

# Fragmentation Uncertainties in Hadron Observables for Top Quark Mass Measurements

Doojin Kim



LHC Top Working Group Meeting at CERN, November 22, 2016

**Gennaro Corcella, Roberto Franceschini and DK, to appear soon**

# Top Mass Measurement: Status

- ❑ Precision top mass measurement: **consistency check** of SM, higgs **vacuum stability** etc.
- ❑ Measurement at  $\lesssim 0.5\%$   $\Rightarrow$  precision QCD
- ❑ Top quark mass (ATLAS + CMS preliminary LHC<sub>top</sub>WG, Aug. 2016)

ATLAS comb. (06/2016):  **$172.84 \pm 0.34 \pm 0.61$**

CMS comb. (09/2015):  **$172.44 \pm 0.13 \pm 0.47$**

- ❑ Statistical uncertainty: not a problem since LHC is a top factory, **systematics-limited**
- ❑ Systematics – experimental uncertainty: jet energy scaling, etc.
- ❑ Systematics – theory uncertainty: due to poor theoretical understanding, etc.



# Measurement Techniques

## □ From standard/conventional approaches to alternative ones

- ❖ Template method [ATLAS, Eur. Phys. J. C72 (2012)],
  - ❖ Ideogram method [CMS PAS TOP 14-001],
  - ❖ Matrix element method [DØ, arXiv:1501.07912]
  - ❖ Cross sections [ATLAS, Eur. Phys. K. C74 (2014), CONF 2014-053]
  - ❖ Endpoint method [CMS PAS TOP 11-027]
  - ❖ *b*-jet energy-peak method [CMS PAS TOP-15-002]
  - ❖ Solvability method [DK, Matchev and Shyamsundar, in progress]
  - ❖  $J/\psi$  method [CMS PAS TOP 15-014]
  - ❖ *B*-hadron 2D-decay length [CMS PAS TOP 12-030]
  - ❖ Leptonic final state [CMS PAS TOP 16-002]
  - ❖ ***B*-meson observables** [Franceschini and DK, in progress]
  - ❖ Many more which I can't exhaust
- } SM top assumed
- } Kinematics-based

# Measurement Techniques

## □ From standard/conventional approaches to alternative ones

- ❖ Template method [ATLAS, Eur. Phys. J. C72 (2012)],
- ❖ Ideogram method [CMS PAS TOP 14-001],
- ❖ Matrix element method [DØ, arXiv:1501.07912]
- ❖ Cross sections [ATLAS, Eur. Phys. K. C74 (2014), CONF 2014-053]
- ❖ Endpoint method [CMS PAS TOP 11-027]
- ❖ *b*-jet energy-peak method [CMS PAS TOP-15-002]
- ❖ Solvability method [DK, Matchev and Shyamsundar, in progress]
- ❖  $J/\psi$  method [CMS PAS TOP 15-014]
- ❖ *B*-hadron 2D-decay length [CMS PAS TOP 12-030]
- ❖ Leptonic final state [CMS PAS TOP 16-002]
- ❖ ***B*-meson observables** [Franceschini and DK, in progress]
- ❖ Many more which I can't exhaust

SM top  
assumed

Kinematics-  
based

Jets in the  
final state  
→ JES

# Measurement Techniques

## □ From standard/conventional approaches to alternative ones

- ❖ Template method [ATLAS, Eur. Phys. J. C72 (2012)],
- ❖ Ideogram method [CMS PAS TOP 14-001],
- ❖ Matrix element method [DØ, arXiv:1501.07912]
- ❖ Cross sections [ATLAS, Eur. Phys. K. C74 (2014), CONF 2014-053]
- ❖ Endpoint method [CMS PAS TOP 11-027]
- ❖ *b*-jet energy-peak method [CMS PAS TOP-15-002]
- ❖ Solvability method [DK, Matchev and Shyamsundar, in progress]
- ❖  $J/\psi$  method [CMS PAS TOP 15-014]
- ❖ *B*-hadron 2D-decay length [CMS PAS TOP 12-030]
- ❖ Leptonic final state [CMS PAS TOP 16-002]
- ❖ ***B*-meson observables** [Franceschini and DK, in progress]
- ❖ Many more which I can't exhaust

SM top  
assumed

Kinematics-  
based

Jets in the  
final state  
→ JES

No jetty objects in  
the final state → no  
JES, Th. uncertainty

# Why Different Strategy?

- ❑ Different methods have different sensitivity to systematics
  - ❖ **complementary** to one another
- ❑ **Good exercise/testbed** for new physics signature
  - ❖ pair-produced mother particles, invisible particles, multi-step decays, etc.
- ❑ (Potentially) a new handle in search for new physics, e.g.,  $b$  partner searches

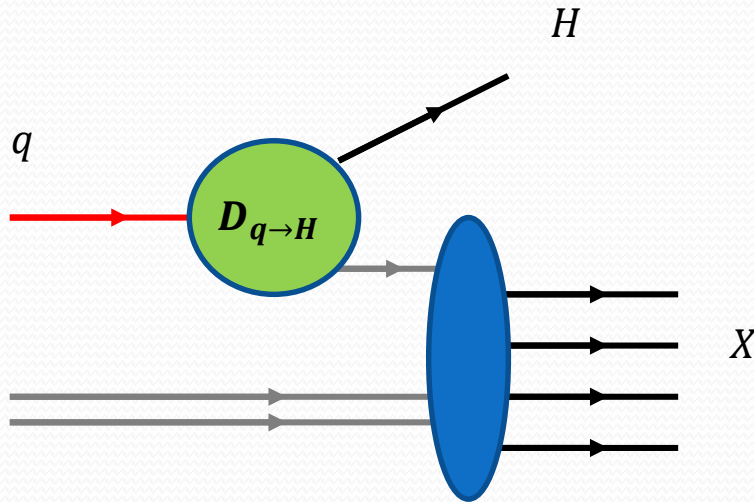


# Why Different Strategy?

- ❑ Different methods have different sensitivity to systematics
  - ❖ **complementary** to one another
- ❑ **Good exercise/testbed** for new physics signature
  - ❖ pair-produced mother particles, invisible particles, multi-step decays, etc.
- ❑ (Potentially) a new handle in search for new physics, e.g.,  $b$  partner searches
  
- ❑ **Crucial** to understand the transformation from a quark to hadrons, but **challenging** because it is governed by non-perturbative QCD



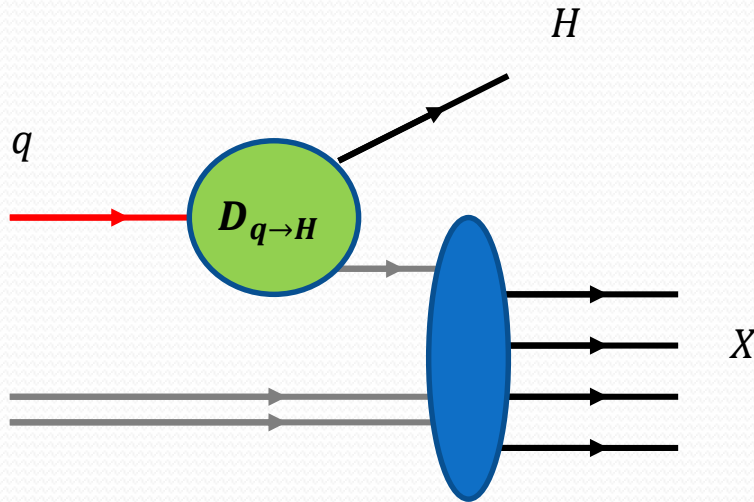
# Filling the Gap



- ❑ Computing *fragmentation function*,  $D_{q \rightarrow H}(z)$
- ❑ Precision data available at LEP (arXiv: 1102.4748, hep-ex/01120282) and SLC (hep-ex/0202031)

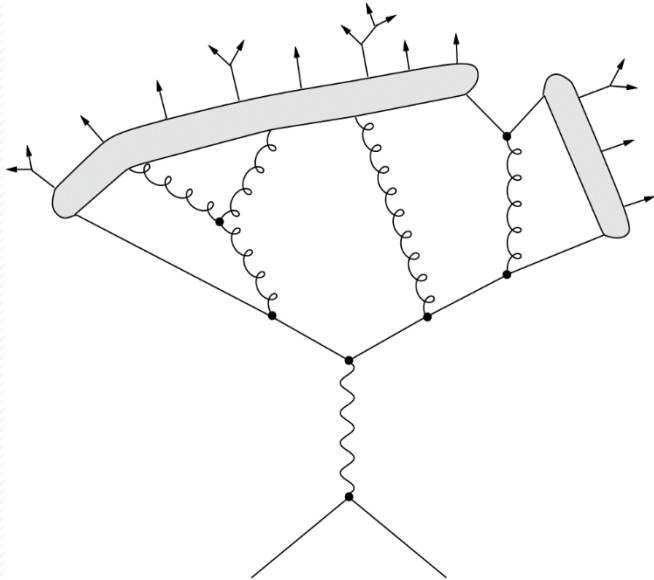


# Filling the Gap



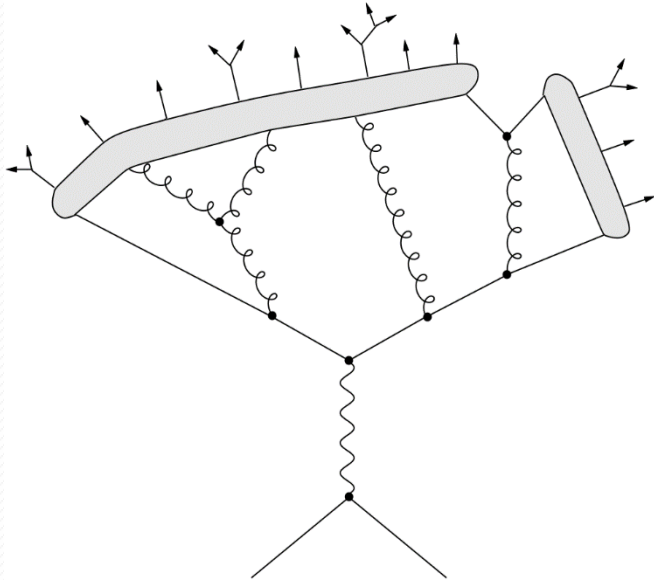
- ❑ Computing *fragmentation function*,  $D_{q \rightarrow H}(z)$
- ❑ Precision data available at LEP (arXiv: 1102.4748, hep-ex/01120282) and SLC (hep-ex/0202031)
- ❑ Higher order corrections necessary (including resummation sometimes)
- ❑ Relying on factorization of the cross section to a very high accuracy
- ❑ Not clear to apply lepton collider data to hadron colliders

# Filling the Gap



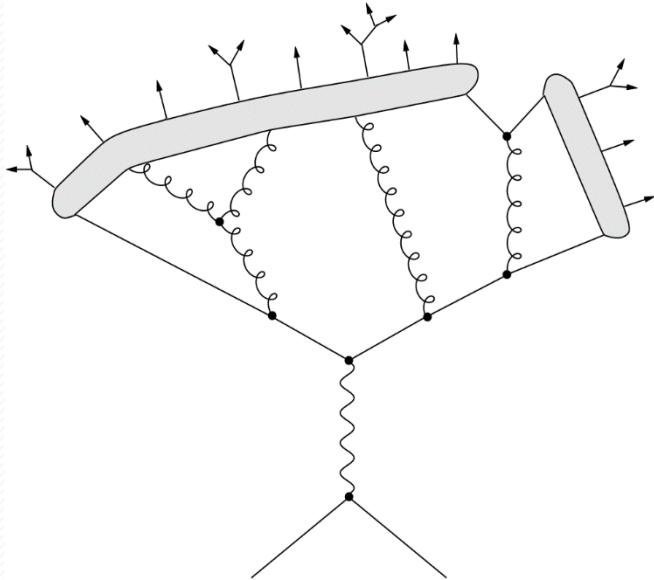
- ❑ Employing *hadronization model* with phenomenological parameters
- ❑ “Tuning” of the parameters to reproduce the available data

# Filling the Gap



- ❑ Employing *hadronization model* with phenomenological parameters
- ❑ “Tuning” of the parameters to reproduce the available data
- ❑ Not obvious that the tuned model describes the future data
- ❑ Should be test at hadron collider environment

# Filling the Gap



- ❑ Employing *hadronization model* with phenomenological parameters
- ❑ “Tuning” of the parameters to reproduce the available data
- ❑ Not obvious that the tuned model describes the future data
- ❑ Should be test at hadron collider environment

## ❑ Our approach/goal

- top quark mass *sensitivity to parameters*,
- *what should be constrained* to achieve better precision

# “Tuning” of PYTHIA8 Parameters

**A study of the sensitivity to the PYTHIA8 parton shower parameters of  $t\bar{t}$  production measurements in  $pp$  collisions at  $\sqrt{s} = 7$  TeV with the ATLAS experiment at the LHC**

The ATLAS Collaboration

## Abstract

Various measurements of  $t\bar{t}$  observables, performed by the ATLAS experiment in  $pp$  collisions at  $\sqrt{s} = 7$  TeV, are used to constrain the initial- and final-state radiation parameters of the PYTHIA8 Monte Carlo generator. The resulting tunes are compared to previous tunes to the  $Z$  boson transverse momentum at the LHC, and to the LEP event shapes in  $Z$  boson hadronic decays. Such a comparison provides a test of the universality of the parton shower model. The tune of PYTHIA8 to the  $t\bar{t}$  measurements is applied to the next-to-leading-order generators MadGraph5\_aMC@NLO and POWHEG, and additional parameters of these generators are tuned to the  $t\bar{t}$  data. For the first time in the context of parton shower tuning in Monte Carlo simulations, the correlation of the experimental uncertainties has been used to constrain the parameters of the Monte Carlo models.

ATL-PHYS-PUB-2015-007  
25 March 2015



# “Tuning” of PYTHIA8 Parameters

**A study of the sensitivity to the PYTHIA8 parton shower parameters of  $t\bar{t}$  production measurements in  $pp$  collisions at  $\sqrt{s} = 7$  TeV with the ATLAS experiment at the LHC**

The ATLAS Collaboration

## Abstract

Various measurements of  $t\bar{t}$  observables, performed by the ATLAS experiment in  $pp$  collisions at  $\sqrt{s} = 7$  TeV, are used to constrain the initial- and final-state radiation parameters of the PYTHIA8 Monte Carlo generator. The resulting tunes are compared to previous tunes to the Z boson transverse momentum at the LHC, and to the LEP event shapes in Z boson hadronic decays. Such a comparison provides a test of the universality of the parton shower model. The tune of PYTHIA8 to the  $t\bar{t}$  measurements is applied to the next-to-leading-order generators MadGraph5\_aMC@NLO and POWHEG, and additional parameters of these generators are tuned to the  $t\bar{t}$  data. For the first time in the context of parton shower tuning in Monte Carlo simulations, the correlation of the experimental uncertainties has been used to constrain the parameters of the Monte Carlo models.

ATL-PHYS-PUB-2015-007  
25 March 2015



# “Tuning” of PYTHIA8 Parameters

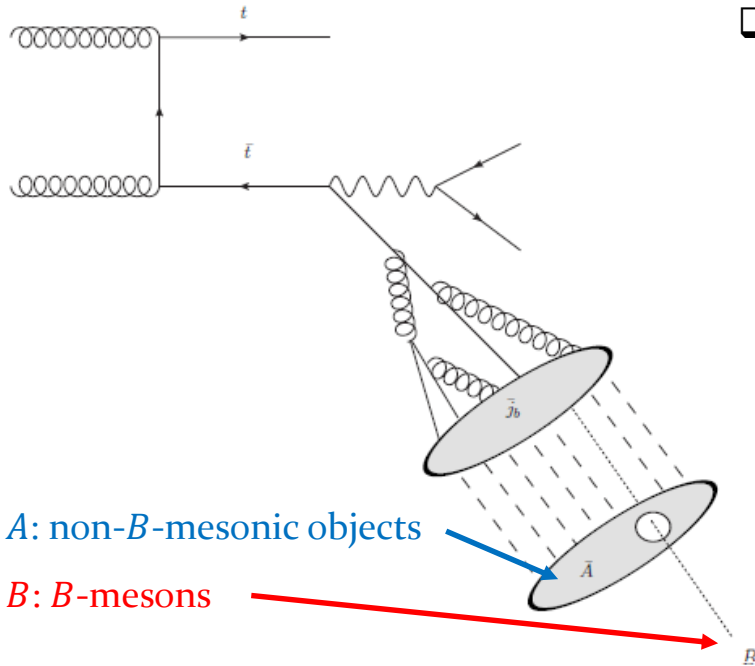
**A study of the sensitivity to the PYTHIA8 parton shower parameters of  $t\bar{t}$  production measurements in  $pp$  collisions at  $\sqrt{s} = 7$  TeV with the ATLAS experiment at the LHC**

**See also Efe Yazgan’s talk, and  
ATL-PHYS-PUB-2016-020,  
CMS-PAS-TOP-16-021**

Various measurements of  $t\bar{t}$  observables, performed by the ATLAS experiment in  $pp$  collisions at  $\sqrt{s} = 7$  TeV, are used to constrain the initial- and final-state radiation parameters of the PYTHIA8 Monte Carlo generator. The resulting tunes are compared to previous tunes to the Z boson transverse momentum at the LHC, and to the LEP event shapes in Z boson hadronic decays. Such a comparison provides a test of the universality of the parton shower model. The tune of PYTHIA8 to the  $t\bar{t}$  measurements is applied to the next-to-leading-order generators MadGraph5\_aMC@NLO and PowHEG, and additional parameters of these generators are tuned to the  $t\bar{t}$  data. For the first time in the context of parton shower tuning in Monte Carlo simulations, the correlation of the experimental uncertainties has been used to constrain the parameters of the Monte Carlo models.

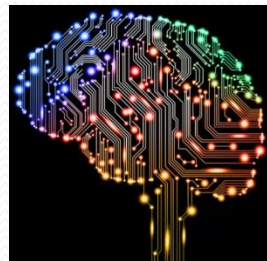


# Strategy in a Nut-shell



A: non- $B$ -mesonic objects

B:  $B$ -mesons

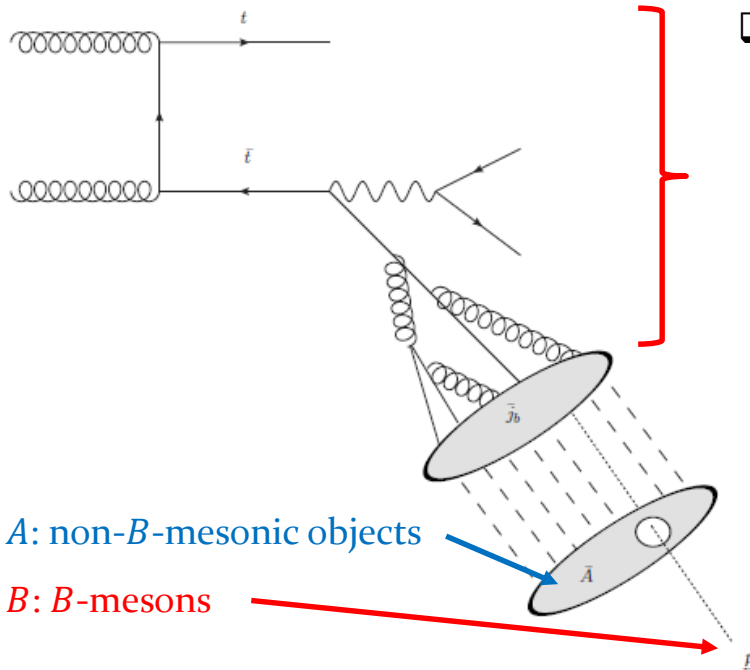


□ For a given input top mass,

- 1) set relevant parameters (next slide),
- 2) generate, shower, and hadronize leptonic  $t\bar{t}$  events using PYTHIA 8.2.19,
- 3) find anti- $k_t$  jets using FastJet,
- 4) find jets containing a  $B$ -meson as a constituent, and extract its information,
- 5) evaluate various  $B$ -meson observables (next-to-next slide) along with leptons depending on observables: Mellin moments, peak/endpoint,
- 6) Correlate them with input top masses and find sensitivity measures (defined later),
- 7) Repeat 1) through 6) for other parameter sets

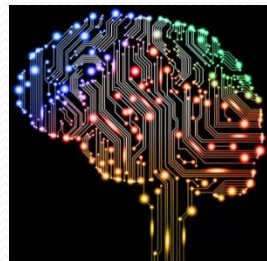


# Strategy in a Nut-shell



A: non- $B$ -mesonic objects

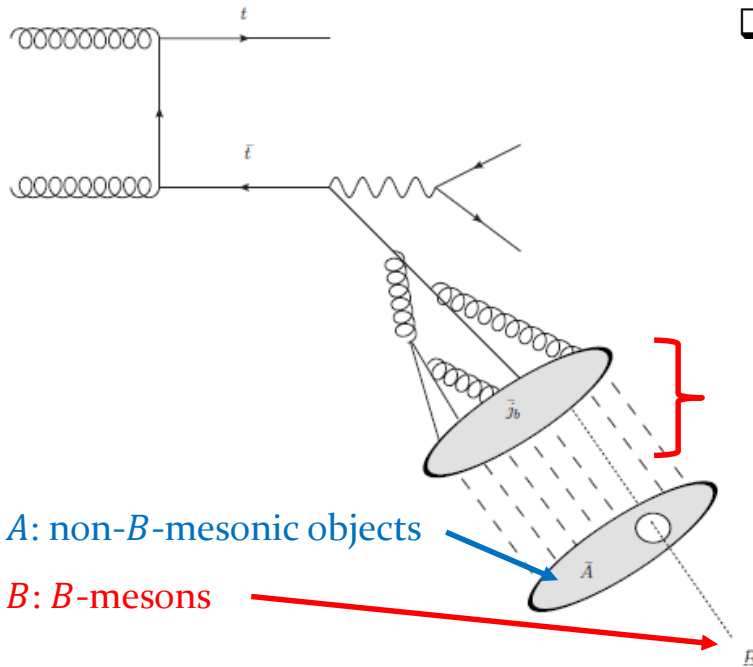
B:  $B$ -mesons



□ For a given input top mass,

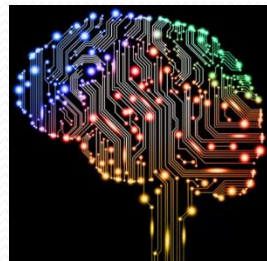
- 1) set relevant parameters (next slide),
- 2) generate, shower, and hadronize leptonic  $t\bar{t}$  events using PYTHIA 8.2.19,
- 3) find anti- $k_t$  jets using FastJet,
- 4) find jets containing a  $B$ -meson as a constituent, and extract its information,
- 5) evaluate various  $B$ -meson observables (next-to-next slide) along with leptons depending on observables: Mellin moments, peak/endpoint,
- 6) Correlate them with input top masses and find sensitivity measures (defined later),
- 7) Repeat 1) through 6) for other parameter sets

# Strategy in a Nut-shell



A: non- $B$ -mesonic objects

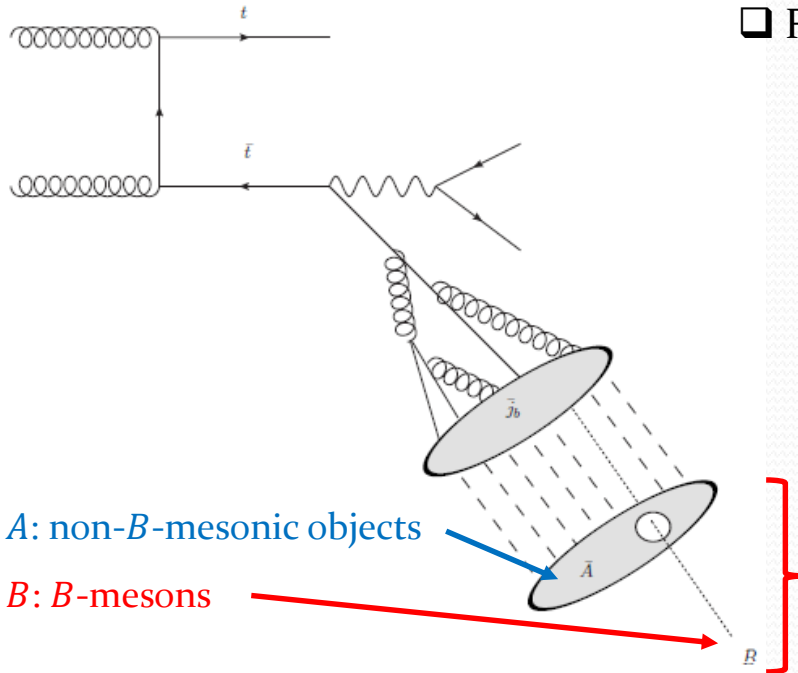
B:  $B$ -mesons



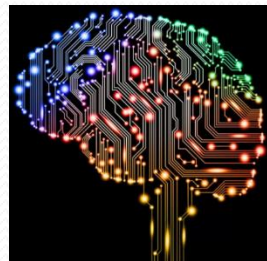
□ For a given input top mass,

- 1) set relevant parameters (next slide),
- 2) generate, shower, and hadronize leptonic  $t\bar{t}$  events using PYTHIA 8.2.19,
- 3) find anti- $k_t$  jets using FastJet,
- 4) find jets containing a  $B$ -meson as a constituent, and extract its information,
- 5) evaluate various  $B$ -meson observables (next-to-next slide) along with leptons depending on observables: Mellin moments, peak/endpoint,
- 6) Correlate them with input top masses and find sensitivity measures (defined later),
- 7) Repeat 1) through 6) for other parameter sets

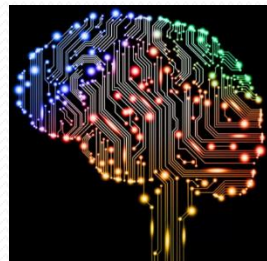
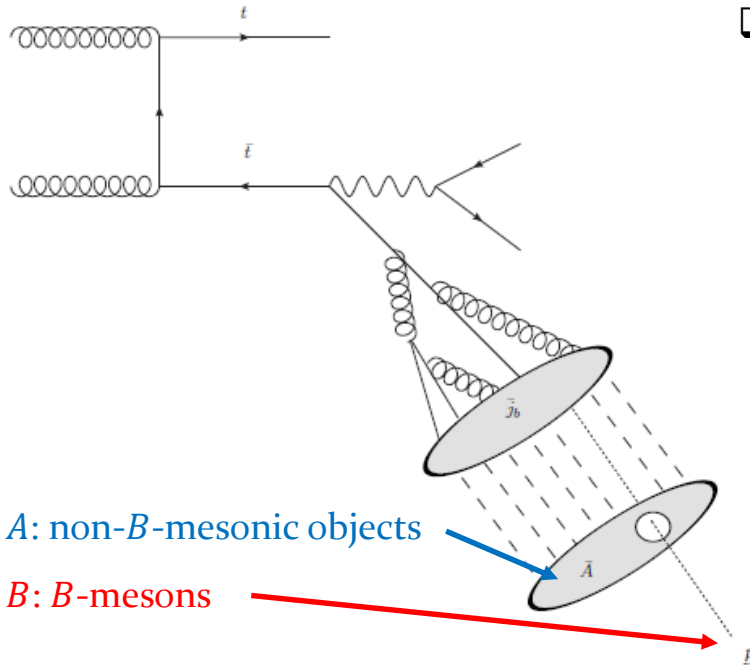
# Strategy in a Nut-shell



- For a given input top mass,
  - 1) set relevant parameters (next slide),
  - 2) generate, shower, and hadronize leptonic  $t\bar{t}$  events using PYTHIA 8.2.19,
  - 3) find anti- $k_t$  jets using FastJet,
  - 4) find jets containing a  $B$ -meson as a constituent, and extract its information,
  - 5) evaluate various  $B$ -meson observables (next-to-next slide) along with leptons depending on observables: Mellin moments, peak/endpoint,
  - 6) Correlate them with input top masses and find sensitivity measures (defined later),
  - 7) Repeat 1) through 6) for other parameter sets

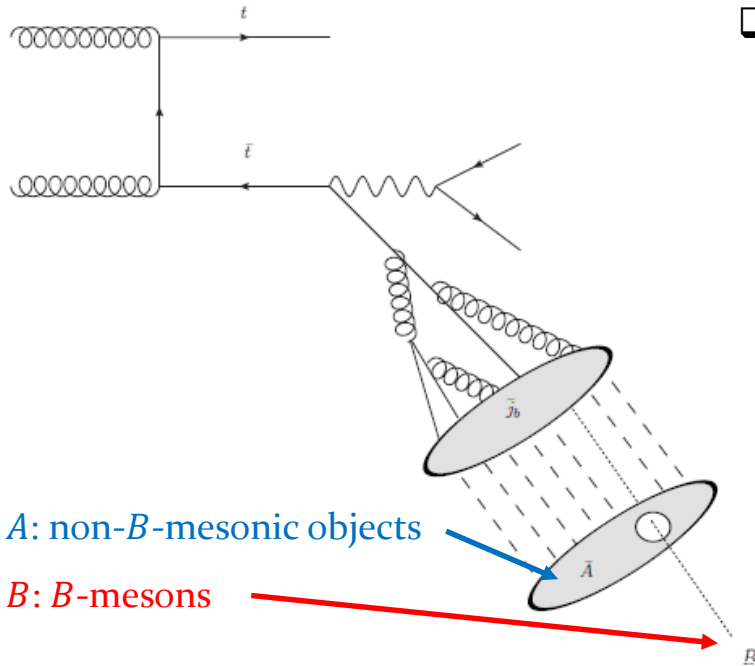


# Strategy in a Nut-shell



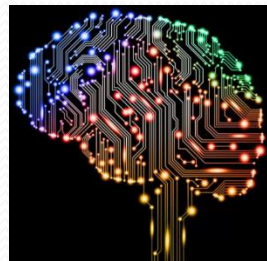
- For a given input top mass,
  - 1) set relevant parameters (next slide),
  - 2) generate, shower, and hadronize leptonic  $t\bar{t}$  events using PYTHIA 8.2.19,
  - 3) find anti- $k_t$  jets using FastJet,
  - 4) find jets containing a *B*-meson as a constituent, and extract its information,
  - 5) evaluate various *B*-meson observables (next-to-next slide) along with leptons depending on observables: Mellin moments, peak/endpoint,
  - 6) Correlate them with input top masses and find sensitivity measures (defined later),
  - 7) Repeat 1) through 6) for other parameter sets

# Strategy in a Nut-shell



A: non- $B$ -mesonic objects

B:  $B$ -mesons



□ For a given input top mass,

- 1) set relevant parameters (next slide),
- 2) generate, shower, and hadronize leptonic  $t\bar{t}$  events using PYTHIA 8.2.19,
- 3) find anti- $k_t$  jets using FastJet,
- 4) find jets containing a  $B$ -meson as a constituent, and extract its information,
- 5) evaluate various  $B$ -meson observables (next-to-next slide) along with leptons depending on observables: Mellin moments, peak/endpoint,
- 6) Correlate them with input top masses and find sensitivity measures (defined later),
- 7) Repeat 1) through 6) for other parameter sets

# B-meson-related Parameters

- ❑ Fully leptonic LO  $t\bar{t}$  at the LHC 13 TeV with NNPDF2.3 QCD+QED LO and anti- $k_t$  jets of  $R = 0.5$
- ❑ Input top quark mass varies from 170 GeV to 180 GeV by an interval of 1 GeV

Parameter	PYTHIA8 setting	Variation range
$r_b$ in the Bowler modification for heavy quarks	StringZ:rFactB	0.713 – 0.813
$a$ parameter in the non-standard Lund ansatz for $b$ quarks	StringZ:aNonstandardB	0.54 – 0.82
$b$ parameter in the non-standard Lund ansatz for $b$ quarks	StringZ:bNonstandardB	0.78 – 1.18
$p_{T,\min}^{\text{FSR}}$ [GeV]	TimeShower:pTmin	0.25 – 1.00
recoiler switch	TimeShower:recoilToColoured	on or off
$\alpha_s^{\text{FSR}}(m_Z)$	TimeShower:alphaSvalue	0.1092 – 0.1638

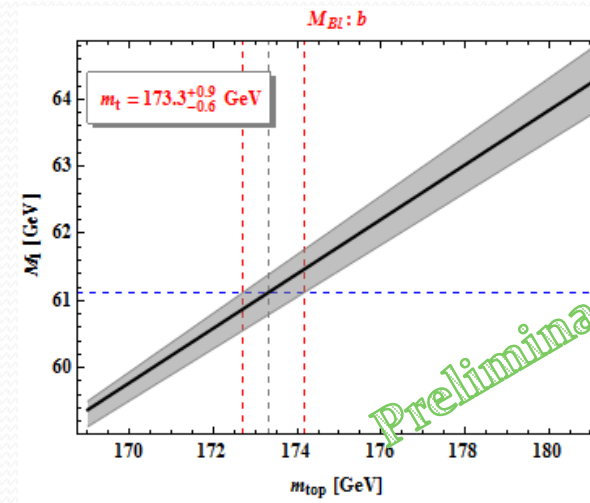
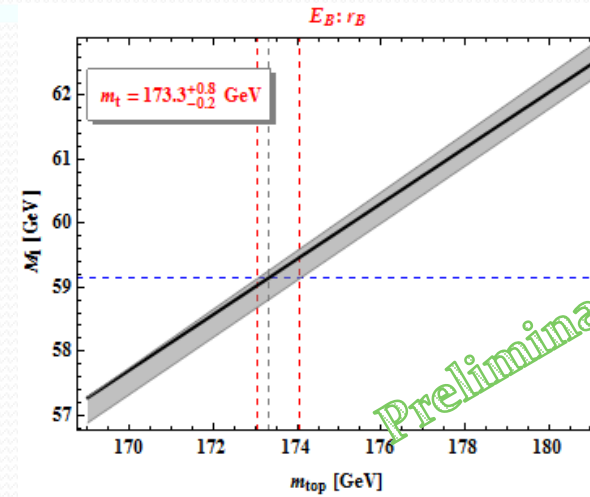
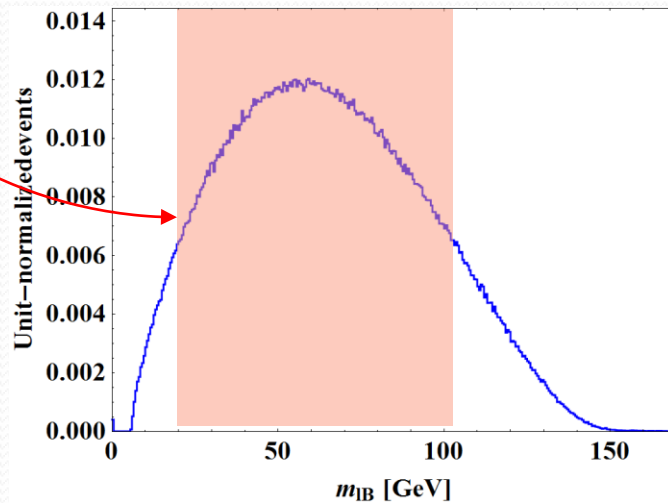
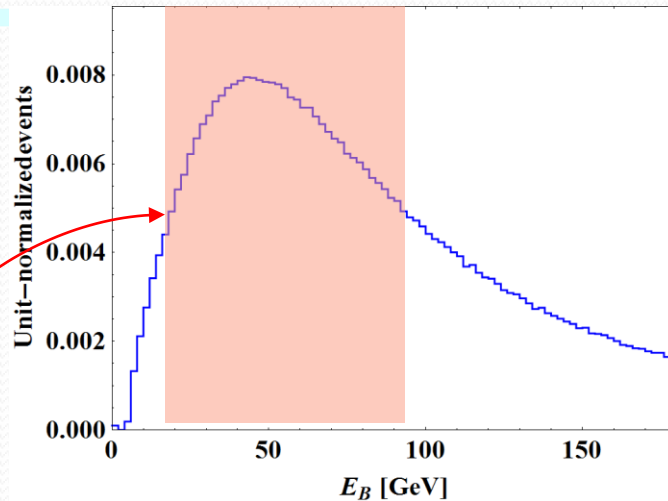
# B-meson Observables

Observable	Mellin moment	Peak/Endpoint	Features
$E_B$	V	V	<ul style="list-style-type: none"> <li>Expecting inheritance of “invariance” property of the energy-peak in the b-jet energy spectrum</li> </ul>
$E_{B_1} + E_{B_2}$	V		<ul style="list-style-type: none"> <li>Two B-meson tagging required</li> </ul>
$P_{T,B}$	V		
$P_{T,B_1} + P_{T,B_2}$	V		<ul style="list-style-type: none"> <li>Two B-meson tagging required</li> </ul>
$M_{B\ell}$	V	V	<ul style="list-style-type: none"> <li>True pairing (theory-level)</li> <li>Experimental observable paring: the smaller in each combination</li> </ul>
$M_{T2}$	V (only for $(B\ell)$ subsystem)	V	<ul style="list-style-type: none"> <li><math>(B)</math> and <math>(B\ell)</math> subsystems</li> <li>True assignment (theory-level) for the <math>(B\ell)</math> subsystems</li> <li>Experimental observable paring for the <math>(B\ell)</math> subsystems: the smaller of the two possible assignments</li> </ul>
$M_{T2,\perp}$	V (only for $(B\ell)$ subsystem)	V	<ul style="list-style-type: none"> <li>ISR-free observables</li> <li><math>(B)</math> and <math>(B\ell)</math> subsystems</li> </ul>

# Results: Mellin Moment

$$\mathcal{M}_1 = \int_{FWHM} dx xf(x)$$

FWHM



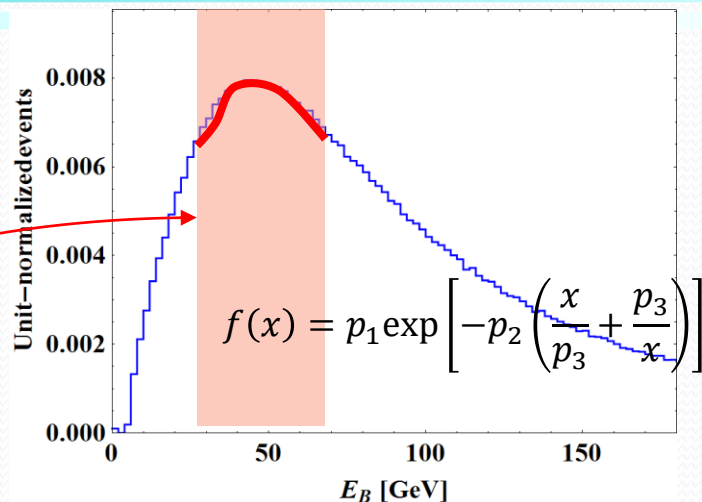
Preliminary

Preliminary

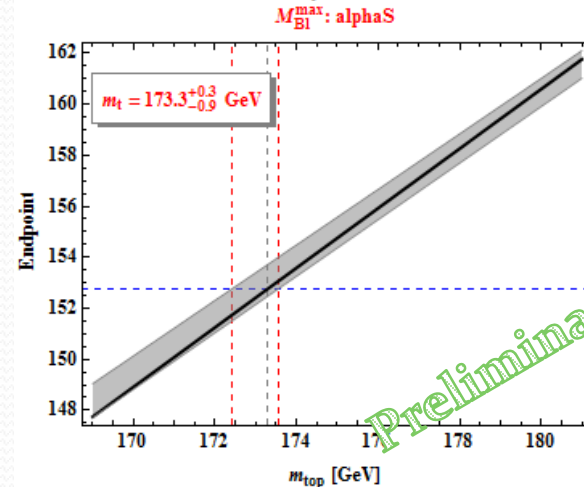
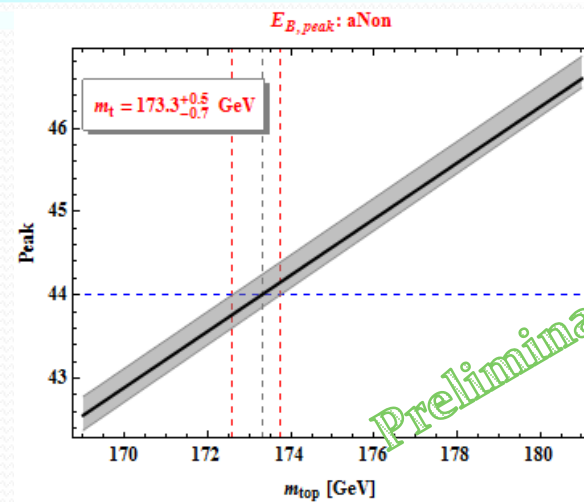
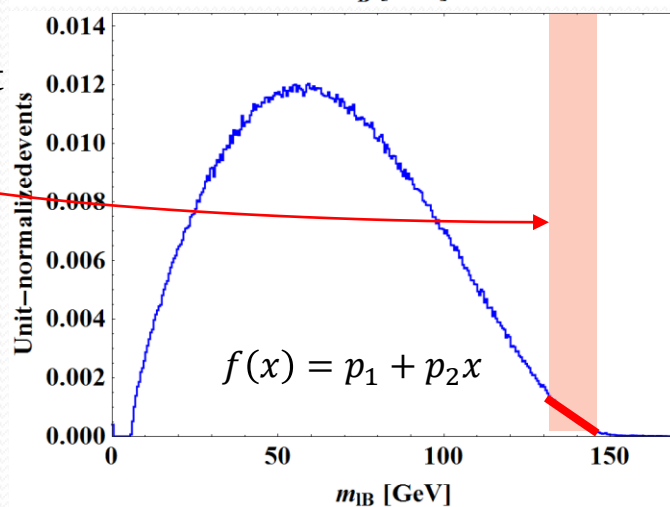


# Results: Peak/Endpoint

Full width at  $\frac{3}{4}$  height  
( $\chi^2/\text{dof} \sim 1$ )



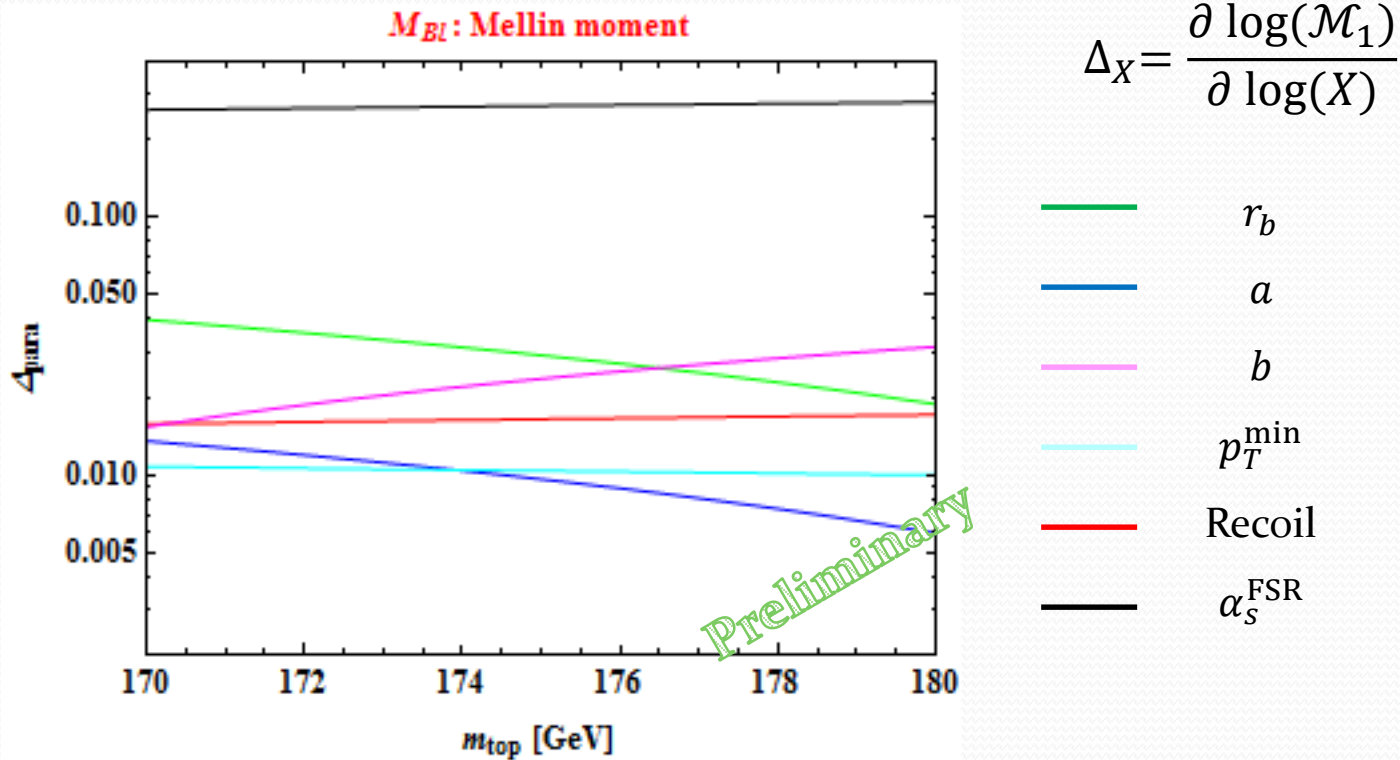
$\sim 5\%$  from the endpoint  
( $\chi^2/\text{dof} \sim 1$ )



Preliminary

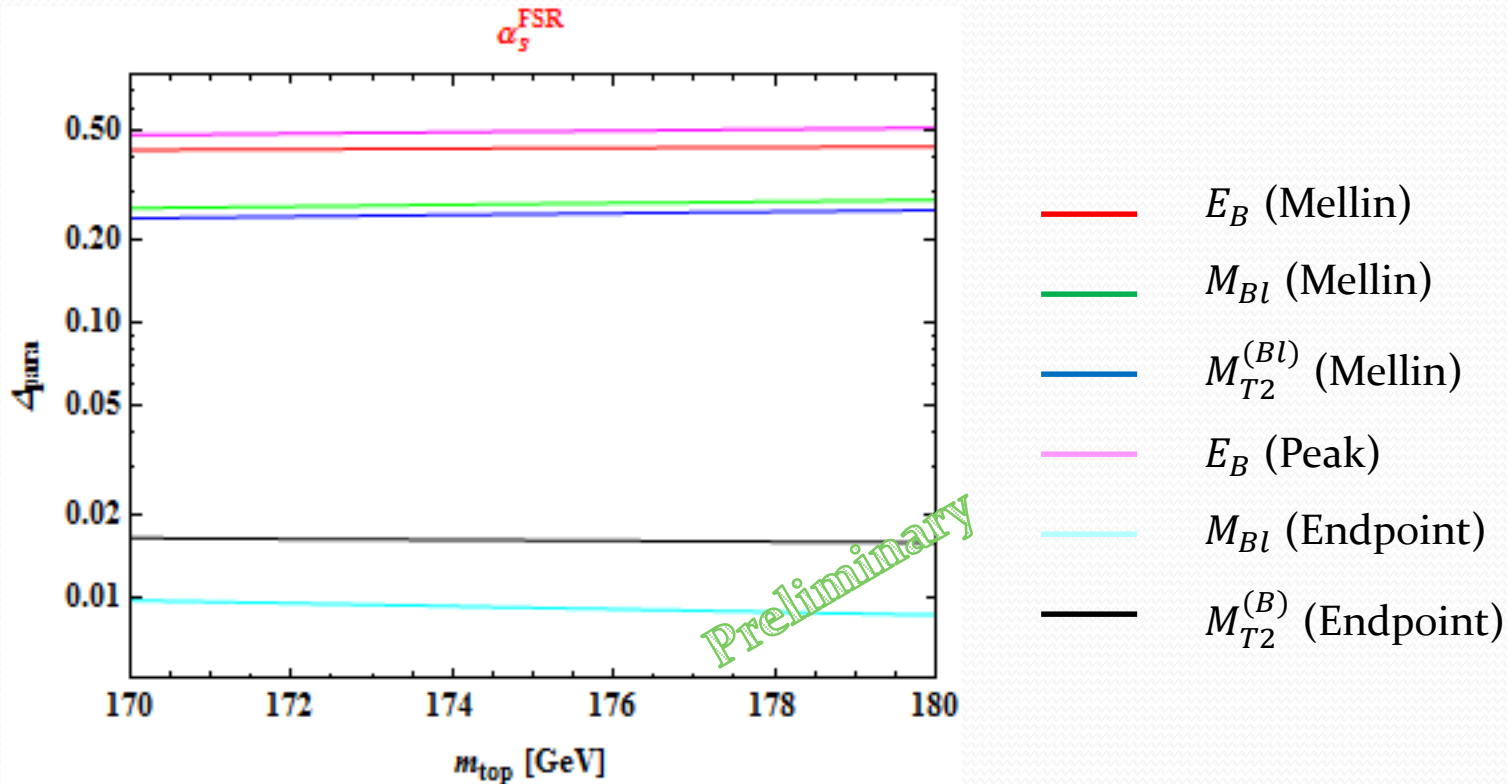
Preliminary

# Results: Parameter Sensitivity



- For many observables, top quark mass is **most sensitive to  $\alpha_s^{\text{FSR}}$**  (not surprisingly).
  - (Roughly) the more chance to have FSR, the softer distributions are

# Results: Parameter Sensitivity

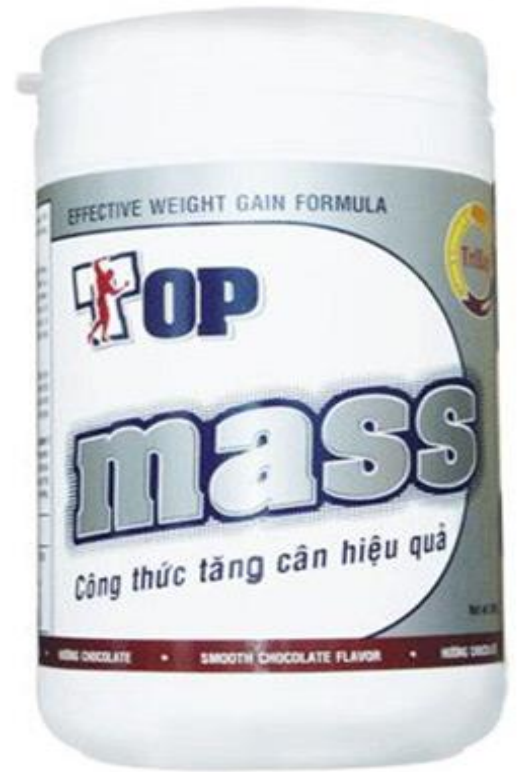


□ Endpoint observables are **less sensitive to  $\alpha_s^{FSR}$** .

→ (Roughly) the endpoints are protected from FSR (with local data fits).

# Conclusions

- ❑ Different methods for top quark mass measurement: the more the messier? the more the merrier?!
  - ❖ Different sensitivity to systematics, complementary to one another, good exercise for BSM scenarios
- ❑  $B$ -meson observable method
  - ❖ Non-jetty nature
  - ❖ Most sensitive to  $\alpha_s^{\text{FSR}}$ , so a better “tune” reduces the theoretical uncertainty of top mass in  $B$ -meson observables
  - ❖  $\alpha_s^{\text{FSR}}$  should be constrained at 1-2% level, while the others at 10-20% to achieve  $\sim 0.5\%$  precision in  $m_t$  ( $\alpha_s^{\text{FSR}} \rightarrow r_b \rightarrow \dots$ )
  - ❖  $m_t^{\text{ext}}|_{m_t^{\text{in}}=173.3 \text{ GeV}} = 173.3^{+0.6}_{-1.3} \text{ GeV}$  ( $M_{B1}/M_{T2}^{(B)}$  endpoints)
  - ❖  $m_t^{\text{ext}}|_{m_t^{\text{in}}=173.3 \text{ GeV}} = 173.3^{+0.5}_{-0.8} \text{ GeV}$  ( $M_{T2}^{(B\ell)}$  endpoint w/ negligible sensitivity to  $\alpha_s^{\text{FSR}}$ )
- ❑ The same exercises with HERWIG will be available.





**Thank you!**

# B-meson Decay

□ Fully reconstructible with tracks

**J/ψ modes**  $b \xrightarrow{\text{few } 10^{-3}} J/\psi + X \xrightarrow{10^{-1}} \ell^+ \ell^- + X$

➤  $B_s^0 \rightarrow J/\psi \phi \rightarrow \mu^- \mu^+ K^- K^+$  (1106.4048)       $B^0 \rightarrow J/\psi K_S^0 \rightarrow \mu^- \mu^+ \pi^- \pi^+$  (1104.2892)

➤  $B^+ \rightarrow J/\psi K^+ \rightarrow \mu^- \mu^+ K^+$  (1101.0131, 1309.6920)       $\Lambda_b \rightarrow J/\psi \Lambda \rightarrow \mu^- \mu^+ p \pi^-$  (1205.0594)

**D modes**

➤  $B^0 \xrightarrow{3 \times 10^{-3}} D^- \pi^+ \xrightarrow{10^{-2}} K_S^0 \pi^- \pi^+$ ,  $B^0 \xrightarrow{3 \times 10^{-3}} D^- \pi^+ \xrightarrow{10^{-2}} K^- \pi^+ \pi^- \pi^+$ ,

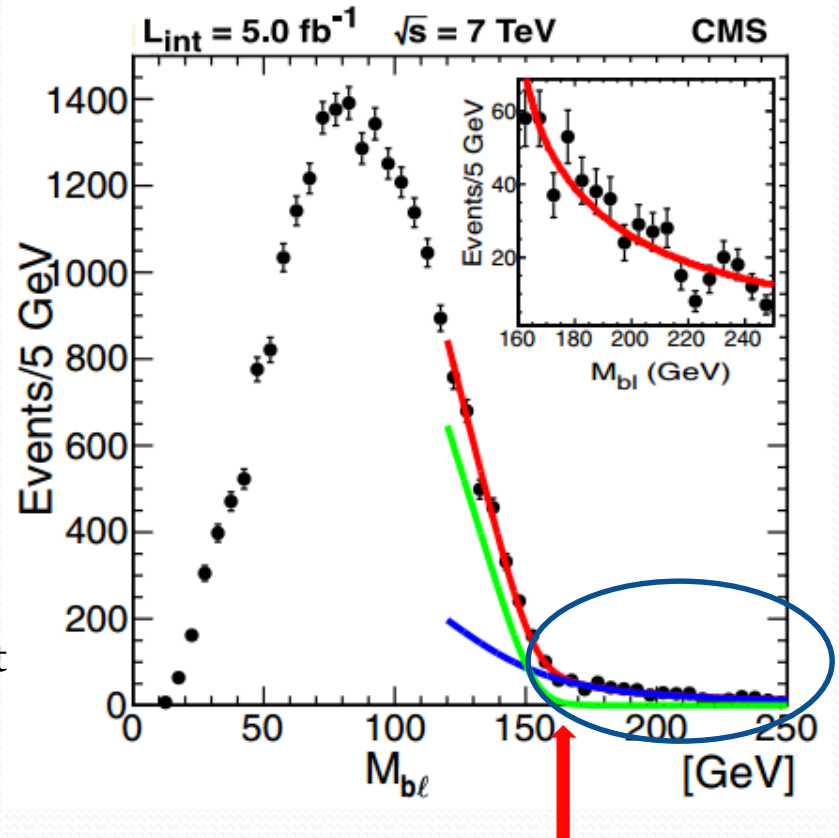
$B^0 \xrightarrow{3 \times 10^{-3}} D^- \pi^+ \xrightarrow{3 \times 10^{-2}} K_S^0 \pi^+ \pi^- \pi^+$

➤  $B^- \xrightarrow{5 \times 10^{-3}} D^0 \pi^- \xrightarrow{4 \times 10^{-2}} K^- \pi^+ \pi^-$ ,  $B^- \xrightarrow{5 \times 10^{-3}} D^0 \pi^- \xrightarrow{2 \times 10^{-2}} K^{*-} (892) \pi^+ \pi^- \rightarrow K_S^0 \pi^+ \pi^- \pi^+$ ,

$B^- \xrightarrow{5 \times 10^{-3}} D^0 \pi^- \xrightarrow{6 \times 10^{-3}} K_S^0 \rho^0 \pi^-$ ,  $B^- \xrightarrow{5 \times 10^{-3}} D^0 \pi^- \xrightarrow{5 \times 10^{-3}} K^- \pi^+ \rho^0 \pi^-$

# Endpoint Method

- ❑ Three observables:  $m_{b\ell}$ ,  $b$  and  $\ell$  subsystem  
 $M_{T2}$  endpoints [CMS-TOP-11-027]
- ❑ Endpoint extraction: local data fit (around expected endpoints)
  - ❖ (Detector response-convoluted)  
Kinked-line function (for signal) + background model function
- ❑ Shift of endpoints [Alioli et al, (2012)]
  - ❖ Width effect, NLO correction, ... (might give a systematic bias)



# Endpoint Method

Fit quantity	Constraint		
	None	$m_\nu = 0$	$m_\nu = 0$ and $M_W = 80.4$ GeV
$m_\nu^2$ (GeV <sup>2</sup> )	$-556 \pm 473 \pm 622$	(0)	(0)
$M_W$ (GeV)	$72 \pm 7 \pm 9$	$80.7 \pm 1.1 \pm 0.6$	(80.4)
$M_t$ (GeV)	$163 \pm 10 \pm 11$	$174.0 \pm 0.9^{+1.7}_{-2.1}$	$173.9 \pm 0.9^{+1.7}_{-2.1}$

- ❖ **Simultaneous measurement** of three mass parameters
- ❖ **Good exercise** for mass determination of new particles

- ❖ **Best top mass** measurement (by endpoint method) assuming  $m_W$  and  $m_\nu$  well-measured



# $b$ -jet Energy-peak Method

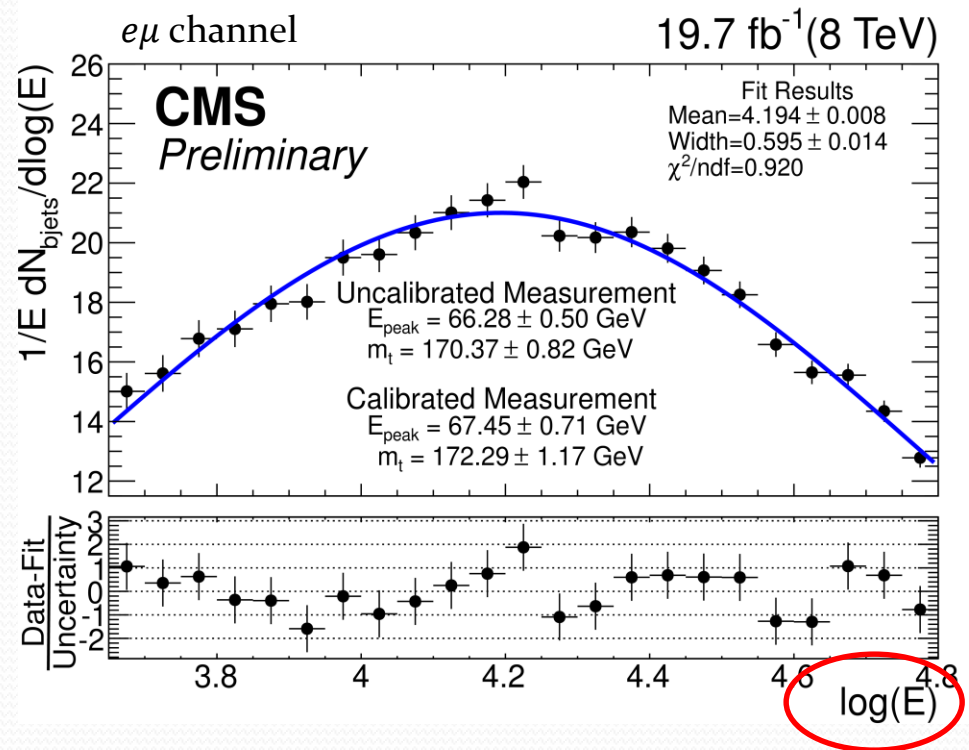
- Based on the **LO** observation
- Dileptonic final state (cleanest channel)

$$t\bar{t} \rightarrow b\bar{b}e^{\pm}\mu^{\mp} + \nu_e\nu_{\mu}$$

- Energy spectrum should be symmetric w.r.t.  $E_b^*$  in  $\log E$ 
  - Gaussian fit near the peak region [CMS PAS TOP-15-002]

- Best-fit top mass**

$$m_t = 172.29 \pm 1.17 \pm 2.66 \text{ GeV}$$



# $b$ -jet Energy-peak Method

## Systematic uncertainties

Source of uncertainty	$\delta E_{peak}$ (GeV)	$\delta m_t$ (GeV)
Experimental uncertainties		
Jet energy scale	0.74	1.23
b jet energy scale	0.13	0.22
Jet energy resolution	0.18	0.30
Pile-up	0.02	0.03
b-tagging efficiency	0.12	0.20
Lepton efficiency	0.02	0.03
Fit calibration	0.14	0.24
Backgrounds	0.21	0.34
Modeling of hard scattering process		
Generator modeling	0.91	1.50
Renormalization and factorization scales	0.13	0.22
ME-PS matching threshold	0.24	0.39
Top $p_T$ reweighting	0.91	1.50
PDFs	0.13	0.22
Modeling of non-perturbative QCD		
Underlying event	0.22	0.35
Color reconnection	0.38	0.62
Total	1.62	2.66

- ❑ Statistical uncertainty of 1.17 GeV will be under control (more statistics coming up).
- ❑ Experimental uncertainty (mostly from JES) will be under control.
- ❑ Theoretical uncertainty is from modeling of hard scattering processes.
- ❑ Any chance to improve/understand the systematic uncertainty?
  - ❖ Study of **higher-order effects**