2016 CERN Theory Retreat

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- 04.11.2016
- **the L'Adagio Hotel in St.Genis**

Asia -> Europe -> US -> Europe(CERN)

CERN (Oct. 2015 – present)
Geneva, Switzerland

Korea-CERN Fellow

UCLA (2012-2015)
Los Angeles, US

Work with Graciela Gelmini

SNU (2005-2010)
Seoul, Korea

PhD. Supervised by
Jihn E. Kim

MultiDark
Multimessenger Approach
for Dark Matter Detection

IFT/UAM (2010-2012)
Madrid, Spain

MultiDark Fellow

Research Interests

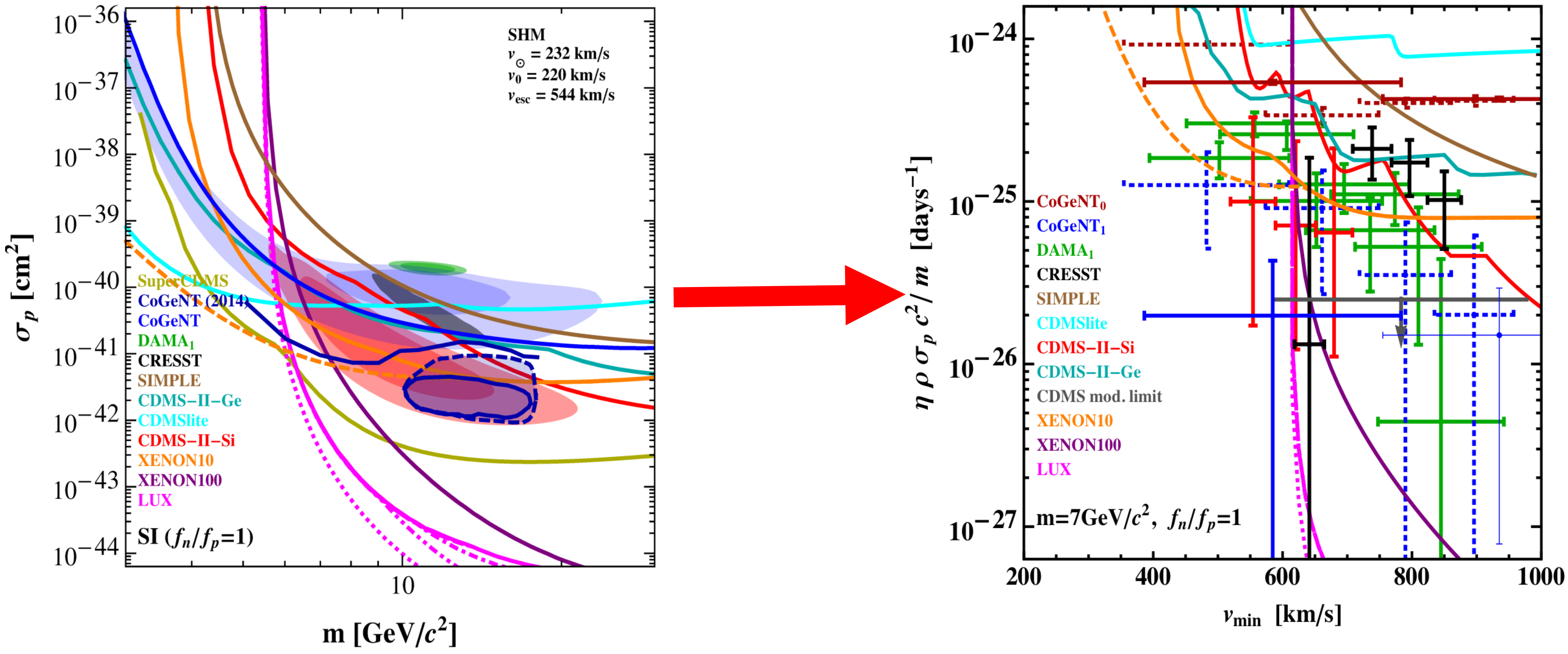
- **Dark Matter Phenomenology**
  - Model building (SUSY, Non-SUSY, WIMP, axion...)
  - Indirect Detection
  - Direct Detection (DD)
- **Recent Research Focus**
  - Developing DD analysis method
  - Halo-independent analysis of Direct Detection data

JCAP10(2013)026, JCAP10(2013)048, JCAP03(2014)014, JCAP06(2014)002, JCAP07(2014)028,JCAP08(2015)046, JCAP11(2015)038, JCAP11(2015)038, and JCAP 1610 (2016)029

In collaboration with P. Gondolo (Utah), G. Gelmini (UCLA), E. del Nobile (Padova), S. Witte (UCLA), A. Georgescue (UCLA, now in a tech company)

From the next slide ...

What is Halo-Independent Analysis (I)

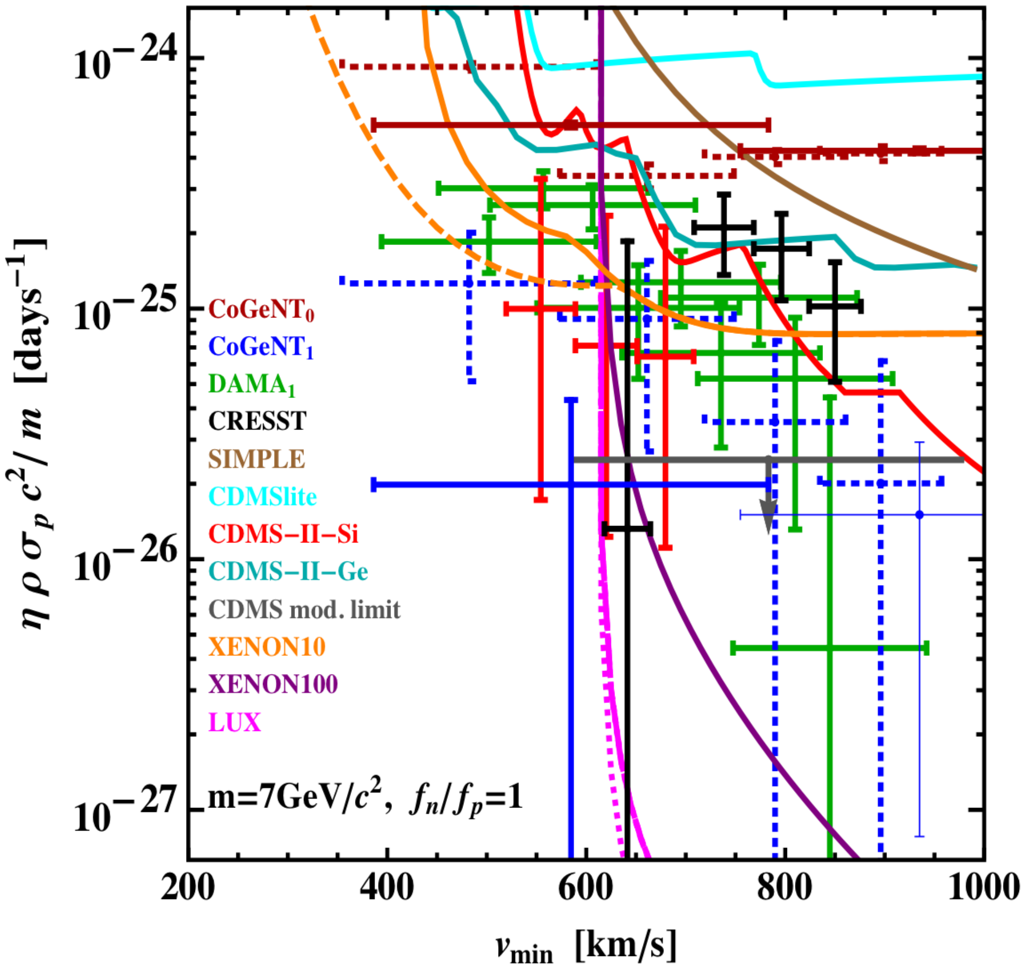


What is Halo-Independent Analysis (II)

- **1) A way of presenting DD data**

  (WIMP signal interpretation, or Constraint on WIMP model)

- **2) Testing (in)compatibility independent of Halo model**



Incremental Refinement of the method

Halo-independent method (HI)

- (diff. cross section) ~ 1/v^2
- Elastic scattering
- A set of crosses ; measurement
- Line ; upper limit

→

Generalized HI

- (diff. cross section) ~ any
- Elastic and inelastic scattering
- A set of crosses ; measurement
- Line ; upper limit

→

Extended HI

- Unbinned data
- Construction of "piecewise confidence band"

→

Global likelihood HI

- Unbinned data + arb. # of binned data
- "piecewise confidence band"
- "plausibility region"

1. HI &Generalized HI

Halo-Independent Analysis (II)

$$\tilde{\eta}(v_{\min}) \equiv \frac{\rho\sigma}{m}\int_{v_{\min}}^{\infty} \mathrm{d}v \frac{f(v)}{v}$$

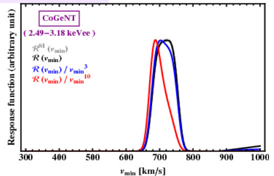
$$R_{[E'_1,E'_2]} = \sum_T \xi_T \int_{E'_2}^{E'_2} dE' \epsilon(E') G_T(E_R, E') \times \int_0^\infty dE_R v^2 \frac{d\sigma_T}{dE_R}(E_R, v)\frac{\rho\eta(E_R)}{m}$$

$$= \sum_T \xi_T \int_{E'_2}^{E'_2} dE' \epsilon(E') G_T(E_R, E') \times \int_0^\infty dv_{\min} \frac{4\mu^2 v_{\min}}{m_T} v^2 \frac{d\sigma_T}{dE_R}(E_R, v)\tilde{\eta}(v_{\min})$$

- * Target-independent quantity

$$R_{[E'_1,E'_2]} = \int_0^\infty \mathrm{d}v_{\min}\, \tilde{\eta}(v_{\min}, t)\, \mathcal{R}_{[E'_1,E'_2]}(v_{\min})$$

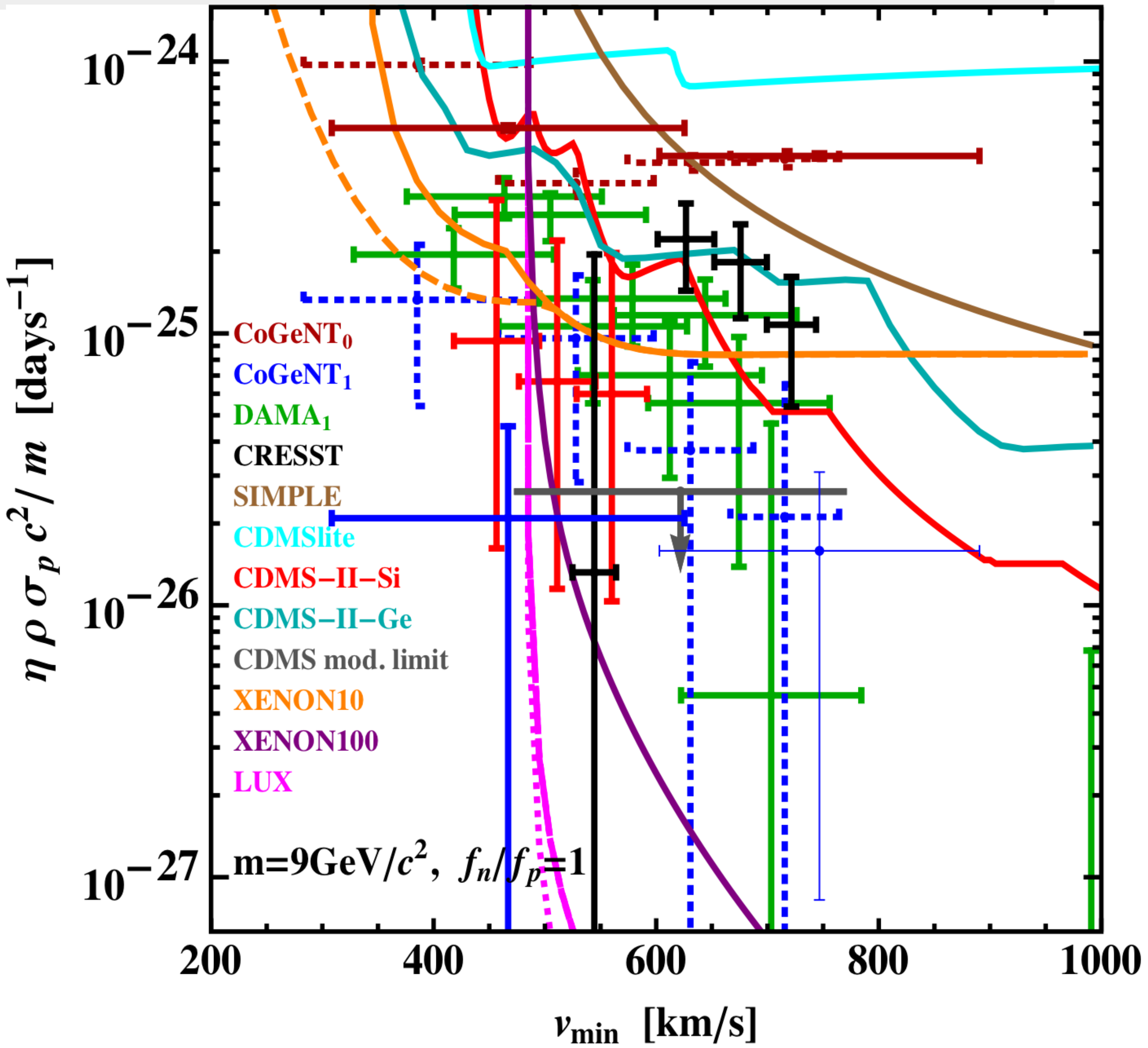
- Halo Function
- Detector Response Function



** vmin is a dummy variable, and halo-function is detector-agnostic **

HI Result

- IC-SI WIMP
- mass = 9GeV



HI result

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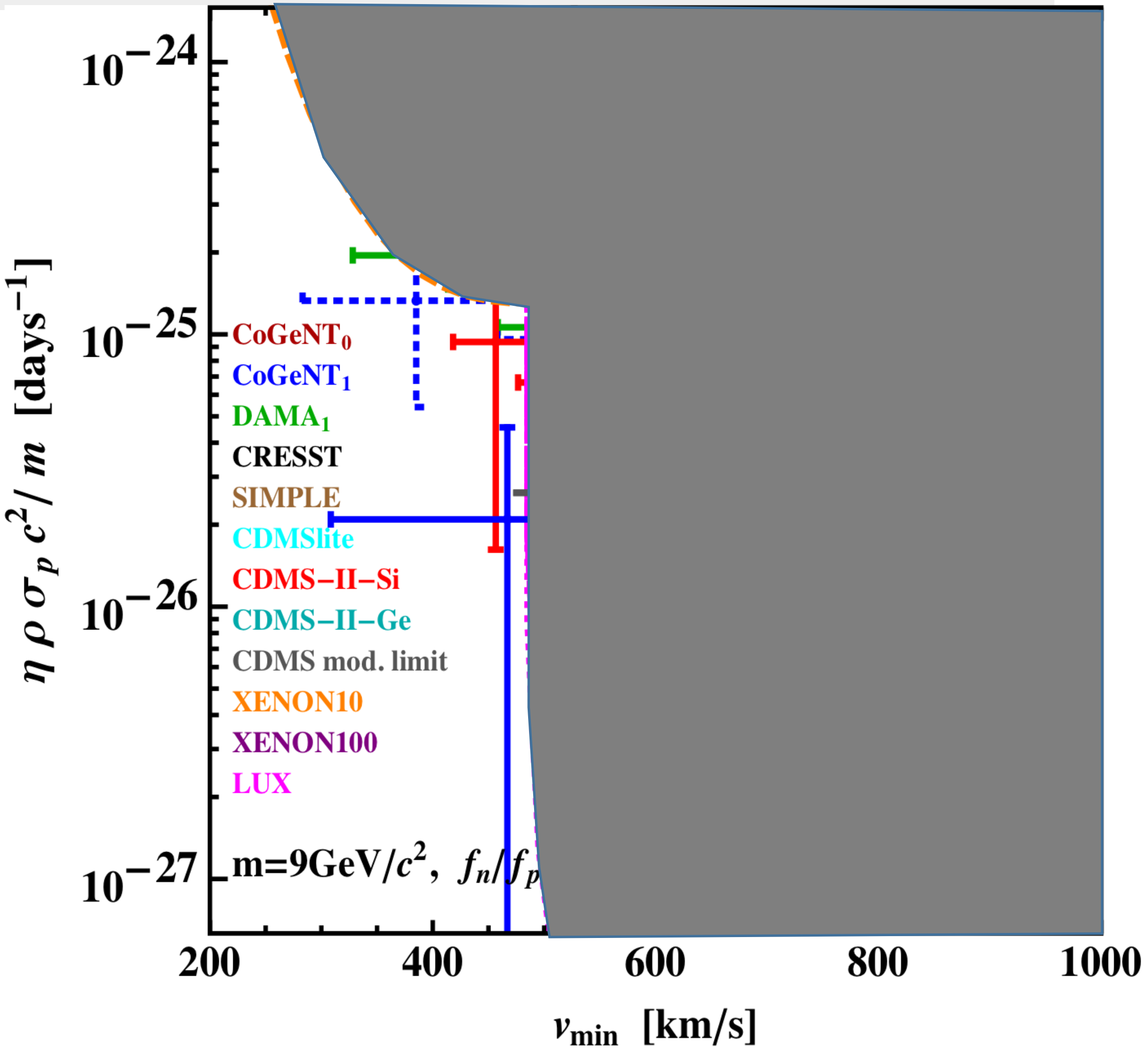


Generalized HI(I)

- Obstacles
- \- If the differential cross section has multiple terms having different velocity dependence, we cannot apply the original halo-independent method. For example,
- $$\frac{d\sigma_T}{dE_R} \sim Av^m + Bv^n$$
- requires two different halo-functions

$$\eta^{(m)}(v_{\rm min}) \equiv \int_{v_{\rm min}} dv \frac{f(v)}{v^{1-m}}$$

$$\eta^{(n)}(v_{\rm min}) \equiv \int_{v_{\rm min}} dv \frac{f(v)}{v^{1-n}}$$

→

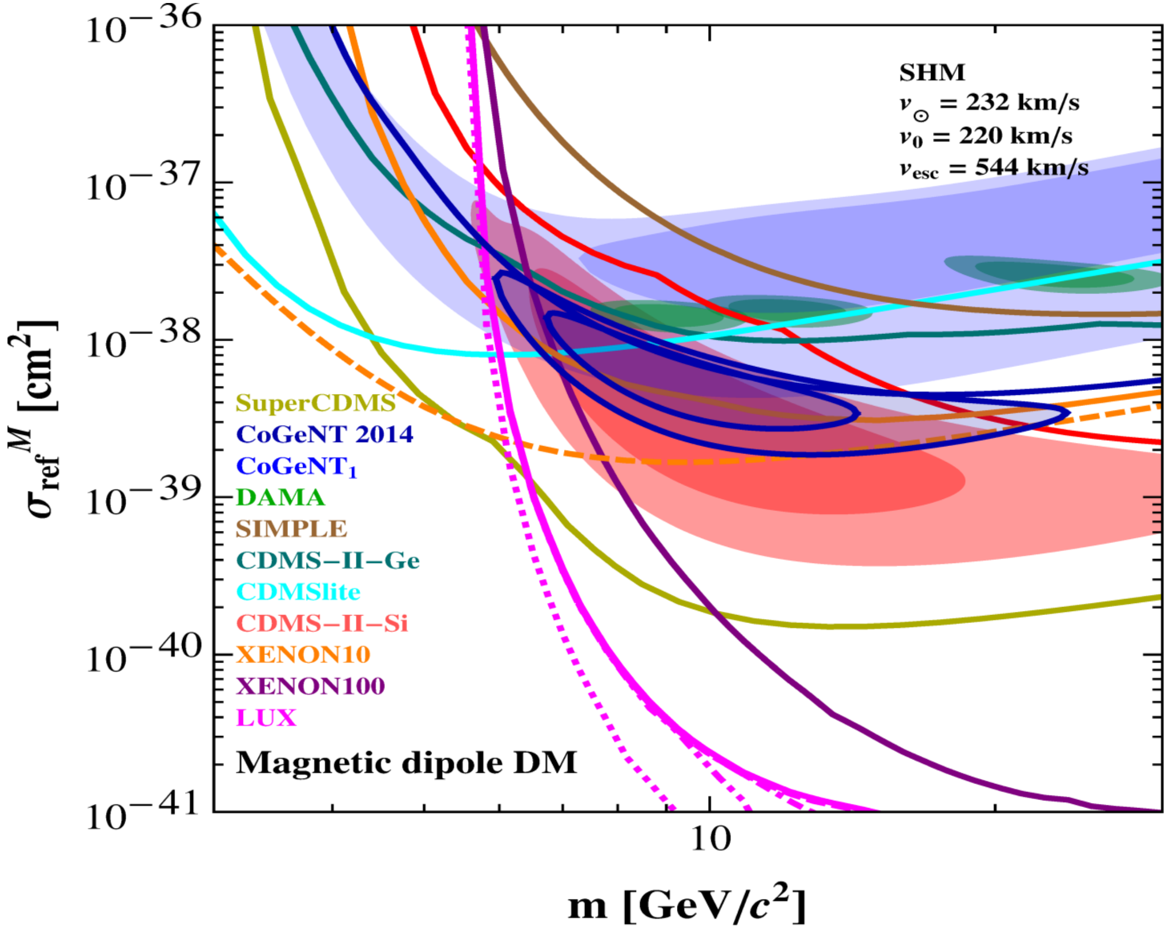
$$R_{[E_1',E_2']} = \int_0^\infty \mathrm{d}v_{\rm min} \tilde{\eta}(v_{\rm min}, t) \mathcal{R}_{[E_1',E_2']}(v_{\rm min})$$

Integration-by-parts Trick ! (ask a question for a detail)

SHM analysis of Magnetic DM

$$\frac{\mathrm{d}\sigma_T}{\mathrm{d}E_\mathrm{R}} = \sigma^M_\mathrm{ref} \frac{m_T}{\mu_T^2} \frac{1}{v^2} \left[Z^2 \left(\frac{v^2}{v^2_\mathrm{min}} - 1 + \frac{\mu_T^2}{m^2} \right) F^2_{\mathrm{E},T}(\mathbf{q}^2) + 2 \frac{\lambda_T^2}{\lambda_N^2} \frac{\mu_T^2}{m_N^2} \left(\frac{J_T+1}{3J_T} \right) F^2_{\mathrm{M},T}(\mathbf{q}^2) \right]$$

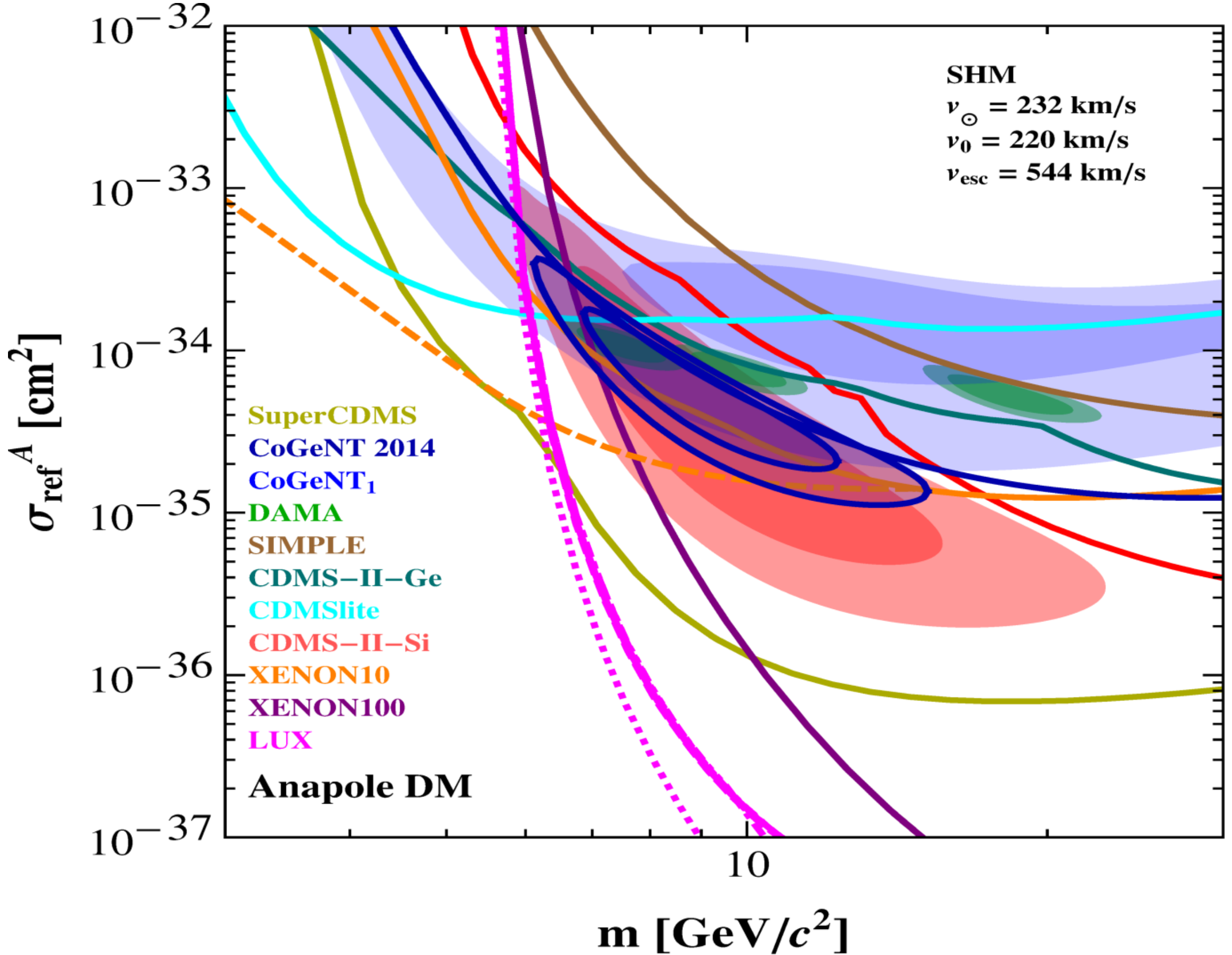
- Magnetic Dipole DM



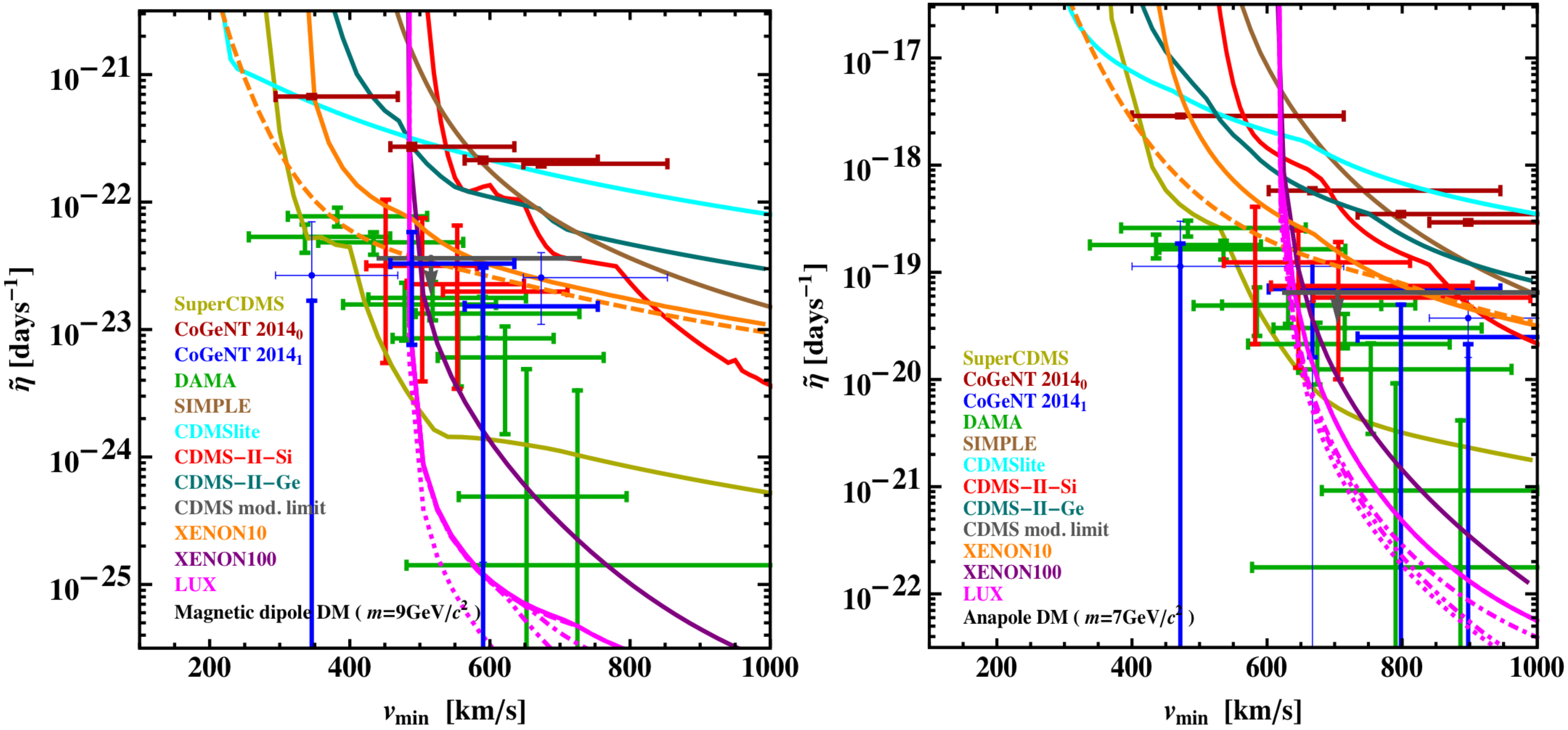
SHM analysis of Anapole DM

$$\frac{d\sigma_T}{dE_R} = \sigma^A_{\rm ref} \frac{m_T}{\mu_N^2} \frac{v_{\rm min}^2}{v^2} \left[Z^2 \left(\frac{v^2}{v_{\rm min}^2} - 1 \right) F^2_{{\rm E},T}(\mathbf{q}^2) + 2 \frac{\lambda_T^2}{\lambda_N^2} \frac{\mu_T^2}{m_N^2} \left(\frac{J_T + 1}{3 J_T} \right) F^2_{{\rm M},T}(\mathbf{q}^2) \right]$$

- Anapole DM



Results (MDM & ADM)



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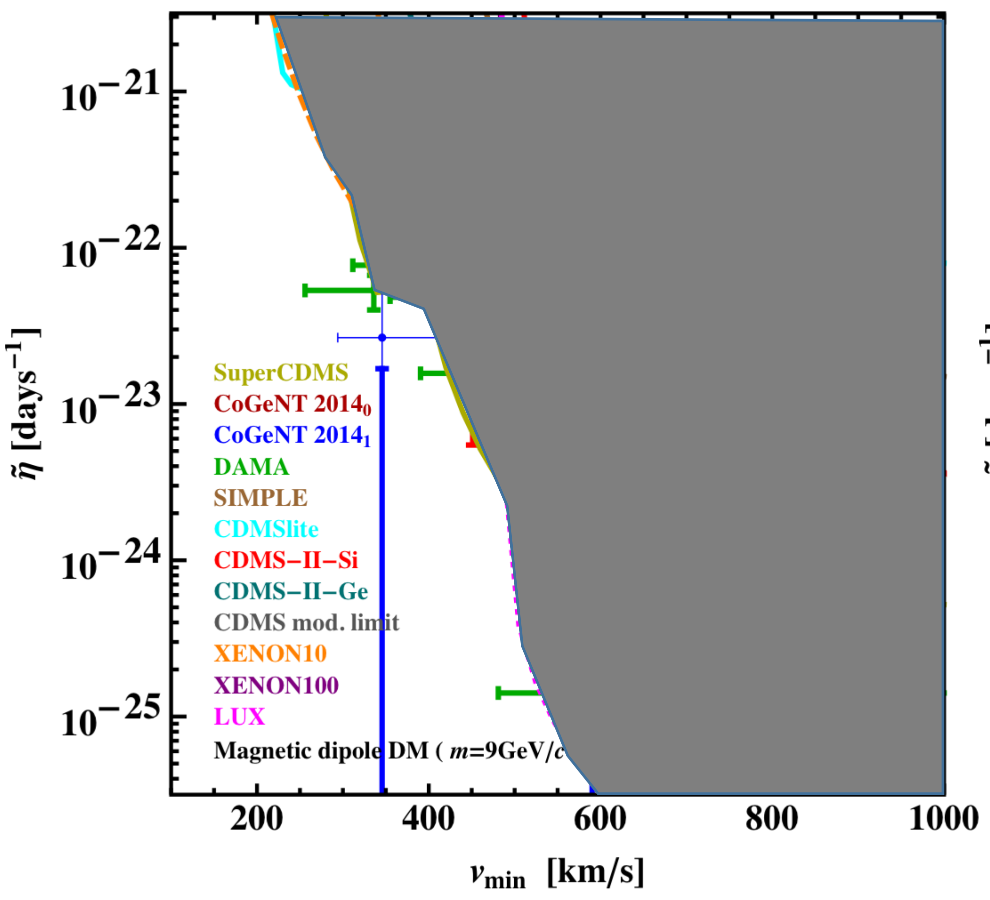
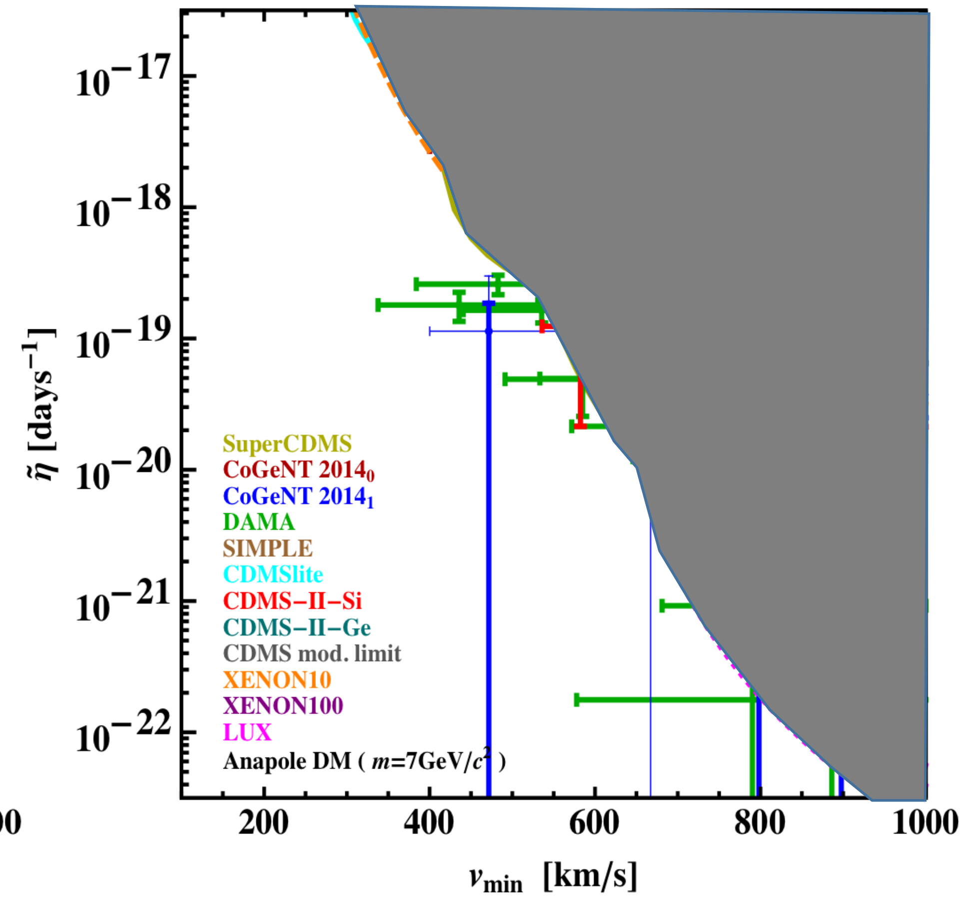


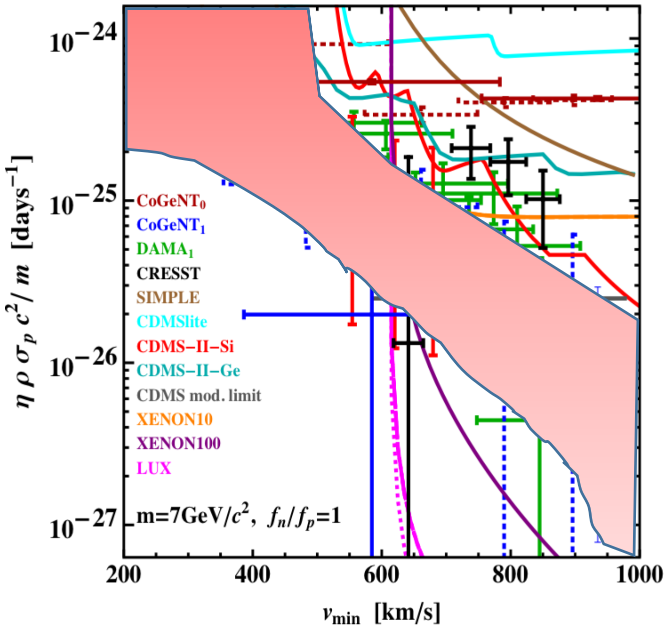


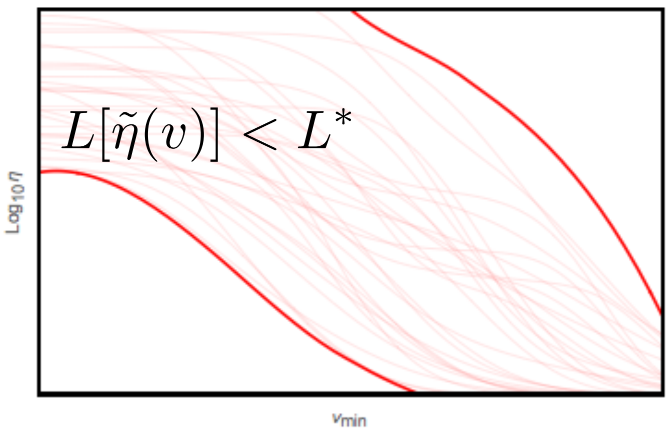
2. Extended HI

Ambiguity in the interpretation of the result

- The Assessment of Compatibility using the “crosses” and “upper limit” was never be quantitative.
- A statistical interpretation of the “cross” is not clear
- Forming a band in the eta-vmin space maybe helpful.
- - Natural candidate : The envelope of the family of all possible curves satisfying a criterion
- - Practically difficult
- - Statistical meaning is not clear. (What is the correct Degrees of Freedom?)

Idea : The Band!





- Enveloping boundary of all possible halo-functions satisfying certain criterion, $L[\tilde{\eta}(v)] < L^*$

Chi-square :

$$L \equiv -2 \ln \mathcal{L}$$

- **Problem :**
- **- Practically impossible**
- **- What L* to choose? Coverage?**

Nice property of Extended Likelihood

- The Extended Likelihood (Barlow, 1990)

$$\mathcal{L}[\tilde{\eta}(v_{\min})] = e^{-N_E[\tilde{\eta}]} \prod_{a=1}^{N_O} MT \left. \frac{\mathrm{d}R_{tot}}{\mathrm{d}E'} \right|_{E'=E'_a}$$

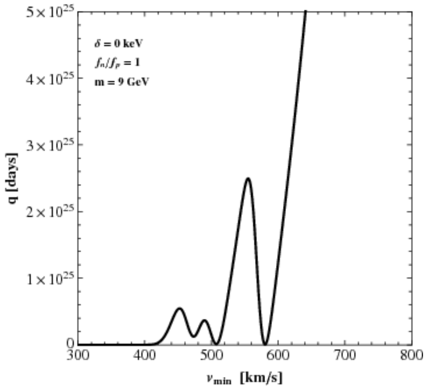
- Exponential pre-factor
- Joint probability

- The likelihood for unbinned data (e.g., 3 events in CDMS-II-Si)
- The unique solution to the maximization of the likelihood.
- The exponential term plays a role.
- Maximization is given by a multi-step functions whose number of steps are at most the number of events.
- (Fox, Kahn, McCollough. 2014, Gelmini,Georgescu,Gondolo&JHH 2015, Gelmini, JHH&Witte 2016 (Global likelihood))
- **The N_O-Step Theorem + The (N_O+1)-Step Theorem**

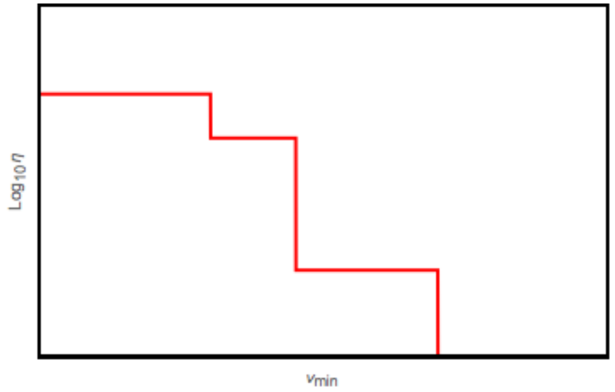
"N_O step Theorem"

- KKT conditions for the halo-function:

$$
\begin{aligned}
&(I) \quad q(v_{\min}) = \int_{v_\delta}^{v_{\min}} \mathrm{d}v \frac{\delta L}{\delta \tilde{\eta}(v)}, \\
&(II) \quad q(v_{\min}) \geq 0, \\
&(III) \quad \forall \epsilon > 0, \quad \tilde{\eta}(v_{\min}+\epsilon) \leq \tilde{\eta}(v_{\min}), \text{ and} \\
&(IV) \quad q(v_{\min}) \lim_{\epsilon \to +0} \frac{\tilde{\eta}(v_{\min}+\epsilon) - \tilde{\eta}(v_{\min})}{\epsilon} = 0
\end{aligned}
$$



- **"N_O step Theorem"** : The halo-function maximizing the extended likelihood is in the form of a multi-step function, whose number of steps is at most N_O = # of observed events.
- **The problem reduction :**
- \- Infinite dimensional -> finite dimensional



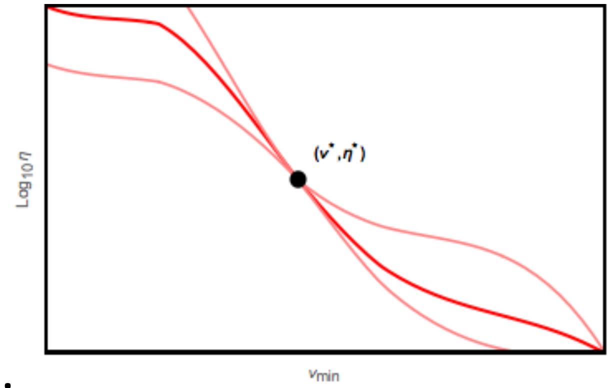
- Fox, Kahn, McCollough. 2014
- Gelmini,Georgescu,Gondolo&JHH 2015
- Gelmini, JHH&Witte 2016 (Global likelihood))

"(N_O+1) step Theorem"

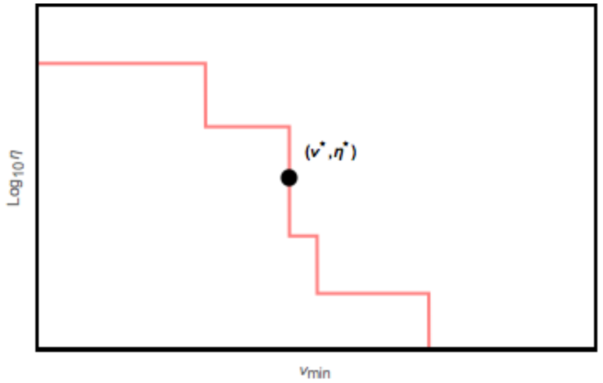
- A slight modification of the problem:

 - add one more constraint, namely

$$\tilde{\eta}(v^*) = \tilde{\eta}^*$$



- **"(N_O+1) step Theorem"** : The halo-function maximizing the extended likelihood subject to the above constraint is in the form of (N_O+1) step function.



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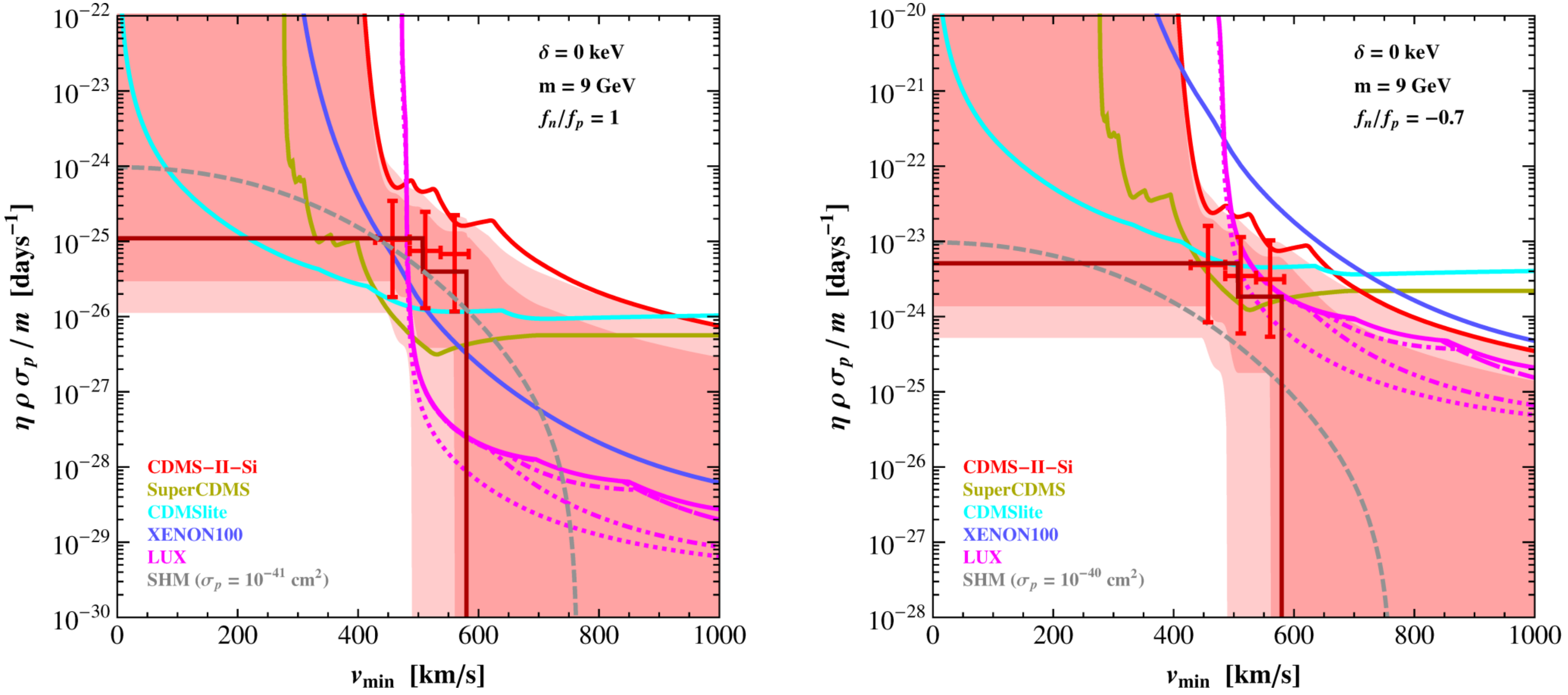
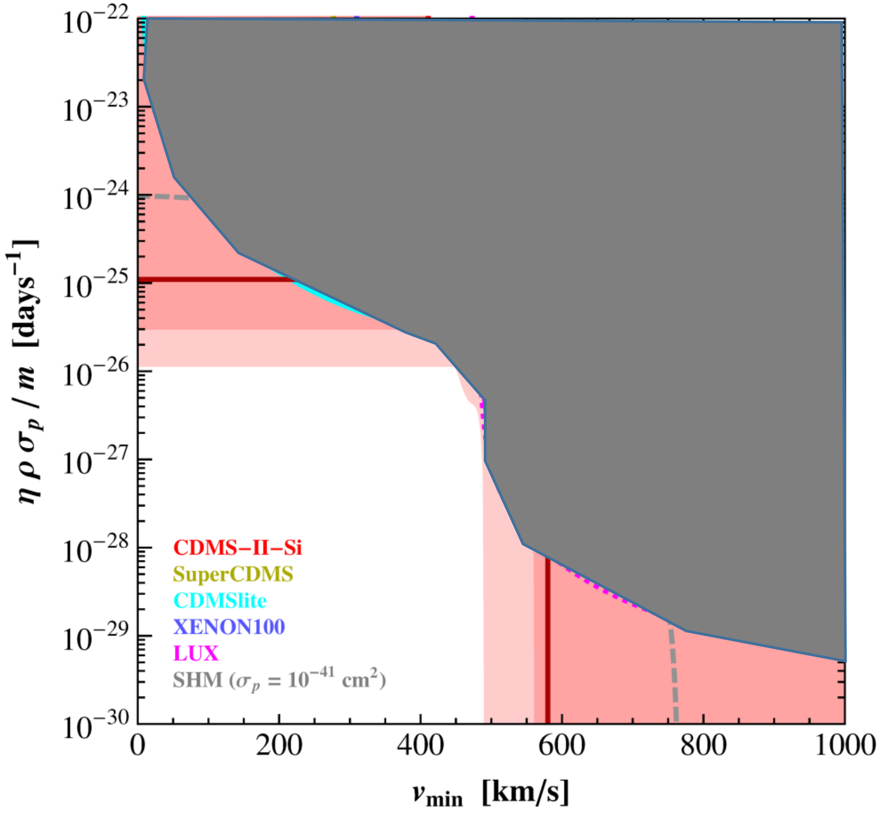
EHI results

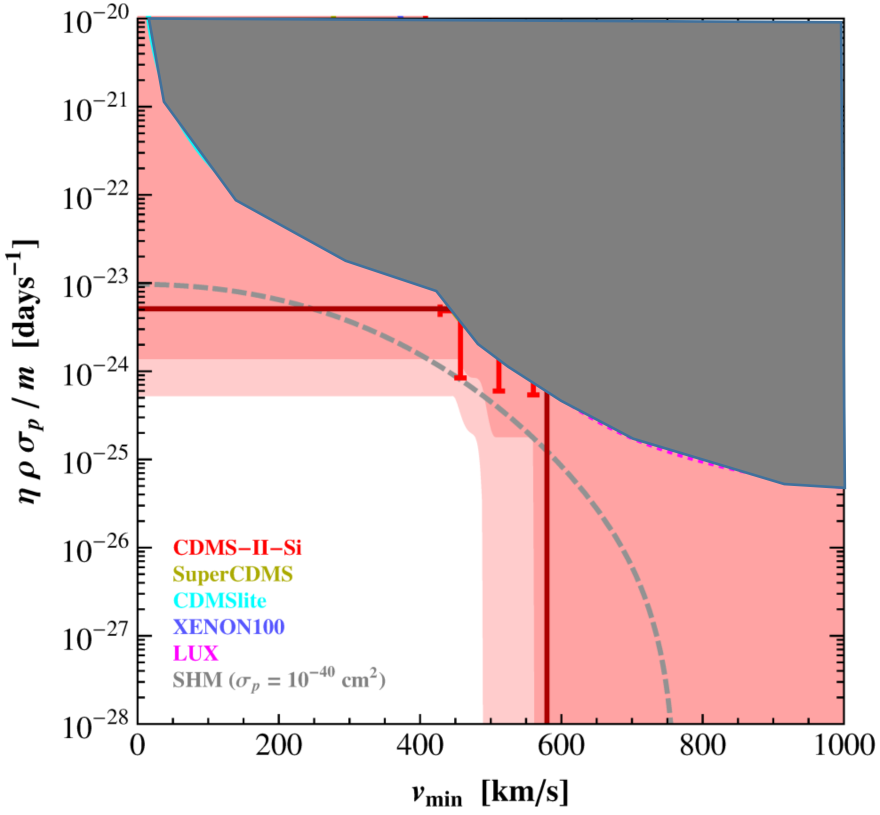


δ = 0 keV
m = 9 GeV
f_n/f_p = 1
f_n/f_p = −0.7
CDMS−II−Si
SuperCDMS
CDMSlite
XENON100
LUX
SHM ($\sigma_p = 10^{-41}$ cm^2)
SHM ($\sigma_p = 10^{-40}$ cm^2)
$\eta\,\rho\,\sigma_p\,/\,m$ [days^{-1}]
v_{min} [km/s]

EHI results



10^{-22}
10^{-23}
10^{-24}
10^{-25}
10^{-26}
10^{-27}
10^{-28}
10^{-29}
10^{-30}
0
200
400
600
800
1000
$\eta\, \rho\, \sigma_p \,/\, m$ [days^{-1}]
$v_{\rm min}$ [km/s]
CDMS−II−Si
SuperCDMS
CDMSlite
XENON100
LUX
SHM (σ_p = 10^{-41} cm^2)



10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-25}
10^{-26}
10^{-27}
10^{-28}
0
200
400
600
800
1000
$\eta\, \rho\, \sigma_p \,/\, m$ [days^{-1}]
$v_{\rm min}$ [km/s]
CDMS−II−Si
SuperCDMS
CDMSlite
XENON100
LUX
SHM (σ_p = 10^{-40} cm^2)

3. Global likelihood HI

Global EHI

- Global likelihood

$$L_G[\tilde{\eta}] \equiv -2\ln\mathcal{L}_{\rm EHI} - 2\ln\sum_i \mathcal{L}_i^{\rm bin}$$

- Can we construct the band?
  - Yes, but now with (N_O + N_bin) steps
- Is it meaningful by itself?
  - Sometimes, but not always
  - There always exists a band, regardless of “poor quality of fit” and/or “mutual disagreement” among the data.

Goodness-of-Fit

* Global likelihood

$$\mathcal{L}_G = \prod_i^{N_{\rm exp}} \mathcal{L}_i$$

- Compatibility measure

  - Goodness-of-fit

$$\mathrm{GoF} \equiv \min[-2\ln\mathcal{L}_G]$$

  - Parameter goodness-of-fit

$$\mathrm{PG} \equiv -2\ln\mathcal{L}_G^{\rm max} + 2\sum_i^{N_{\rm exp}} \ln\mathcal{L}_i^{\rm max}$$

  - Constrained parameter goodness-of-fit

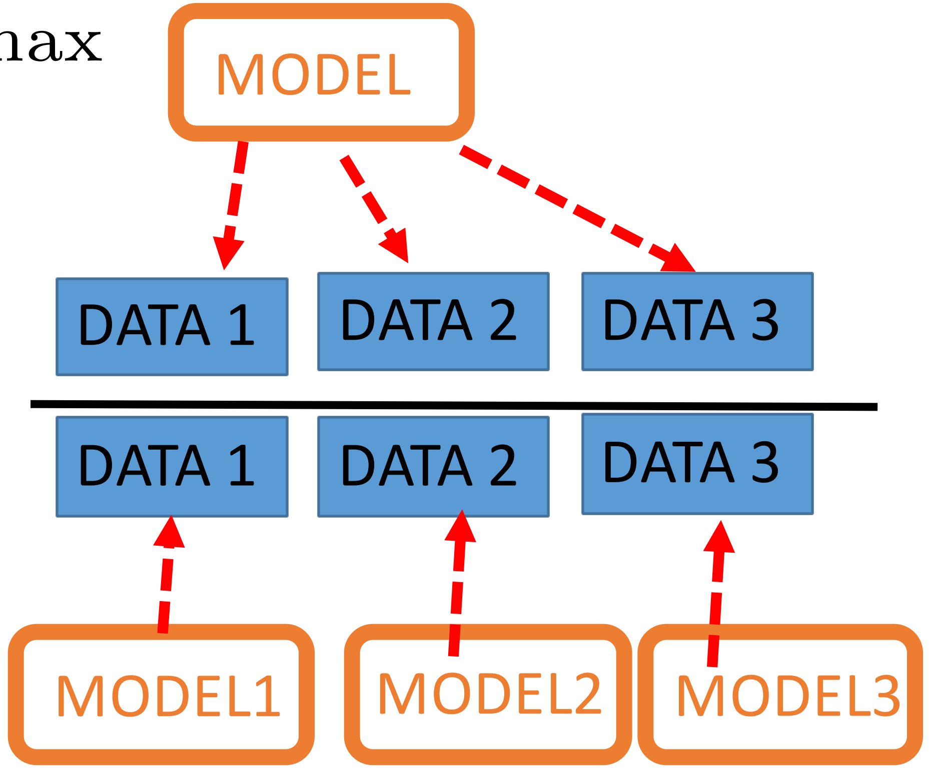
$$\mathrm{cPG} \equiv -2\ln\mathcal{L}_G^{\rm max,c}(v^*, \tilde{\eta}^*) + 2\sum_i^{N_{\rm exp}} \ln\mathcal{L}_i^{\rm max,c}(v^*, \tilde{\eta}^*)$$

Parameter Goodness-of-Fit

$$\mathrm{PG} \equiv -2\ln\mathcal{L}_G^{\max} + 2\sum_i^{N_{\rm exp}} \ln\mathcal{L}_i^{\max}$$

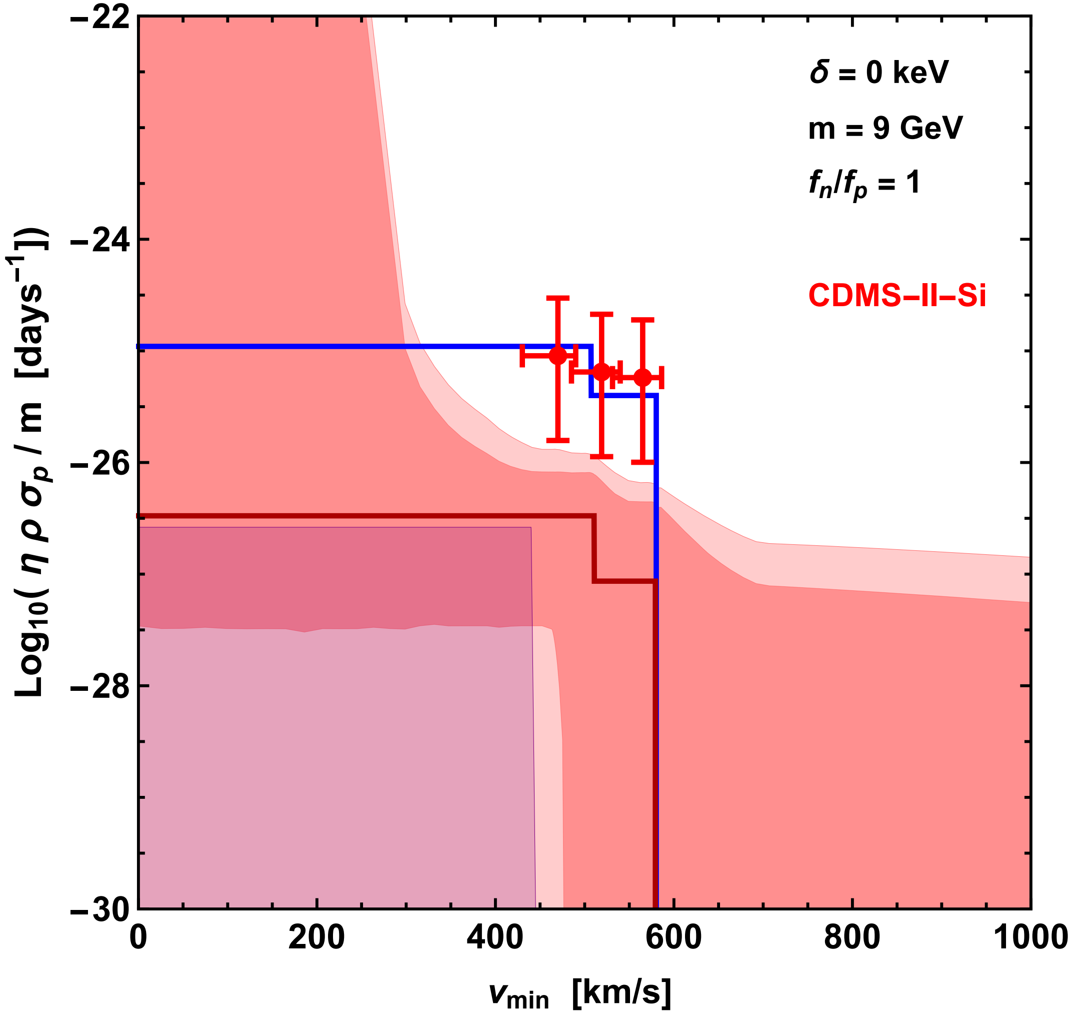
$$= -2\ln\frac{\mathcal{L}_G^{\max}}{\prod_i \mathcal{L}_i^{\max}}$$

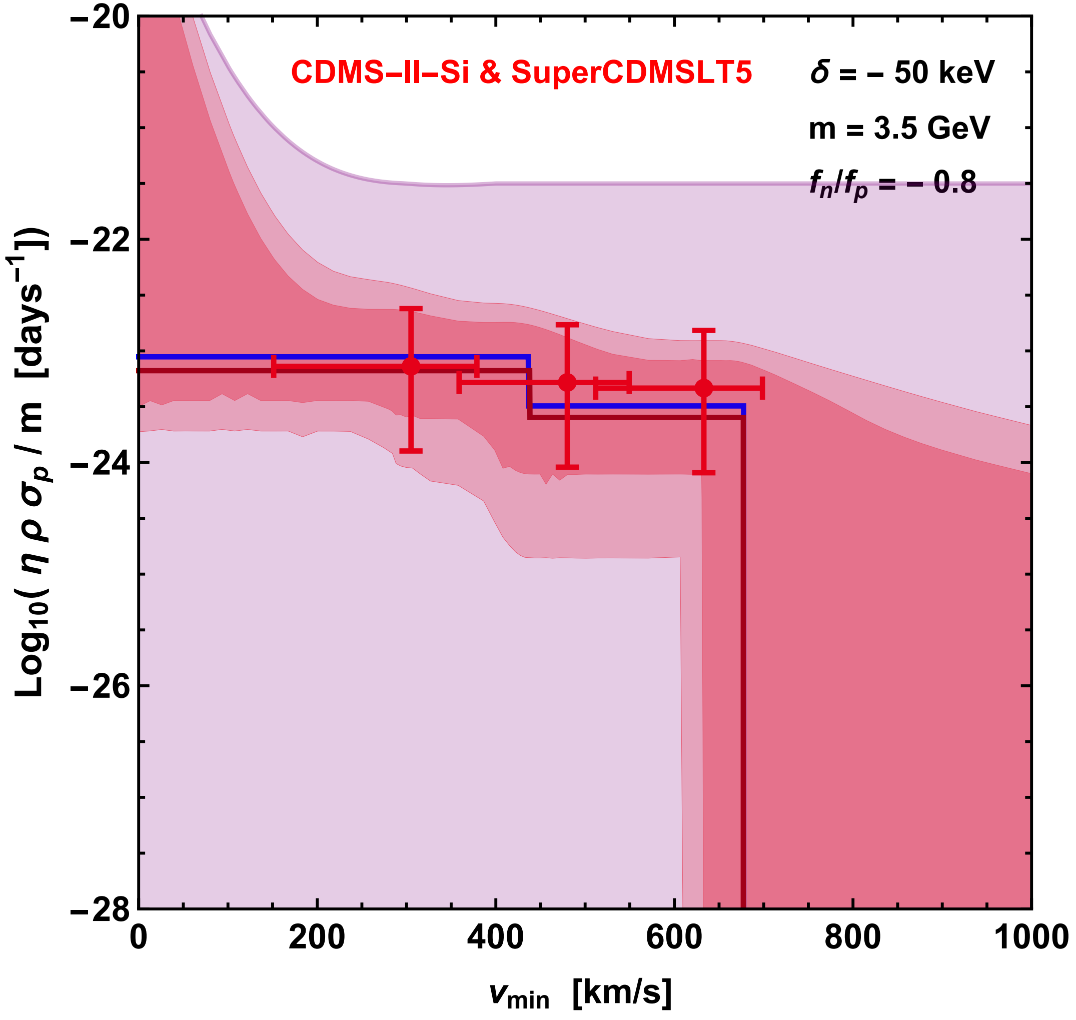
=>



* A test on the (somewhat trivial) hypothesis that all the experiments were performed in the same Universe

Global EHI 1 (SuperCDMS + CDMS-II-Si)





The Band and Region

- Piecewise confidence BAND

- A collection of confidence interval
- Lives in ”parameter space”
- Always exists
- Connected band

- Plausibility REGION

- A set of hypotheses which are not rejected by the data
- Lives in ”hypothesis space”, where each hypothesis is parameterized by pair of variable, (v*,eta*)
- May not exist
- Possibly disconnected region

* Complementary role

Conclusion

- Halo-Independent analysis is possible, and can be used for 1) visualizing data, and 2) measuring compatibility.
- Generalized HI can be used to test EFT interactions and inelastic WIMP
- Two types of regions, “confidence band” and “plausibility region” play complementary role.

Future work

- Application to annual modulation (no-monotonicity)
- Combining with background independent method => generalizing Yellin's method
- Application to an exotic scenario (e.g. 2->3 scattering)

Thank you for your attention!

경청해 주셔서 감사합니다!

Backup slides

Generalized Halo-Independent Method (II)

- **Integration-by-parts Trick**
- - The original expression for the differential rate

$$\frac{dR}{dE'} = \epsilon(E')\frac{\rho}{m}\sum_T \xi_T G_T(E_R, E') \int_0^\infty dE_R \int_{v_{\min}(E_R)}^\infty dv \frac{f(v)}{v} v^2 \frac{d\sigma_T}{dE_R}$$
$$= \epsilon(E')\frac{\rho}{m}\sum_T \xi_T G_T(E_R, E') \int_0^\infty dE_R v^2 \frac{d\sigma_T}{dE_R}\eta(E_R)$$

- is from interchanging the order of integrations
- - Instead we can use the integration-by-parts to get any
- desired form of halo function.
- - In many cases, they give the same results, but not always.

“N_O step Theorem” (Backup)

- KKT conditions for the halo-function:

$$(I) \quad q(v_{\min}) = \int_{v_\delta}^{v_{\min}} \mathrm{d}v \frac{\delta L}{\delta \tilde{\eta}(v)},$$

$$(II) \quad q(v_{\min}) \geq 0,$$

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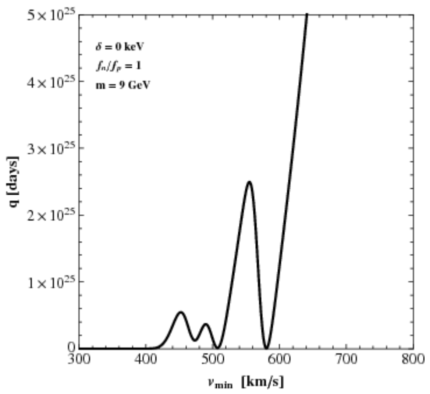
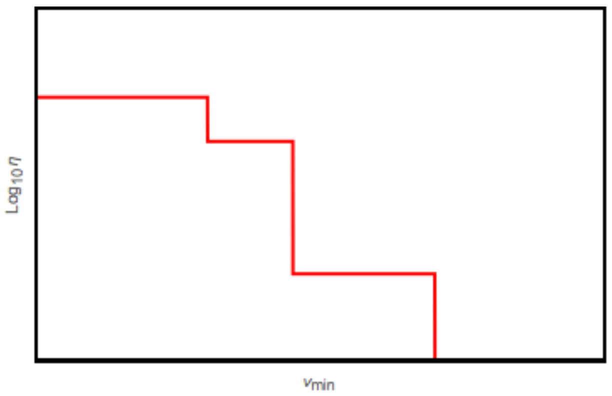


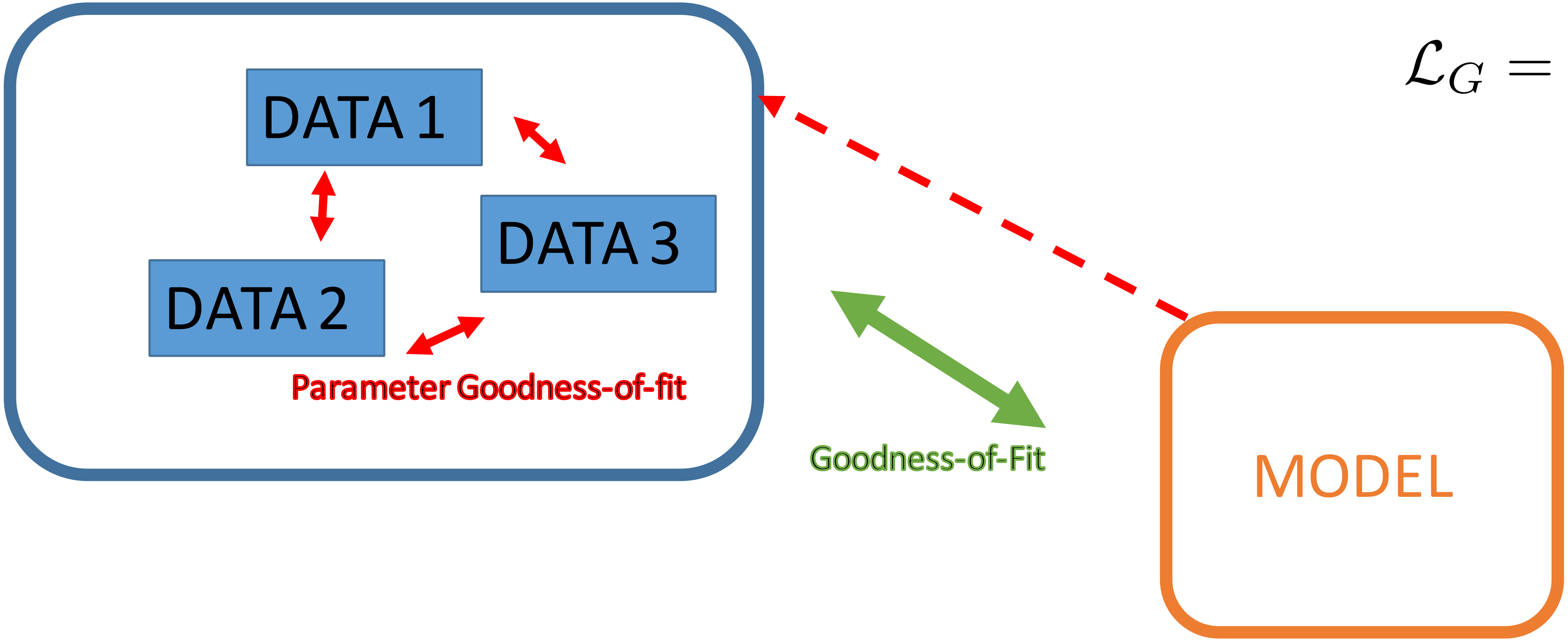
- **“N_O step Theorem”** : The halo-function maximizing the extended likelihood is in the form of a multi-step function, whose number of steps is at most N_O = # of observed events.
- **The problem reduction :**
- - Infinite dimensional -> finite dimensional



(Mutual) Compatibility

* Global likelihood

$$\mathcal{L}_G = \prod_{i}^{N_{\rm exp}} \mathcal{L}_i$$

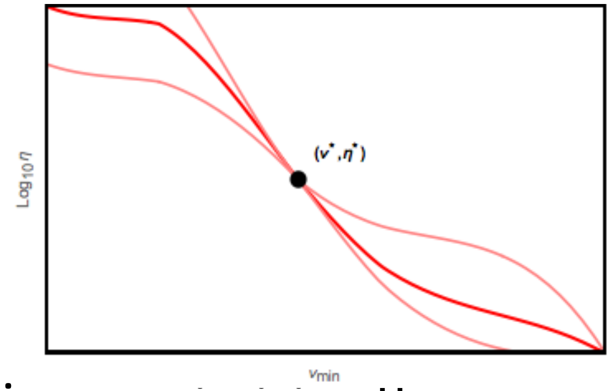


“(N_O+1) step Theorem” (Backup)

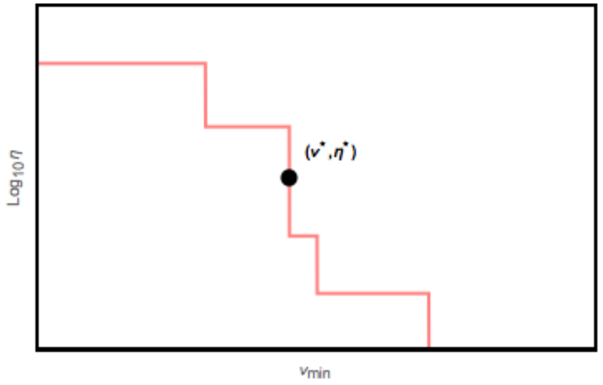
- A slight modification of the problem:

- add one more constraint, namely

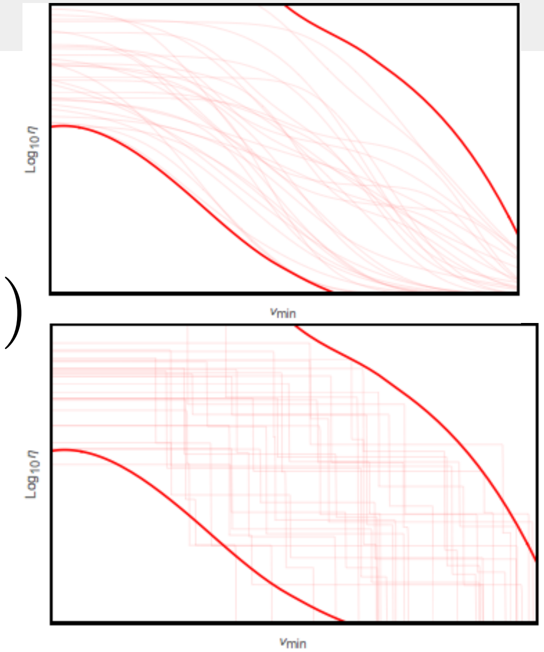
$$\tilde{\eta}(v^*) = \tilde{\eta}^*$$



- **“(N_O+1) step Theorem”** : The halo-function maximizing the extended likelihood subject to the above constraint is in the form of (N_O+1) step function.



The band : pointwise confidence interval (Backup)



- N_O step Theorem:
  - One can compute

$$L_{\min} \equiv \max_{\tilde{\eta}(v)} \left(-2 \ln \mathcal{L}[\tilde{\eta}(v)]\right) \quad \text{and} \quad \tilde{\eta}_{\mathrm{BF}}(v)$$

- (N_O+1) step Theorem:
  - One can compute

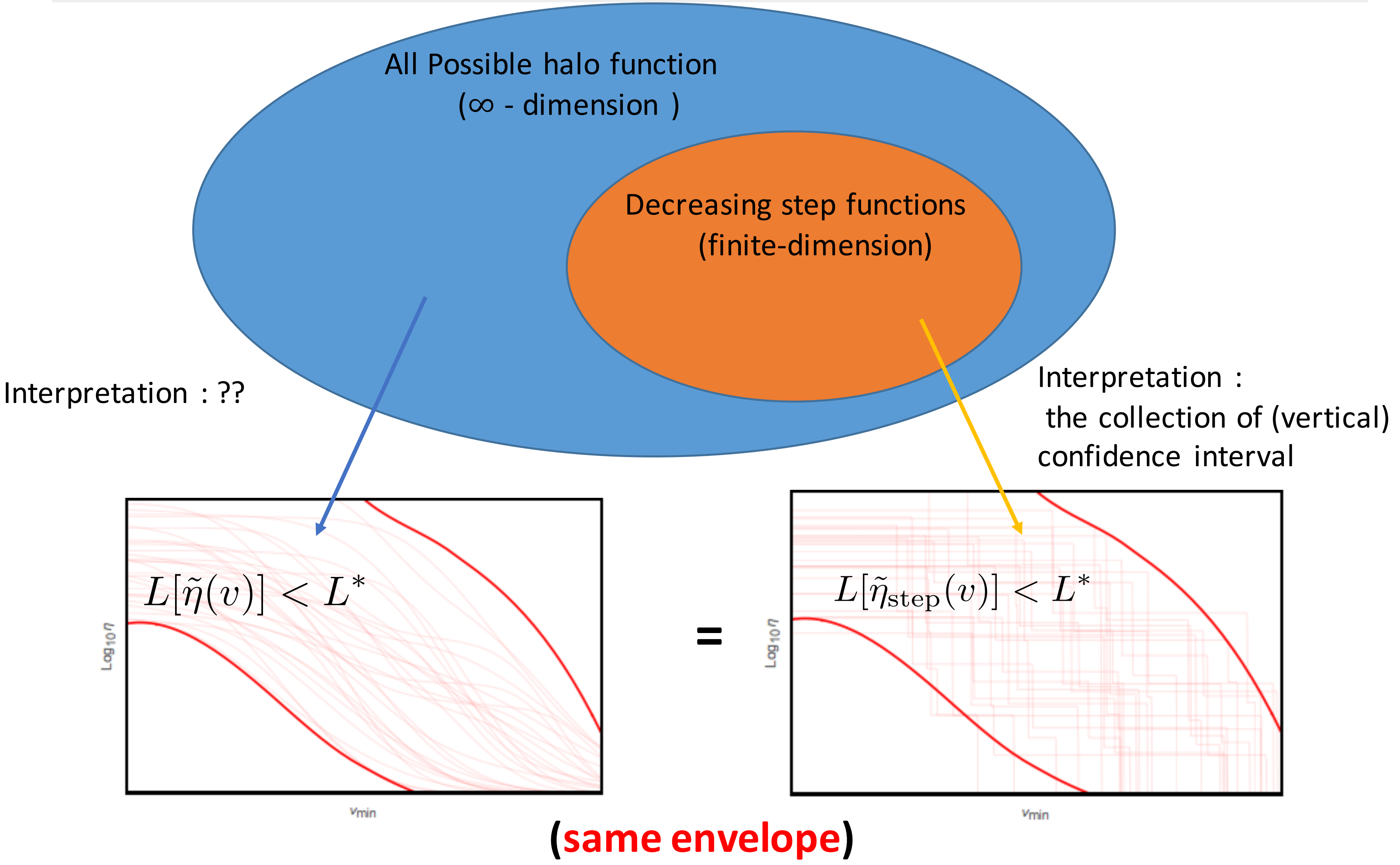
$$\Delta L^{c}_{\min}(v^*, \tilde{\eta}^*) \equiv L^{c}_{\min}(v^*, \tilde{\eta}^*) - L_{\min}$$

** With discretized vmin variable ➔ Profile likelihood ratio with 1 remaining d.o.f.

➔ **We can apply Wilks Theorem!!** ➔ "pointwise confidence interval"

$$\Delta L^{c,k}_{\min}(\tilde{\eta}^*) = -2 \ln \left[\frac{\mathcal{L}(\hat{\hat{\tilde{\eta}}}_0, \ldots, \hat{\hat{\tilde{\eta}}}_{k-1}, \tilde{\eta}_k = \tilde{\eta}^*, \hat{\hat{\tilde{\eta}}}_{k+1}, \ldots, \hat{\hat{\tilde{\eta}}}_{K-1})}{\mathcal{L}(\hat{\tilde{\eta}}_0, \ldots, \hat{\tilde{\eta}}_k, \ldots, \hat{\tilde{\eta}}_{K-1})} \right]$$

Constructing the band (Backup)



Parameter Goodness-of-Fit

$$\mathrm{PG} \equiv -2\ln\mathcal{L}_G^{\max} + 2\sum_i^{N_{\exp}} \ln\mathcal{L}_i^{\max}$$

$$= -2\ln\frac{\mathcal{L}_G^{\max}}{\prod_i \mathcal{L}_i^{\max}}$$

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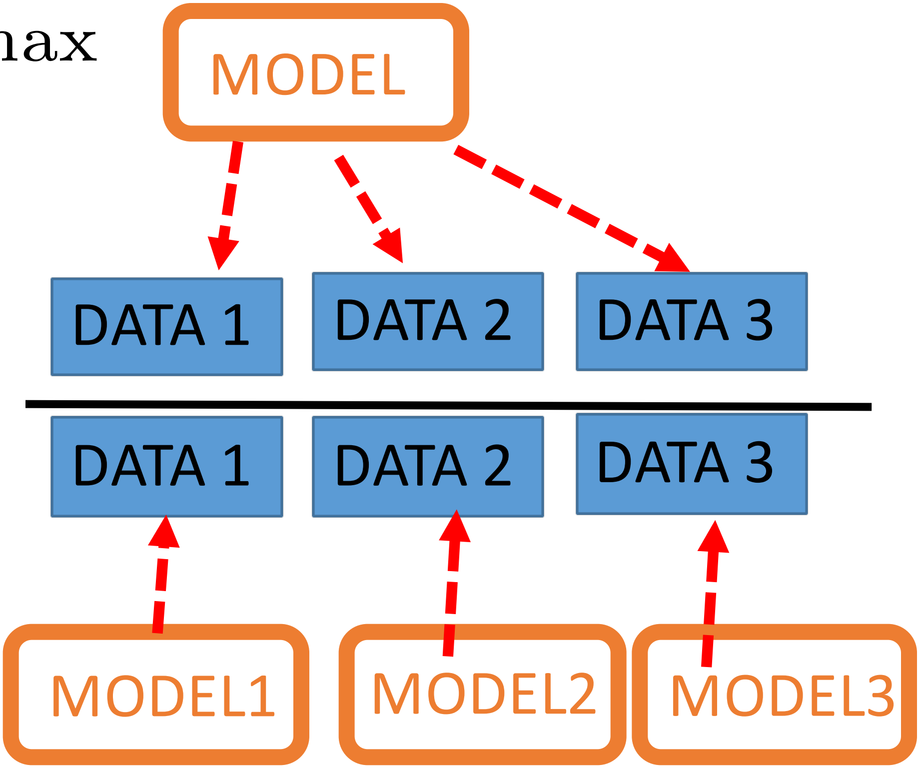

* A test on the (somewhat trivial) hypothesis that all the experiments were performed in the same Universe