Statistical Reconstruction of Net Baryon-Number Cumulants

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ALICE Collaboration
CERN

Jun 28, 2016

ALICE

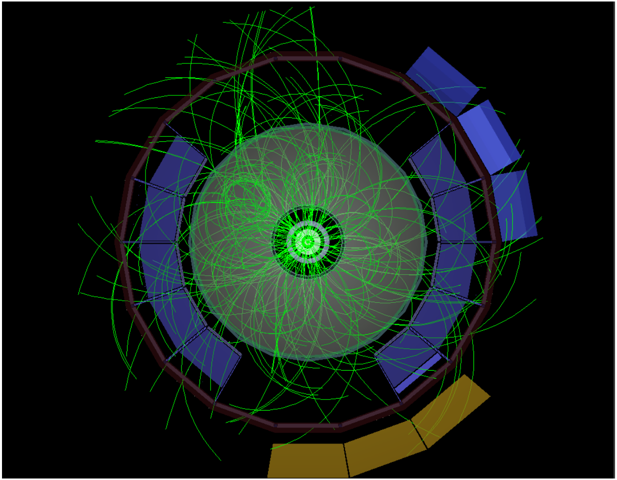

Figure: ALICE heavy-ion collision, Nov. 6, 2011

Why?

- Net Baryon Number: Conserved, taken from multiplicities measured by ALICE
- Statistical mechanics: fluctuations in conserved quantities related to thermodynamic properties of QGP

$$< \delta B_{net}^2 >= T^2 \frac{\partial^2}{\partial \mu_B^2} \log(Z) \sim \xi^2$$

- Measurements related to correlation length ξ help distinguish QGP and identify phase transitions

Why Haven't We Done This Already?

- To measure fluctuations, we use cumulants (alternate form of moments)
- But imperfect detector misses some particles, drastically changing distribution shape
- Simple correction (multiply every multiplicity by 1/acceptance) doesn't help very much

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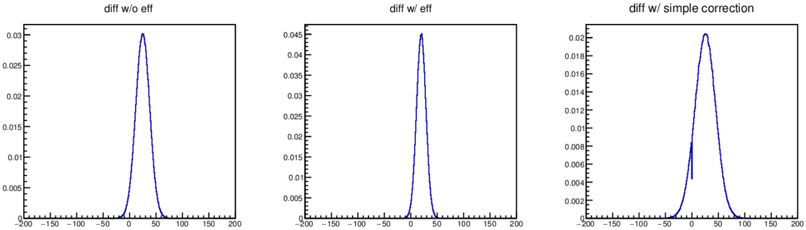

How Do We Fix This?

- Solution proposed by Adam Bzdak and Volker Koch (2012) involving factorial moments:

$$f_{ik} = \sum_{n_1=i}^{\infty} \sum_{n_2=k}^{\infty} p(n_1, n_2) \frac{n_1!}{(n_1 - i)!} \frac{n_2!}{(n_2 - k)!}$$

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$$\begin{aligned} K_6 = {} & N - 120K_1^6 + F_{06} + 15F_{05} + 65F_{04} + 90F_{03} + 31F_{02} - 2F_{11} - 30F_{12} - 80F_{13} - 45F_{14} - 6F_{15} + 31F_{20} - 30F_{21} \\ & + 30F_{22} + 30F_{23} + 15F_{24} + 90F_{30} - 80F_{31} + 30F_{32} - 20F_{33} + 65F_{40} - 45F_{41} + 15F_{42} + 15F_{50} - 6F_{51} + F_{60} \\ & + 360K_1^4(N + F_{02} - 2F_{11} + F_{20}) - 270K_1^2(N + F_{02} - 2F_{11} + F_{20})^2 + 30(N + F_{02} - 2F_{11} + F_{20})^3 - 120K_1^3(K_1 \\ & - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30}) + 120K_1(N + F_{02} - 2F_{11} + F_{20})(K_1 - F_{03} - 3F_{02} + 3F_{12} \\ & + 3F_{20} - 3F_{21} + F_{30}) - 10(K_1 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30})^2 + 30K_1^2(N + F_{04} + 6F_{03} + 7F_{02} - 2F_{11} \\ & - 6F_{12} - 4F_{13} + 7F_{20} - 6F_{21} + 6F_{22} + 6F_{30} - 4F_{31} + F_{40}) - 15(N + F_{02} - 2F_{11} + F_{20})(N + F_{04} + 6F_{03} \\ & + 7F_{02} - 2F_{11} - 6F_{12} - 4F_{13} + 7F_{20} - 6F_{21} + 6F_{22} + 6F_{30} - 4F_{31} + F_{40}) - 6K_1(K_1 - F_{05} - 10F_{04} - 25F_{03} - 15F_{02} \\ & + 15F_{12} + 20F_{13} + 5F_{14} + 15F_{20} - 15F_{21} - 10F_{23} + 25F_{30} - 20F_{31} + 10F_{32} + 10F_{40} - 5F_{41} + F_{50}). \end{aligned} \quad \text{(A6)}$$

It Works! Sort Of

- Correction method applied, errors determined via bootstrap method
- Not perfect yet (numerical inaccuracies), but much better than naive correction

No.	Physics:	Measured:	Simple Corrected:	Full Corrected:
1	24.9986	20.0206	25.2375	25.2375
1Er	0.0304437	0.0197374	0.0434017	0.0434017
2	175.118	79.4425	384.721	172.993
2Er	0.518099	0.255682	1.23334	0.886822
3	-7.93023	7.70124	-39.9084	21.2168
3Er	11.1291	3.55195	38.8565	20.5743
4	-761.465	-105.837	7575.86	410.018
4Er	321.182	61.219	1423.53	474.251
5	28622	4403.94	-212514	1515.16
5Er	9998.84	1266.44	63316	16604.1
6	1.20209e+06	117445	-1.67079e+07	97542.5
6Er	399683	30128.9	3.20187e+06	609543

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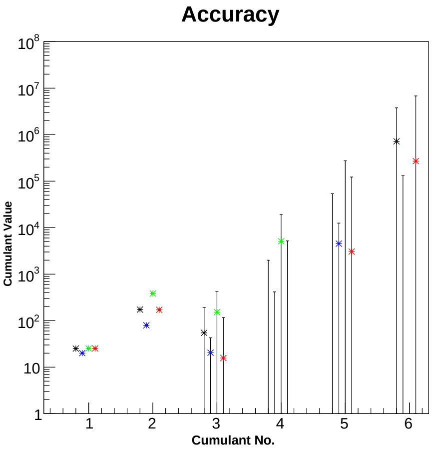

Figure: Black = Physics, Blue = Measured, Green = Simple Corrected, Red = Full Corrected

Further Work

- Use different error estimation method (delta theorem)
- Increase numerical accuracy in taking cumulants and factorial moments
- p_T-dependent acceptance
- Local (as opposed to global) corrections