

Harmonic Lasing in XFELs: results from FLASH

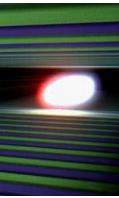
E. Schneidmiller and M. Yurkov

8th Hard X-ray FEL Collaboration Meeting
Pohang, Korea

October 26, 2016

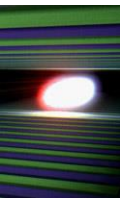


HELMHOLTZ
| ASSOCIATION



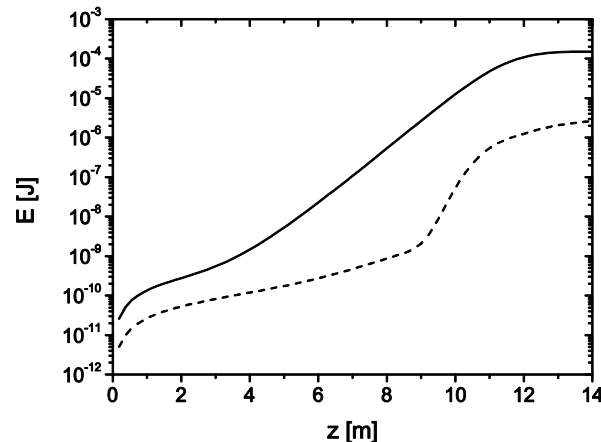
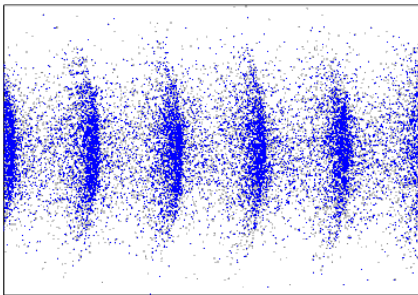
- In a planar undulator ($K \sim 1$ or $K > 1$) the odd harmonics can be radiated on-axis (widely used in SR sources)
- For coherent emission a mechanism is required to create coherent microbunching at harmonic frequencies
- There are two basic mechanisms in FELs:
 - Nonlinear harmonic generation
 - Harmonic lasing

We consider SASE process in a baseline XFEL undulator



- When lasing at the fundamental frequency approaches saturation, the density modulation becomes nonlinear (contains higher harmonics)
- Odd harmonics are radiated then on-axis
- Well-known process, studied in many papers

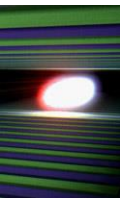
Microbunching at saturation



1st: solid
3rd: dash

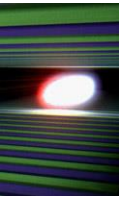
3rd harmonic is driven
by the fundamental

Occurs whenever an FEL reaches saturation; studied and used at FLASH, LCLS etc.

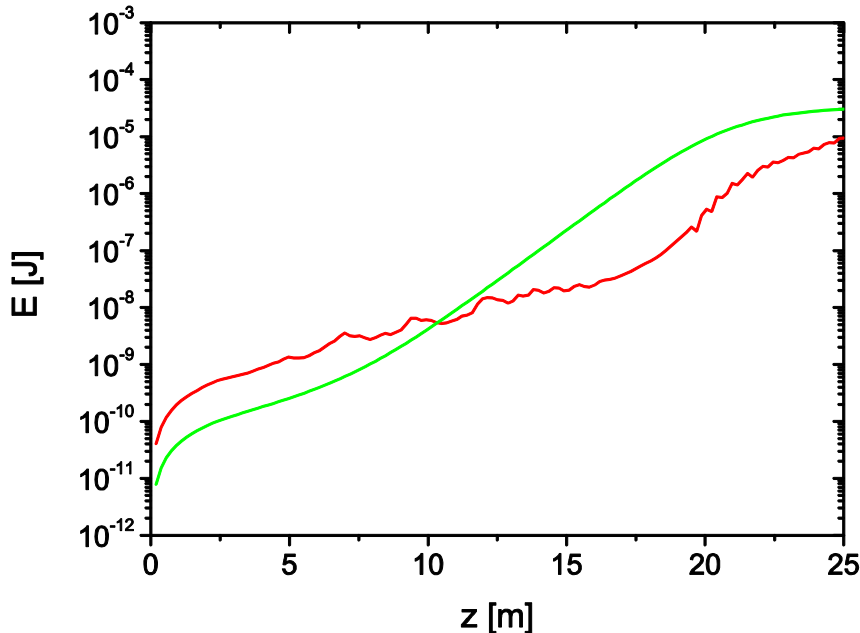


- Power of 3rd harmonic is about 1% of saturation power of the fundamental (and much smaller for higher harmonics)
- Relative bandwidth is approximately the same (contrary to $1/h$ in the case of spontaneous emission)
- Shot-to-shot intensity fluctuations are much stronger
- Transverse coherence is worse

In short, nonlinear harmonics are much less brilliant and less stable than the fundamental

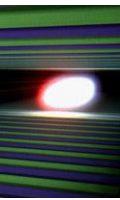


- Harmonic lasing is an FEL instability developing independently of the fundamental (in linear regime)
- We have to disrupt the fundamental to let a harmonic saturate



1st: red
3rd: green

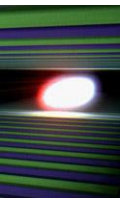
the fundamental is
disrupted by phase shifters



- Saturation efficiency of h -th harmonic scales as $\sim \lambda_w / (hL_{\text{sat}})$
- Relative rms bandwidth scales as $\sim \lambda_w / (hL_{\text{sat}})$
- Shot-to-shot intensity fluctuations are comparable (the same statistics)
- Good transverse coherence

Brilliance is comparable to that of the fundamental!

Harmonic lasing: the history



- First theoretical consideration for low-gain FELs more than 30 years ago ([Colson, 1981](#))
- Several successful experiments with FEL oscillators in infrared range (1988-2010)

- High-gain FELs:

1D theory of harmonic lasing:

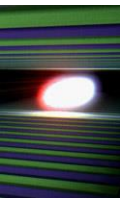
[Murphy, Pellegrini, Bonifacio, 1985](#)

[Bonifacio, De Salvo, Pierini, 1990](#)

[McNeil et al., 2005](#)

3D theory (everything included):

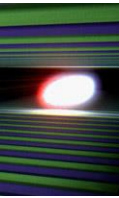
[Z. Huang and K.-J. Kim, 2000](#)



- Eigenvalue equation for calculation of gain length of harmonic lasing including all important effects: emittance, betatron motion, diffraction of radiation, energy spread etc.
- Numerical example for LCLS: **NO harmonic lasing**.
Reason: too large emittance and energy spread anticipated at that time.

$$\begin{aligned}
 \bar{\Phi}_{nm}(p) = & -\frac{h^2 A_{JJ}^2}{A_{JJ1}^2 (2i h B \hat{\Lambda} - p^2)} \int_0^\infty d p' p' \bar{\Phi}_{nm}(p') \\
 & \times \int_0^\infty d \zeta \frac{\zeta}{(1 - i h B \hat{k}_\beta^2 \zeta / 2)^2} \exp \left[-\frac{h^2 \hat{\Lambda}_T^2 \zeta^2}{2} - (\hat{\Lambda} + i \hat{C}) \zeta \right] \\
 & \times \exp \left[-\frac{p^2 + p'^2}{4(1 - i h B \hat{k}_\beta^2 \zeta / 2)} \right] I_n \left[\frac{p p' \cos(\hat{k}_\beta \zeta)}{2(1 - i h B \hat{k}_\beta^2 \zeta / 2)} \right].
 \end{aligned}$$

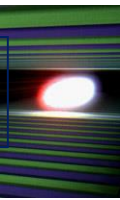
Our revision in 2012



- Found simple parametrization of the gain length and upgraded FEL code FAST
- Could then analyze parameter space (with optimistic conclusions)
- Proposed new methods for suppression of the fundamental
- Discovered qualitatively new effect of anomalously strong harmonic lasing for thin electron beams
- Suggested method for improvement of spectral brightness (later called HLSS FEL)
- Considered practical applications to different facilities

Our conclusion: the option must be seriously considered!

E. Schneidmiller and M. Yurkov, *Phys. Rev. ST-AB* 15(2012)080702



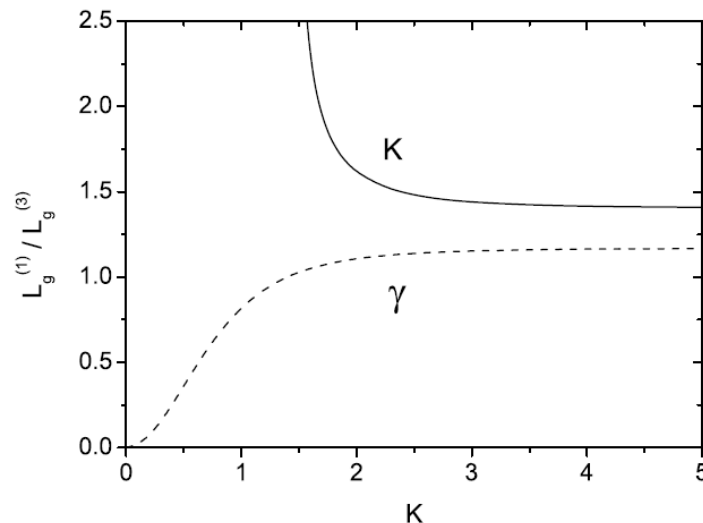
A. Harmonic lasing (with the suppressed fundamental) at some WL vs fundamental lasing at the same WL with reduced K.

$$\frac{L_g^{(1K)}}{L_g^{(h)}} = \frac{h^{1/2} K A_{JJh}(K)}{K_{re} A_{JJ1}(K_{re})} \quad K_{re}^2 = \frac{1 + K^2}{h} - 1$$

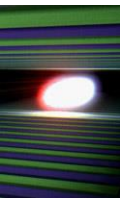
B. Harmonic lasing (with the suppressed fundamental) at some WL vs fundamental lasing at the same WL with increased beam energy.

$$\frac{L_g^{(1\gamma)}}{L_g^{(h)}} = \frac{h^{5/6} A_{JJh}(K)}{A_{JJ1}(K)}$$

Negligible energy spread, beta is optimized in all cases

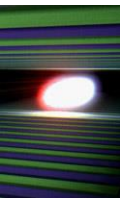


Better than in 1D!

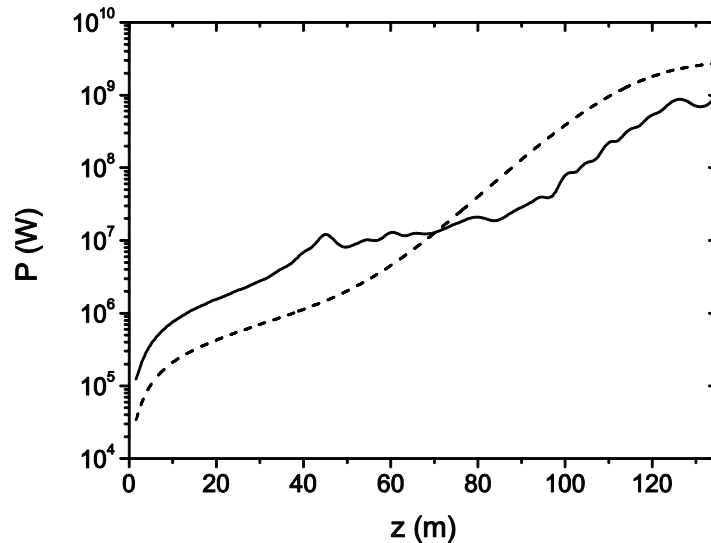


- Phase shifters
- Spectral filtering
- Switching between 3rd and 5th harmonics

Example for the European FEL



3rd harmonic lasing at 62 keV (0.2 A). Beam parameters for 100 pC from s2e (quantum diffusion in the undulator added), energy 17.5 GeV. With 20 pC bunch one can even reach 100 keV.

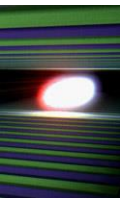


1st: solid

3rd: dash

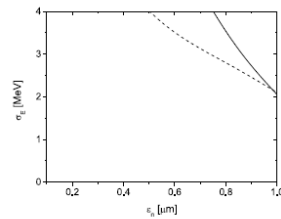
bandwidth is 2×10^{-4} (FWHM)

There are plans for MID instrument (A. Madsen);
users are interested; MAC recommended.

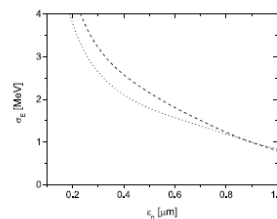


It is expected to have 7 GeV in CW mode and 10 GeV in long pulse mode with 35% duty factor.

1 A



7 GeV

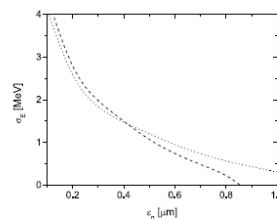
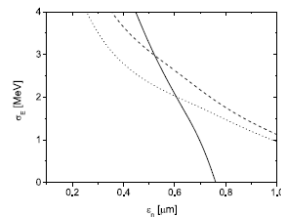


1st: solid

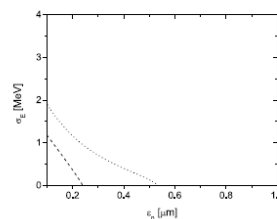
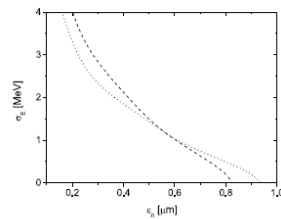
3rd: dash

5th: dot

0.75 A

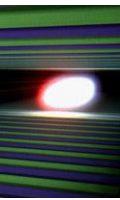


0.5 A



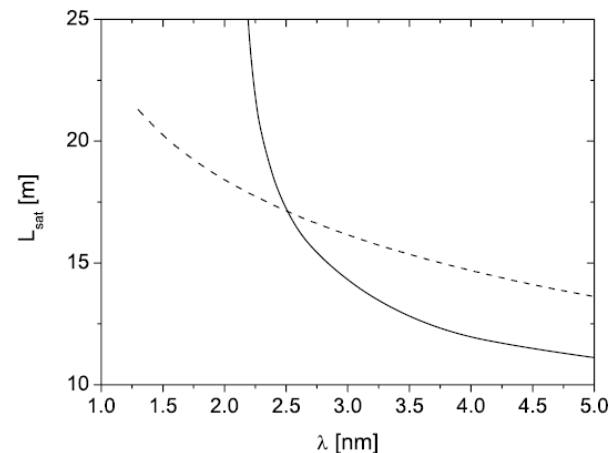
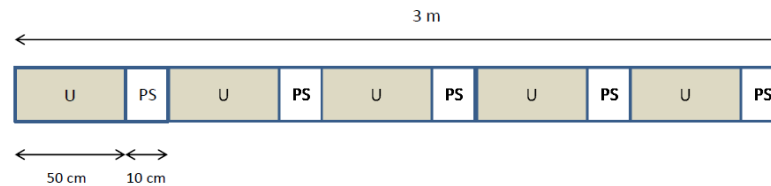
Brinkmann, Schneidmiller,
Sekutowicz, Yurkov, NIMA 768(2014)20

Possible upgrade of FLASH



Lasing down to 1.3 nm is desirable. Making use of 3rd harmonic lasing we can reach this WL with present accelerator energy of 1.25 GeV.

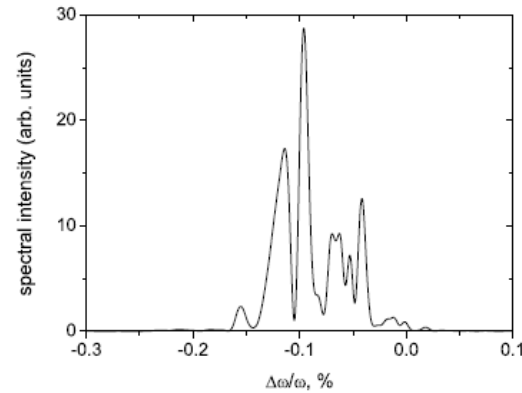
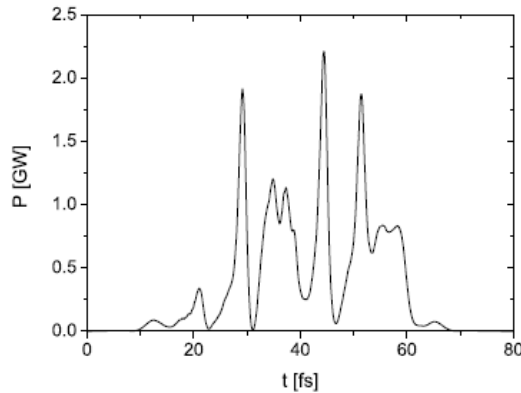
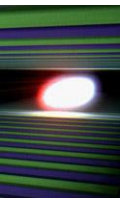
Electron beam	Value
Energy	1.25 GeV
Charge	150 pC
Peak current	2.5 kA
Rms normalized slice emittance	0.5 μm
Rms slice energy spread	250 keV
Rms pulse duration	24 fs
Undulator	Value
Period	2.3 cm
Minimum gap	9 mm
K_{rms} (at minimum gap)	1
Beta-function	7 m
Net magnetic length	25 m
Total length	34.5 m



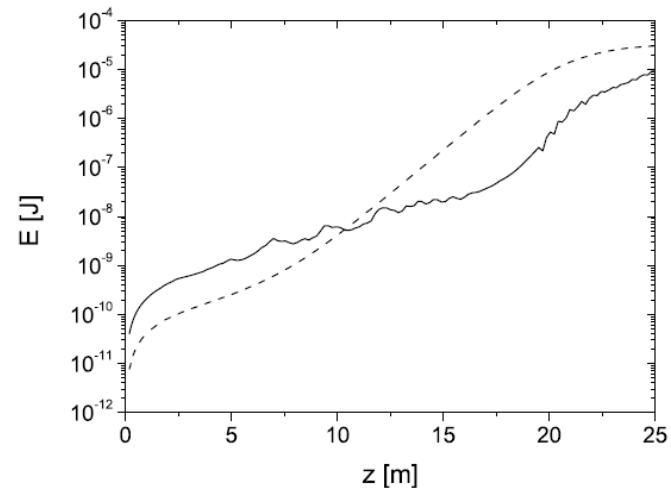
1st: solid
3rd: dash

Schneidmiller, Yurkov, NIMA 717(2013)20

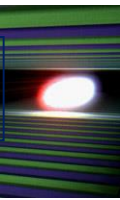
Possible upgrade of FLASH (cont'd)



Wavelength	1.3 nm
Averaged peak power	1 GW
Pulse energy	30 μ J
Shot-to-shot fluctuations	< 10 %
Pulse duration (FWHM)	30 fs
Bandwidth (FWHM)	0.1 %
Angular divergence (FWHM)	10 μ rad
Peak brilliance	10^{31} ph./s mrad ² mm ² 0.1% BW)

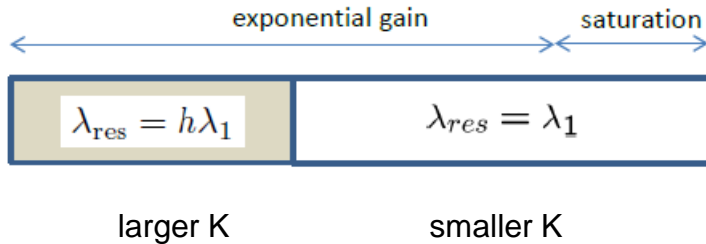


1st: solid
3rd: dash



We proposed a simple **trick for improvement of spectral brightness** in a gap-tunable undulator: harmonic lasing in linear regime (with narrow bandwidth) in the first part of the undulator, then reducing K and reaching saturation at the fundamental. Then we have high power and narrow BW.

E. Schneidmiller and M. Yurkov, Phys. Rev. ST-AB 15(2012)080702

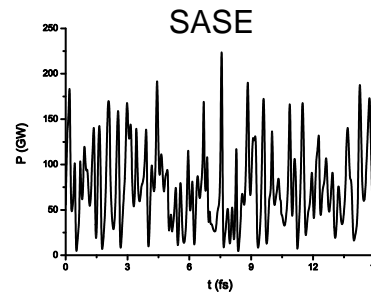
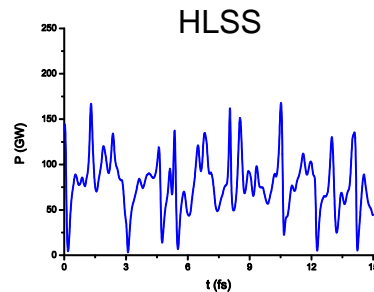
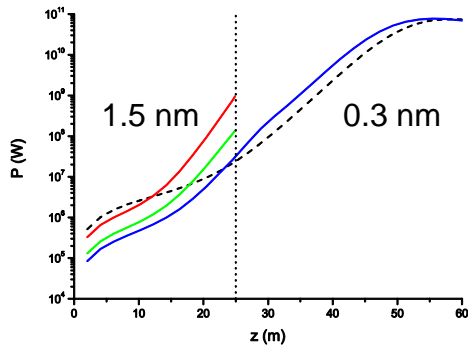


Bandwidth reduction factor:

$$R \simeq h \frac{\sqrt{L_w^{(1)} L_{sat,h}}}{L_{sat,1}}$$

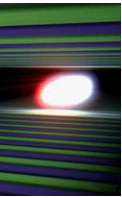
Typically $R = 0.6-0.9 h$

The fundamental and all harmonics have to stay well below saturation in the first part of the undulator. Use of phase shifters in the first undulator is optional.

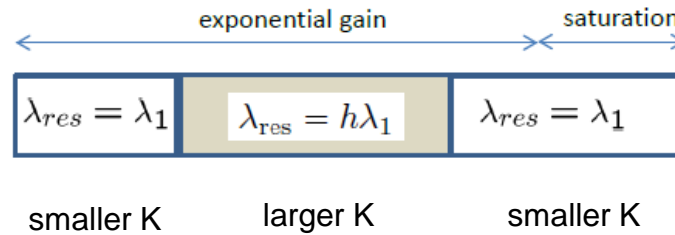


E. Schneidmiller and
M. Yurkov, FEL'13

HLSS vs pSASE

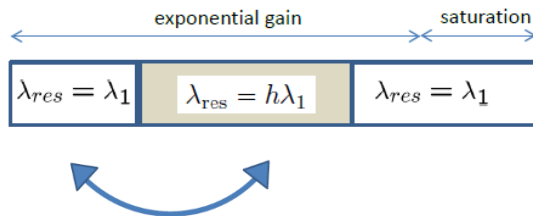


A similar concept (pSASE): D. Xiang et al., PRST-AB 16(2013)010703

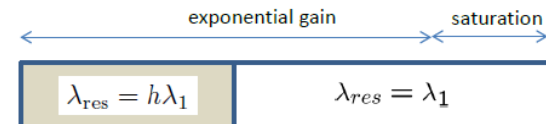


First two sections are linear amplifiers (with large BW and small BW). One can swap them and keep the same properties of radiation in the end (small BW and high power):

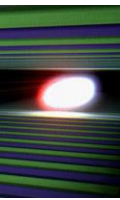
pSASE FEL



HLSS FEL

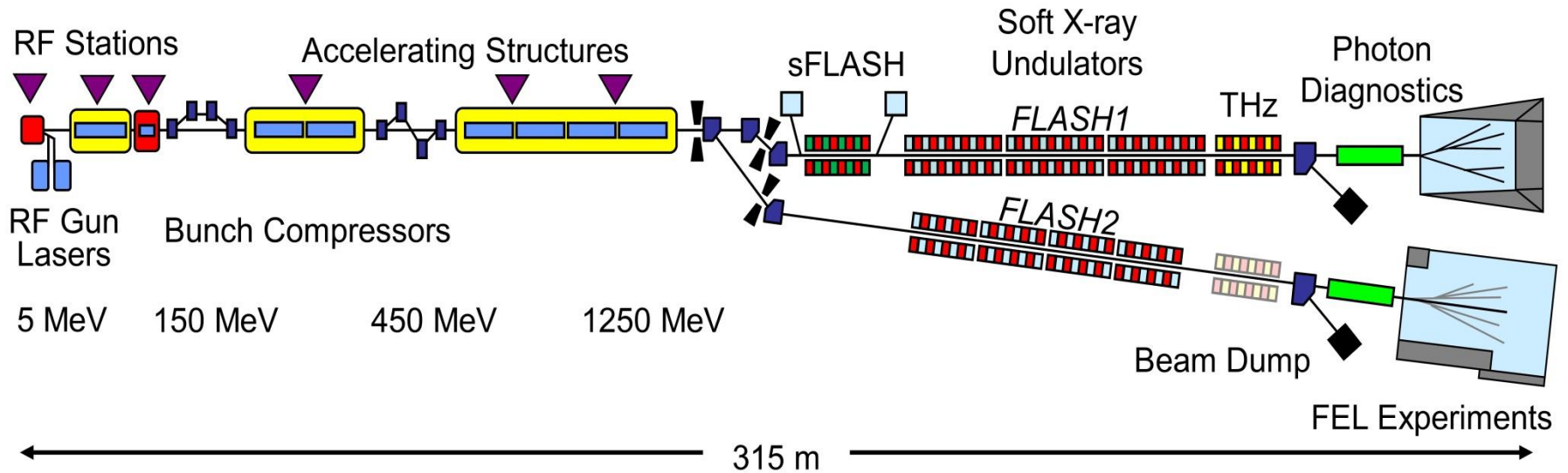
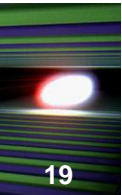


HLSS and post-saturation taper



- Post-saturation taper works better for seeded FELs;
- Coherence length does not have to equal bunch length, even moderate increase is sufficient;
- In self-seeding schemes the saturation length is about twice that of SASE: less space for post-saturation taper;
- HLSS saturates even earlier than SASE: more space for post-saturation taper, more power can be extracted.

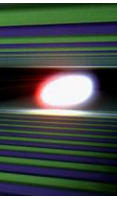
HLSS FEL seems to be the optimal solution for maximizing FEL power.



■ Undulators

■	Period	Length	
■ FLASH1:	2.73 cm	27 m (6 x 4.5 m modules)	fixed gap
■ FLASH2:	3.14 cm	30 m (12 x 2.5 m modules)	variable gap

HLSS at FLASH2: 7 nm (May 1, 2016)



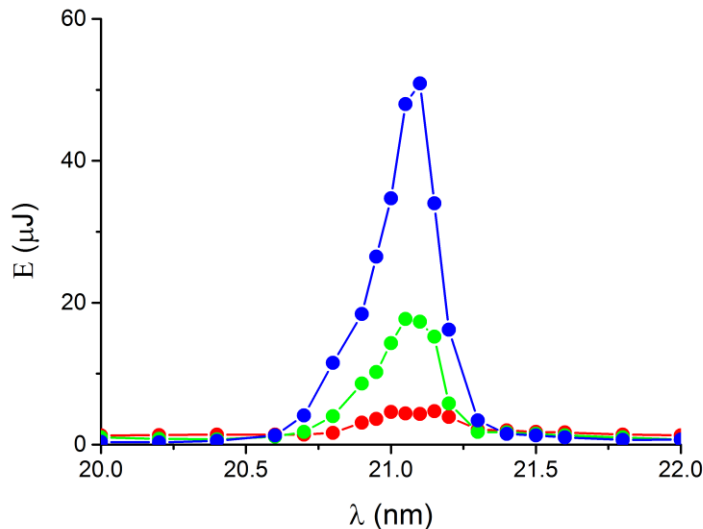
← exponential gain → saturation (actually, no saturation)



3 undulators
21 nm

7 undulators
7 nm

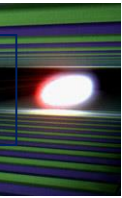
K-scan of the undulators: only 1st (red);
1st and 2nd (green); 1st, 2nd and 3rd (blue)



- Normal SASE at 7 nm in 10 undulators: 12 uJ (exponential gain)
- Detuning first (first two, first three) undulator sections: sharp intensity drop
- Coming close to 21 nm: sharp increase, resonant behavior
- With 3 undulators we have 51 uJ instead of 12 uJ; gain length of the 3rd harmonic is shorter than that of the fundamental at 7 nm!
- Nonlinear harmonic generation in the first part is absolutely excluded: pulse energy at 21 nm after 3 undulators was 40 nJ (but about 200 uJ at saturation): 4 orders of magnitude
- Results can only be explained by 3rd harmonic lasing at 7 nm

E. Schneidmiller and M. Yurkov, Proc. IPAC2016, MOPOW009

a. HLSS at FLASH2: 11 nm (June 6-7, 2016)



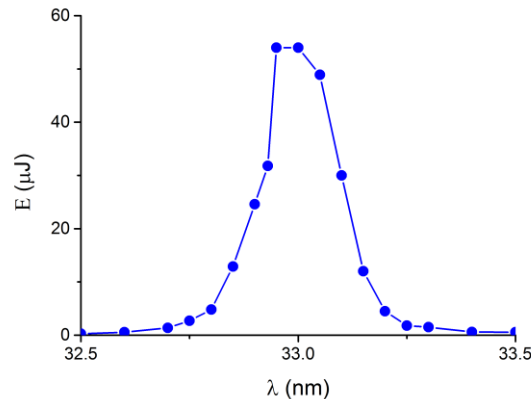
← exponential gain → saturation (actually, no saturation)



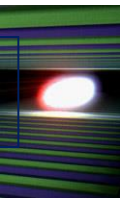
4 undulators
33 nm

6 undulators
11 nm

K-scan of the first 4 undulators



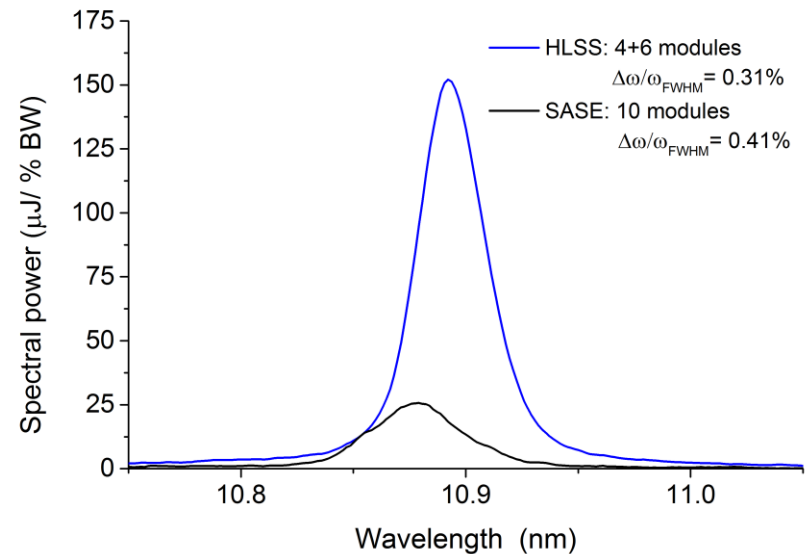
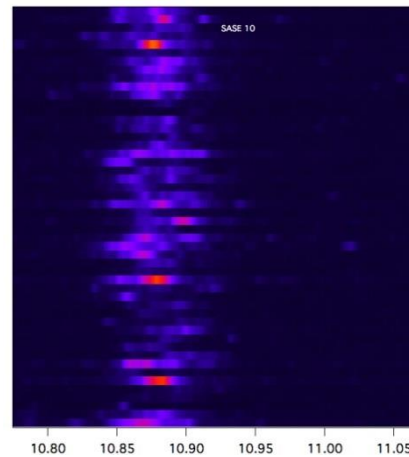
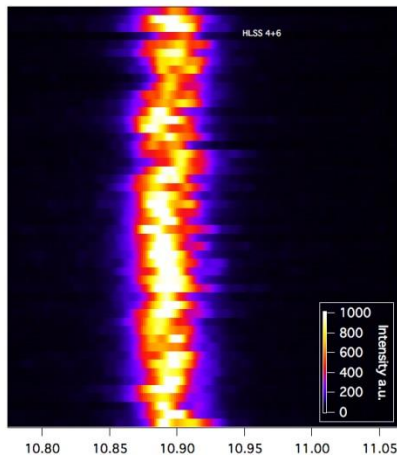
- Lower electron energy
- Normal SASE at 11 nm with 10 undulators and HLSS with 4+6 undulators
- Nonlinear harmonic generation in the first part is excluded: three orders below saturation
- An attempt to see bandwidth reduction



Spectral measurements

HLSS (4+6)

SASE (10)



Expectations

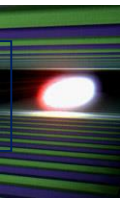
$$R \simeq h \frac{\sqrt{L_w^{(1)} L_{sat,h}}}{L_{sat,1}}$$

$R = 1.7$

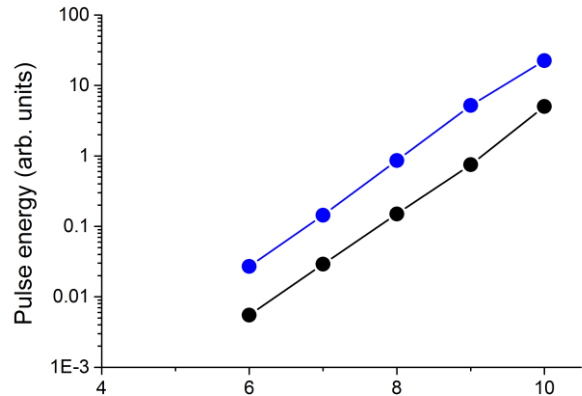
Measured: $R = 1.3$

Energy chirp!

Many thanks to M. Kuhlmann



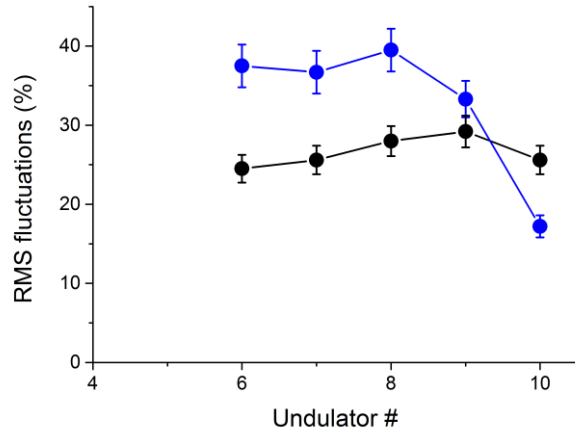
Statistical determination of an increase of the coherence time



$$M_l \propto 1/L^{coh}$$

$$M_l = 1/\sigma^2$$

$$\frac{L_{HLSS}^{coh}}{L_{SASE}^{coh}} = \frac{\sigma_{HLSS}^2}{\sigma_{SASE}^2} \simeq 1.8$$



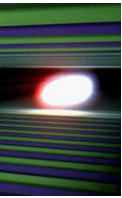
Expectations

$$R \simeq h \frac{\sqrt{L_w^{(1)} L_{sat,h}}}{L_{sat,1}}$$

R = 1.7

SASE (black) and HLSS (blue)

HLSS at FLASH2: 4.5 nm (Sep. 18, 2016)

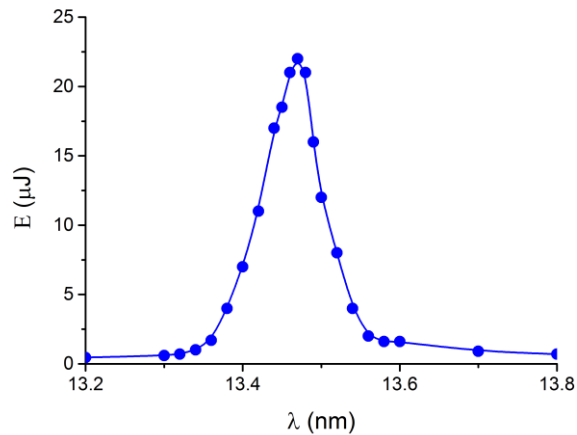


← exponential gain → saturation →

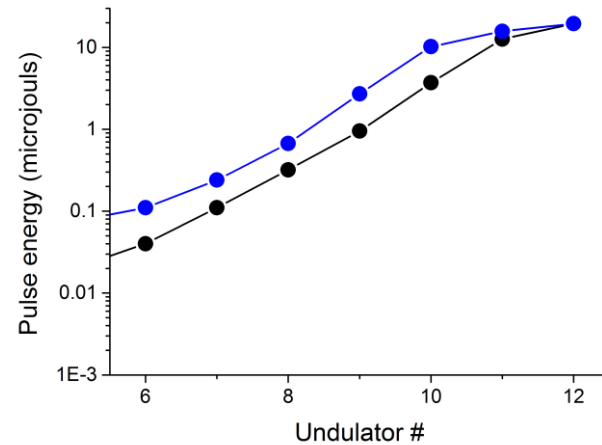
$\lambda_{\text{res}} = h\lambda_1$	$\lambda_{\text{res}} = \lambda_1$
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3 undulators
13.5 nm

9 undulators
4.5 nm

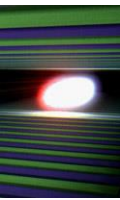


K-scan of the first 3 undulators

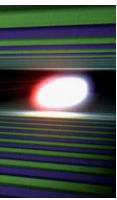


Gain curve:

SASE (black) and HLSS (blue)

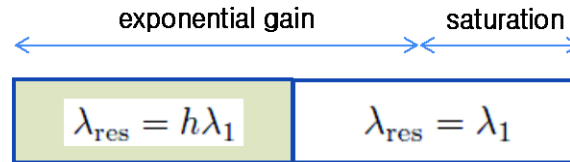
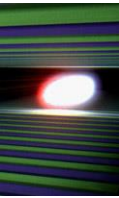


- Harmonic lasing is an interesting option for the European XFEL and FLASH;
- Main application I: extension of photon energy range (60-100 keV for the European XFEL, also CW upgrade; FLASH up to 1 keV);
- Main application II: bandwidth reduction and brilliance increase (HLSS) + Terawatt option;
- Successful demonstration of HLSS principle at FLASH2;
- First evidence of harmonic lasing in a high-gain FEL and at a short wavelength (4.5 nm) paves the way for its applications in X-ray FEL facilities.



Backup slides

HLSS at FLASH2: simulations



No tapering (log scale)

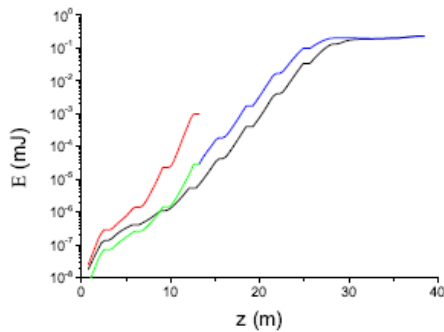


Figure 2: FEL pulse energy versus undulator length. In the first part of the undulator (tuned to the resonance with 39 nm) the first (red) and the third (green) harmonics are shown. The third harmonic continues to get amplified in the second part of the undulator (now as the fundamental) tuned to 13 nm (shown in blue). A reference case of lasing at 13 nm on the fundamental in the whole undulator with constant K-value is shown in black.

Tapering (linear scale)

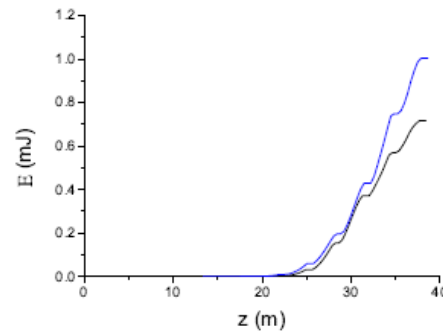


Figure 3: FEL pulse energy versus undulator length when the post-saturation taper is applied. HLSS case is shown in blue, and the SASE case - in black.

Spectral density

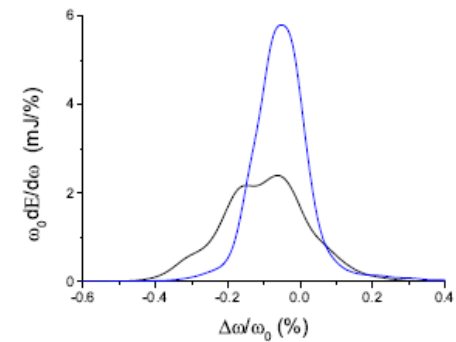
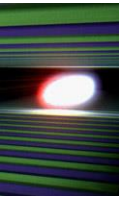


Figure 4: Spectral density of the radiation energy for HLSS FEL configuration (blue) and for SASE FEL (black).

$$R \simeq h \frac{\sqrt{L_w^{(1)} L_{\text{sat},h}}}{L_{\text{sat},1}}$$

$R = 0.6 h = 1.8$



Generalization of formulas from Saldin, Schneidmiller and Yurkov, Opt. Commun. 235(2004)415

$$L_g \simeq L_{g0} (1 + \delta) \quad \text{field gain length}$$

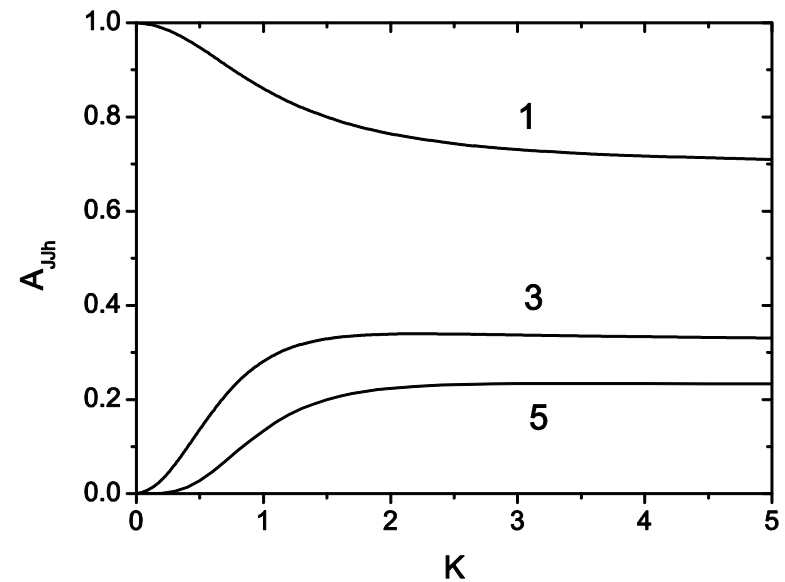
$$L_{g0} = 1.67 \left(\frac{I_A}{I} \right)^{1/2} \frac{(\epsilon_n \lambda_w)^{5/6}}{\lambda_h^{2/3}} \frac{(1 + K^2)^{1/3}}{h^{5/6} K A_{JJh}}$$

$$\delta = 131 \frac{I_A}{I} \frac{\epsilon_n^{5/4}}{\lambda_h^{1/8} \lambda_w^{9/8}} \frac{h^{9/8} \sigma_\gamma^2}{(K A_{JJh})^2 (1 + K^2)^{1/8}}$$

$$\beta_{\text{opt}} \simeq 11.2 \left(\frac{I_A}{I} \right)^{1/2} \frac{\epsilon_n^{3/2} \lambda_w^{1/2}}{\lambda_h h^{1/2} K A_{JJh}} (1 + 8\delta)^{-1/3}$$

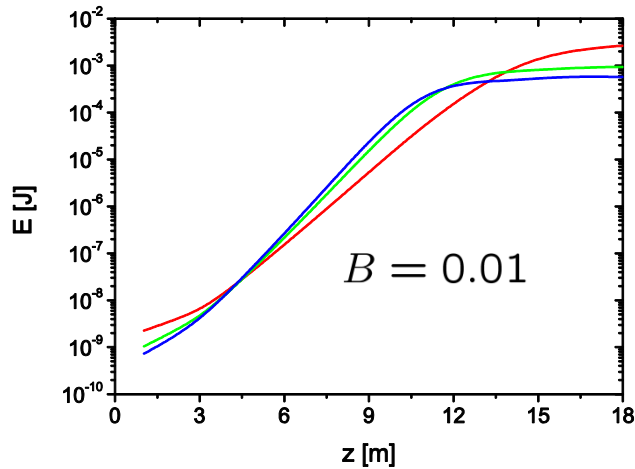
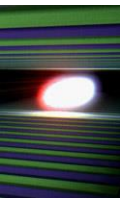
$$L_g(\beta) \simeq L_g(\beta_{\text{opt}}) \left[1 + \frac{(\beta - \beta_{\text{opt}})^2 (1 + 8\delta)}{4\beta_{\text{opt}}^2} \right]^{1/6} \quad \text{for } \beta > \beta_{\text{opt}} \quad \text{new also for the fundamental}$$

$$A_{JJh}(K) = J_{(h-1)/2} \left(\frac{hK^2}{2(1+K^2)} \right) - J_{(h+1)/2} \left(\frac{hK^2}{2(1+K^2)} \right)$$

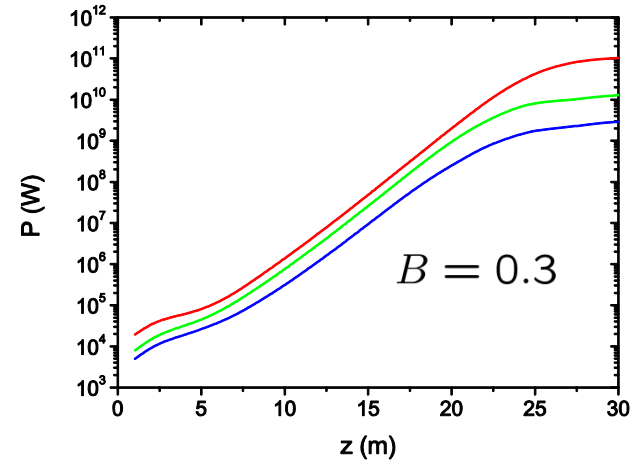


$$2\pi\epsilon/\lambda \sim 1 \quad \text{or} \quad 2\pi\epsilon/\lambda \gg 1$$

Anomalous harmonic lasing

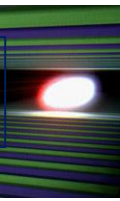


1st: red
3rd: green
5th: blue



XFEL.EU: fundamental at 4.5 nm, beam energy 10.5 GeV, slice parameters for 100 pC from s2e, energy spread is 1 MeV

One can use this effect (in pump-probe experiments or for multi-user operation) or find ways to suppress it (if disturbs).

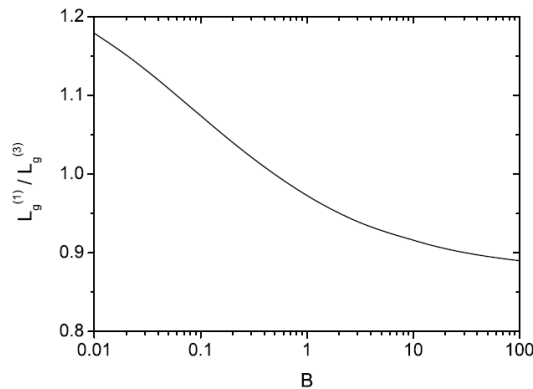


The case $2\pi\epsilon/\lambda \sim 1$ is typical for hard X-ray beamlines

If the same beam is used to drive a soft X-ray undulator (like SASE3 of XFEL.EU), the case $2\pi\epsilon/\lambda \ll 1$ is automatically achieved

For a reasonable beta-function one deals then with a small diffraction parameter $B = 4\pi\epsilon\beta\Gamma/\lambda$ ($L_g \propto 1/\Gamma$)

If the diffraction parameter is sufficiently small and K is sufficiently large, harmonics can grow faster than the fundamental!



$$3D: \quad \Gamma \propto (A_{JJ}^2 \omega^2)^{1/2}$$

$$1D: \quad \Gamma_{1D} \propto (A_{JJ}^2 \omega)^{1/3}$$

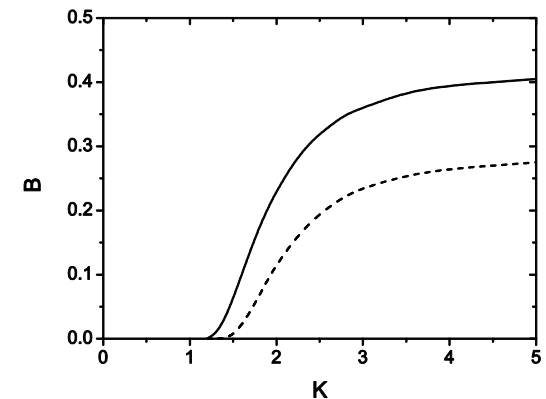
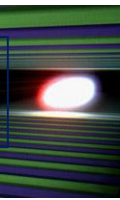


Fig. 4. Ratio of gain lengths for lasing at the fundamental wavelength and at the third harmonic versus diffraction parameter of the fundamental wavelength for large values of the undulator parameter K .



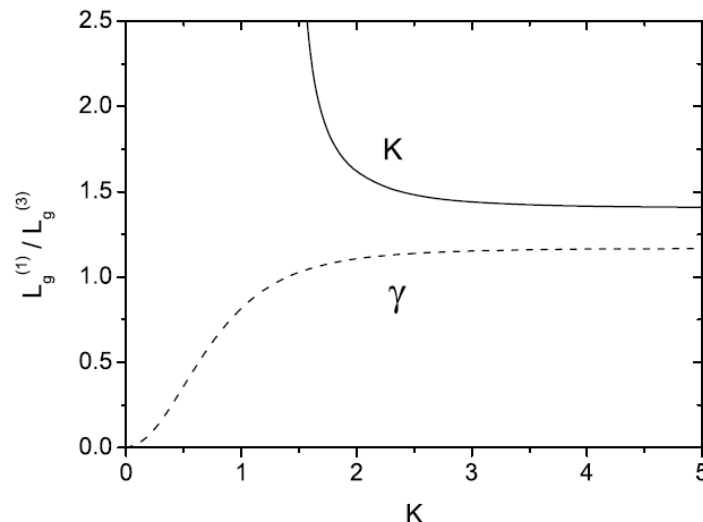
A. Harmonic lasing (with the suppressed fundamental) at some WL vs fundamental lasing at the same WL with reduced K.

$$\frac{L_g^{(1K)}}{L_g^{(h)}} = \frac{h^{1/2} K A_{JJh}(K)}{K_{re} A_{JJ1}(K_{re})} \quad K_{re}^2 = \frac{1 + K^2}{h} - 1$$

B. Harmonic lasing (with the suppressed fundamental) at some WL vs fundamental lasing at the same WL with increased beam energy.

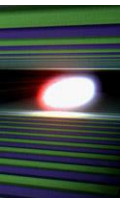
$$\frac{L_g^{(1\gamma)}}{L_g^{(h)}} = \frac{h^{5/6} A_{JJh}(K)}{A_{JJ1}(K)}$$

Negligible energy spread, beta is optimized in all cases



Better than in 1D!

Gain length of harmonic lasing



Generalization of formulas from Saldin, Schneidmiller and Yurkov, Opt. Commun. 235(2004)415

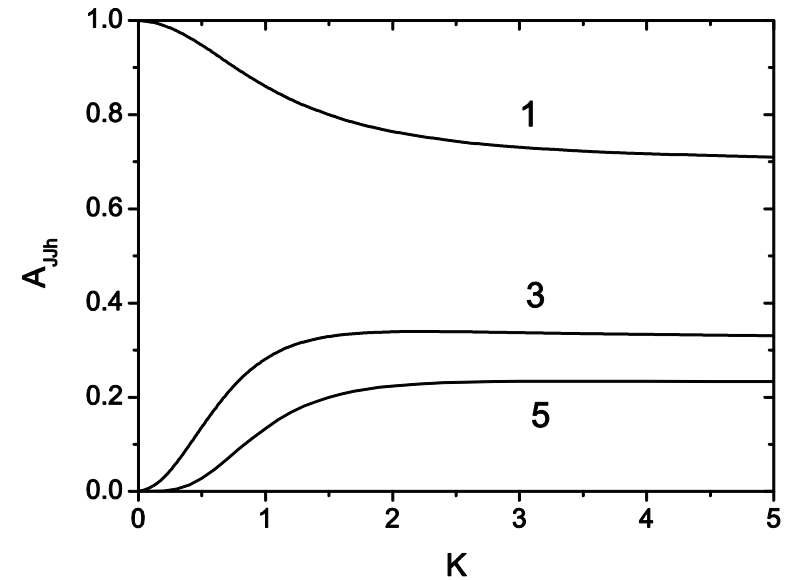
$$L_g \simeq L_{g0} (1 + \delta) \quad \text{field gain length}$$

$$L_{g0} = 1.67 \left(\frac{I_A}{I} \right)^{1/2} \frac{(\epsilon_n \lambda_w)^{5/6}}{\lambda_h^{2/3}} \frac{(1 + K^2)^{1/3}}{h^{5/6} K A_{JJh}}$$

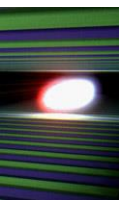
$$\delta = 131 \frac{I_A}{I} \frac{\epsilon_n^{5/4}}{\lambda_h^{1/8} \lambda_w^{9/8}} \frac{h^{9/8} \sigma_\gamma^2}{(K A_{JJh})^2 (1 + K^2)^{1/8}}$$

$$\beta_{\text{opt}} \simeq 11.2 \left(\frac{I_A}{I} \right)^{1/2} \frac{\epsilon_n^{3/2} \lambda_w^{1/2}}{\lambda_h h^{1/2} K A_{JJh}} (1 + 8\delta)^{-1/3}$$

$$L_g(\beta) \simeq L_g(\beta_{\text{opt}}) \left[1 + \frac{(\beta - \beta_{\text{opt}})^2 (1 + 8\delta)}{4\beta_{\text{opt}}^2} \right]^{1/6} \quad \text{for } \beta > \beta_{\text{opt}} \quad \text{new also for the fundamental}$$



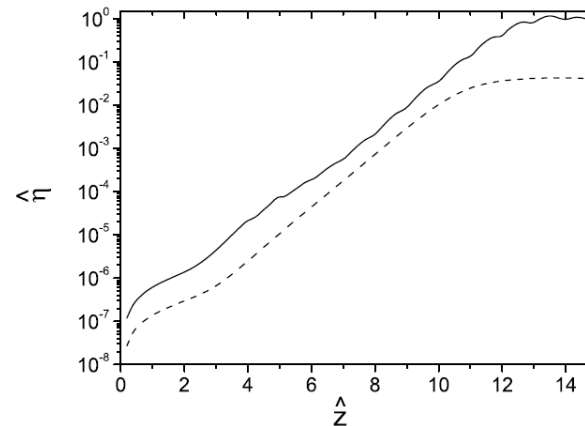
$$2\pi\epsilon/\lambda \sim 1 \quad \text{or} \quad 2\pi\epsilon/\lambda \gg 1$$



The method is proposed by [McNeil et al., 2005](#)
[Parisi et al., 2005](#)

If phase shifters are tuned such that the phase delay is $2\pi/3$ (or $4\pi/3$) for the fundamental, then its amplification is disrupted. At the same time the phase shift is equal to 2π for the third harmonic, i.e. it continues to get amplified without being affected by phase shifters.

Consecutive use of the same phase shifters, as proposed in [McNeil et al., 2005](#) works well for a monochromatic seed but not for SASE. There is a frequency shift depending on number and magnitude of phase shifts.



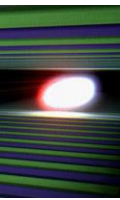
1D simulations

1st: solid

3rd: dash

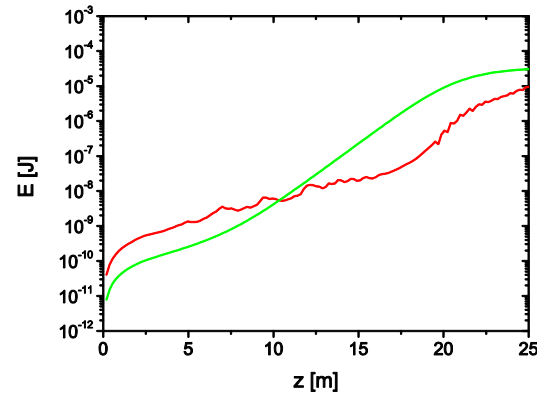
A better method (alternation of phase shifts $2\pi/3$ and $4\pi/3$) is proposed in [Parisi et al., 2005](#)

Still not good enough!

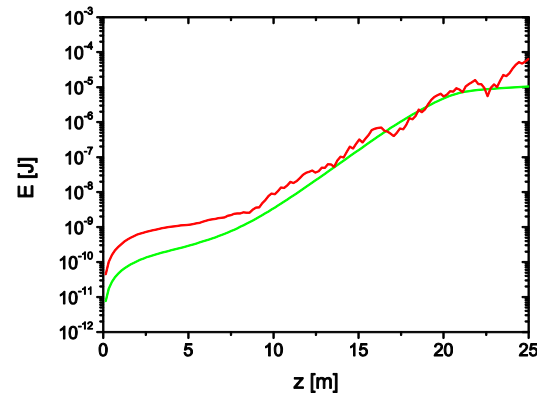


Piecewise use of phase shifters with the strength $2\pi/3$ and $4\pi/3$ suggested in Schneidmiller and Yurkov, PRST-AB 15(2012)080702, also NIMA 717(2013)37

Random distribution of phase shifters with the strength $2\pi/3$ and $4\pi/3$ suggested by Z. Huang, G. Marcus et al. (unpublished)

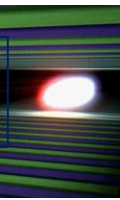


1st: red
3rd: green



Proposal for FLASH upgrade, 1 keV;
49 phase shifters

Conclusion: one needs a lot of phase shifters and a fancy distribution.



Harmonics are more sensitive to energy spread due to a higher mobility of particles (larger R56' in the undulator)

However, a reserve in gain length in the case of no energy spread lets harmonics be competitive with the fundamental also when the energy spread effects are significant.

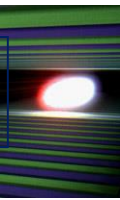
XFEL.EU: Lasing at 1 A with 0.5 nC (current 5 kA, emittance 0.7 μm)

Changing K

Changing energy

	1st	3rd
Energy	10.5 GeV	10.5 GeV
K_{rms}	1.05	2.3
Field L_g (no en. sp.)	10.4 m	6.9 m
Allowed en. sp.	2.8 MeV	

	1st	3rd
Energy	17.5 GeV	10.5 GeV
K_{rms}	2.2	2.3
Field L_g (no en. sp.)	7.9 m	6.9 m
Allowed en. sp.	1.3 MeV	



Higher harmonics are doing better for a large K and no energy spread

At some point there is a cutoff due to energy spread

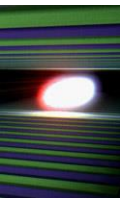
There are technical issues (undulator field errors, undulator wakefields etc.)

The 5th harmonic lasing can still be considered practical in many cases

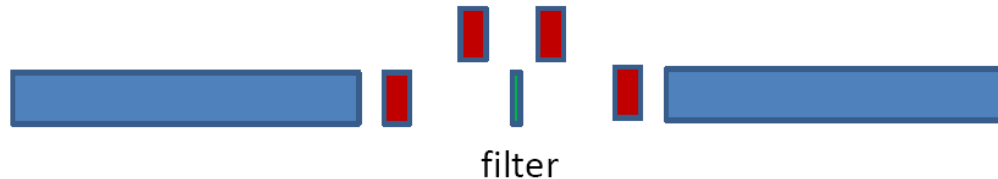
Lasing at 1 A with 0.5 nC (current 5 kA, emittance 0.7 um)

Changing K

	1st	5th
Energy	10.5 GeV	10.5 GeV
K_{rms}	1.05	3.1
Field L_g (no en. sp.)	10.4 m	5.8 m
Allowed en. sp.	2.3 MeV	



In the middle of the undulator the electron beam trajectory deviates from a straight line (chicane or closed bump), and a filter is inserted.



Transmitted intensity scales as $\exp(-\mu d)$, where d is the thickness, and the coefficient μ depends on frequency as $a \exp(-b\omega)$. Very efficient high-pass filter due to the double exponential suppression.

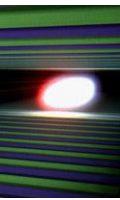
Beam modulations are smeared through the chicane due to R56.

Entrance of the second part of the undulator: no modulations, and only 3rd harmonic radiation.

Can be combined with a self-seeding setup: just add the filter!

If one filter is not sufficient: use two filters or a combination with phase shifters.

Switching between 3rd and 5th harmonics



K is large enough, the 3rd and the 5th harmonics have about the same gain length:

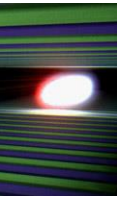


There can be more pieces:



R. Brinkmann, E. Schneidmiller, J. Sekutowicz, M. Yurkov, NIMA 768(2014)20

HLSS vs “standard” self-seeding and SASE



■ Bandwidth:	self-seeding	HLSS	SASE
■ Power (previous slide):	HLSS	self-seeding	SASE (?)
■ Intrinsic stability:	SASE	HLSS	self-seeding
■ Sensitivity to machine jitters:	SASE	HLSS	self-seeding
■ Setup time:	SASE	HLSS	self-seeding

HLSS FEL promises a mild monochromatization, the highest power, a reasonable stability and robustness.

And ... HLSS is free!