Intra-Beam scattering studies for CLIC damping rings

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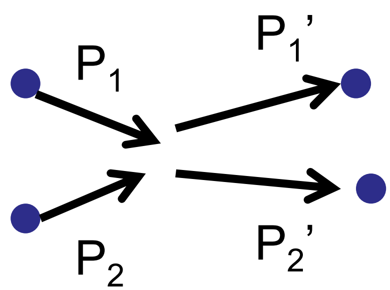
Intra-Beam Scattering

IBS is the effect due to multiple Coulomb scattering between charged particles in the beam



- Only Particles at the same position interact
- Only the momenta of the particles change

All the scattering events produce a variation of the particle distribution of the beam:

$$\rho(\mathbf{r}, \mathbf{p}, t), \qquad N_b = \int \rho(\mathbf{r}, \mathbf{p}, t)\, d^3\mathbf{p} d^3\mathbf{r}$$

Considering only IBS, the evolution of the distribution function is given by:

$$\frac{\partial \rho}{\partial t}(\mathbf{r}, \mathbf{p}, t) \stackrel{IBS}{=} \int \left(\frac{\partial N}{\partial t}(\mathbf{r}, t, \mathbf{p}^{'} \to \mathbf{p}) - \frac{\partial N}{\partial t}(\mathbf{r}, t, \mathbf{p} \to \mathbf{p}^{'}) \right) d^3\mathbf{p}^{'}$$

Also mean values of functions of the particle coordinates change:

$$\epsilon_k = \beta_k r_k^{'\,2} + 2\alpha_k r_k \, r_k^{'} + \gamma_k r_k^2$$

$$\epsilon_{k,RMS}(t) = \frac{< \epsilon_k >_t}{2} = \frac{1}{2} \int \epsilon_k(r_k, r_k^{'}) \, \rho(\mathbf{r}, \mathbf{r}^{'}, t) \, d^3\mathbf{r}^{'} d^3\mathbf{r}$$

It would be possible to follow the evolution of the beam parameters if the evolution of the distribution $\rho(\mathbf{r}, \mathbf{p}, t)$ was known.

The effect of the IBS on the beam emittance is evaluated by means of the growth rates:

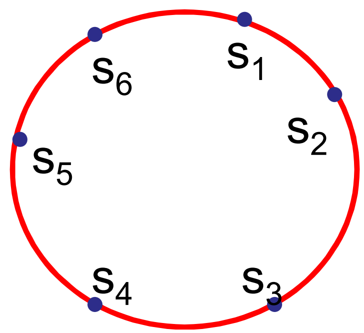
$$\frac{1}{\tau_{k,IBS}} = \frac{1}{\epsilon_{k,RMS}} \frac{d}{dt} \frac{<\epsilon_k>}{2} \qquad k = x, z, s$$

Growth rates are calculated at specific points on the lattice.

The Bjorken-Mtingwa formulation gives, for the growth rates:

$$\frac{1}{\tau_k} = \frac{{r_0}^2 c N (\log)}{8\pi\gamma^4\beta^3\epsilon_x\epsilon_z\epsilon_s} \int_0^\infty \frac{\lambda^{1/2}\, d\lambda}{|L+\lambda I|^{1/2}} \left\{ TrL^{(k)}\, Tr(L+\lambda I)^{-1} - 3\, TrL^{(k)} (L+\lambda I)^{-1} \right\}$$

Growth rates are calculated at different points of the lattice and then averaged over the ring:



$$\frac{1}{\tau} = \sum_{k=1}^{N} \frac{S_{k+1} - S_k}{S} \frac{1}{\tau_k} \qquad S = \sum_{k=1}^{N} S_k$$

Considering only the contribution of IBS, the evolution of the emittance is given by :

$$\frac{d\epsilon_{k,RMS}}{dt} = \frac{\epsilon_{k,RMS}}{\tau_{k,IBS}}$$

Taking into consideration also radiation damping and quantum excitation, the emittance evolves according to the equation:

$$\frac{d\epsilon_{k,RMS}}{dt} = -\frac{2}{\tau_k}(\epsilon_{k,RMS} - \epsilon_{k,0,RMS}) + \frac{2\,\epsilon_{k,RMS}}{\tau_{k,IBS}}$$

For the CLIC damping rings the parameters are:

- Damping time = 1.5 , 1.5 , 0.8 ms
- IBS Growth time = 1.9, 6, 3.9 ms
- Horizontal emittance (No IBS) = 89 nm
- Horizontal emittance (IBS) = 461 nm
- Vertical emittance (No IBS) = 3.0 nm
- Vertical emittance (IBS) = 4.4 nm
- Longitudinal emittance (No IBS) = 2256 eV s
- Longitudinal emittance (IBS) =3460 eV s

In classical IBS theory $\tau_{k,IBS}$ is calculated assuming:

Bjorken-Mtingwa:

- Coulomb Scattering transition rate from QED
- Gaussian Distribution

Piwinsky-Martini:

- Rutherford cross section
- Gaussian Distribution

- How do $\tau_{k,IBS}$ change if the distribution is not Gaussian ?
- Is it possible to follow the evolution of the particle distribution in the DR ?

MOCAC Code

Developed by P. R. Zenkevich and A. E. Bolshakov, *ITEP, Moscow, Russia*

- **MAIN PROGRAM**
  - **Intra-beam scattering**
  - **Electron cooling**
  - **Multiple scattering and energy loss on residual gas**
  - **Beam losses because of single interactions**
  - **Transformation to momentum-coordinate space**
  - **Stochastic cooling**
  - **Internal target**

- Not dedicated to IBS study.
- Rather complicated structure
- Not modified since 2006

It needs to be elaborated

MOCAC Code: test

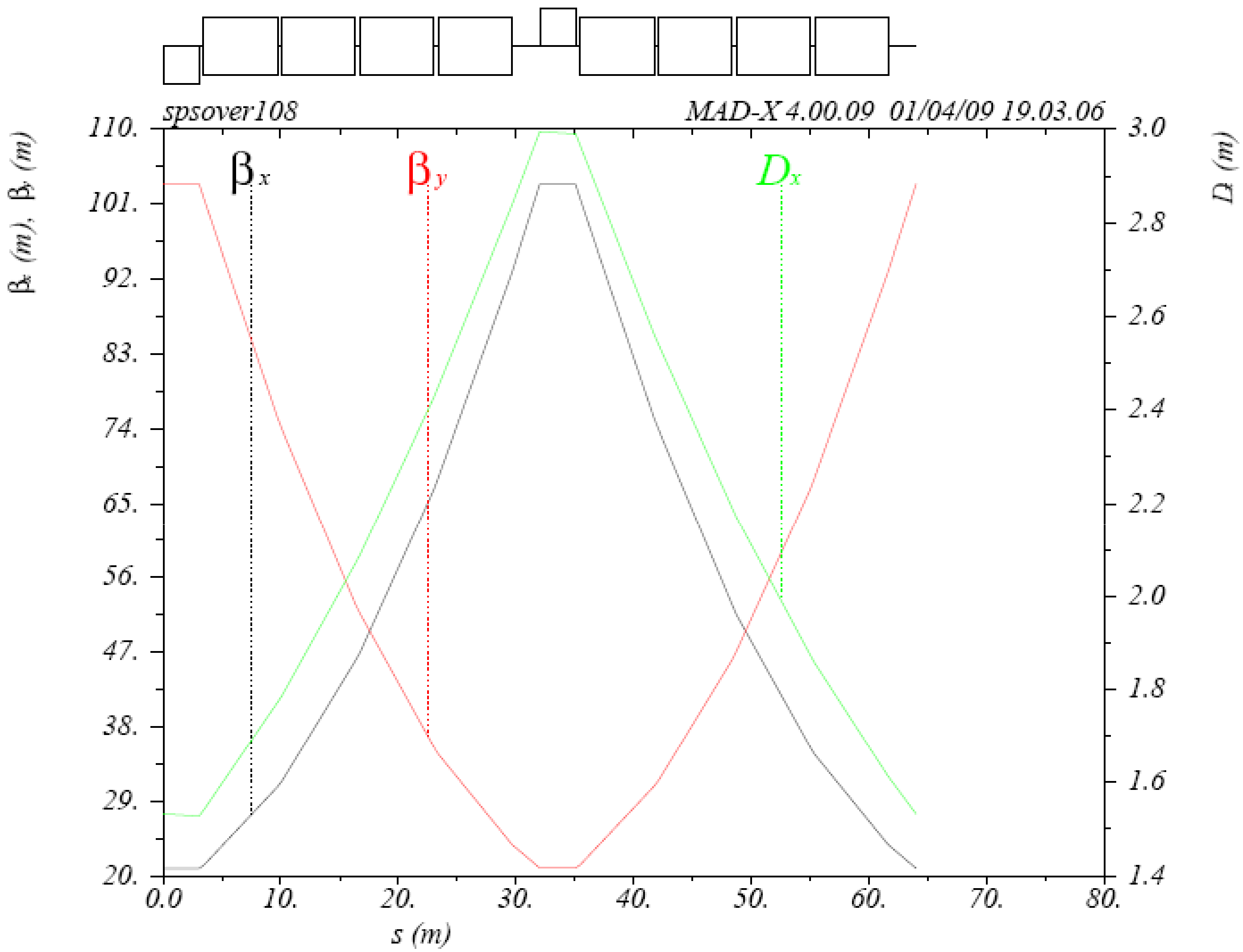
Due to the complicated structure of the code it was not easy to fit the parameters of the program for the test.

29-janv-09

SPS 1/108 lattice: p=26 GeV/c, N=1E14 (1 bunch), eHn=eVn=0.5 um, lbunch=1 ns, dpp=2.5E-3

	1/TL [s-1]	1/TX [s-1]	1/TY [s-1]	Cb(Log)
MADX: BJORKEN-MTINGWA Coulomb log computed	2,91E-03	1,22E-01	1,51E-02	15,742
Mathematica: BJORKEN-MTINGWA Coulomb log computed	3,05E-03	1,30E-01	1,65E-02	15,747
Mathematica: BJORKEN-MTINGWA Coulomb log=20	3,87E-03	1,65E-01	2,09E-02	20,000
Mathematica: PIWINSKI	3,81E-03	1,62E-01	2,28E-02	
MOCAC	-2,70E+00	2,88E+00	5,06E-01	

A more accurate comparison will be possible after cleaning of the code.


spsover108
MAD-X 4.00.09 01/04/09 19.03.06
βx
βy
Dx
βx (m), βy (m)
Dx (m)
110.
101.
92.
83.
74.
65.
56.
47.
38.
29.
20.
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
0.0
10.
20.
30.
40.
50.
60.
70.
80.
s (m)

MOCAC Code: IBS routine

MOCAC 'tracks' particles through their invariants.

Transversal invariants:

$$\epsilon_x(i) = \beta_x \left(x_i' - D' \frac{\Delta p_i}{p} \right)^2 + 2\alpha_x \left(x_i' - D' \frac{\Delta p_i}{p} \right) \left(x_i - D \frac{\Delta p_i}{p} \right) + \gamma_x \left(x_i - D \frac{\Delta p_i}{p} \right)^2$$

$$\epsilon_z(i) = \beta_z {z'_i}^2 + 2\alpha_z z_i z_i' + \gamma_z {z_i}^2$$

Longitudinal invariant:

$$\epsilon_s(i) = \left(\frac{\Delta p_i}{p} \right)^2 + \frac{(2\pi)^2 \nu_s^2}{(\alpha - \frac{1}{\gamma^2})^2 C^2} \Delta {s_i}^2 \qquad i = 1, \ldots, Num.Part.$$

Phases of the particles are not tracked.

MOCAC Code: IBS routine

The initial distribution of the beam is a randomly generated Gaussian*:

Only the invariants of the particles are generated according to the probability density distribution:

$$p(\epsilon_k) = \frac{1}{<\epsilon_k>} e^{\left(-\frac{\epsilon_k}{<\epsilon_k>}\right)} \qquad <\epsilon_k> = 2\epsilon_{k,RMS}$$

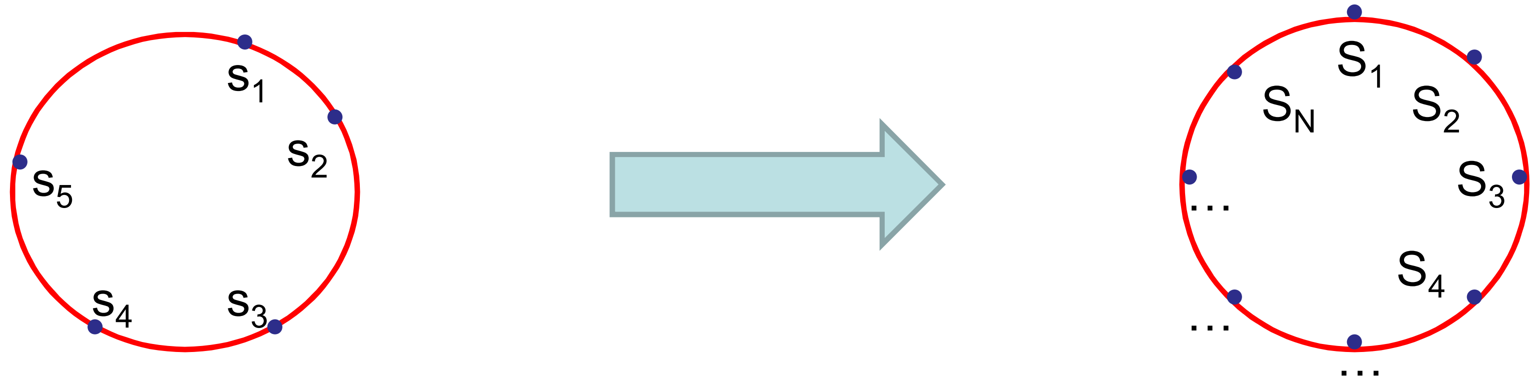
The initial emittances of the beam are given in the input file.

Taking uniformly distributed phases assures a Gaussian distribution of the particles.

MOCAC Code: IBS routine

- Input parameters are read from an input file (ASCII)
- Optical functions of the lattice are read from a MAD 8 output file

- A cubic interpolation of the optical functions is performed to calculate the values of the functions at N equally spaced points of the lattice (total length of the lattice is kept constant)



$\beta_x(s_k), \alpha_x(s_k), \beta_y(s_k), \alpha_y(s_k), D_x(s_k), D'_x(s_k)$ $\beta_x(S_k), \alpha_x(S_k), \beta_y(S_k), \alpha_y(S_k), D_x(S_k), D'_x(S_k)$

MOCAC Code: IBS routine

The total length of the simulation is N_{INJ} x T_{INJ}

T_{INJ} is the injection period (not used for IBS)

Each period T_{INJ} is divided in N_{IBS} sub-periods of length T_{IBS} = T_{INJ} / N_{IBS}

In each sub-period T_{IBS} the scattering routine is run at each of the equally spaced N points of the ring.

The scattering routine is called at intervals ΔT = T_{IBS} / N

After each call of the scattering routine the invariants of every particle are changed according to the momentum change of the particles.

MOCAC Code: Scattering routine

The coordinates and momenta of the particles are generated.

$\phi_x \quad \phi_z \quad \phi_s$ are randomly generated uniformly in [0 2π] for each particle.

$$\frac{\Delta p_i}{p} = \sqrt{\epsilon_s(i)}\cos(\phi_s)$$

$$\Delta s_i = \frac{(\alpha - \frac{1}{\gamma^2})C}{2\pi\nu_s}\sqrt{\epsilon_s(i)}\sin(\phi_s)$$

$$x'_i = -\frac{\sqrt{\epsilon_x(i)}}{\sqrt{\beta_x}}\left[\alpha_x\cos(\phi_x) + \sin(\phi_x)\right] + D'_x\frac{\Delta p_i}{p}$$

$$x_i = \sqrt{\epsilon_x(i)\beta_x}\cos(\phi_x) + D_x\frac{\Delta p_i}{p}$$

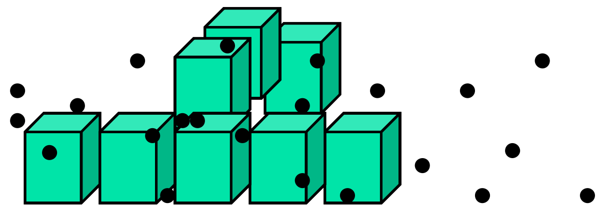
$$z'_i = -\frac{\sqrt{\epsilon_z(i)}}{\sqrt{\beta_z}}\left[\alpha_z\cos(\phi_z) + \sin(\phi_z)\right]$$

$$z_i = \sqrt{\epsilon_z(i)\beta_z}\cos(\phi_z)$$

The scattering routine implements the Binary Collision Model
(T. TAKIZUKA and H. ABE, Journal of Computational Physics 25, 1977)

MOCAC Code: Scattering routine (BCM)

Particles are grouped in geometric cells.

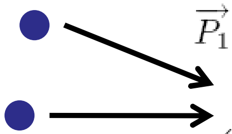


The density inside each cell is considered uniform $\rho = N_p/V$
Particles in the same cell are considered at the same point in space and then can scatter.

The particles inside each cell are randomly coupled and a 'collision' between the particles of each couple is simulated.

MOCAC Code: Collision simulation

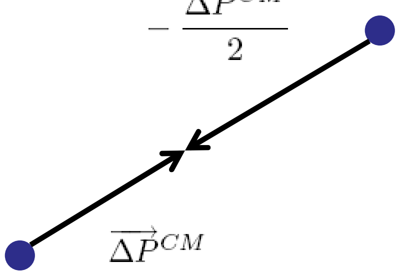
The collision is simulated in the center of mass frame, which is assumed to be the beam rest frame, for any couple of particles.



$$\overrightarrow{P_1} = \left({x'}_1, {y'}_1, \frac{\Delta p_1}{p} \right)$$

$$\overrightarrow{P_2} = \left({x'}_2, {y'}_2, \frac{\Delta p_2}{p} \right)$$

$$\overrightarrow{\Delta P} = \left({x'}_2 - {x'}_1, {y'}_2 - {y'}_2, \frac{\Delta p_2}{p} - \frac{\Delta p_1}{p} \right)$$



$$-\frac{\overrightarrow{\Delta P}^{CM}}{2}$$

$$\frac{\overrightarrow{\Delta P}^{CM}}{2}$$

$$\overrightarrow{\Delta P}^{CM} = \left({x'}_2 - {x'}_1, {y'}_2 - {y'}_2, \frac{1}{\gamma} \left(\frac{\Delta p_2}{p} - \frac{\Delta p_1}{p} \right) \right)$$

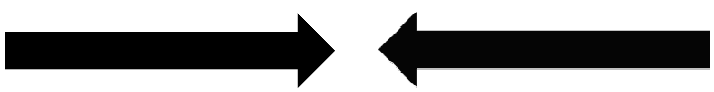
MOCAC Code: Collision simulation

Operating the following rotation of the beam frame:

$$R = \begin{pmatrix} \frac{\Delta P_x{}^{CM}\,\Delta P_s{}^{CM}}{|\overrightarrow{\Delta P}^{CM}|\,|\overrightarrow{\Delta P_\perp}^{CM}|} & \frac{\Delta P_z{}^{CM}\,\Delta P_s{}^{CM}}{|\overrightarrow{\Delta P}^{CM}|\,|\overrightarrow{\Delta P_\perp}^{CM}|} & -\frac{|\overrightarrow{\Delta P_\perp}^{CM}|}{|\overrightarrow{\Delta P}^{CM}|} \\ -\frac{\Delta P_z{}^{CM}}{|\overrightarrow{\Delta P_\perp}^{CM}|} & \frac{\Delta P_x{}^{CM}}{|\overrightarrow{\Delta P_\perp}^{CM}|} & 0 \\ \frac{\Delta P_x{}^{CM}}{|\overrightarrow{\Delta P}^{CM}|} & \frac{\Delta P_z{}^{CM}}{|\overrightarrow{\Delta P}^{CM}|} & \frac{\Delta P_s{}^{CM}}{|\overrightarrow{\Delta P}^{CM}|} \end{pmatrix}$$

we are lead to the simplest case:

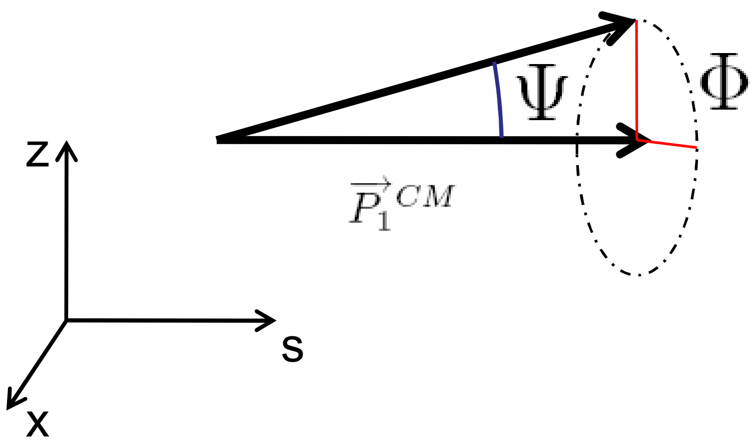
$$\left(0, 0, \frac{|\overrightarrow{\Delta P}^{CM}|}{2}\right) \qquad \left(0, 0, -\frac{|\overrightarrow{\Delta P}^{CM}|}{2}\right)$$



MOCAC Code: Collision simulation

The momentum change can be now expressed by the angles Ψ and Φ

$$\overrightarrow{P_1'}^{CM} = \overrightarrow{P_1}^{CM} + \overrightarrow{\Delta P_1}^{CM}$$



Φ is uniformly distributed in [0,2π]

$$\overrightarrow{\Delta P_1}^{CM} = \left(\frac{|\overrightarrow{\Delta P}^{CM}|}{2} \cos(\Phi)\sin(\Psi), \frac{|\overrightarrow{\Delta P}^{CM}|}{2} \sin(\Phi)\sin(\Psi), \frac{|\overrightarrow{\Delta P}^{CM}|}{2}(\cos(\Psi) - 1) \right)$$

MOCAC Code: Collision simulation

The scattering angle is determined by:

$$\Psi = 2r_i\sqrt{\frac{8\pi c\rho L_c\Delta T}{\gamma^5\beta^3\left|\overrightarrow{\Delta P}^{CM}\right|^3}} \qquad L_c = \log\left(\frac{\Delta z\beta^2\gamma^2\left|\overrightarrow{\Delta P}^{CM}\right|^2}{2r_i}\right)$$

There is a discrepancy between this formula and *P. Zenkevich, NIM A, 577* (2007):

$$\sin\left(\frac{\Psi}{2}\right) = r_i\sqrt{\frac{4\pi c\rho L_c\Delta T}{\gamma^5\beta^3 N\left|\overrightarrow{\Delta P}^{CM}\right|^{3/2}}}$$

To be clarified by Zenkevich.

MOCAC Code: Collision simulation

Averaging the momentum change over all the particles we should get:

$$\overrightarrow{F} = \frac{d < \Delta\overrightarrow{P_1} >}{dt} = -\frac{2\pi c {r_i}^2}{\gamma^5 \beta^3} \left\langle L_c \frac{\overrightarrow{P_1} - \overrightarrow{P_2}}{\left|\overrightarrow{P_1} - \overrightarrow{P_2}\right|^3} \right\rangle$$

$\overrightarrow{F}$ is the friction force in the Fokker-Planck equation:

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial P_m}(F_m \rho) + \frac{1}{2}\frac{\partial^2}{\partial P_m \partial P_{m'}}(D_{m,m'}\rho)$$

MOCAC Code: Collision simulation

By transforming back $\overrightarrow{\Delta P_1}^{CM}$ to the laboratory frame it is possible to compute:

$$\overrightarrow{P_1'}^{LAB} = \overrightarrow{P_1}^{LAB} + \overrightarrow{\Delta P_1}^{LAB} \qquad \overrightarrow{P_2'}^{LAB} = \overrightarrow{P_2}^{LAB} + \overrightarrow{\Delta P_2}^{LAB}$$

Using the momentum conservation: $\overrightarrow{\Delta P_2}^{LAB} = -\overrightarrow{\Delta P_1}^{LAB}$

The new invariants for the 2 particles are then calculated.

MOCAC Code: Possible changes

- Complete the cleaning of the code and run new tests (in progress)
- Read the optical function from a MADX output file (short time)
- Tracking the particles using the phase advances (not an issue)
- Replace the scattering dynamics with a dynamics compatible with the classical IBS theory (been examined, feasible-medium time)
- Implement the damping effect in the code
(under investigation by Zenkevich and Bolshakov – will be needed after IBS investigations completed)

Conclusions

MOCAC contains good ideas to investigate the evolution of the beam distribution in the Damping Ring

- It is possible to include radiation damping
- It is possible to use different scattering dynamics
- It is compatible with MAD X

But it needs to be made compatible with the classical IBS calculations

THANKS.

The End