Regression with Gaussian Processes

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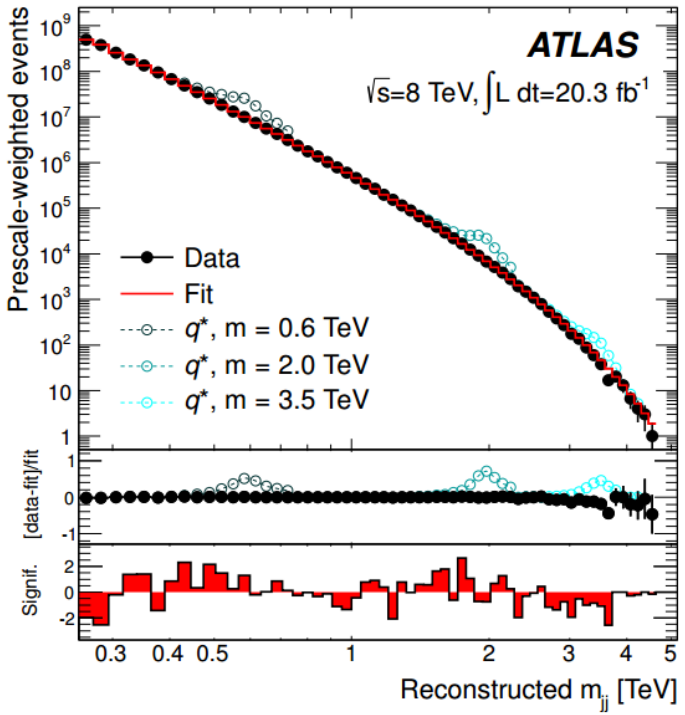

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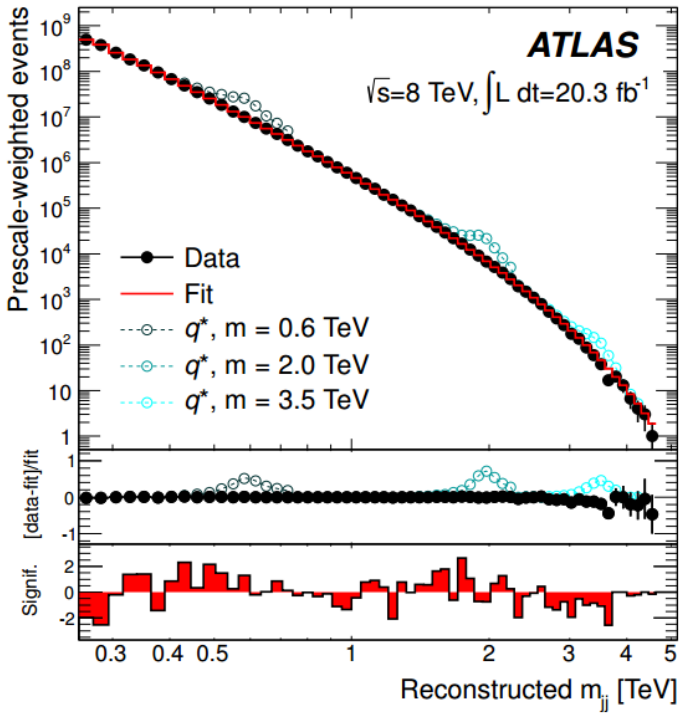

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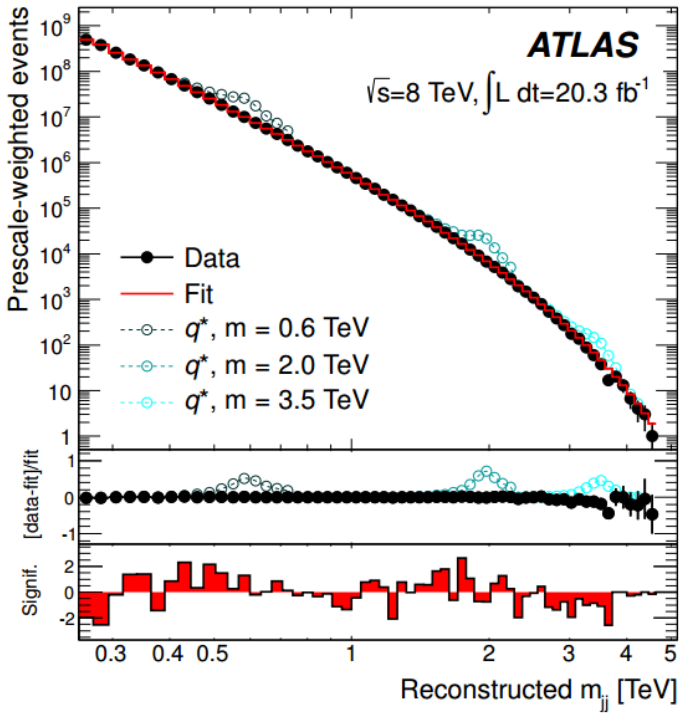
- Doesn't work well at high luminosity



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- Choose $A(x) = Ce^{-\frac{x}{d}}$ and $\ell(x) = Lx^p$
- Four parameters C, d, L, p

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- When fitting for background + signal:
  - For simplicity, assume signals are Gaussian (in practice, use other shapes)
  - Allowed to subtract off Gaussian before fitting
  - Parameters of Gaussian (height, width, mean) are optimized with kernel parameters

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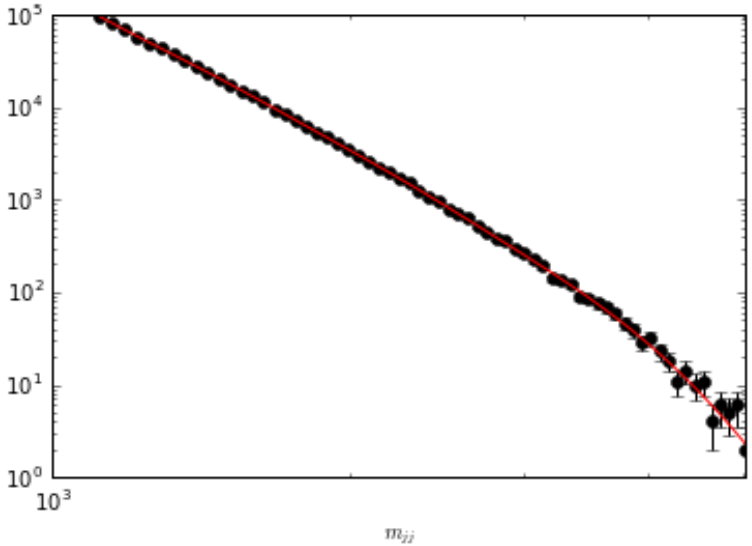
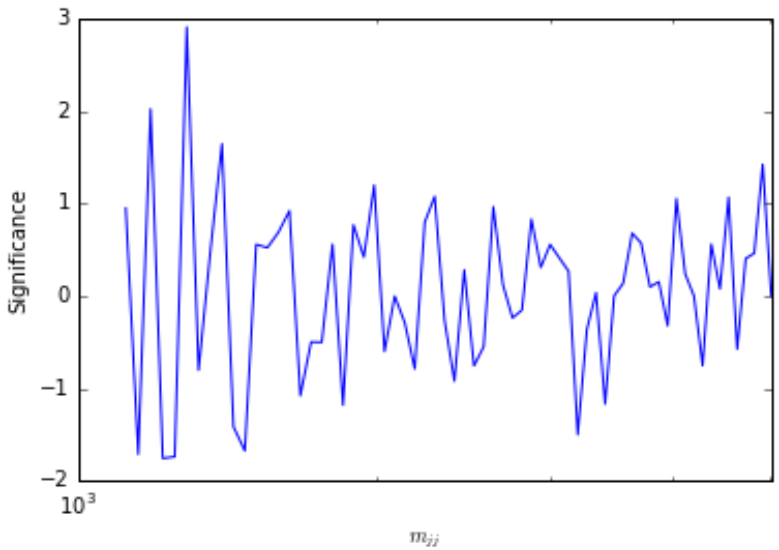
- Two measures of effectiveness:
  - χ^2 value = sum of squares of statistical fluctuations
    - Good value is approximately equal to degrees of freedom
    - Can also look at fluctuations at each point individually
  - Inject signals of varying heights, and compare to fitted signals
    - Should exhibit linear relationship

Results

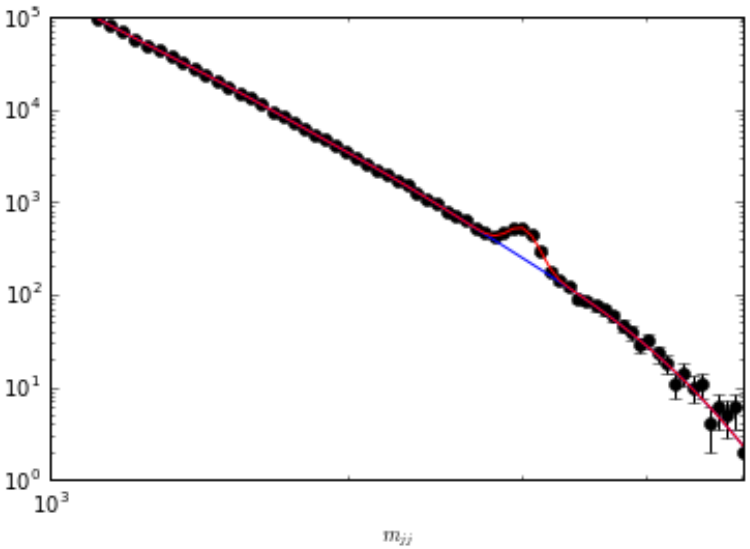
Background fit and significances ($\chi^2 = 60.16$ for 62 d.o.f.)

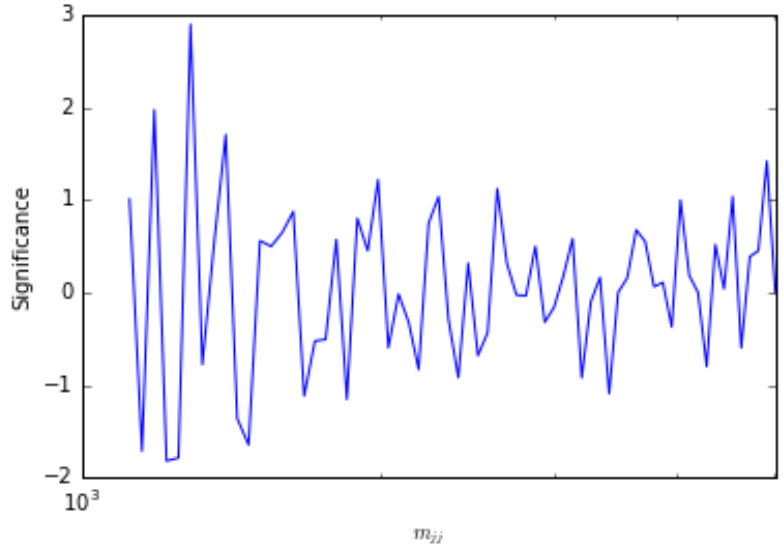




Results

Fit and significances with $N = 1000$ signal at 3000 GeV
(Fitted signal of $N = 1042$, $\chi^2 = 57.89$ for 59 d.o.f.)





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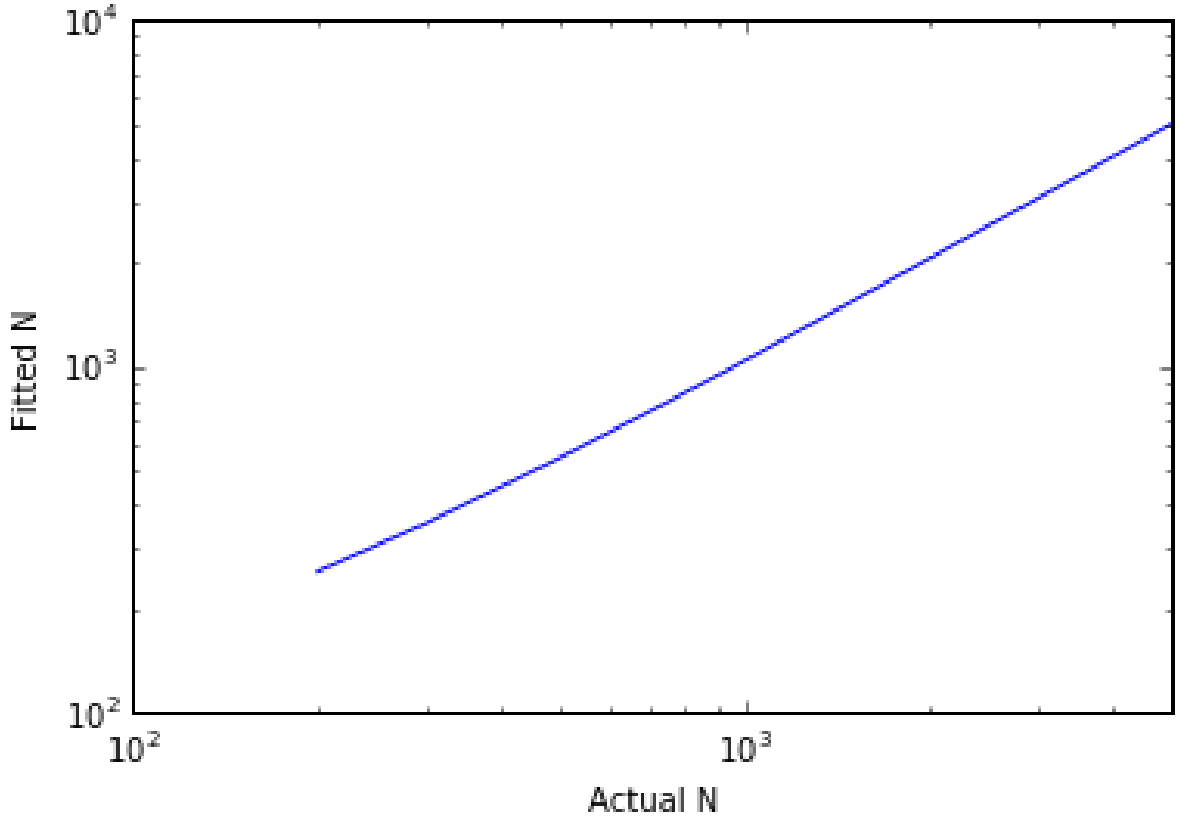
Comparison of injected and fitted signals at 3000 GeV



Future Research

- More tests of effectiveness:
  - Test how well background-only fit will fit background+signal
  - Test on perturbations of data/other data sets
- Test against existing methods

Acknowledgements

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- Daniel Whiteson and University of California, Irvine for providing me with this project and mentorship
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Getting Close with Fellow Summer Students