# $F_L$ structure function in DGLAP/BFKL approach

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# Outline

Unified DGLAP/BFKL approach

High energy vs collinear factorization

Gluon kinematics

Comparison with the HERA data

# Why $F_L$ ?

- Vanishes in the parton model: quarks spin 1/2, transverse momentum limited.
- Should be non-zero in QCD: gluon radiative corrections, transverse momentum grows with $Q^2$
- Directly sensitive to the gluon density.
- Large higher twist effects (observed at large x).
  - Extremely important to see what happens at low x and low $Q^2$ $\longrightarrow$ H1 talk!

# Unified DGLAP/BFKL approach

- Simplest resummed model: set of two integral equations for the unintegrated gluon density and quark sea density.

J.Kwiecinski,
A.D.Martin,
A.S.

- Gluons:
  - BFKL with kinematical constraint
  - DGLAP splitting function
  - running coupling

$$
\begin{aligned}
f(x,k^2) \;=\; & \tilde{f}^{(0)}(x,k^2) + \overline{\alpha}_S(k^2)\, k^2 \int_x^1 \frac{dz}{z} \int_{k_0^2} \frac{dk'^2}{k'^2} \left\{ \frac{f\left(\frac{x}{z},k'^2\right)\Theta\left(\frac{k^2}{z}-k'^2\right) - f\left(\frac{x}{z},k^2\right)}{|k'^2-k^2|} + \frac{f\left(\frac{x}{z},k^2\right)}{[4k'^4+k^4]^{\frac{1}{2}}} \right\} \\
& + \overline{\alpha}_S(k^2) \int_x^1 \frac{dz}{z} \left(\frac{z}{6} P_{gg}(z) - 1\right) \int_{k_0^2}^{k^2} \frac{dk'^2}{k'^2} f\left(\frac{x}{z},k'^2\right) + \frac{\alpha_S(k^2)}{2\pi} \int_x^1 dz P_{gq}(z) \Sigma\left(\frac{x}{z},k^2\right) .
\end{aligned}
$$

non-perturbative input

$$
\tilde{f}^{(0)}(x,k^2) \;=\; \frac{\alpha_S(k^2)}{2\pi} \int_x^1 dz\, P_{gg}(z)\, \frac{x}{z} g\left(\frac{x}{z},k_0^2\right) .
$$

# Unified DGLAP/BFKL approach

- Quarks:
  - $k_T$ factorization theorem
  - Three different regions for quark and gluon momenta
    - Non-perturbative (soft)
    - Strongly ordered (low gluon momenta)
    - Perturbative (high gluon momenta)
  - Momentum sum rule

$$\Sigma(x,k^2) \;=\; S^{(\mathrm{soft})}(x) \;+\; \sum_q \int_x^a \frac{dz}{z}\, S_q^{(\mathrm{box})}(z,k'^2=0,k^2;m_q^2)\frac{x}{z}\, g\left(\frac{x}{z},k_0^2\right) \;+\; V(x,k^2)$$
$$+\; \sum_q \int_{k_0^2}^{\infty} \frac{dk'^2}{k'^2} \int_x^1 \frac{dz}{z}\, S_q^{(\mathrm{box})}(z,k'^2,k^2;m_q^2) f\left(\frac{x}{z},k'^2\right) \;+\; \int_{k_0^2}^{k^2} \frac{dk'^2}{k'^2}\, \frac{\alpha_S(k'^2)}{2\pi} \int_x^1 dz\, P_{qq}(z) S_{uds}\left(\frac{x}{z},k'^2\right) \;,$$

non-perturbative, soft terms $\quad S^{(\mathrm{soft})}(x) = S_u^P + S_d^P + S_s^P \;, \qquad S_u^P = S_d^P = 2S_s^P = C_P x^{-0.08}(1-x)^8 \,.$

# $F_L$ calculated from $k_T$ factorization

high momenta

$$F_L(x,Q^2) = \frac{Q^4}{\pi^2}\sum_q e_q^2 \int \frac{dk^2}{k^4}\,\theta(k^2>k_0^2)\int_0^1 d\beta \int d^2\boldsymbol{\kappa'}\,\alpha_s(\mu^2)\,\beta^2(1-\beta)^2\left(\frac{1}{D_{1q}}-\frac{1}{D_{2q}}\right)^2 f(x_g,k^2)$$

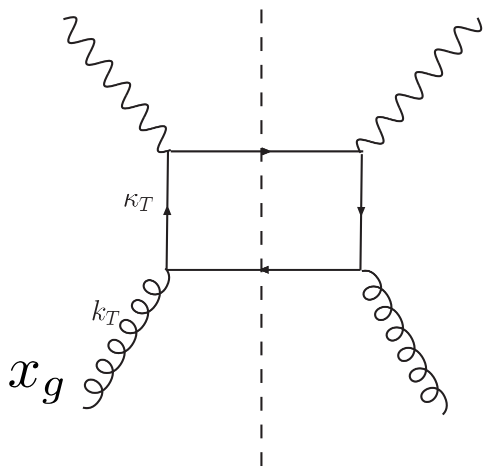
$$+\ \frac{\alpha_s(Q^2)}{\pi}\left\{\frac{4}{3}\int_x^1\frac{dy}{y}\left(\frac{x}{y}\right)^2 F_2(y,Q^2) + \sum_q e_q^2\int_x^1\frac{dy}{y}\left(\frac{x}{y}\right)^2\left(1-\frac{x}{y}\right) yg(y,k_0^2)\right\}.$$

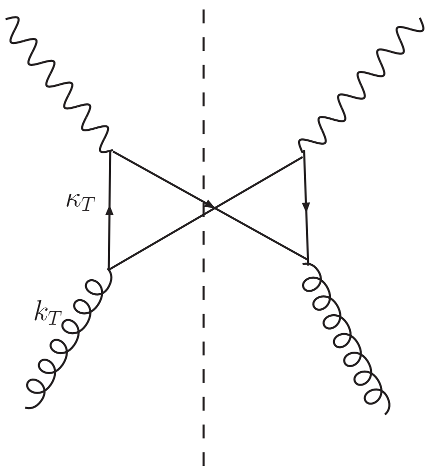
quarks

non-perturbative

$$\frac{x}{z} \equiv x_g \equiv x\left(1+\frac{\kappa'^2+m_q^2}{\beta(1-\beta)Q^2}+\frac{k^2}{Q^2}\right).$$

exact gluon kinematics



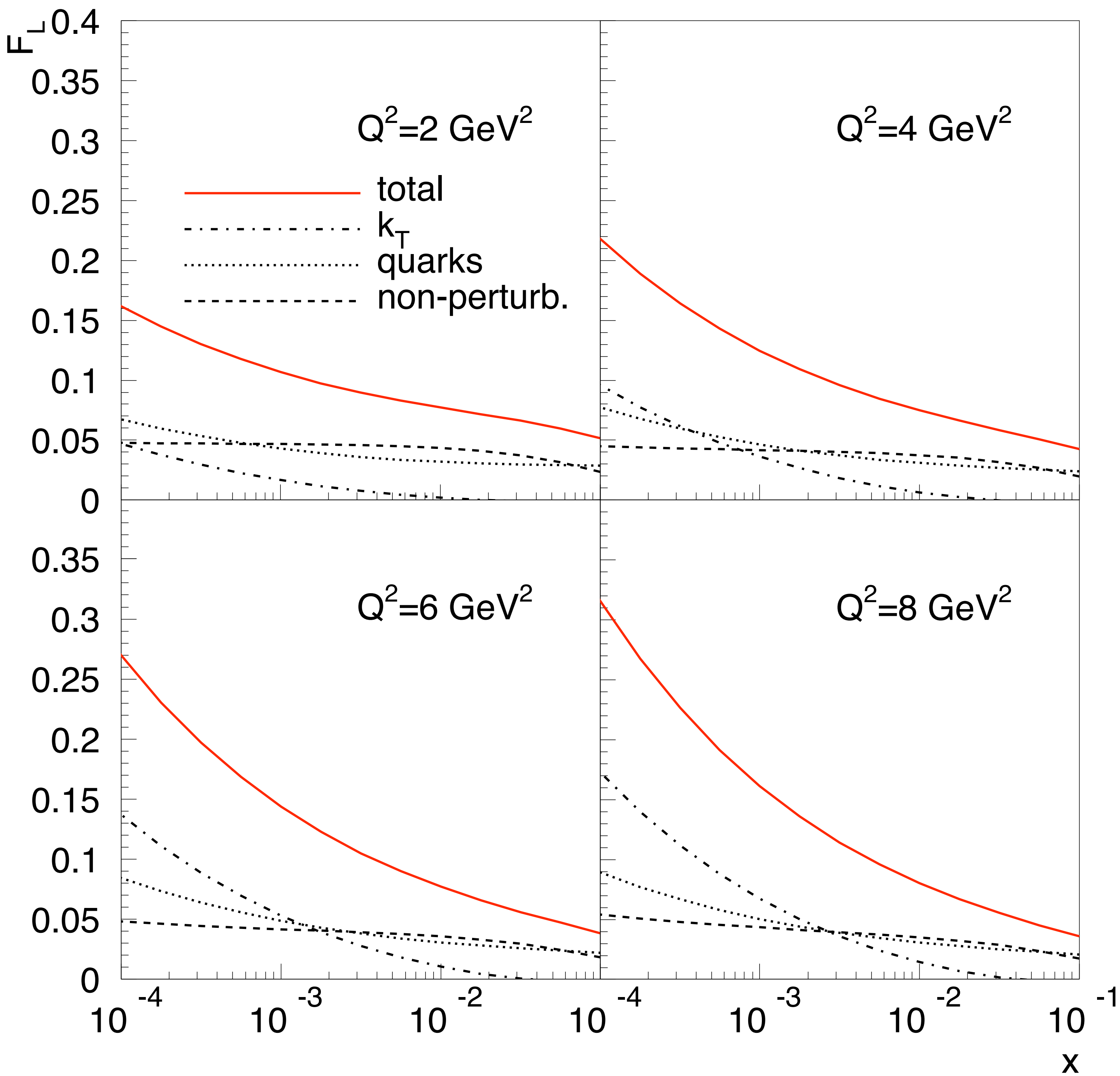




Charm quark density generated dynamically through the boson-gluon fusion with exact kinematics

non-perturbative gluon parametrization

$$yg(y,k_0^2) = N(1-y)^\beta\ ,$$

# Different contributions to $F_L$




Non-perturbative (low gluon momenta) term nearly constant.

Quark contribution non-negligible for lowest $Q^2$ and higher $x > 0.01$

Gluon contribution (high momenta) strongly suppressed for $x > 0.01$

Kinematics very important in this regime ( and not only!)

# $k_T$ factorization vs collinear

$$F_L(x,Q^2) = 2\frac{Q^4}{\pi^2}\sum_q e_q^2 \int \frac{dk^2}{k^4}\int_0^1 d\beta \int d^2\boldsymbol{\kappa'}\,\alpha_s(\mu^2)\,\beta^2\,(1-\beta)^2\,\frac{1}{2}\left(\frac{1}{D_{1q}}-\frac{1}{D_{2q}}\right)^2 f\left(\frac{x}{z},k^2\right)$$

$k^2 \ll Q^2$

on-shell approximation

$$F_L^{(\mathrm{on-shell})}(x,Q^2) = 2\sum_q e_q^2\left[J_q^{(1)} - 2\,\frac{m_q^2}{Q^2}J_q^{(2)}\right]$$

$$J_q^{(1)} = \frac{\alpha_s}{\pi}\int_{\bar{x}_q}^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2\left(1-\frac{x}{y}\right)\sqrt{1-\frac{4m_q^2 x}{Q^2(y-x)}}\,yg(y,Q^2)$$

$$J_q^{(2)} = \frac{\alpha_s}{\pi}\int_{\bar{x}_q}^1 \frac{dy}{y}\left(\frac{x}{y}\right)^3 \ln\left[\frac{1+\sqrt{1-\frac{4m_q^2 x}{Q^2(y-x)}}}{1-\sqrt{1-\frac{4m_q^2 x}{Q^2(y-x)}}}\right] yg(y,Q^2)$$

Gluon kinematics crucial in obtaining the collinear approximation

$$\frac{x}{z} \equiv x_g \equiv x\left(1 + \frac{\kappa'^2 + m_q^2}{\beta(1-\beta)Q^2} + \frac{k^2}{Q^2}\right).$$

$$y \equiv x\left(1+\frac{\kappa'^2+m_q^2}{\beta(1-\beta)Q^2}\right)$$

# $k_T$ factorization vs dipole model

$$F_L(x,Q^2) = 2\frac{Q^4}{\pi^2}\sum_q e_q^2 \int \frac{dk^2}{k^4}\int_0^1 d\beta \int d^2\boldsymbol{\kappa'}\, \alpha_s(\mu^2)\, \beta^2\, (1-\beta)^2\, \frac{1}{2}\left(\frac{1}{D_{1q}} - \frac{1}{D_{2q}}\right)^2 f\left(\frac{x}{z}, k^2\right)$$

small x approximation replaces the argument of the gluon density: gluon is longitudinally soft

$$x_g = \frac{x}{z} \longrightarrow x$$

$$\sigma_L = \frac{\alpha_{em}}{\pi}\sum_q e_q^2 \int d^2\boldsymbol{r} \int_0^1 d\beta\, 4Q^2\beta^2\,(1-\beta)^2 K_0^2(\overline{Q}r) \times \int \frac{d^2\boldsymbol{k}}{k^4}\,\alpha_s f(x,k^2)(1-e^{-i\boldsymbol{r}\cdot\boldsymbol{k}})(1-e^{i\boldsymbol{r}\cdot\boldsymbol{k}})\;.$$

dipole cross section

$$\hat{\sigma}(x,\boldsymbol{r}) \equiv \frac{2\pi}{3}\int \frac{d^2\boldsymbol{k}}{k^4}\alpha_s f(x,k^2)(1-e^{-i\boldsymbol{r}\cdot\boldsymbol{k}})(1-e^{i\boldsymbol{r}\cdot\boldsymbol{k}})$$

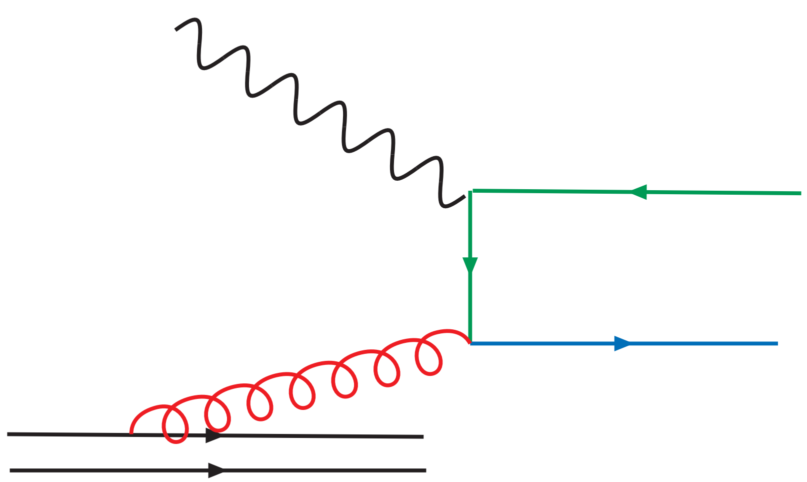
# $k_T$ factorization vs collinear approach and dipole model

$k_T$ factorization
+ exact kinematics

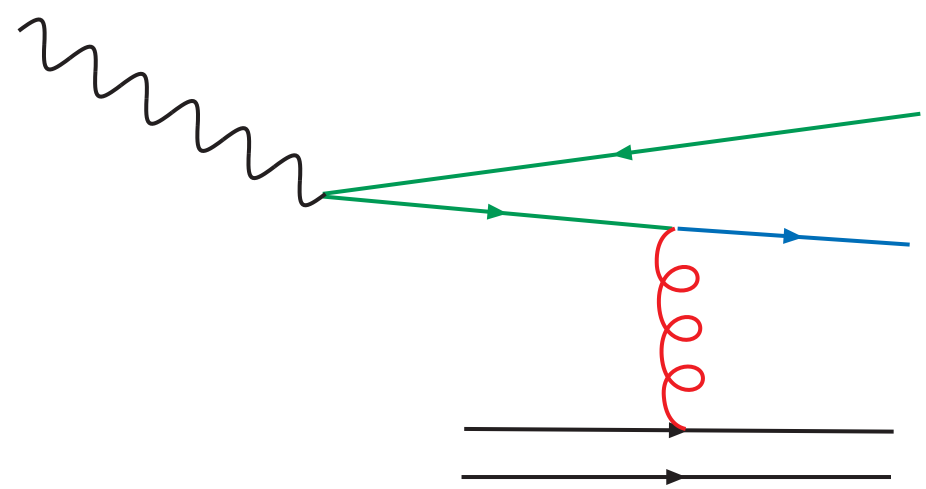
$k^2 \ll Q^2$

$x_g = \frac{x}{z} \longrightarrow x$

collinear approach



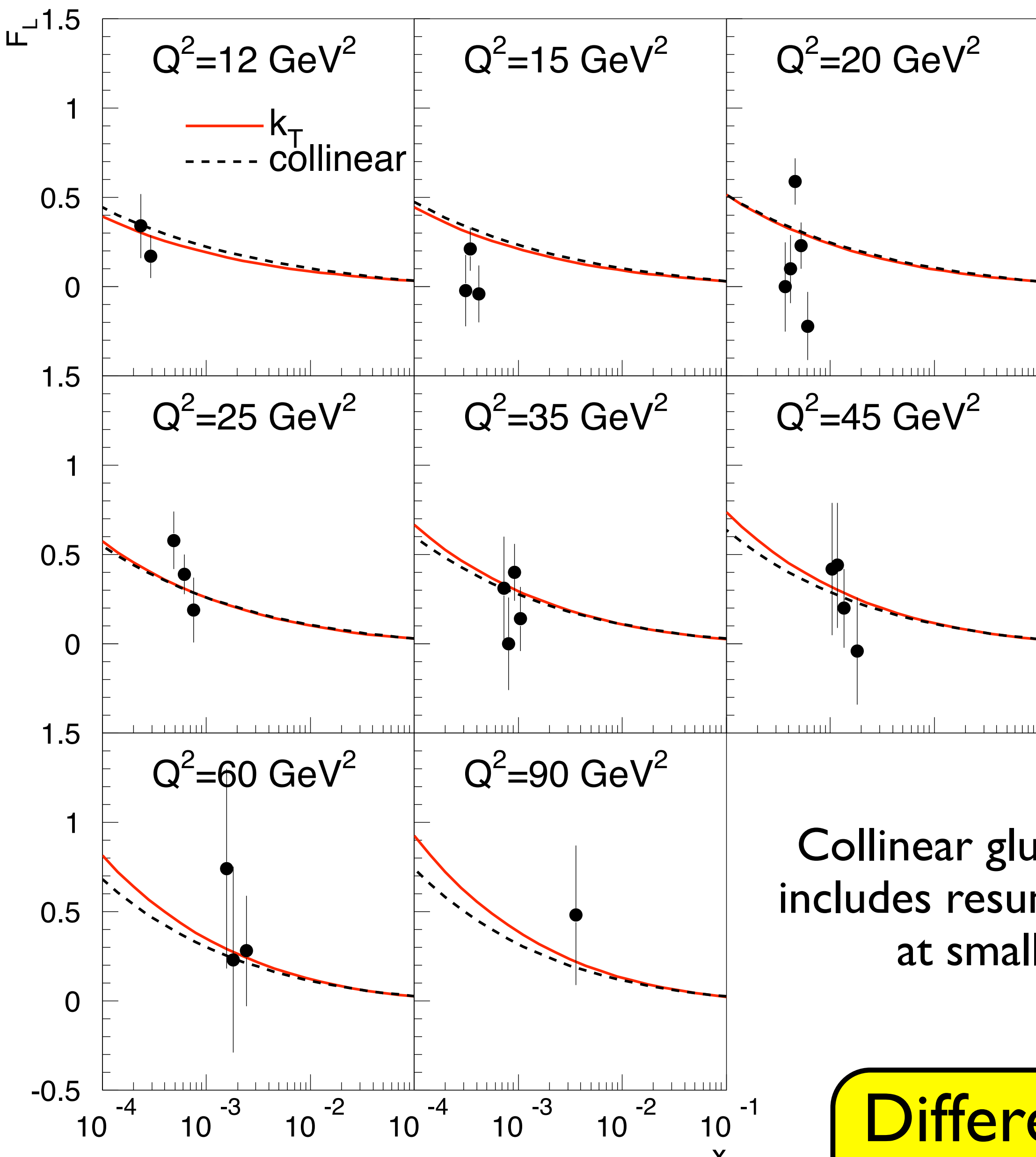
dipole model



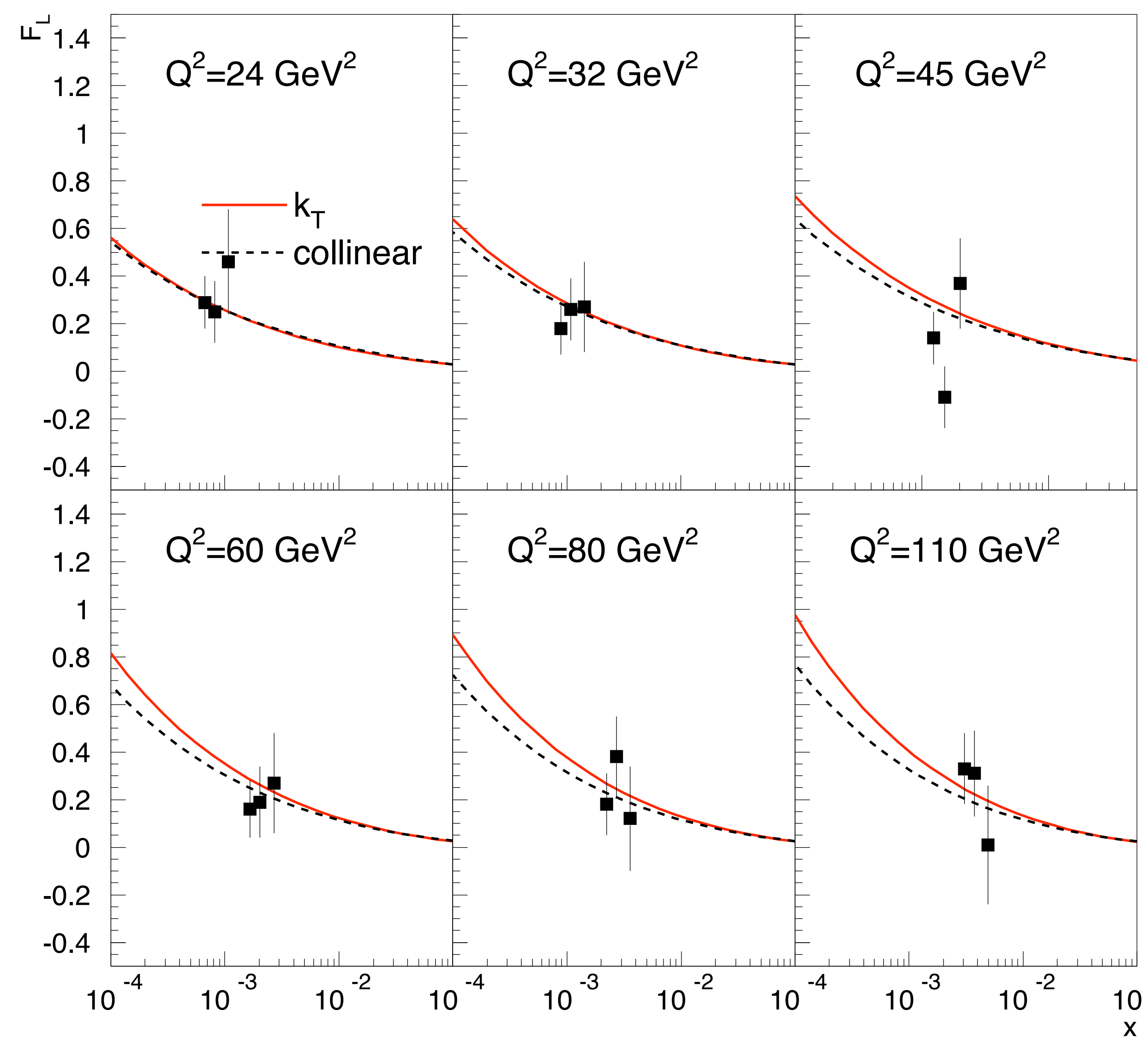
$k_T$ factorization contains both limits

# $k_T$ vs collinear

## H1




## ZEUS



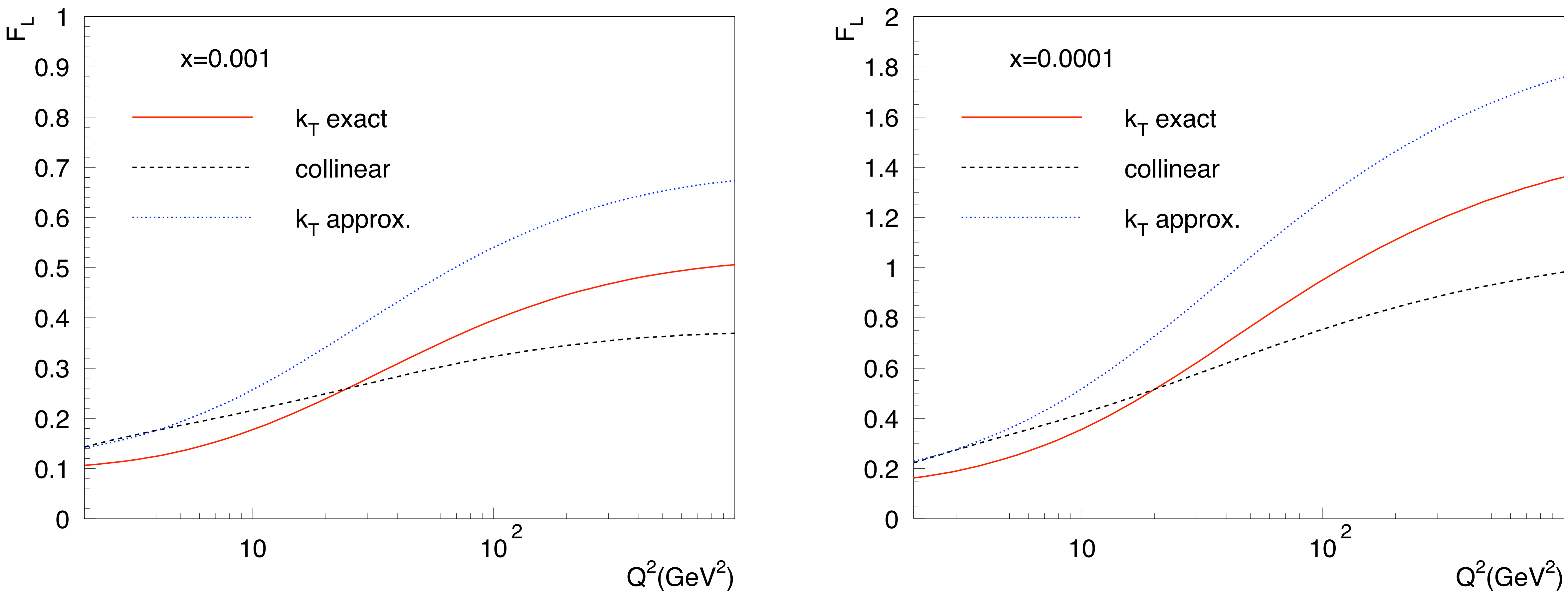

Collinear gluon also includes resummation at small x

**Differences between $k_T$ and collinear approaches are small for $F_L$ in this kinematic regime.**

This is different than $F_2$, where differences are typically large.

# $k_T$ vs collinear

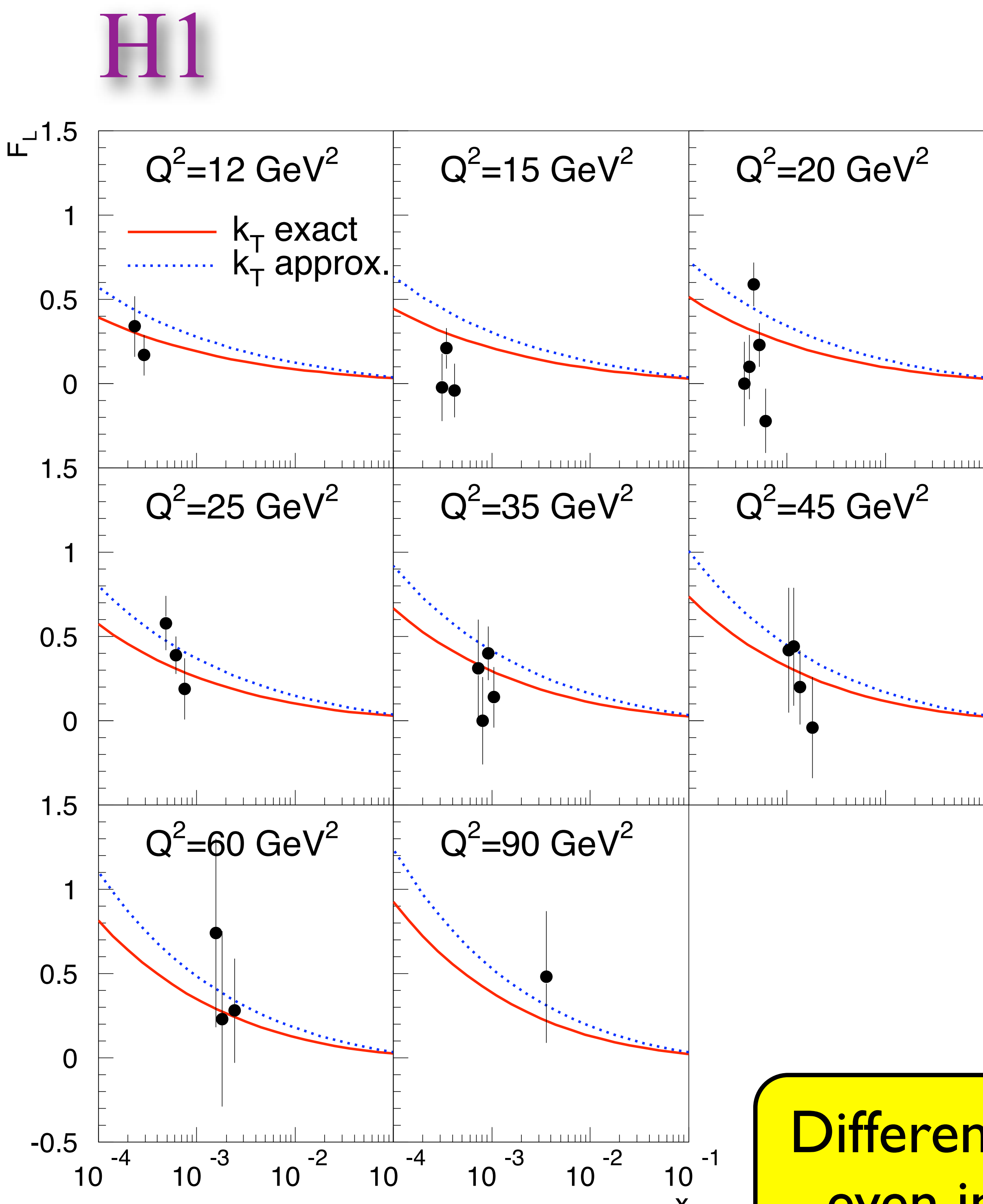
## $Q^2$ dependence

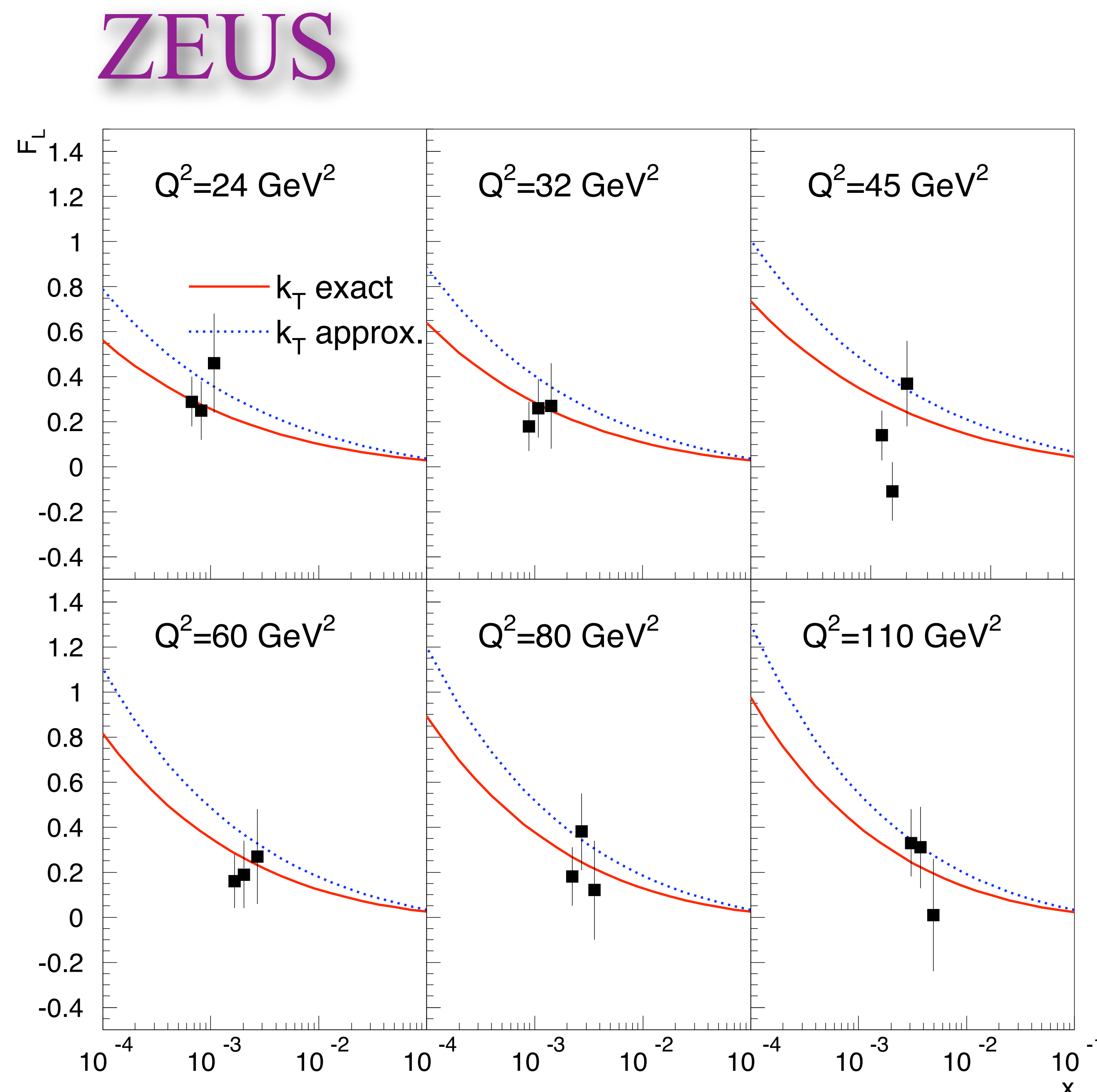



Differences between $k_T$ and collinear approaches are small for $F_L$ in this kinematic regime.

However, $Q^2$ dependence different for the $k_T$ factorization and collinear.

# $k_T$ vs dipole kinematics



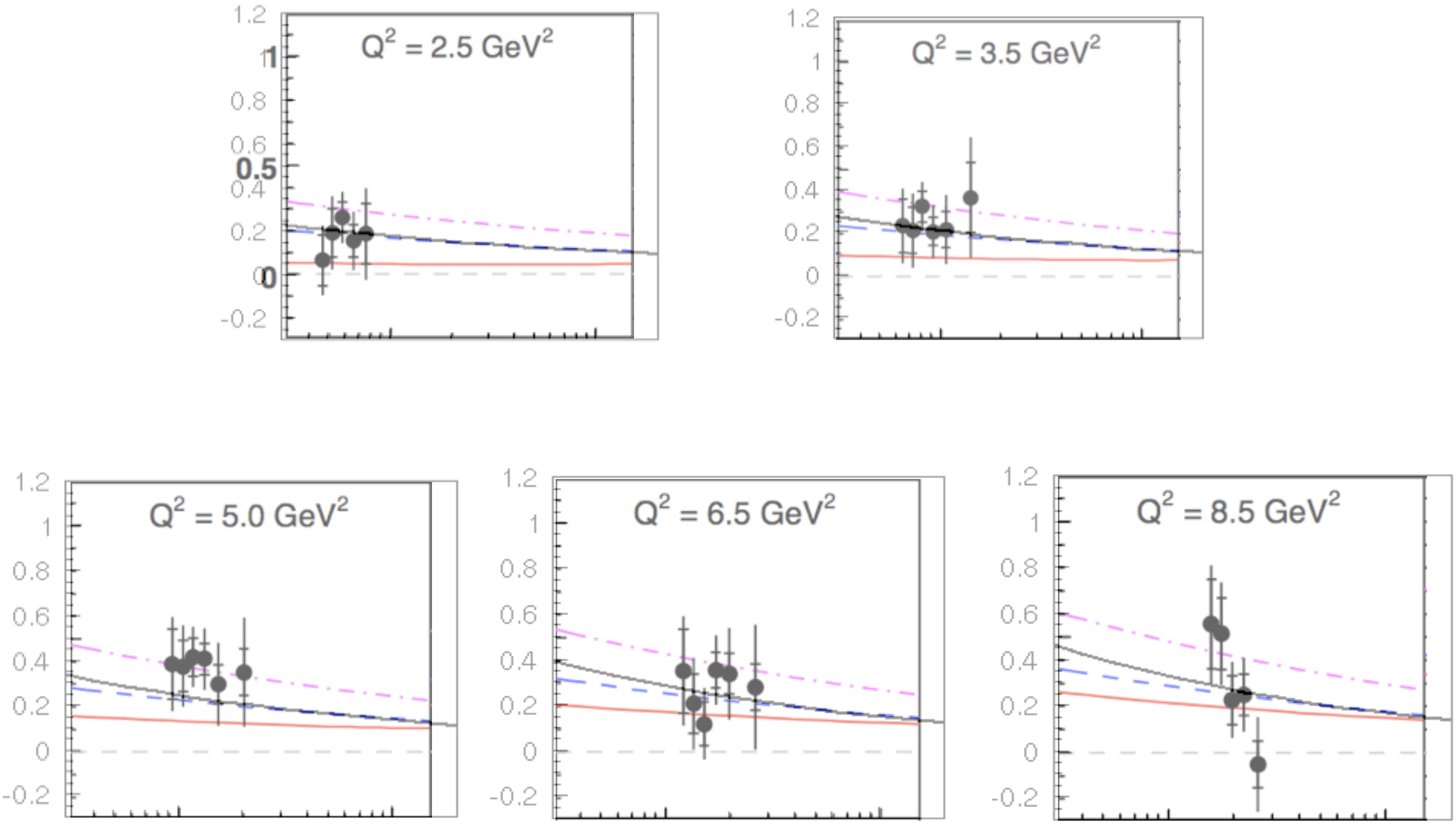

Differences are non-negligible even in the small x regime.

effective x of the gluon

$$x_g \simeq 5.7\,x\ ,$$

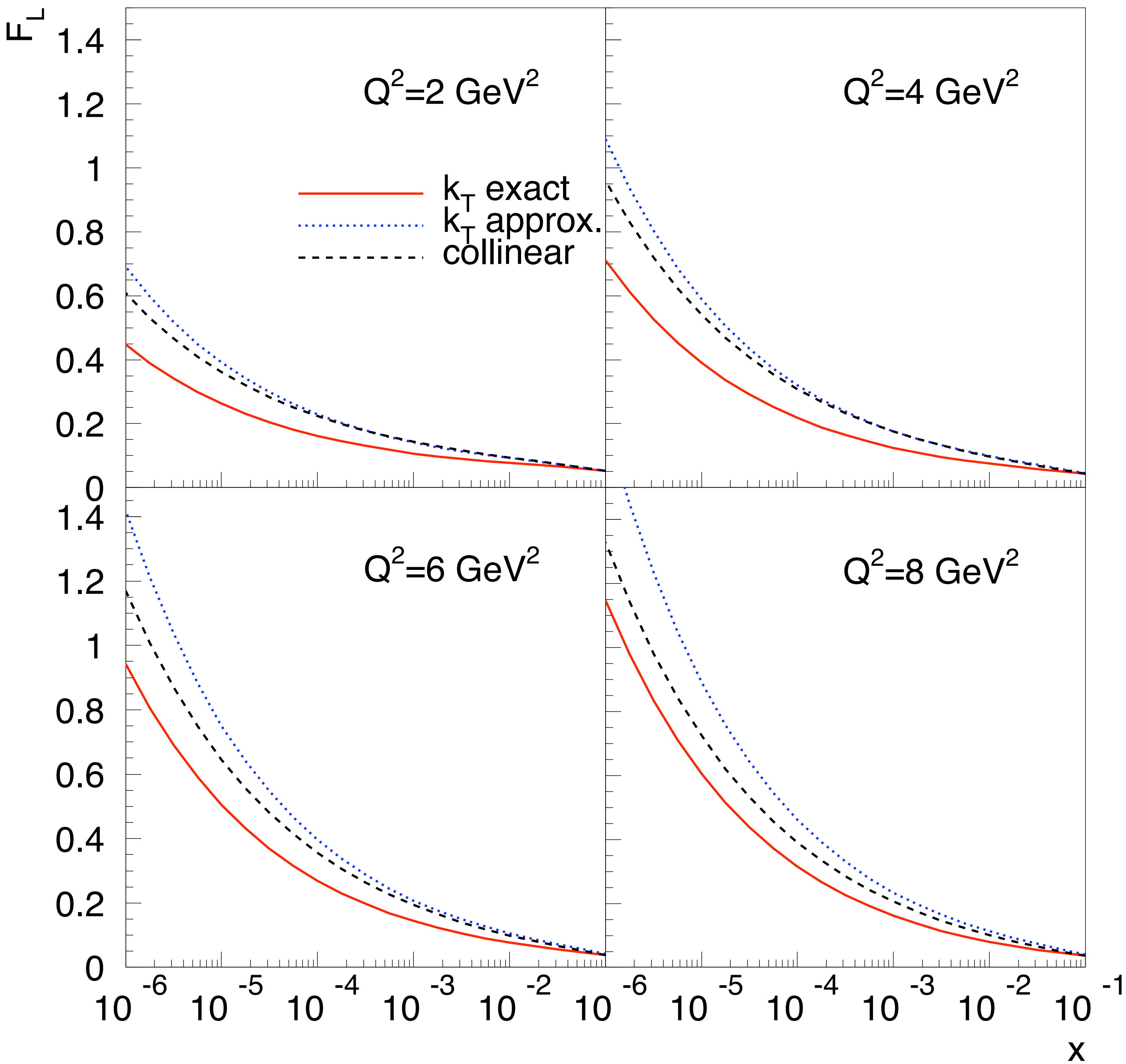
# Low $Q^2$ region

Comparison with the H1 preliminary low Q2 data




Good agreement with the data at low Q2 too.

# Extrapolations to lower x: LHeC




Kinematic effects important even for lowest values of x.

Can compensate for the difference by changing the normalization of the gluon density.

Using approximate (dipole-like) kinematics, forces the gluon density to be smaller.

This is important if one wants to study saturation effects.

# Summary:

- Unintegrated gluon distribution from unified DGLAP/BFKL.
- High energy factorization with exact gluon kinematics contains both collinear and dipole approach.
- In the HERA kinematics regime collinear and $k_T$ factorization give similar results for $F_L$ . Agreement with HERA data.
- Kinematic effects significant, dipole model underestimates the gluon density. Effective $x_g \simeq 5.7x$

# Outlook:

- Unified approach close to Ciafaloni-Colferai-Salam-Stasto resummation approach. Extend to obtain unintegrated gluon distribution function from CCSS resummation.
- Applications of unintegrated resummed densities at LHC.