

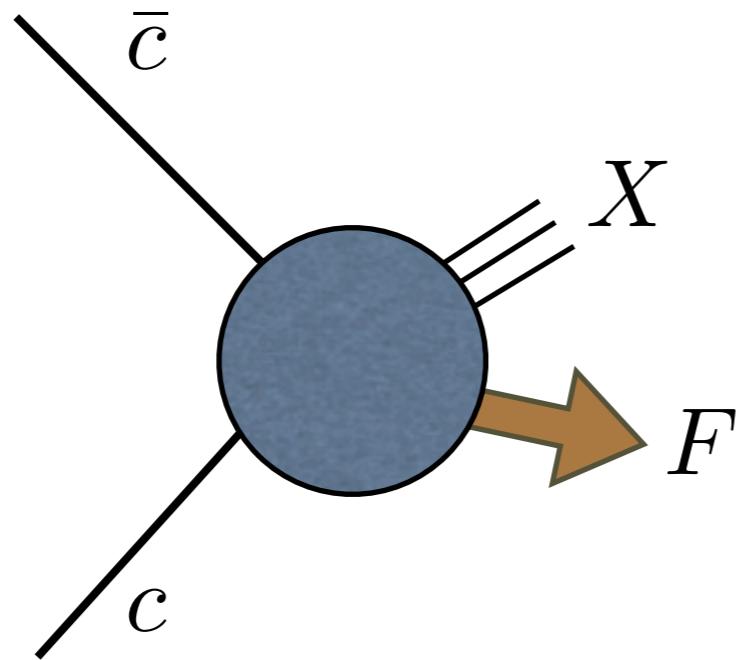
# Transverse momentum resummation at hadron colliders

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DIS2009, Madrid, april 28, 2009

# Introduction

The production of colourless high-mass systems in hadronic collisions is important for physics studies within and beyond the standard model



$$F = Z/\gamma^*, W, H, WW, ZZ\dots$$

$F$  is a system of colourless particles of invariant mass  $M$

**Since  $F$  is colourless the LO process is either  $q\bar{q}$  annihilation or  $gg$  fusion**

Well known examples are vector boson and Higgs boson production, vector boson pairs...

→ It is important to have reliable theoretical predictions

# Transverse momentum spectrum

Among the various distribution an important role is played by the transverse momentum spectrum of the high mass system

In the case of Z boson production the spectrum is precisely measured

→ Uncertainties on the W spectrum directly affect W mass

In the case of Higgs boson production a precise knowledge of the spectrum can help to find strategies to improve statistical significance

When considering the transverse momentum spectrum it is important to distinguish two regions of transverse momenta

# The region $q_T \sim M$

To have  $q_T \neq 0$  the vector boson has to recoil against at least one parton → the LO is of relative order  $\alpha_S$

NLO corrections are known in many cases

- Vector boson production

K.Ellis, G.Martinelli, R.Petronzio (1983)

P.B.Arnold, M.H. Reno (1989)

R.J.Gonsalves, J.Pawlowski, C.F.Wai (1989)

- Higgs boson production

D. de Florian, Z.Kunszt, MG (1999)

V.Ravindran, J.Smith, V.Van Neerven (2002)

C.Glosser, C.Schmidt (2002)

- WW production

J.Campbell, K.Ellis, G.Zanderighi (2007)

S.Dittmaier, S.Kallweit, P.Uwer (2007)

.....

# The region $q_T \ll M$

The small  $q_T$  region is the most important because it is here that the bulk of the events is expected

When  $q_T \ll M$  large logarithmic corrections of the form  $\alpha_S^n \ln^{2n} M^2/q_T^2$  appear that originate from soft and collinear emission

→ the perturbative expansion becomes not reliable

This is a general problem in this kind of processes

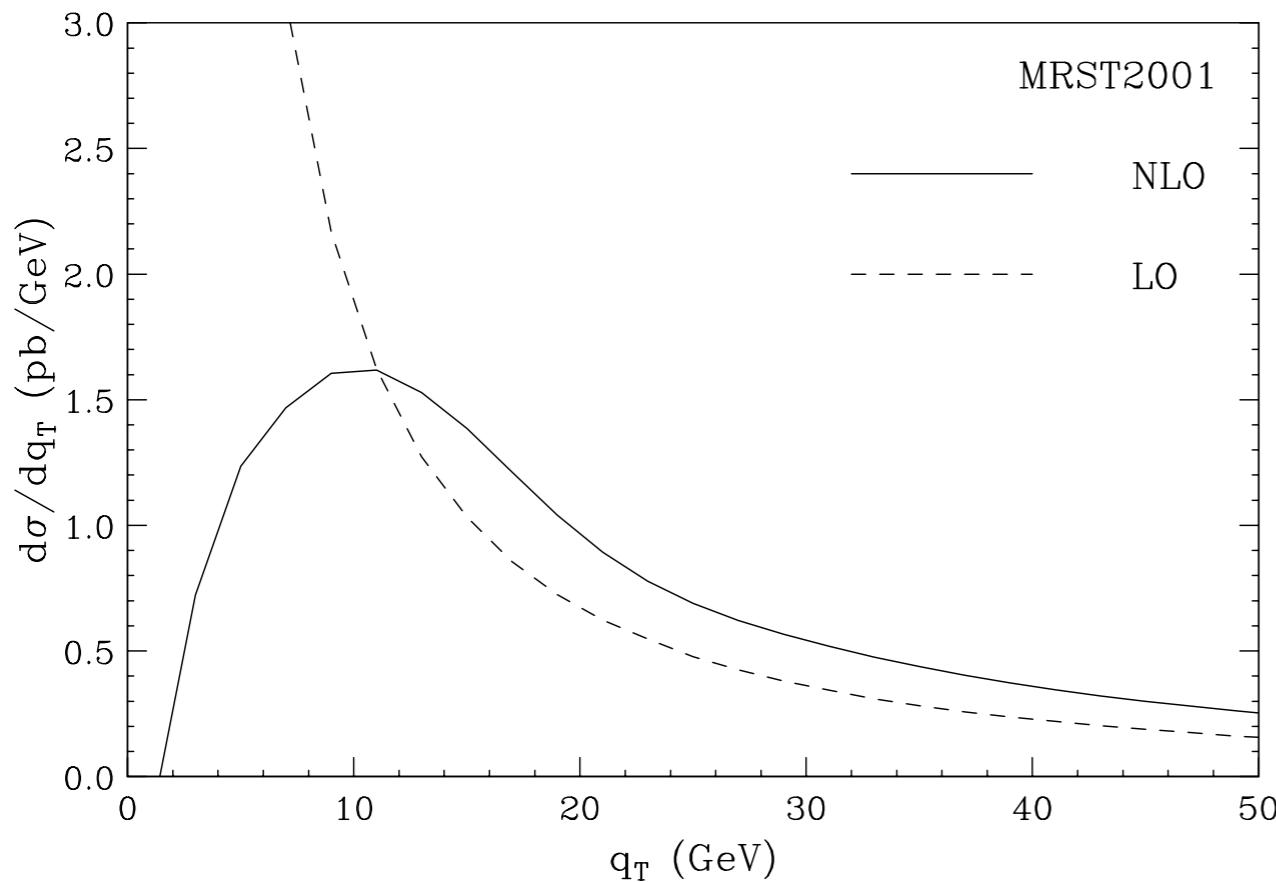
→ **RESUMMATION IS NEEDED**

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LO:  $\frac{d\sigma}{dq_T} \rightarrow +\infty$  as  $q_T \rightarrow 0$

NLO:  $\frac{d\sigma}{dq_T} \rightarrow -\infty$  as  $q_T \rightarrow 0$

This is a general problem in this kind of processes

→ RESUMMATION IS NEEDED

The resummation formalism has been developed in the eighties

Y.Dokshitzer, D.Diakonov, S.I.Troian (1978)

G. Parisi, R. Petronzio (1979)

G. Curci, M.Greco, Y.Srivastava(1979)

J. Collins, D.E. Soper, G. Sterman (1985)

As it is customary in QCD resummations one has to work in a conjugate space in order to allow the kinematics of multiple gluon emission to factorize

In this case, to exactly implement momentum conservation, the resummation has to be performed in impact parameter b-space

The standard (CSS) formalism has several disadvantages:

- The resummation coefficients are process dependent  
D. de Florian, MG (2000)

- The integral over  $b$  involves and extrapolation of the parton distributions to the non-perturbative region

- The resummation effects are large also at small  $b$



- No control on the normalization
- Problems in the matching to the PT result

# Our formalism

A version of the b-space formalism has been proposed that overcomes these problems

Parton distributions factorized at  $\mu_F \sim M$

S.Catani, D. de Florian, MG (2000)

G. Bozzi, S.Catani, D. de Florian, MG(2005)

$$\frac{d\hat{\sigma}_{ac}^{(\text{res.})}}{dq_T^2} = \frac{1}{2} \int_0^\infty db b J_0(bq_T) \mathcal{W}_{ac}(b, M, \hat{s}; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

**process dependent** 

$$\begin{aligned} \mathcal{W}_N^F(b, M; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2) &= \mathcal{H}_N^F(M, \alpha_S(\mu_R^2); M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2) \\ &\times \exp\{\mathcal{G}_N(\alpha_S(\mu_R^2), L; M^2/\mu_R^2, M^2/Q^2)\} \end{aligned}$$

where the large logs  
are organized as:

$$\mathcal{G}_N(\alpha_S, L; M^2/\mu_R^2, M^2/Q^2) = L g^{(1)}(\alpha_S L)$$

**universal** 

$$+ g_N^{(2)}(\alpha_S L; M^2/\mu_R^2, M^2/Q^2) + \alpha_S g_N^{(3)}(\alpha_S L; M^2/\mu_R^2, M^2/Q^2) + \dots$$

with  $L = \ln M^2 b^2 / b_0^2$    $\tilde{L} = \ln(1 + Q^2 b^2 / b_0^2)$  and  $\alpha_S = \alpha_S(\mu_R)$

 **resummation scale**

- The form factor takes the same form as in threshold resummation
- - Unitarity constraint enforces correct total cross section
- Allows a consistent study of perturbative uncertainties

The resummed and fixed order calculations can then be combined to achieve uniform theoretical accuracy over the entire range of  $q_T$

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(\text{res.})}}{dq_T^2} + \frac{d\hat{\sigma}^{(\text{fin.})}}{dq_T^2}$$

The resummed and fixed order calculations can then be combined to achieve uniform theoretical accuracy over the entire range of  $q_T$

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(\text{res.})}}{dq_T^2} - \frac{d\hat{\sigma}^{(\text{fin.})}}{dq_T^2}$$

**standard fixed order result  
minus expansion of  
resummed formula at the  
*same* order**

The resummed and fixed order calculations can then be combined to achieve uniform theoretical accuracy over the entire range of  $q_T$

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(\text{res.})}}{dq_T^2} - \frac{d\hat{\sigma}^{(\text{fin.})}}{dq_T^2}$$

A diagram illustrating the subtraction of the resummed formula from the standard fixed order result. On the left, the equation  $\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(\text{res.})}}{dq_T^2} - \frac{d\hat{\sigma}^{(\text{fin.})}}{dq_T^2}$  is shown. The term  $\frac{d\hat{\sigma}^{(\text{fin.})}}{dq_T^2}$  is circled in red. An orange arrow points from this circled term to the text on the right.

standard fixed order result  
minus expansion of  
resummed formula at the  
*same order*

The calculation can be done at:

- **NLL+LO**: we need the functions  $g^{(1)}$ ,  $g_N^{(2)}$  and the coefficient  $\mathcal{H}_N^{(1)}$  plus the matching at relative order  $\alpha_S$
- **NNLL+NLO**: we also need the function  $g_N^{(3)}$  and the coefficient  $\mathcal{H}_N^{(2)}$  plus the matching at relative order  $\alpha_S^2$

The formalism has been applied so far to **Higgs** and **vector boson** production, **WW** and **ZZ** production, **slepton pair production**

G.Bozzi, S.Catani, D. de Florian, MG(2003,2005)

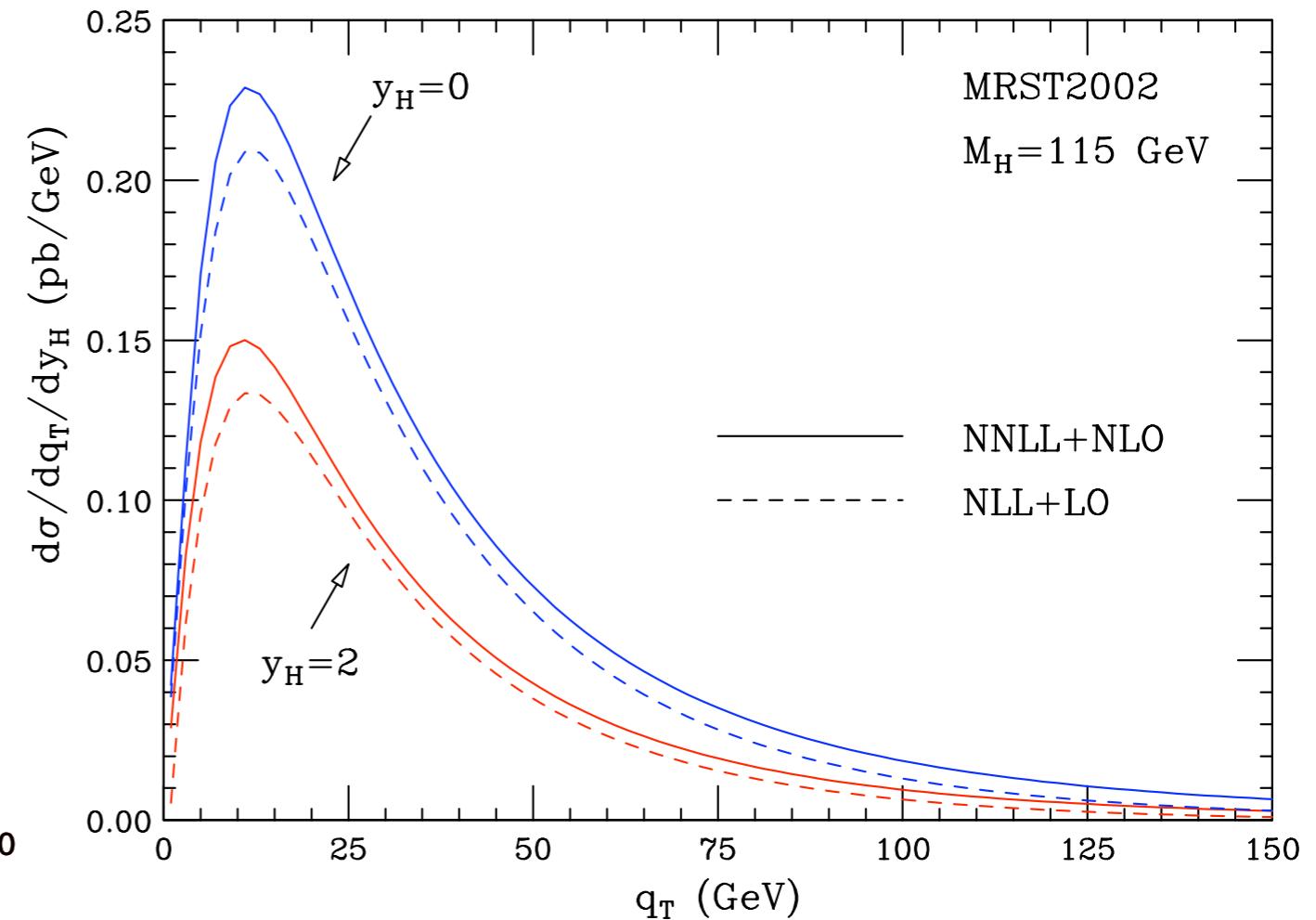
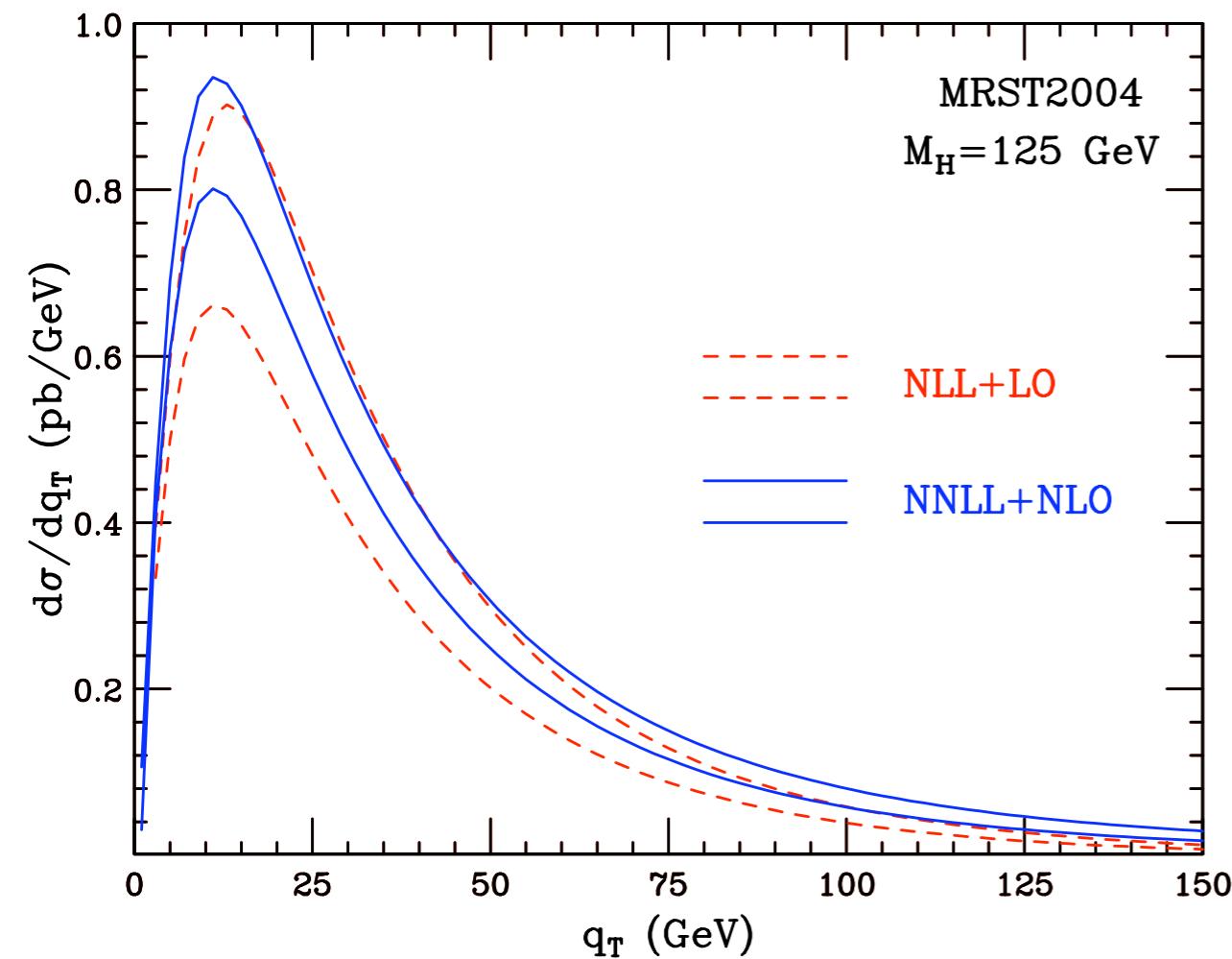
G.Bozzi, B.Fuks, M.Klasen (2006)

MG (2005), R.Frederix, MG (2008)

G.Bozzi, S.Catani, G. Ferrera, D. de Florian, MG (2008)

# Results: Higgs production

G. Bozzi, S.Catani, D. de Florian, MG (2005,2007)

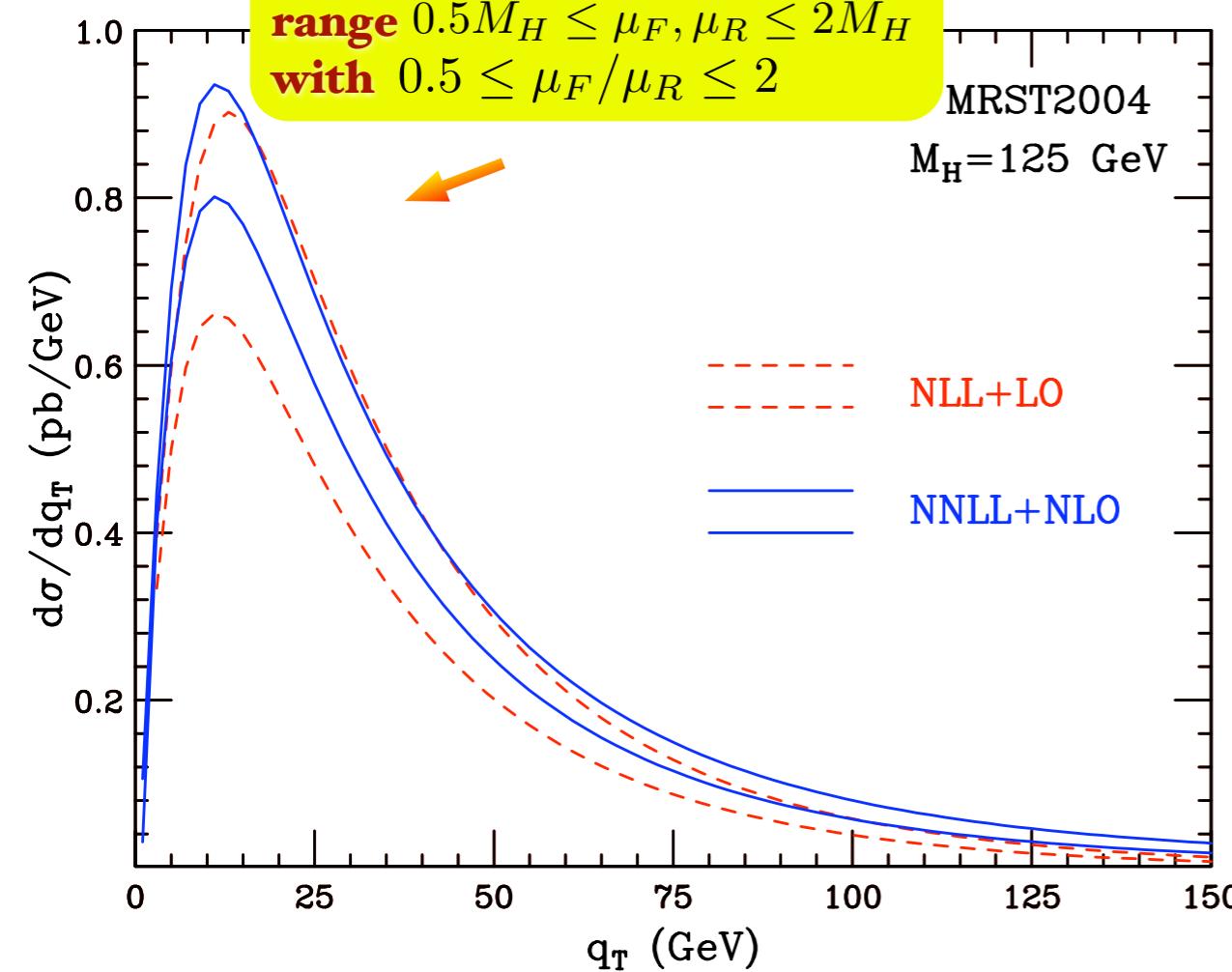


NNLL+NLO and NLL+LO bands overlap: nice convergence of the perturbative resummed result

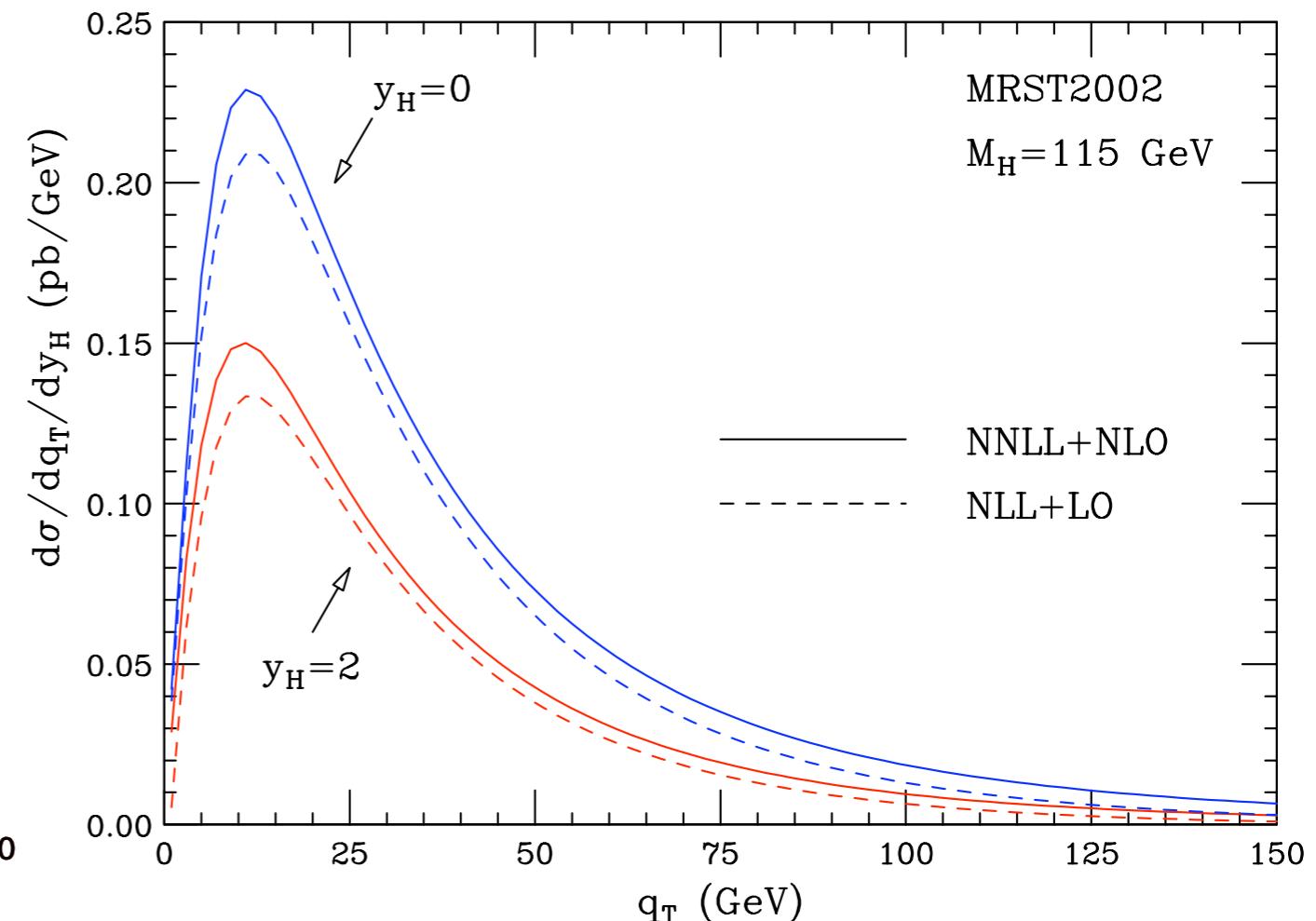
Shape of resummed spectrum mildly dependent on rapidity

# Results: Higgs production

**scale uncertainty computed  
by varying  $\mu_R$  and  $\mu_F$  in the  
range  $0.5M_H \leq \mu_F, \mu_R \leq 2M_H$   
with  $0.5 \leq \mu_F/\mu_R \leq 2$**



G. Bozzi, S.Catani, D. de Florian, MG (2005,2007)

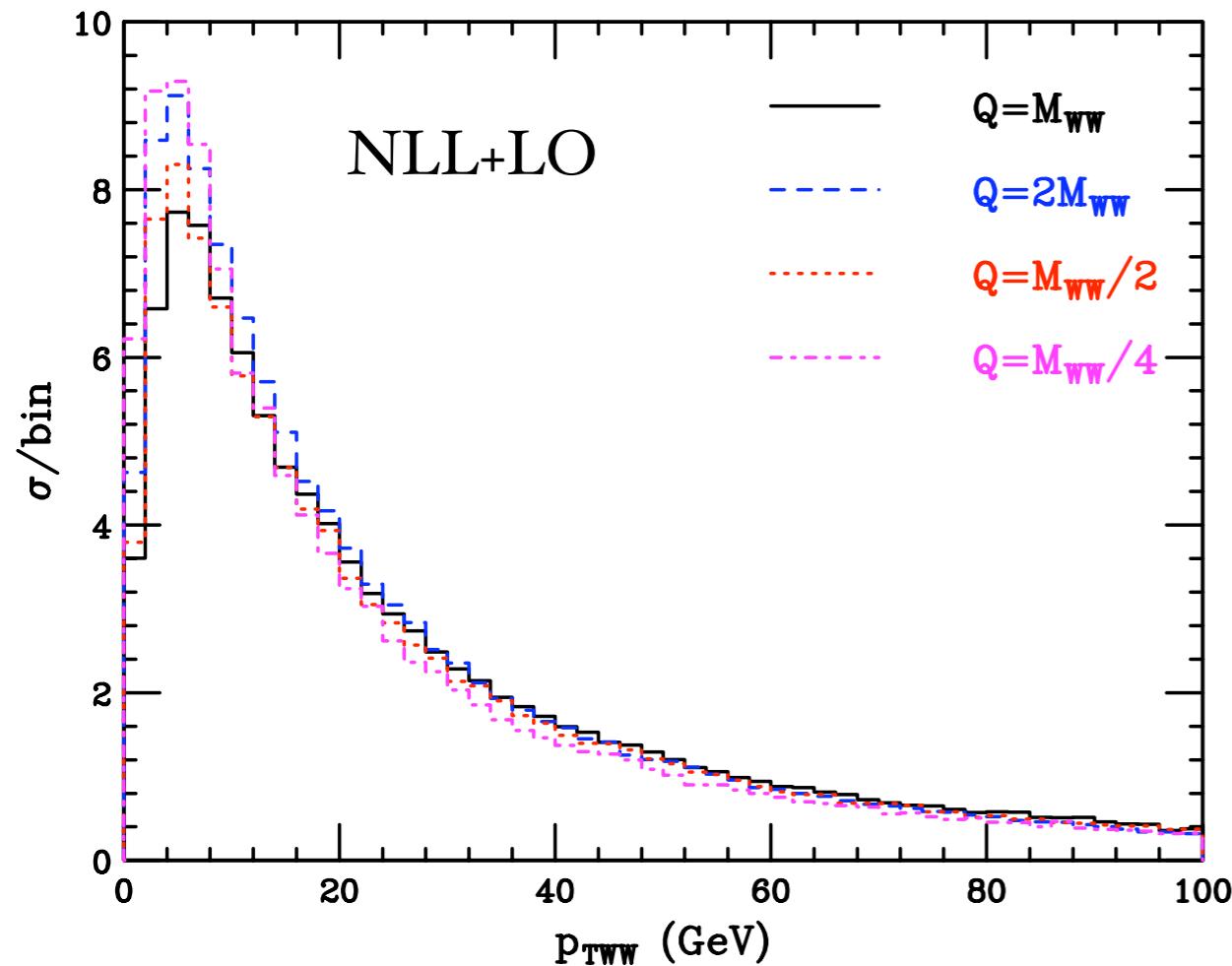


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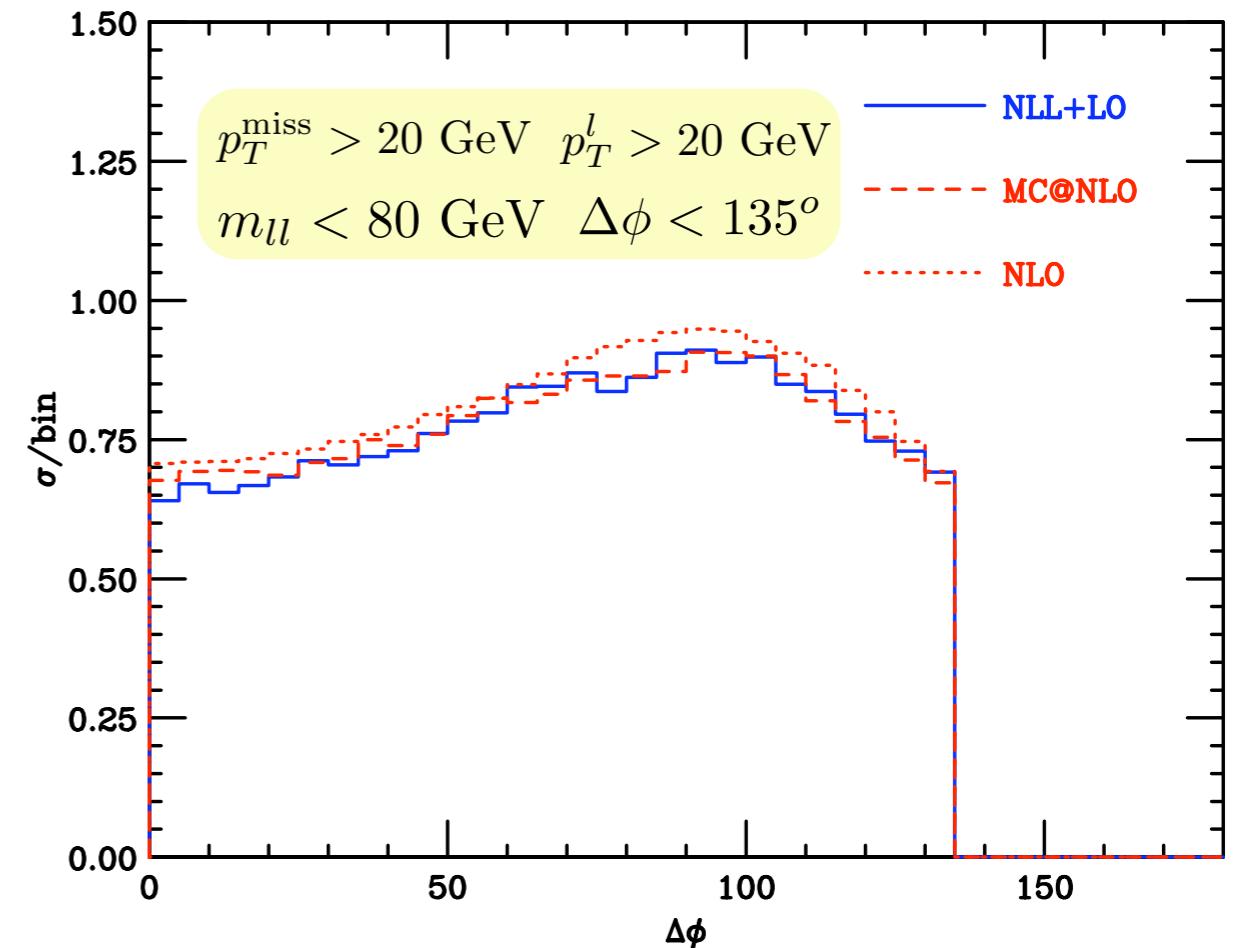
Shape of resummed spectrum mildly dependent on rapidity

# Results: WW production

MG (2005)



Still considerable uncertainty from missing higher order logarithmic contribution



NLO and NLL+LO exactly include spin correlations

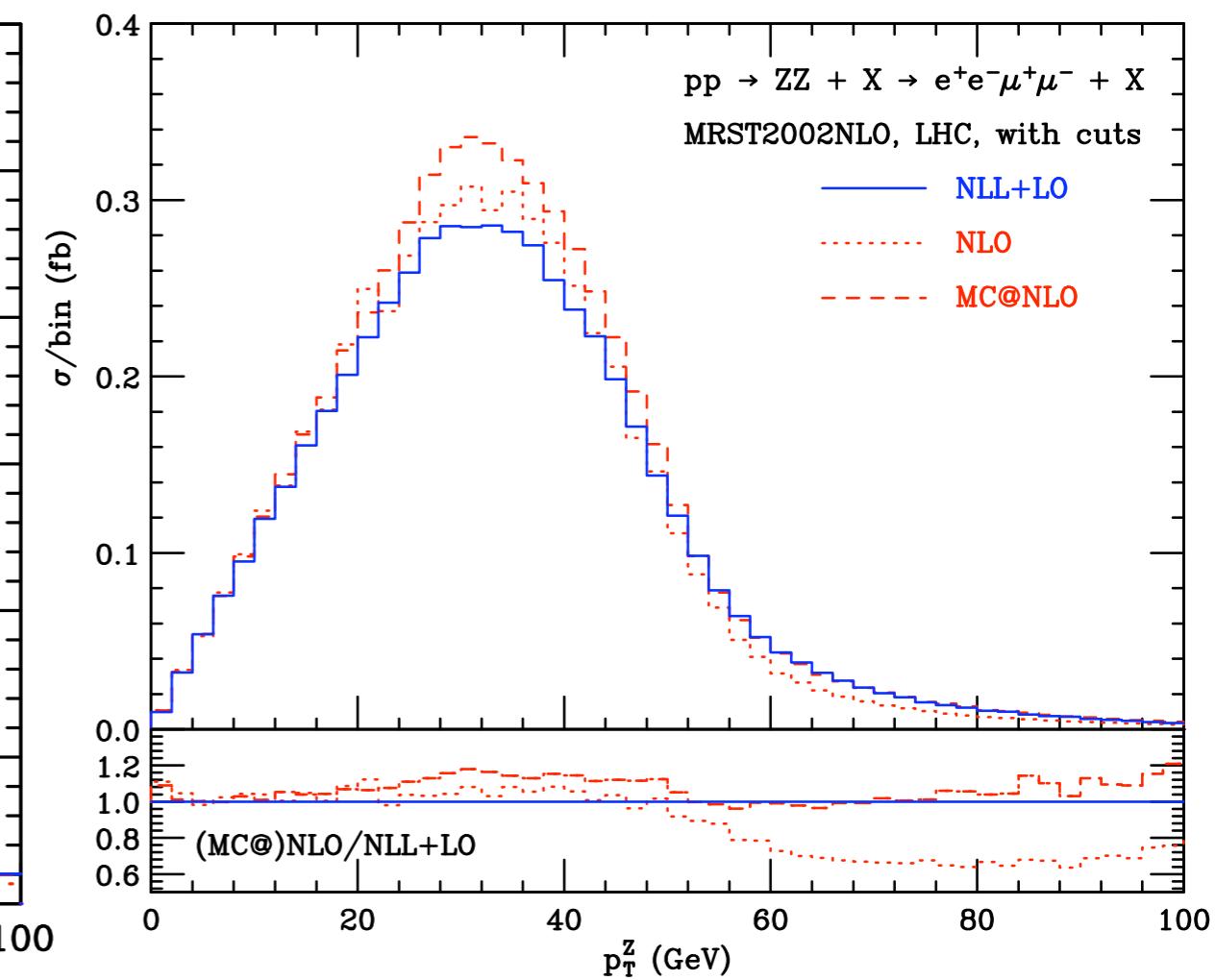
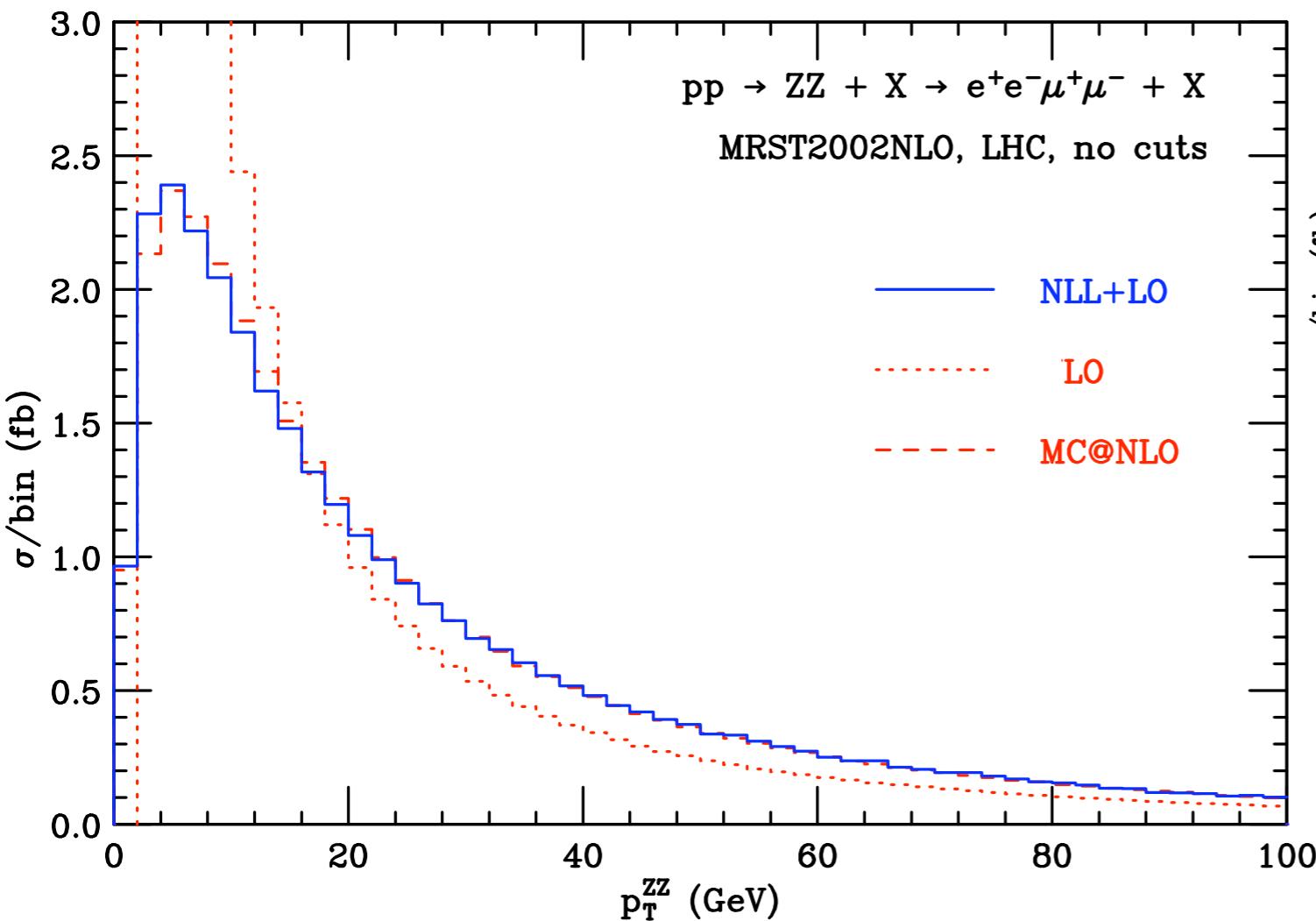
MC@NLO neglects spin correlations in non factorized part of one-loop contribution



this plot shows that the approximation works well

# Results: ZZ production

R.Frederix, MG (2005)



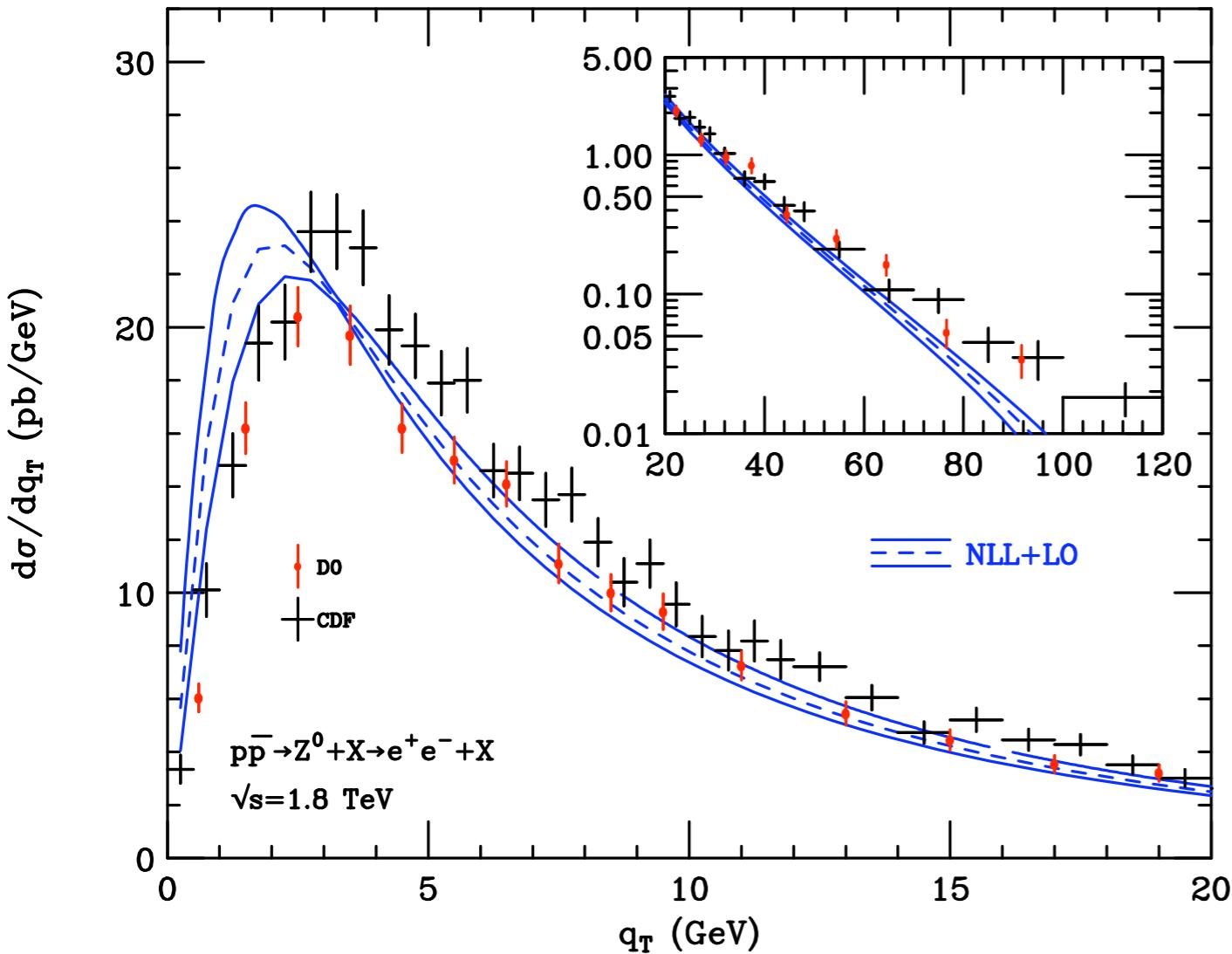
Fixed order spectrum is divergent  
Good agreement between NLL+LO  
and MC@NLO

Resummation  
effects small but  
visible

$p_{T1} > 22 \text{ GeV}$     $p_{T2} > 20 \text{ GeV}$   
 $p_{T3} > 15 \text{ GeV}$     $p_{T4} > 7 \text{ GeV}$   
 $60 \text{ GeV} < M_{ll} < 105 \text{ GeV}$   
 $190 \text{ GeV} < M_{ZZ} < 210 \text{ GeV}$

# Results for Z production

G. Bozzi, S.Catani, G. Ferrera, D. de Florian, MG(2008)



CDF data:  $66 \text{ GeV} < M < 116 \text{ GeV}$

DO data:  $75 \text{ GeV} < M < 105 \text{ GeV}$

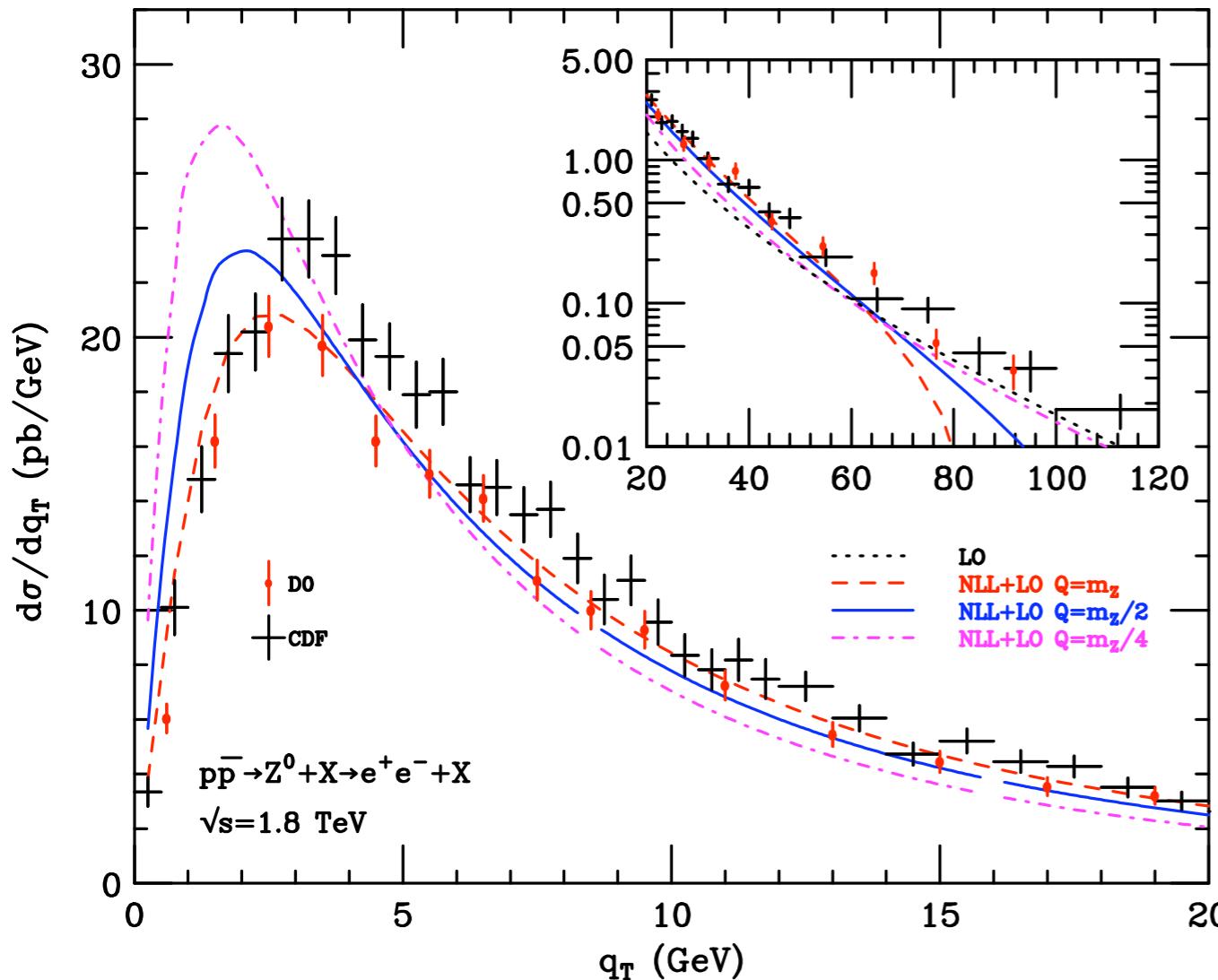
scale uncertainty computed by  
varying  $\mu_R$  and  $\mu_F$  in the range  
 $0.5m_Z \leq \mu_F, \mu_R \leq 2m_Z$  with the  
constraint  $0.5 \leq \mu_F/\mu_R \leq 2$

Scale uncertainty is about  $\pm 5\%$   
at the peak

**Resummed calculation describes reasonably well the data in the region of small and intermediate  $q_T$  even without inclusion of non-perturbative effects**

# Results: Z production

G. Bozzi, S.Catani, G. Ferrera, D. de Florian, MG(2008)



Another source of perturbative uncertainty is due to the missing higher order logarithmic contributions

→ estimated by varying the resummation scale  $Q$  in the range

$$m_Z/4 \leq Q \leq m_Z$$

The uncertainty is about  $\pm 12\%$  in the region of the peak

The uncertainty from resummation scale variations is definitely larger than the one from factorization/renormalization scale variations

→ **NNLL+NLO will be important to reduce these uncertainties**

# Summary & Outlook

- The production of colourless high-mass systems in hadronic collisions is important for physics studies within and beyond the standard model
- When  $q_T$  is much smaller than  $M$  large logarithmic contributions appear that must be resummed to all orders
- I have presented a version of the b-space formalism that has a few attractive features:
  - parton distributions factorized at  $\mu_F \sim M$
  - process dependence embodied in single perturbative factor
  - unitarity constraint enforces correct total cross section
- Applied to a number of important processes: Higgs and vector boson production, WW and ZZ production, slepton pair production
- In the case of vector boson production the accuracy can now be pushed to **NNLL+NLO**, as done for Higgs boson production
  - Hopefully it will help in the W mass measurement