# Correcting Detector Efficiency Effects in Event-by-Event Net-Proton Fluctuations

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# Introduction

- Fluctuations of conserved quantities $Q_i$ (charge, baryon number, strangeness, etc.) related to QGP thermodynamic susceptibilities (aka statistical cumulants):

$$\langle \delta Q_i \delta Q_j \rangle = VT\chi_{ij}$$

$$\chi_{ij}^{(n_i,n_j)} = \frac{1}{VT} \frac{\partial^{n_i}}{\partial(\mu_i/T)^{n_i}} \frac{\partial^{n_j}}{\partial(\mu_j/T)^{n_j}} \log Z$$

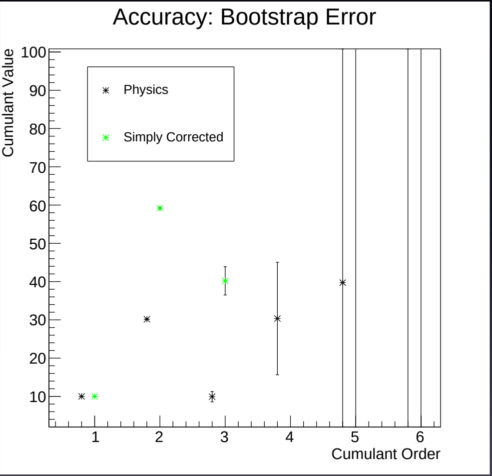
- Near the QGP critical point, susceptibilities deviate significantly from baseline
- Identifying QGP using net baryon-number susceptibilities: $\chi_\mu^{(4)}/\chi_\mu^{(2)} = 1$ for hadronic matter and 1/9 for QGP

# Main Problem

- Problem: Detector imperfections make measuring net-proton cumulants difficult
- Objective: Simulate detector efficiency effects and attempt to correct for them
- Built simple detector simulation (“toy model”)
- Applied and compared various correction methods

# Simple Correction

- Scale measured multiplicity by $\epsilon_p$ and $\epsilon_{\overline{p}}$
- Not expected to work, used as benchmark



# Koch-Bzdak Global Correction



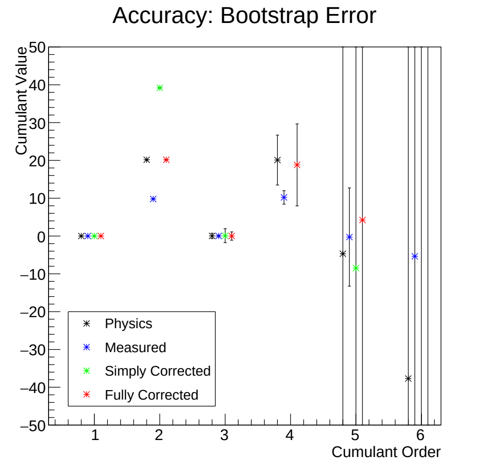
- Method for recovering cumulants developed by A. Bzdak, V. Koch, Phys. Rev. C 044904 (2012)
- Assume binomial losses $\epsilon_p, \epsilon_{\overline{p}}$ (i.e. fixed probability of not finding a track)
- True cumulants $K_n$ given in terms of measured factorial moments $f_{ik}$ of $p$ and $\overline{p}$ multiplicity distributions:

$$f_{ik} = \sum_{n_1=i}^{\infty} \sum_{n_2=k}^{\infty} p(n_1, n_2) \frac{n_1!}{(n_1 - i)!} \frac{n_2!}{(n_2 - k)!}$$

# Koch-Bzdak Global Correction

## Baseline Results

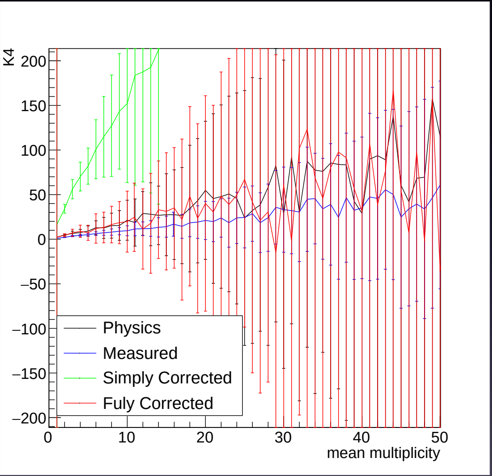
- Accurately reproduces first four cumulants, even with more realistic simulations added (track-merging error, uncertainty in efficiency)
- Uncertainty grows with cumulant order



# Koch-Bzdak Global Correction

## Multiplicity Dependence

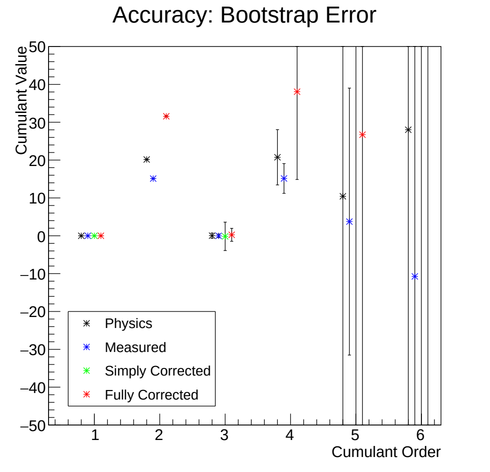
- Mean multiplicity scan for $K_4$
- Uncertainty also grows with multiplicity (small errors propagate more)



# Koch-Bzdak Global Correction

## $p_T$ Dependence

- $p_T$ dependent efficiency violates assumption of method
- Global correction is unable to return correct cumulants under $p_T$ dependent efficiency
- Another method needed



# Koch-Bzdak Local Correction



- Extends global method by introducing "local factorial moments":

$$a_{ik}(x_1, ..., x_i; \overline{x}_1, ..., \overline{x}_k) =$$
$$\langle n(x_1)[n(x_2) - \delta_{x_1,x_2}]...[n(x_i) - \delta_{x_1,x_i} - ... - \delta_{x_{i-1},x_i}]$$
$$\overline{n}(\overline{x}_1)[\overline{n}(\overline{x}_2) - \delta_{\overline{x}_1,\overline{x}_2}]...[\overline{n}(\overline{x}_k) - \delta_{\overline{x_1},\overline{x_2}} - ... - \delta_{\overline{x_{k-1}},\overline{x_k}}]\rangle$$

- Recover original factorial moments using sum over all permutations of bins:

$$f_{ik} = \sum_{\sigma} a_{ik}$$

# Koch-Bzdak Local Correction

Preliminary Results

- Only 2 multiplicity bins
- Computational intensity means normal trials are uncomputable for now

| No. | Physics: | Measured: | Simple Corrected: | Full Corrected: |
|---|---|---|---|---|
| 1 | -0.3734 | -0.3714 | -0.377695 | -0.7428 |
| 2 | 1.14477 | 1.00966 | 2.11581 | 2.76182 |
| 3 | 0.253036 | 0.160996 | 4.60072 | -4.01797 |
| 4 | -0.786494 | -0.489728 | 9.24985 | 11.4679 |
| 5 | -0.687891 | -0.143919 | -25.1198 | 44.1366 |
| 6 | 2.24385 | 0.9096 | -347.485 | -223.563 |
| 4/2 | -0.687031 | -0.485042 | 4.37178 | 4.15231 |
| 6/2 | 1.96008 | 0.900895 | -164.233 | -80.9477 |

# Ratio Correction

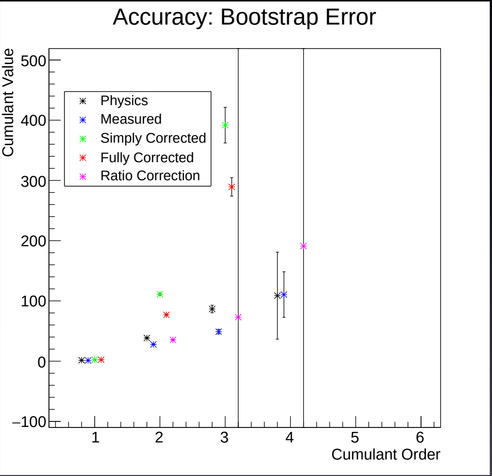
## Method

- Assume ratio between cumulants is independent of underlying distribution
- Measure ratio for Poisson statistics
- Apply ratio to non-Poisson statistics (here, artificially heavy tail)

# Ratio Correction

## Results

- Already used, due to computational simplicity
- Very wide variance, very quickly
- Maybe underlying assumptions are wrong?



# Efficiency Flattening

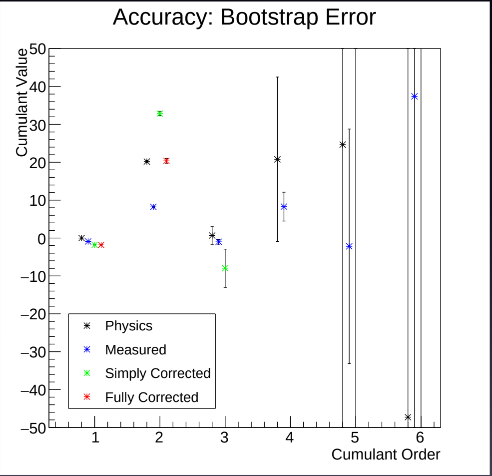
## Method

- Assume prior knowledge of form of $p_T$ dependence
- After data is taken, forcibly reduce efficiency at all $p_T$ to minimum value
- Regain binomial efficiency (and global correction) at the cost of statistics

# Efficiency Flattening

## Results

- Limited success
- Causes under investigation



# Conclusion

- Global correction is accurate for first 4 cumulants assuming no $p_T$ dependence
- Local correction may be accurate if at some point it becomes computable
- Puzzling results:
  - Ratio correction (expected to do poorly) outperformed efficiency flattening (expected to do well)
  - Possibly other assumptions on efficiency flattening, e.g. forcing of Poisson behavior

# Thank You!

# Koch-Bzdak Global Correction

- Examples:

$$K_1 = \frac{\langle n_p \rangle}{\epsilon_p} - \frac{\langle n_{\overline{p}} \rangle}{\epsilon_{\overline{p}}}$$

$$K_2 = N - K_1^2 + F_{02} - 2F_{11} + F_{20}$$

$$\begin{aligned} K_4 = N &- 6K_1^4 + F_{04} + 6F_{03} + 7F_{02} - 2F_{11} - 6F_{12} \\ &- 4F_{13} + 7F_{20} - 6F_{21} + 6F_{22} + 6F_{30} - 4F_{31} \\ &+ 4F_{40} + 12K_1^2(N + F_{02} - 2F_{11} + F_{20}) \\ &- 3(N + F_{02} - 2F_{11} + F_{20})^2 \\ &- 4K_1(K_1 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30}) \end{aligned}$$

- Where:

$$F_{ik} = \frac{f_{ik}}{\epsilon_p^i \epsilon_{\overline{p}}^k} \qquad N = \frac{\langle n_1 \rangle}{\epsilon_p} + \frac{\langle n_2 \rangle}{\epsilon_{\overline{p}}}$$