frac{dk'}{k'}\left|\nu_\omega(k)\right.$$

Matching the solutions at $k=k_{crit}$ determines the phase of oscil.= $\pi/4$

Lipatov 86 ➔ encode the infrared behaviour of QCD by assuming a phase η at k_0, i.e. η determines ω_n, f_n

$$\boxed{\varphi_\omega(k_0) \;\equiv\; 2\int_{k_0}^{k_{\rm crit}} \frac{dk'}{k'}\,|\nu_\omega(k)| \;=\; \left(n-\frac{1}{4}\right)\pi+\eta,}$$





Structure functions in DIS

$$F_2(x, Q^2) = \int_x^1 dz \int \frac{dk}{k} \Phi_{\mathrm{DIS}}(z, Q, k) xg\left(\frac{x}{z}, k\right),$$

Φ_{DIS} *known in QCD*

Φ_P *barely known*

unintegrated gluon density

$$xg(x,k) = \sum_n \int \frac{dk'}{k'} \Phi_p(k') \left(\frac{k'x}{k}\right)^{-\omega_n} k^2 f^*_{\omega_n}(k') f_{\omega_n}(k),$$

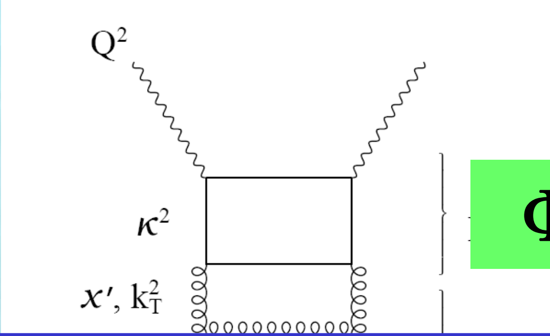
enhancement of leading eigenfun.by $(1/x)^{\omega}$

HERA
LHeC

$$xg(x,k) = \sum_n a_n x^{-\omega_n} k^{(2+\omega_n)} f_{\omega_n}(k).$$

no enhancement of leading eigenfun.

$$\Phi_p(k) = \sum_n a_n k^{(2-\omega_n)} f_{\omega_n}(k),$$



Φ_{DIS}

Kwiecinski, Martin Stasto

$$S_q(x,Q^2) = \frac{Q^2}{4\pi^2}\int\frac{dk^2}{k^4}\int_0^1 d\beta\int d^2\kappa'\alpha_S\left\{[\beta^2+(1-\beta)^2]\left(\frac{\boldsymbol{\kappa}}{D_{1q}}-\frac{\boldsymbol{\kappa}-\boldsymbol{k}}{D_{2q}}\right)^2\right.$$

$$\left.+\ [m_q^2+4Q^2\beta^2(1-\beta)^2]\left(\frac{1}{D_{1q}}-\frac{1}{D_{2q}}\right)^2\right\} f\left(\frac{x}{z},k^2\right)\Theta\left(1-\frac{x}{z}\right)$$

$\kappa' = \kappa-(1-\beta)k$ and

$$D_{1q} = \kappa^2+\beta(1-\beta)Q^2+m_q^2$$

$$D_{2q} = (\boldsymbol{\kappa}-\boldsymbol{k})^2+\beta(1-\beta)Q^2+m_q^2$$

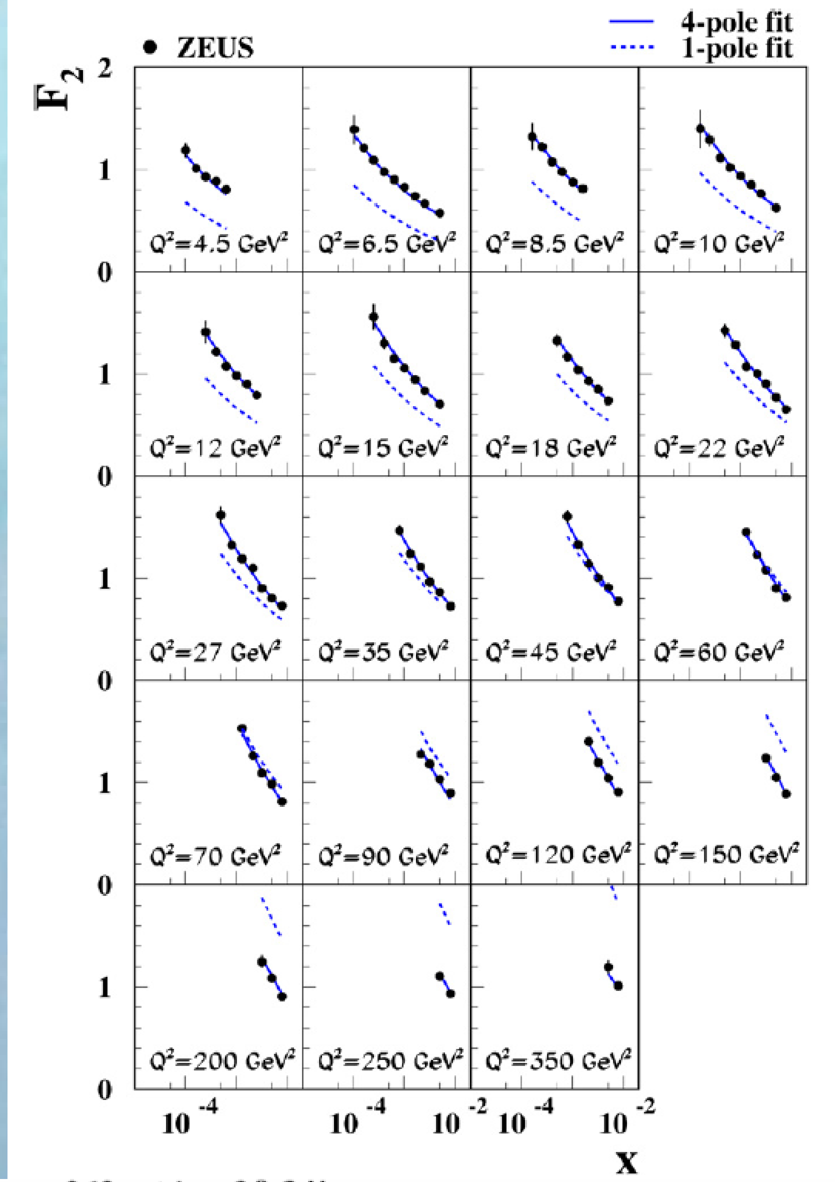
$$z = \left[1+\frac{\kappa'^2+m_q^2}{\beta(1-\beta)Q^2}+\frac{k^2}{Q^2}\right]^{-1}.$$

$$F_2 = \sum_q e_q^2(S_q+V_q),$$

Fit with charm

Correct qualitative behaviour from leading singularity

Excellent fit to data for $x < 10^{-2}$ with 4 poles



J. Ellis [a], H. Kowalski [b], D.A. Ross [a,c]

Physics Letters B 668 (2008) 51–56

The qualities of fits using up to 4 poles, and the corresponding pole residues, assuming $\eta = -0.16\pi$ at $k_0 = 0.3$ GeV

Number of poles	χ^2/N_{df}	a_1	a_2	a_3
1	11 894/101	0.478	–	–
2	1157/100	0.566	−0.98	–
3	167/99	0.707	0.87	3.70
4	83.3/98	0.483	−6.32	−26.0

Problem of the first solution:

Coefficients do not correspond to a reasonable $\Phi_P(k)$
i.e. to a non negative proton form factor

Solution: assume a realistic Φ_P and compute the coefficients from the projection

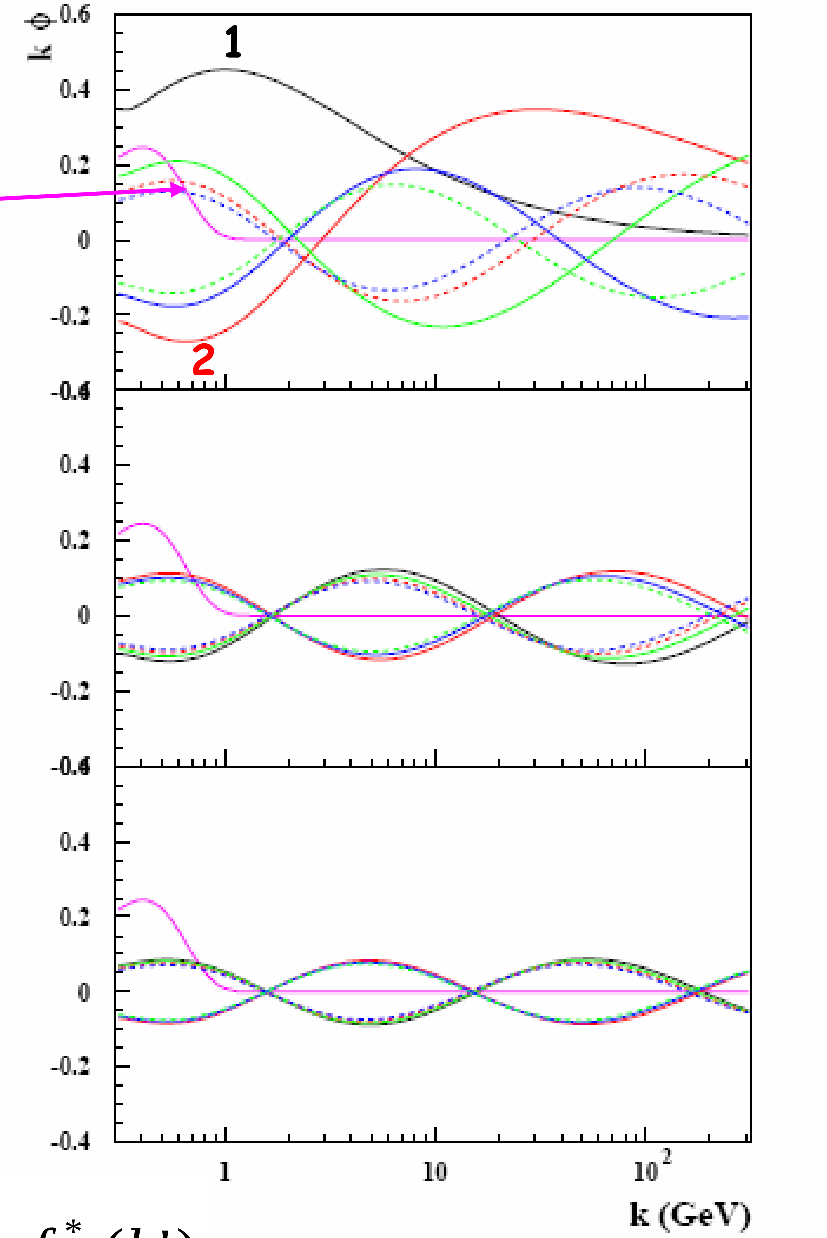
$$a_n = \int \frac{dk'}{k'} \Phi_P(k') k'^{-\omega_n} \; f^*_{\omega_n}(k')$$

➔ many eigenfunction are necessary

DAF Pomeron fit with

$\Phi_p = k^2 \exp(-bk^2)$

Eigenfunctions



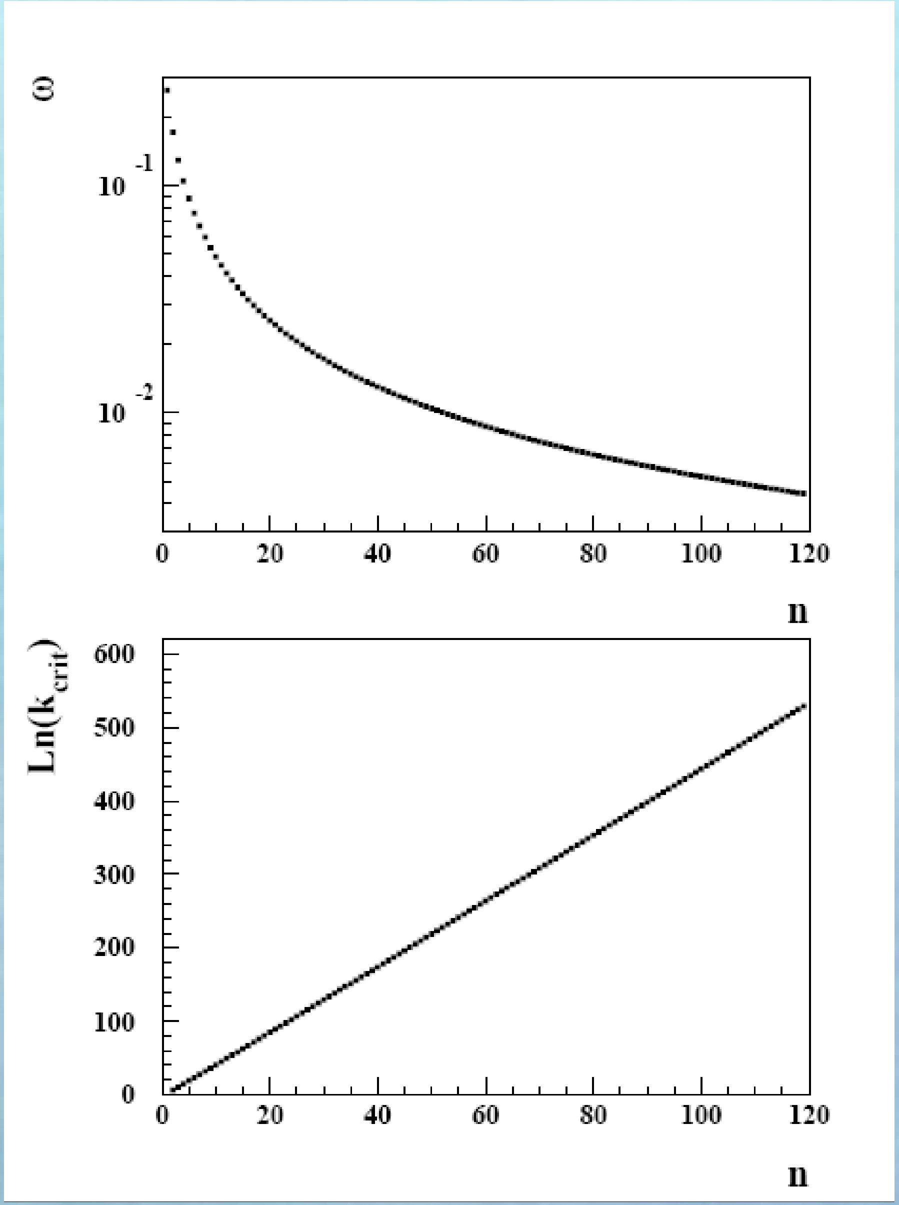
1-7

8-14

15-21

$$a_n = \int \frac{dk'}{k'} \Phi_P(k') k'^{-\omega_n} \; f^*_{\omega_n}(k')$$

ω_n and $\log(k_{crit})$



DAF Pomeron fit with

$\Phi_p = k^2 \exp(-bk^2)*(x_0)^\omega$

$\eta = -0.20*\pi$

$b = 1$ - 10 GeV^{-2}

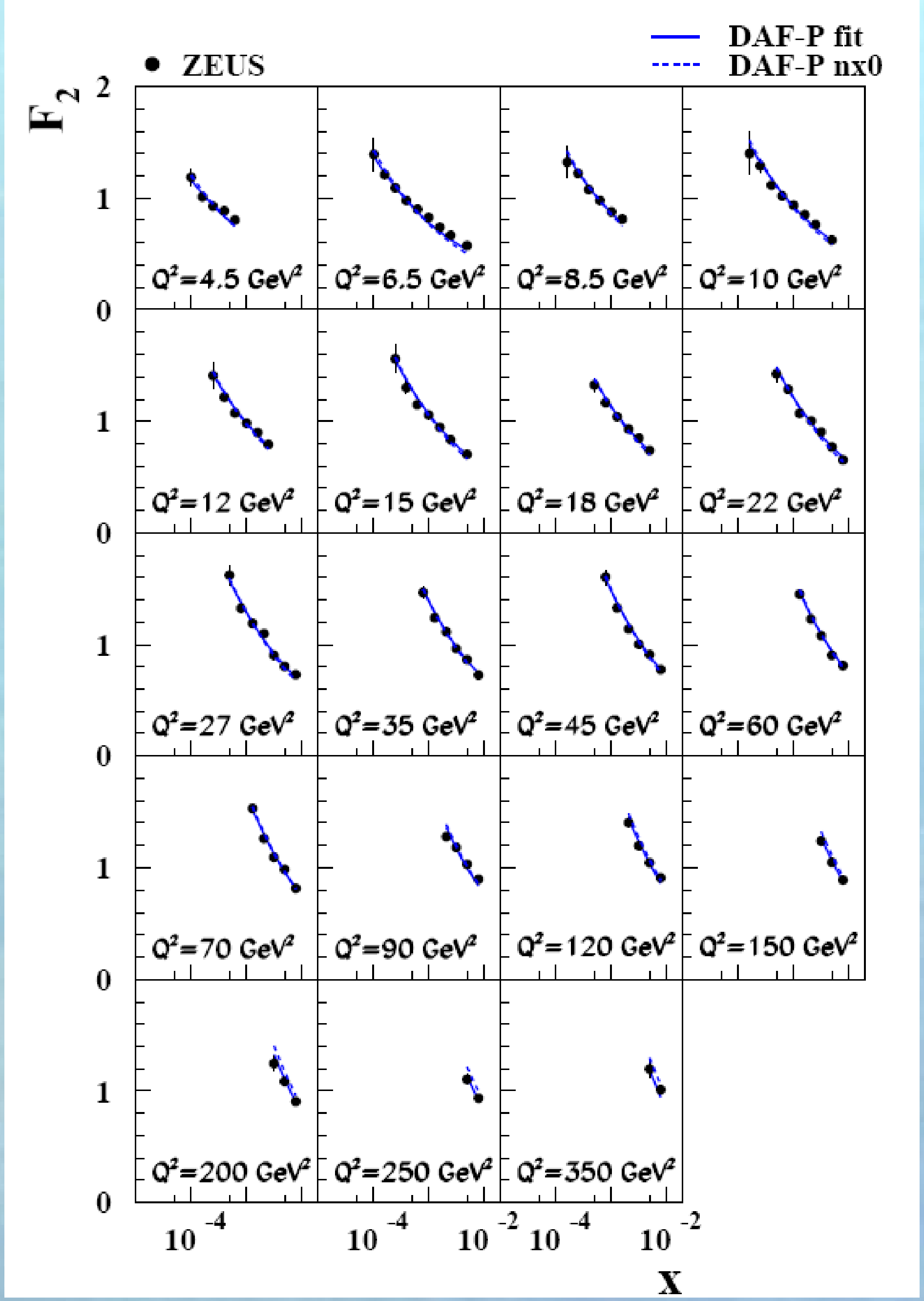
$x_0 \sim 0.2$-0.4

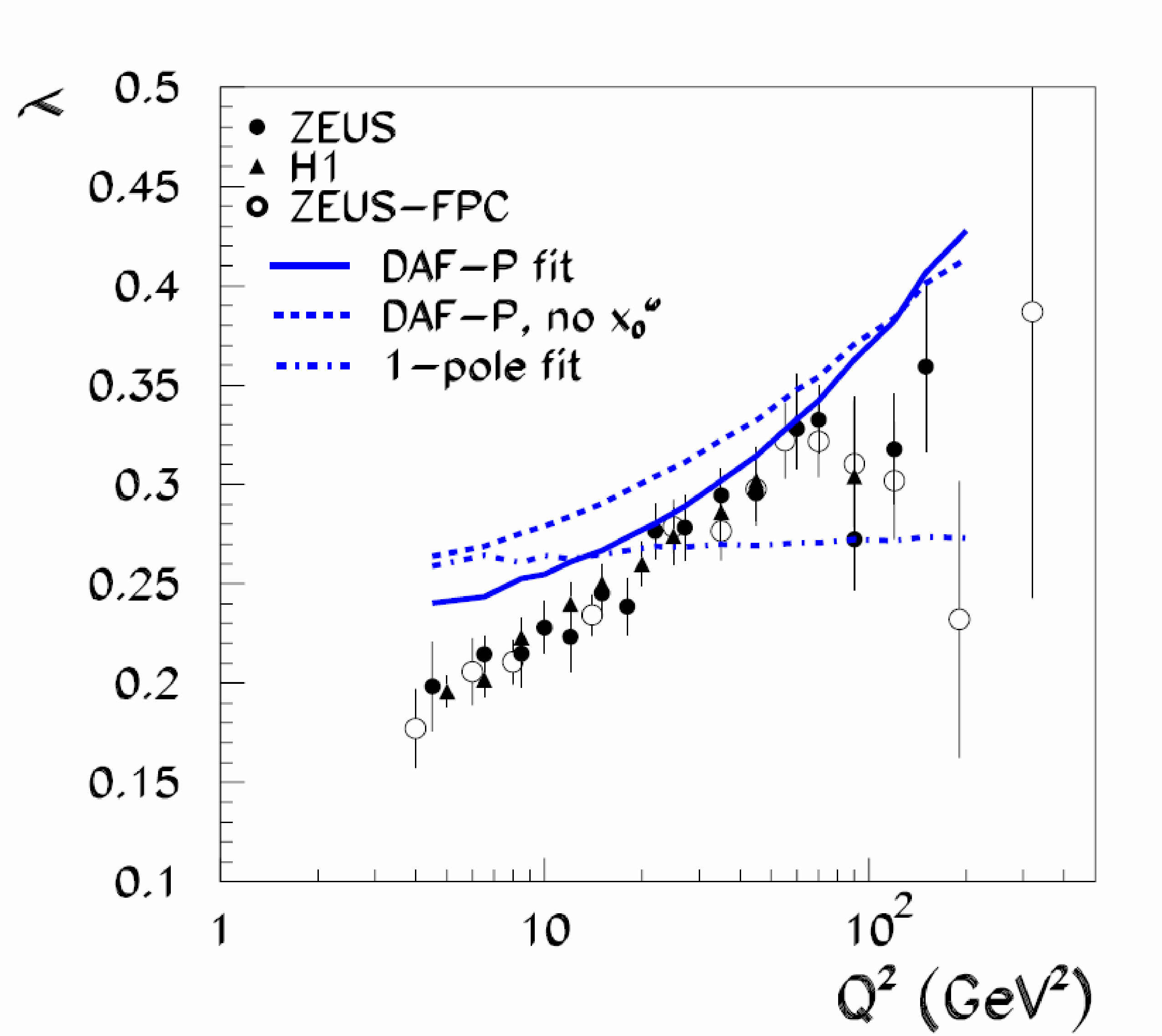
for $x < 0.001$

$\chi^2/ndf = 18/30$

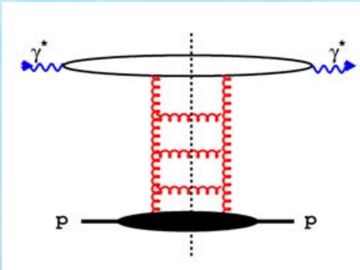
for $x < 0.01$

$\chi^2/ndf = 115/100$

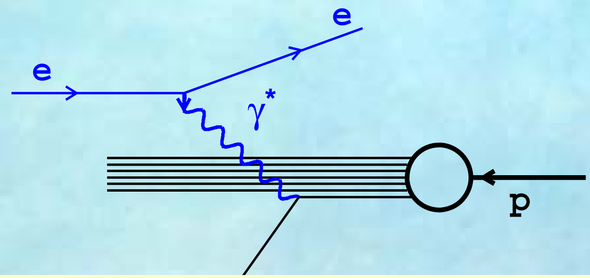




Sum of contributions with small eigenvalues can give a *larger* rate of rise than the leading eigenvalue !!!

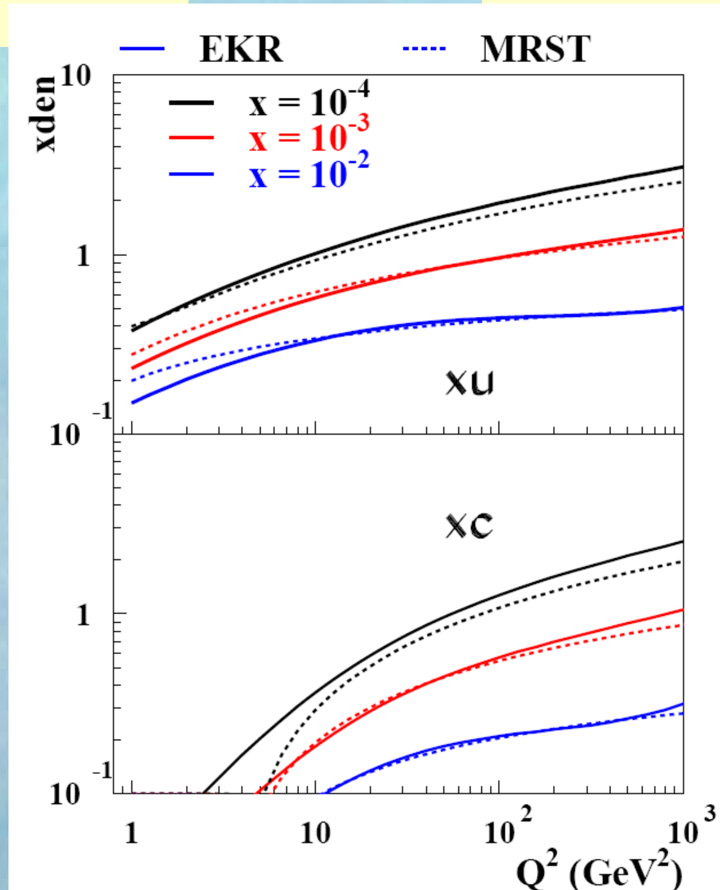


$$F_2(x,Q^2) = x\sum_q e_q^2 q(x,Q^2)$$



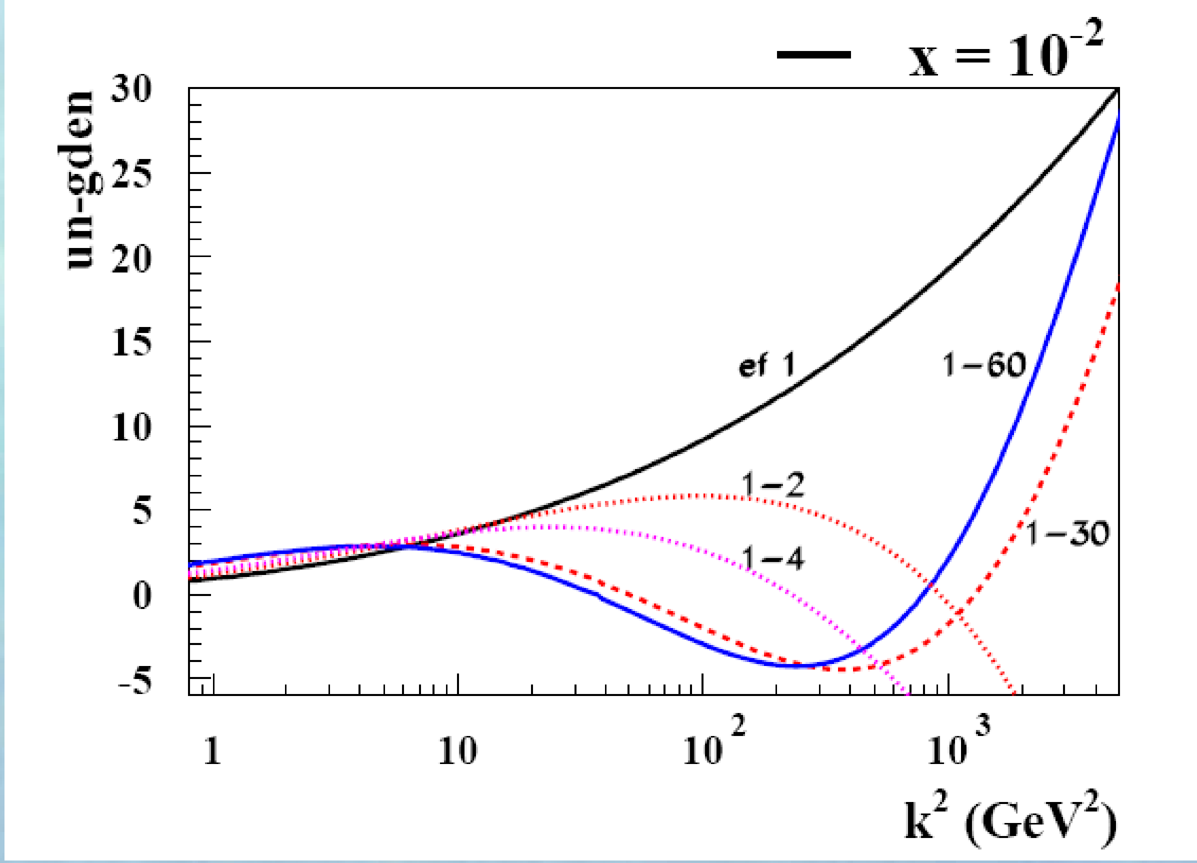
$$q(x,Q) = \int_0^Q \frac{dk}{k}\,\Phi_{DIS}(Q,k)\,xg(x,k)$$

$$q(x,\mu^2) = q_0(x) + \frac{\alpha_s}{\pi}\int_x^1 \frac{d\xi}{\xi} q_0(\xi)\left\{P_{qq}\left(\frac{x}{\xi}\right)\ln\mu/\kappa + \ldots\right.$$



Unintegrated gluon density

$$xg(x,k) = \sum_n \int \frac{dk'}{k'} \Phi_p(k') \left(\frac{k'x}{k}\right)^{-\omega_n} k^2 f^*_{\omega_n}(k') f_{\omega_n}(k),$$

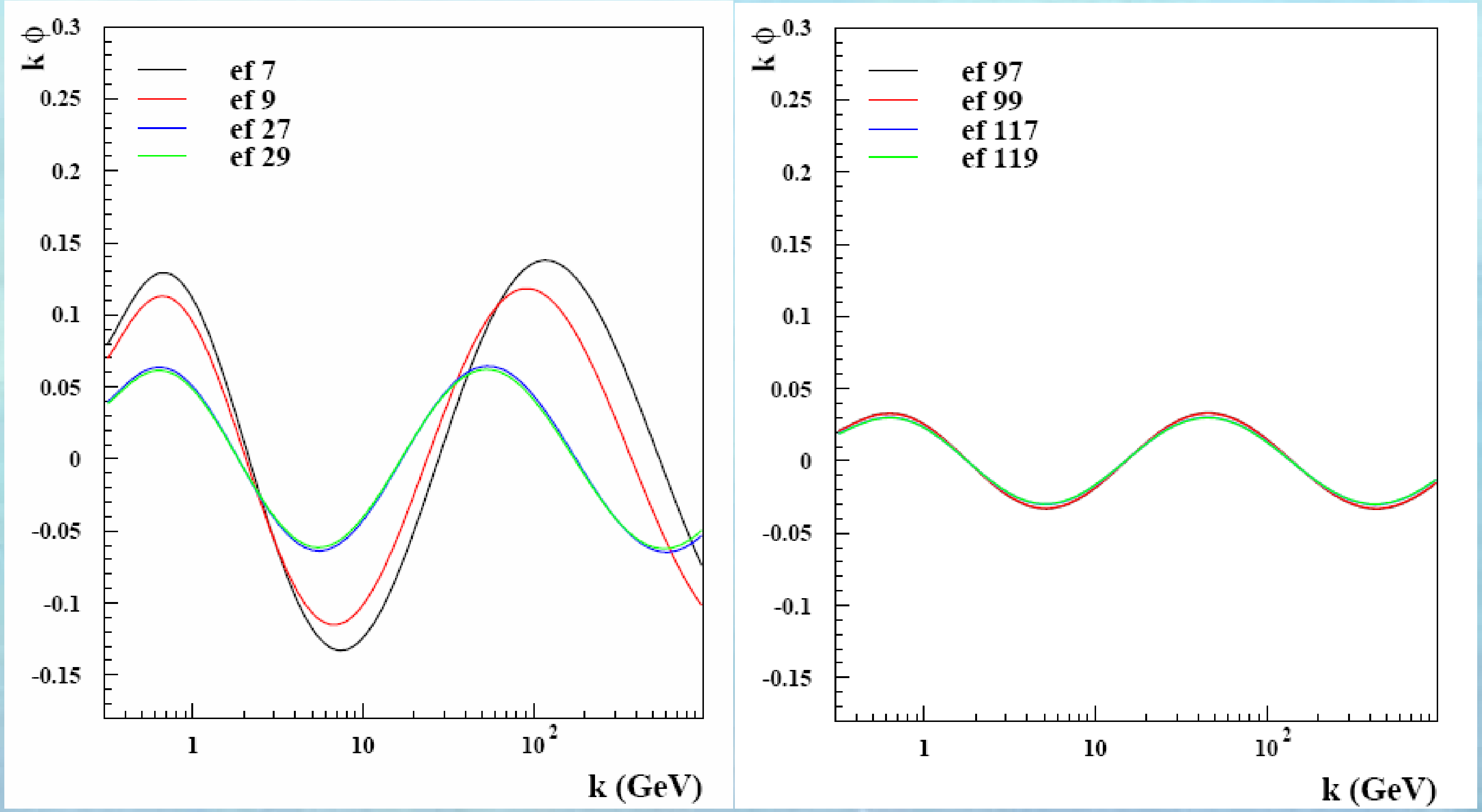


large contribution of subleading terms

Discreet solution is NOT dominated by the leading singularity

Problem: negative unintegrated gluon density

Origin of the problem: eigenfunc. converge at large n to the same function @ low k



Solution: $\eta(k_0)$ becomes a function of n

e.g.

$$\eta_n(k_0) = \eta_0 - (\eta_0 - \eta_{ST})\frac{\omega_n}{\omega_1}$$

$$\eta_n(k_0) = \eta_0 - (\eta_0 - \eta_{ST})\sqrt{\frac{\omega_n}{\omega_1}}$$

Convergence of ef's at large n, near the infrared boundary, allows to find a phase η_0 such that:

$$a_n = 0 = \int \frac{dk'}{k'} \Phi_P(k') k'^{-\omega_n} \; f^*_{\omega_n}(k'), \qquad n > n_0$$

when $\Phi_P(k) = k^2 \exp(-k^2 b), \qquad b = 2\,\mathrm{GeV}^{-2}$

➔ $\eta_0 = -0.74\pi$

with a very weak dependence on *b*

➔ **Contribution of ef's with large n is cancelled**

Lipatov:

Origin of a relations like

$$\eta_n(k_0) = \eta_0 - (\eta_0 - \eta_{ST})\sqrt{\frac{\omega_n}{\omega_1}}$$

could be due to a kind of Schroedinger eq (eigenvalue eq)
for a non-perturbative wave functions ψ

$$\omega_n \psi_n = K \cdot \psi_n$$

$\eta_n(k_0)$ is a phase of ψ_n near pert.to non-pert. boundary

Note, BFKL eq can be cast into the form of Schroedinger eq

➔ non-perturbative and perturbative solutions should join smoothly

DAF-Pomeron fit with shifts

$$\eta_n(k_0) = \eta_0 - (\eta_0 - \eta_{ST})\sqrt{\frac{\omega_n}{\omega_1}}$$

$$\eta_0 = -0.74\pi, \eta_{ST} = -0.02\pi$$

$$\chi^2 / N_{df} = 107/98$$

$$\eta_n(k_0) = \eta_0 - (\eta_0 - \eta_{ST})\frac{\omega_n}{\omega_1}$$

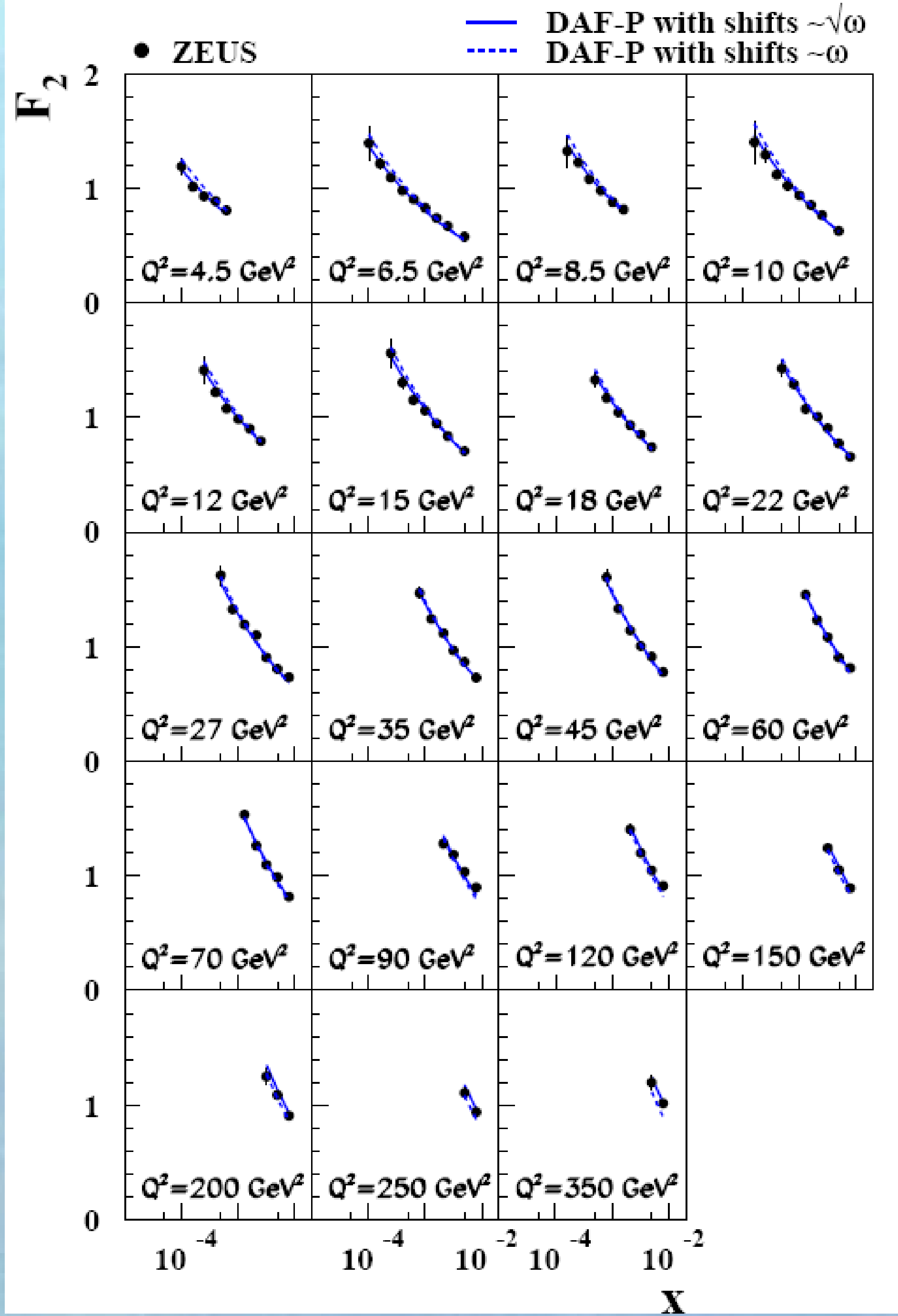
$$\eta_0 = -0.74\pi, \eta_{ST} = -0.02\pi$$

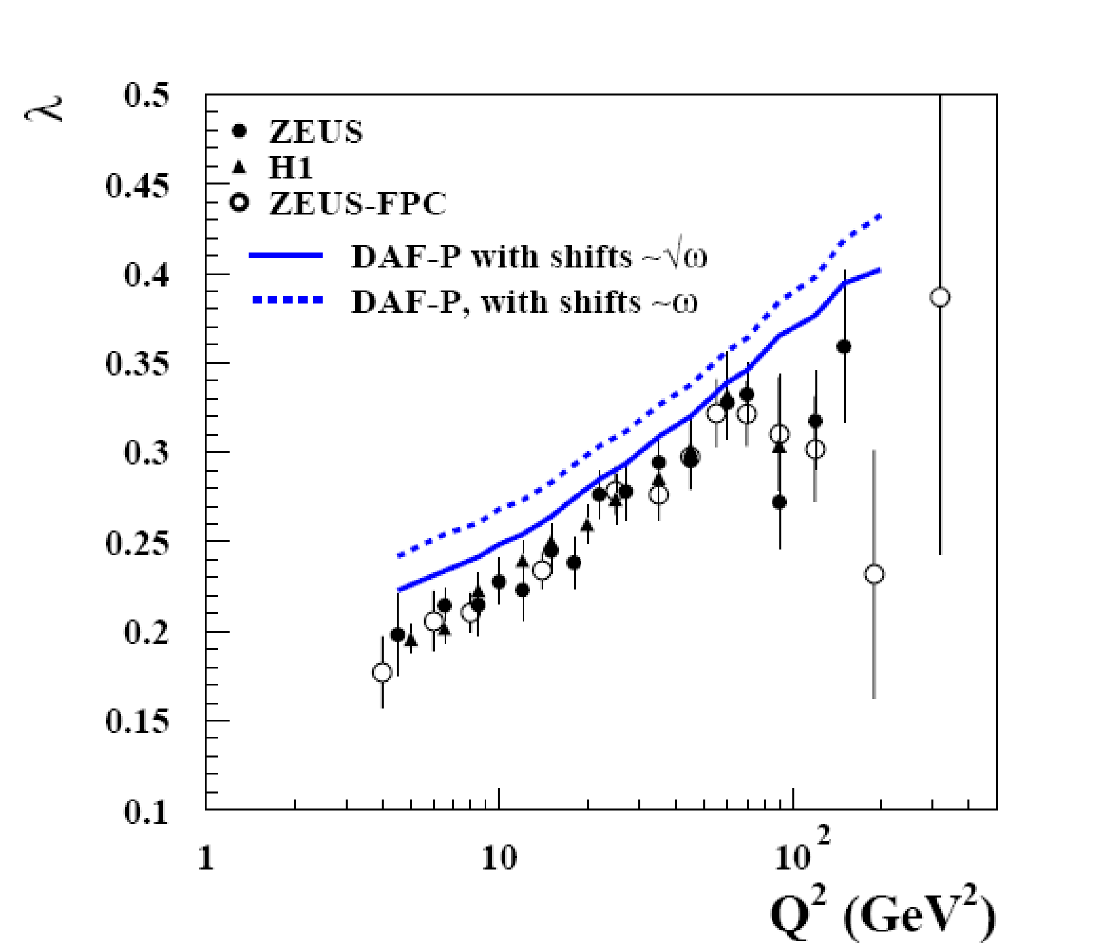
$$\chi^2 / N_{df} = 288/98$$

$$\Phi_p = k^2 \exp(-bk^2)*(x_0)^{\omega}$$

$$b = 2\ \text{GeV}^{-2}$$

$$x_0 \sim 1.0$$





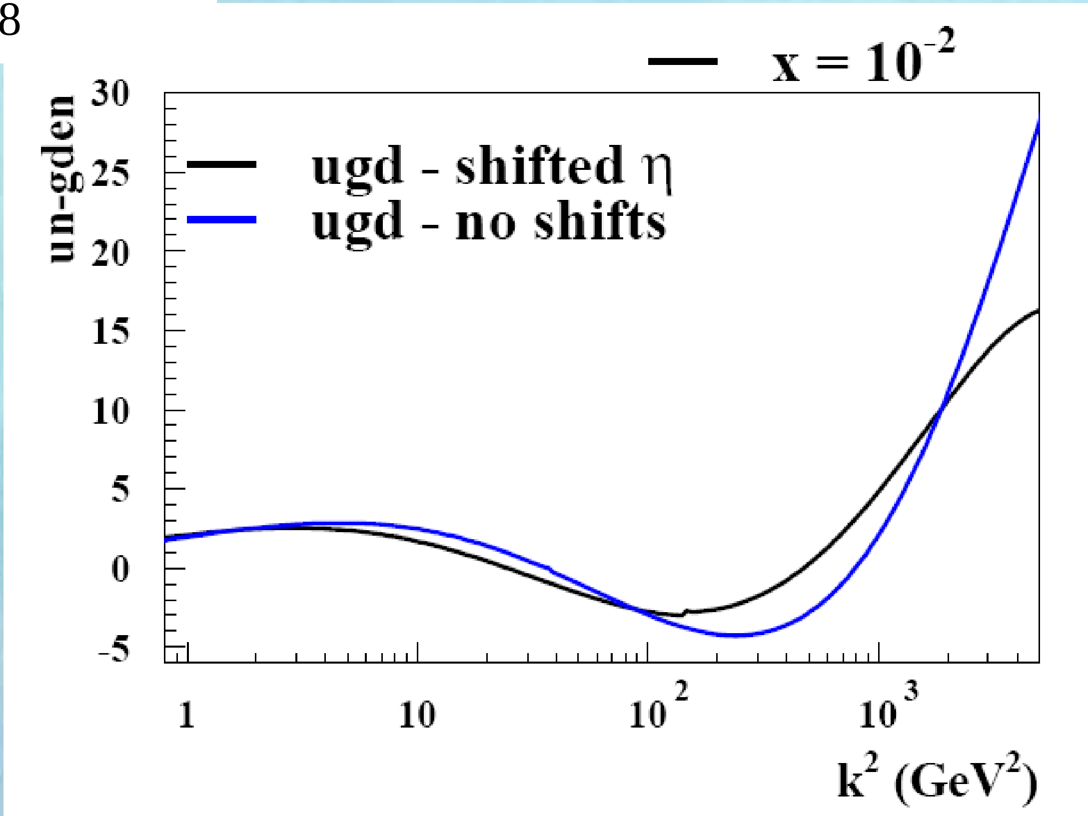
λ
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
ZEUS
H1
ZEUS-FPC
DAF-P with shifts ~√ω
DAF-P, with shifts ~ω
1
10
10^2
Q^2 (GeV^2)

$$\eta_n(k_0) = \eta_0 - (\eta_0 - \eta_{ST})\sqrt{\frac{\omega_n}{\omega_1}}$$

$$\eta_0 = -0.74\pi, \eta_{ST} = -0.02\pi$$

$$\chi^2 / N_{df} = 107/98$$

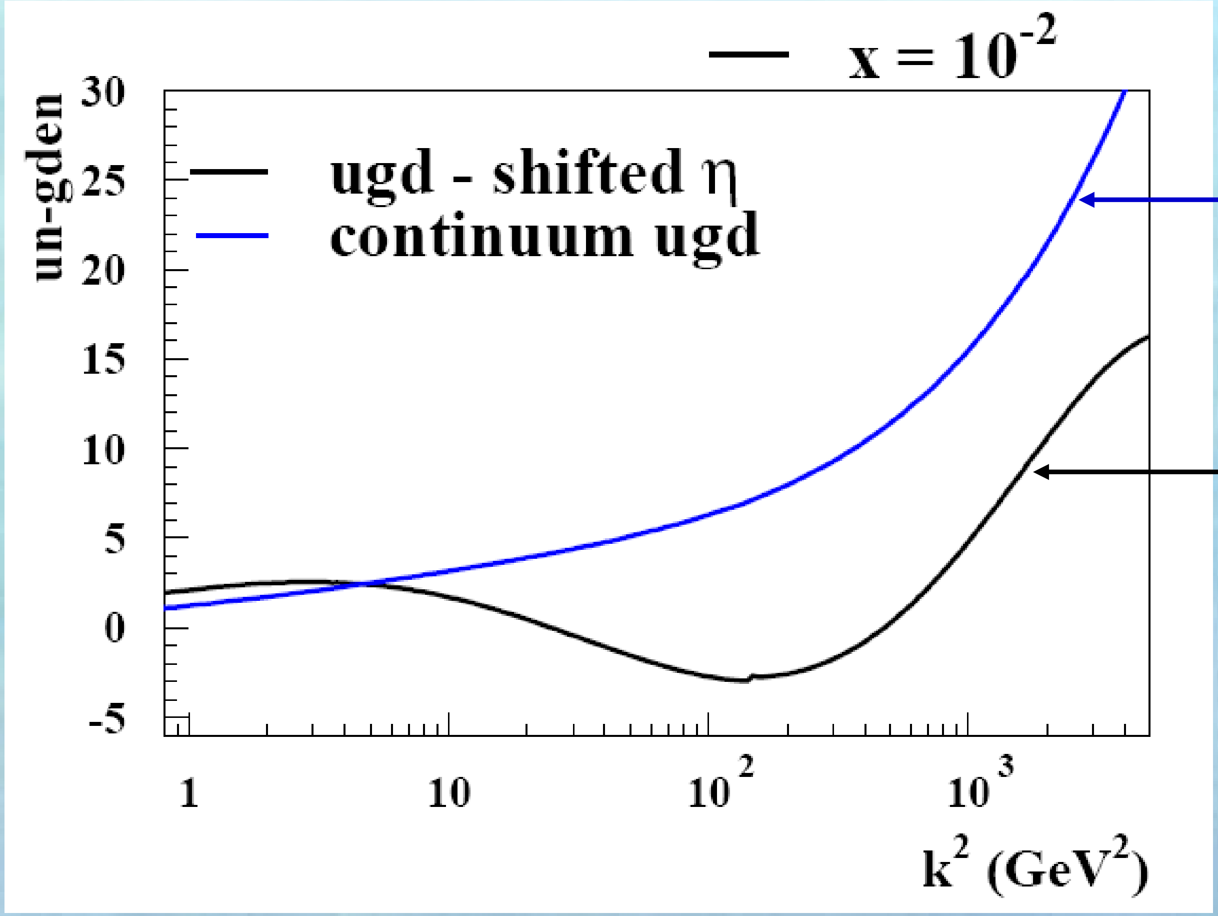
Unintegrated Gluon Desity



the problem of unintegrated gluon density is close to being solved,

we have to investigate systematically the complete space of solution, solvable technical problem

Unintegrated Gluon Desity



DGLAP-like

Discrete Pom

Summary

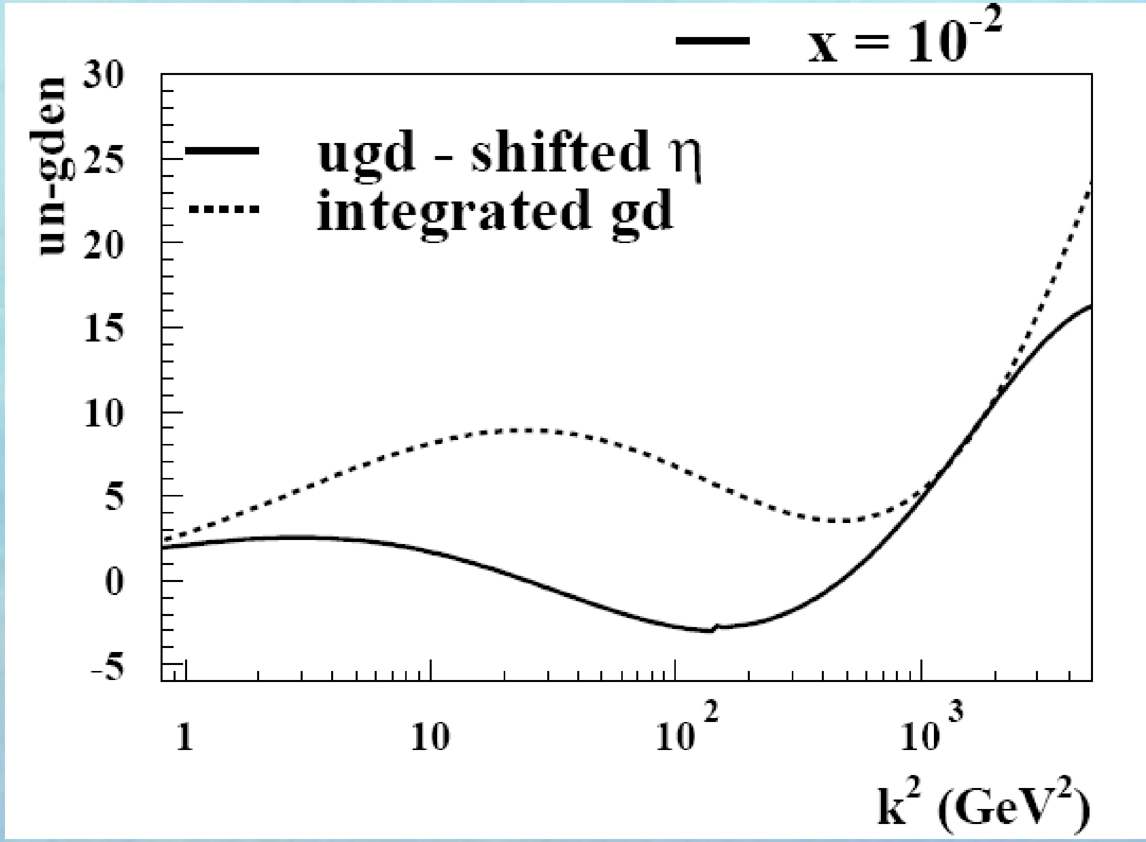
Gluon density obtained from the Discrete BFKL Pomeron describe the low x HERA data very well

Discrete-P gluon density is determined by a set of non-perturbative phases η

$\eta \leftrightarrow \omega$ relation is well determined by data
→ first information about gluons
in the non-perturbative region, confinement?

Discrete-P GD seems to have a very different shape than the DGLAP GD
→different X-section for gluon jets at LHC?,
different F_2 at very low-x at LHeC?
measurement of q dependence at EIC…

END of the Talk


x = 10^-2
un-gden
ugd - shifted η
integrated gd
30
25
20
15
10
5
0
-5
1
10
10^2
10^3
k^2 (GeV^2)

Where Do BFKL and DGLAP Meet

Lipatov, private communication

Unintegrated BFKL gluon density (LO, no running α_s)

$$xg(x,k^2) = \int d\gamma \Phi_p(\gamma) \left(\frac{k^2}{\mu^2}\right)^\gamma x^{-\bar{\alpha_s}\chi(\gamma)} = \int d\gamma \Phi_p(\gamma) \exp(F(\gamma))$$

$\gamma = ½ + i\nu$

Saddle point

$$(F(\gamma))' = (\gamma \ln(k^2/\mu^2) + \bar{\alpha_s} \ln(1/x)\chi(\gamma))' = 0$$

$$\chi(\gamma) = \frac{1}{\gamma} - 2\zeta(3)\gamma^2 + \cdots$$

$$\gamma^2 = \frac{\bar{\alpha} \ln(1/x)}{\ln(k^2/\mu^2)}$$

$$\omega \approx \bar{\alpha_s}/\gamma = \sqrt{\frac{\bar{\alpha_s} \ln(k^2/\mu^2)}{\ln(1/x)}}$$

valid if $\bar{\alpha}(k^2) \ln(1/x) \ll 1,$

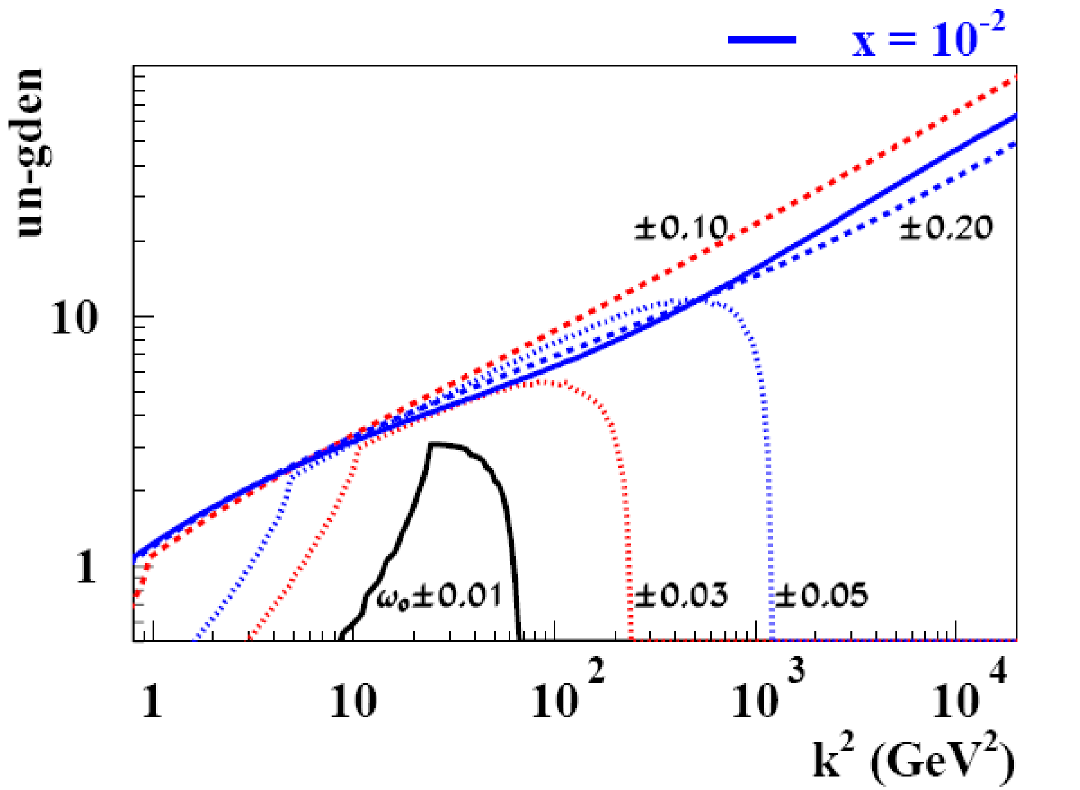
! not fulfilled for HERA or even Higgs at LHC !

equal to DLL limit of DGLAP (LO, no running α_s)

Continuum ugd

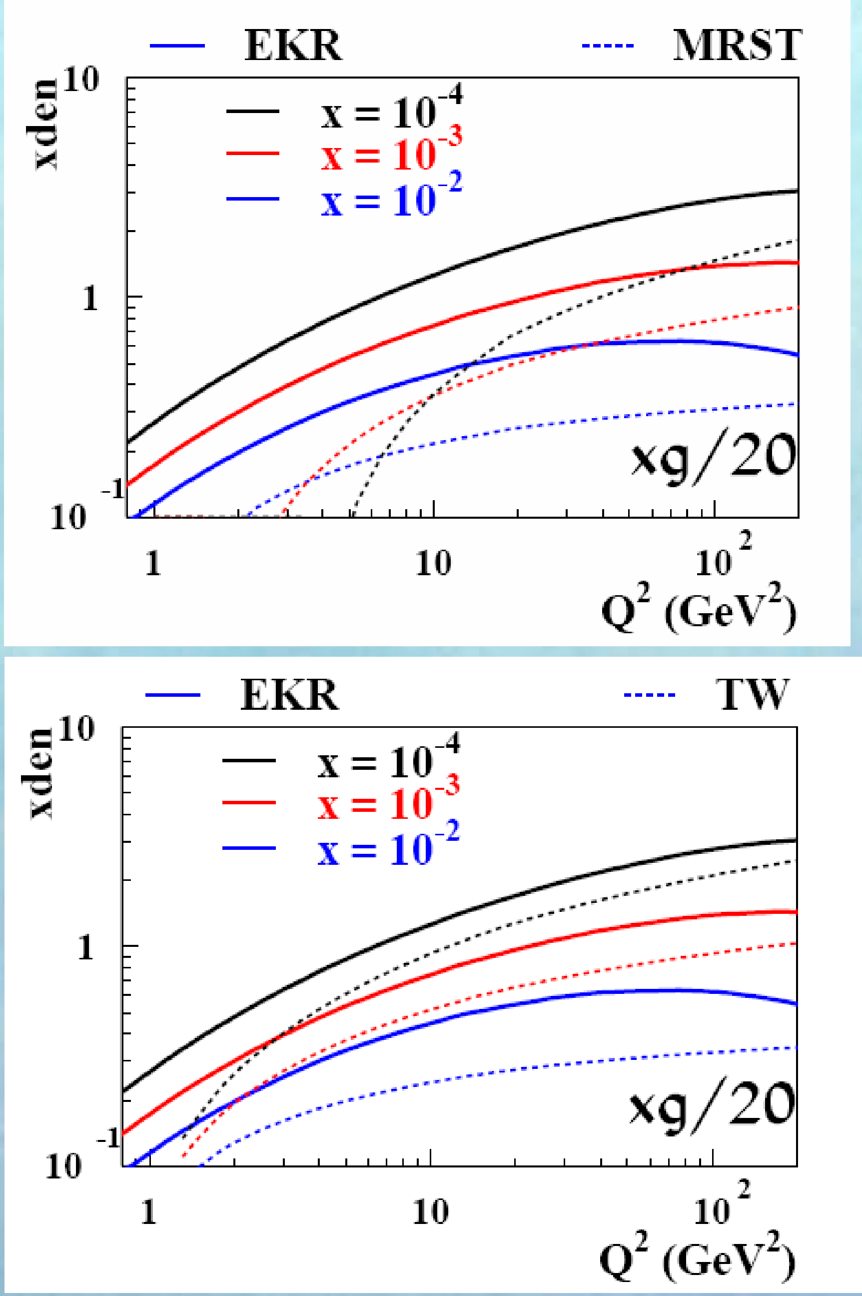
$$\Delta xg(x,k^2)_{continuum} \;=\; k^2\int_{\nu_0}^{\infty} d\nu \frac{dk'}{k'}\Phi(k')\phi_\nu(k')\phi_\nu(k)\left(\frac{xk'}{x_0 k}\right)^{-\omega(\nu,k)} .$$

$$\phi_\nu(k) \;=\; \frac{1}{\sqrt{\pi}k}\sin\left(\nu\ln(k^2/k_0^2)-\eta\pi\right),$$



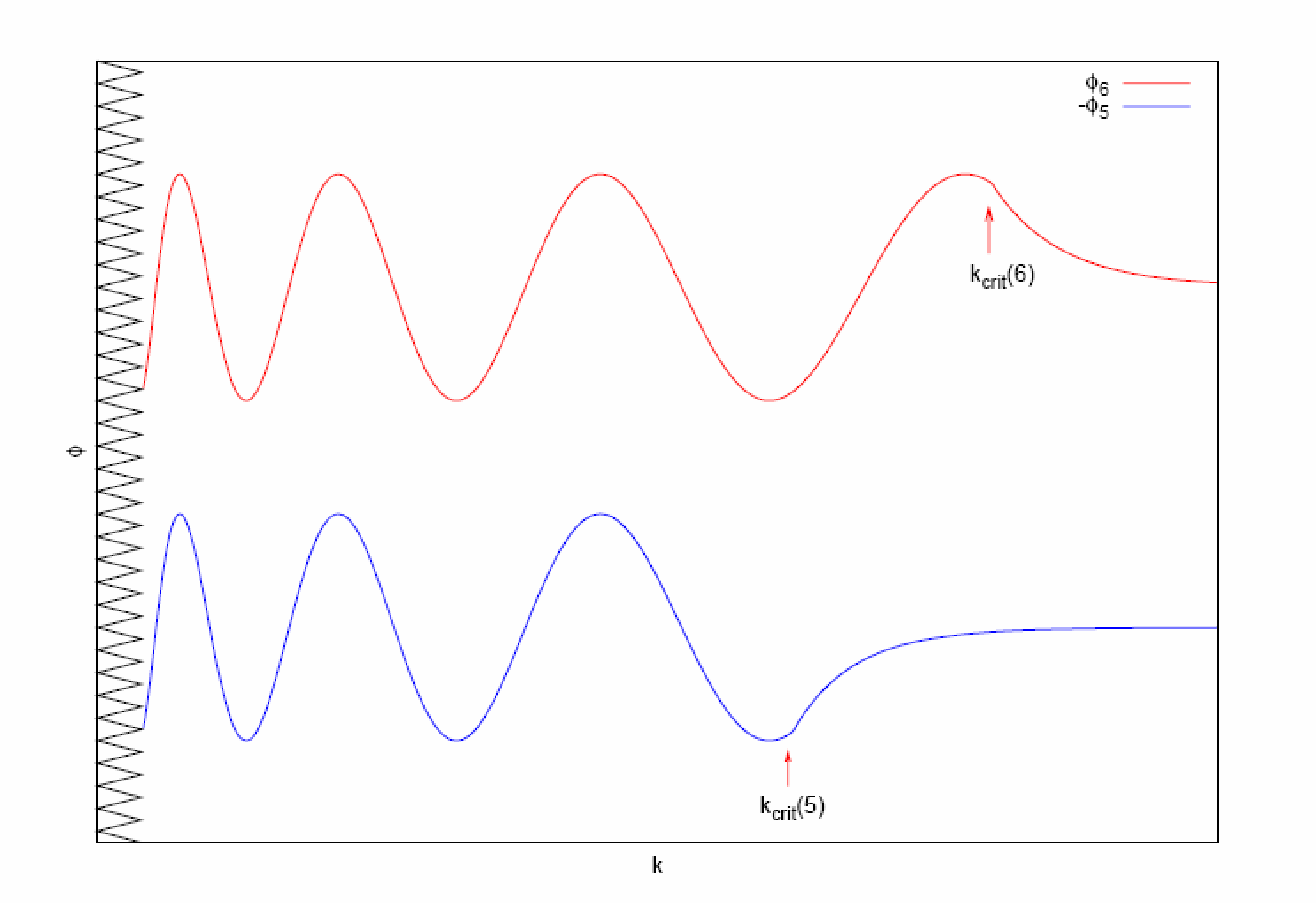
Continuum ugd is dominated by contributions close to the leading ef

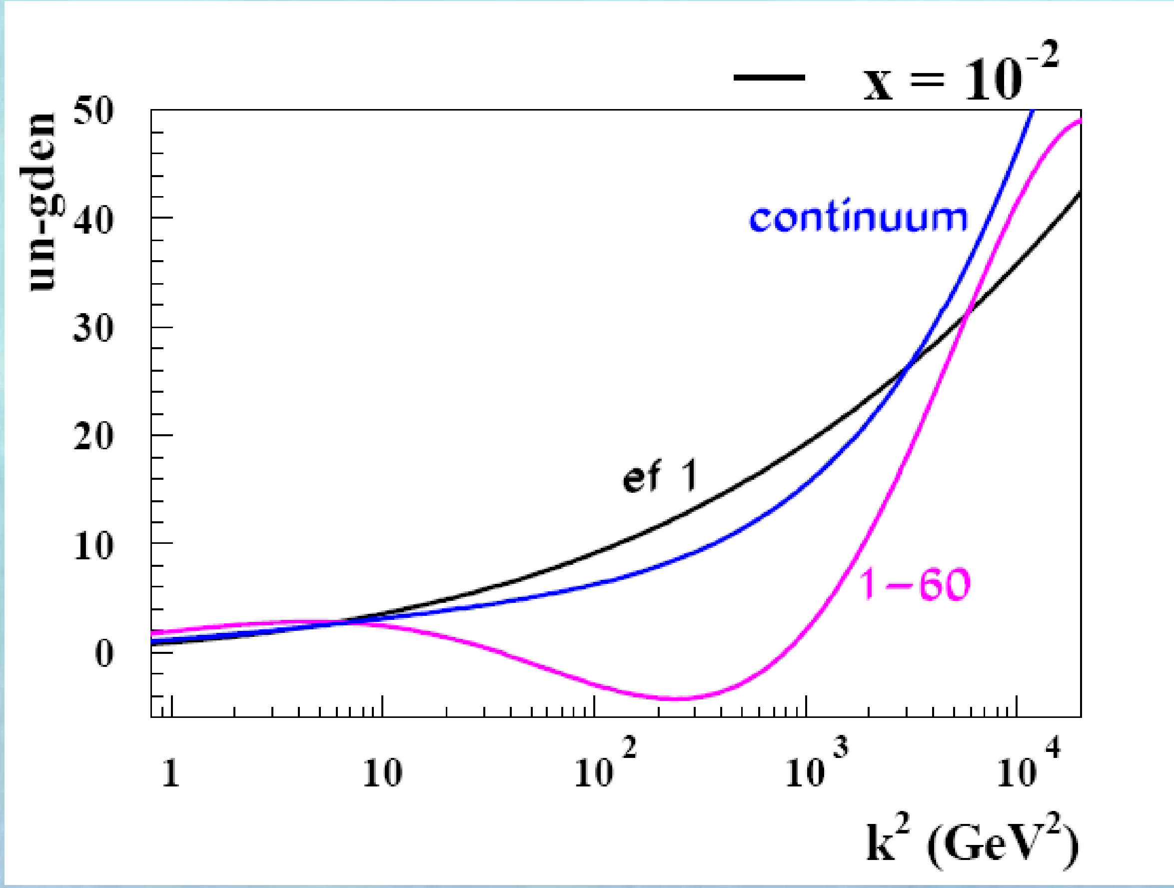
➔ Discreet and continuum solutions are very different



EKR
MRST
xden
$x = 10^{-4}$
$x = 10^{-3}$
$x = 10^{-2}$
xg/20
Q^2 (GeV^2)
EKR
TW
xden
$x = 10^{-4}$
$x = 10^{-3}$
$x = 10^{-2}$
xg/20
Q^2 (GeV^2)

Cartoon





x = 10^-2
un-gden
50
40
30
20
10
0
continuum
ef 1
1−60
1
10
10^2
10^3
10^4
k^2 (GeV^2)

Pomeron and Gauge/String Duality

Brower, Polchinski, Strassler, and Tan, hep-th/0603115

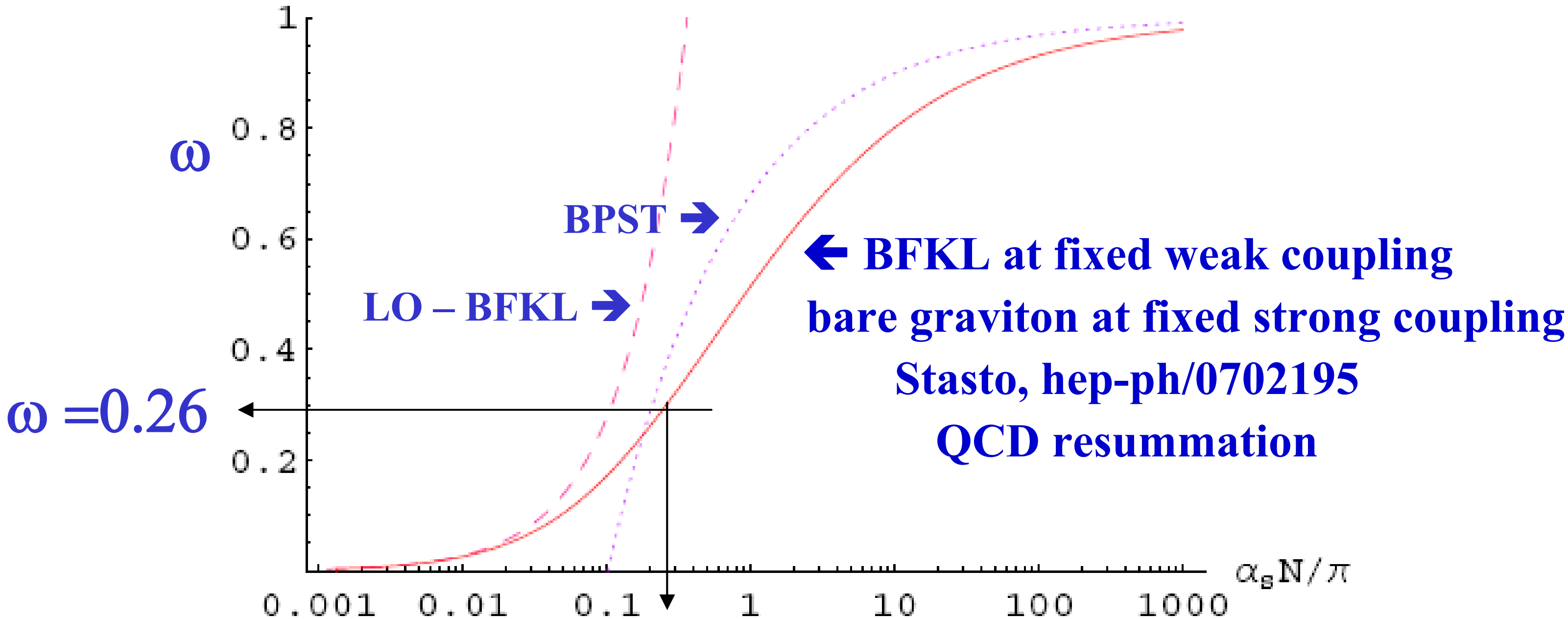
Pomeron is a coherent color-singlet object, build from gluons, with universal properties; it is a closed string propagating in ADS space, when the conformal symmetry is broken at some infrared point in the fifth dimension

$$1+\omega = 2 - \frac{2}{\sqrt{4\pi\alpha_S N}}$$

in ADS/CFT

in N=4 YM SuSy QCD

Kotikov, Lipatov, Onishchenko, Velizhanin, Physt. Lett. B 632, 754 (2006)

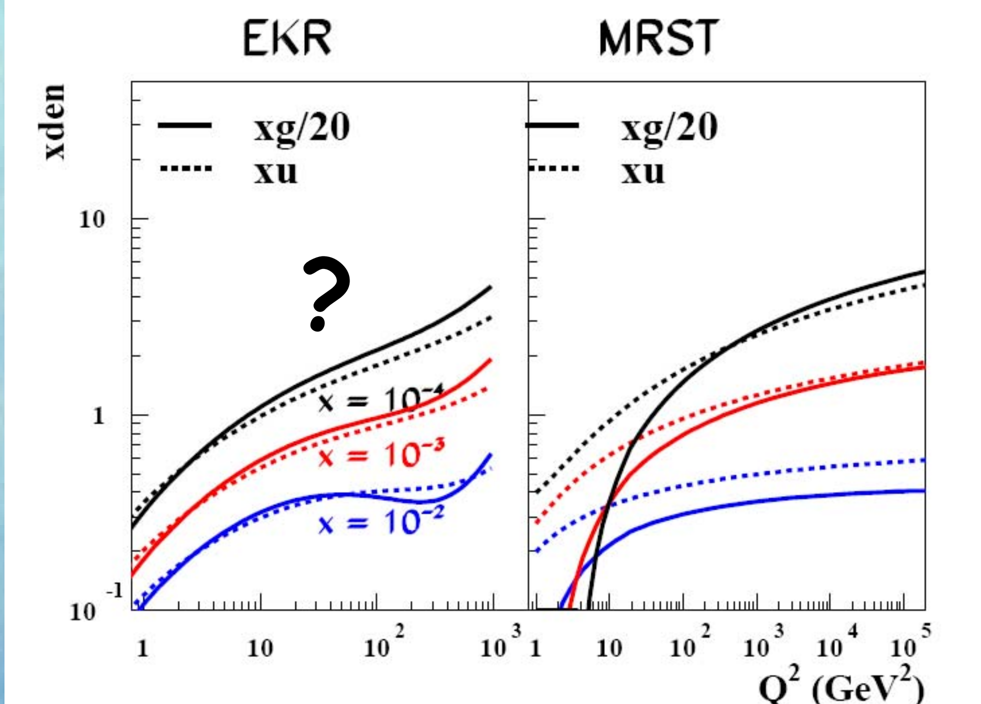


Consequences for LHC

Good knowledge of gluon density around $x \sim 10^{-2}$ and $Q^2 \sim 10000$ GeV^2 is essential for LHC physics (Higgs region)

Large effort is going into precise measurement of W and Z inclusive X-sections ➔ precise determination of sea-quark distributions
➔ precise gluon density

Is the sea-quark ←➔ gluon density relation the same in the DGLAP-like picture (MRST/CTEQ) and DAF-Pomeron?



sea-quark ←➔ gluon relation can be checked by the jets with p_T around 50 GeV

Outline of the talk:

Why Pomeron at HERA?

What is Discreet AF-BFKL Pomeron
Evidence for DAF-Pomeron from HERA data

Relation with DGLAP
MRST ←→ EKR

Pomeron-Graviton Correspondence

Consequences for LHC, EIC, LHeC

Instead of Conclusion

Study of Gluon Density is important because
it is the analog of Black Body Radiation in QED

It seemed hopeless to study pure Gluon Radiation
since it is never free. However, it is becoming free
for a short moment in HEP reactions

HERA has shown that physics processes at low-x are
completely dominated by pure Gluon Density,

Investigation of Gluon Density has a chance to become as
fundamental as Black Body radiation