Strongly intensive observable for multiplicities in forward and backward windows in string model

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Outline

- ◇ The model. Definitions.
- ◇ Small observation windows $\delta\eta_F = \delta\eta_B \equiv \delta\eta \ll 1$
- ◇ Arbitrary pseudorapidity windows width
- ◇ Windows separated in azimuth and rapidity
- ◇ AA vs pp interactions
- ◇ PYTHIA simulation results
- ◇ Conclusions

The model with independent identical strings

1) The number of strings, N, fluctuates event by event
around some mean value, $\langle N\rangle$, with some scaled variance, $\omega_N = D_N/\langle N\rangle$.

Intensive observable does not depends on $\langle N\rangle$.
Strongly intensive observable does not depends on $\langle N\rangle$ and ω_N.

2) The fragmentation of each string contributes event-by-event to
the forward and backward observation rapidity windows, $\delta\eta_F$, and $\delta\eta_B$,
the μ_F and μ_B charged particles correspondingly,
which fluctuate around some mean values, $\langle\mu_F\rangle$ and $\langle\mu_B\rangle$,
with some scaled variances, $\omega_{\mu_F} = D_{\mu_F}/\langle\mu_F\rangle$ and $\omega_{\mu_B} = D_{\mu_B}/\langle\mu_B\rangle$.

The observation rapidity windows are separated by some rapidity interval:
η_{sep} - the distance between the centers of the $\delta\eta_F$ and $\delta\eta_B$.

For symmetric reaction and symmetric observation windows:

$$\delta\eta_F = \delta\eta_B, \qquad \langle\mu_F\rangle = \langle\mu_B\rangle, \qquad \omega_{\mu_F} = \omega_{\mu_B}$$

Two-particle correlation functions

Clear that in this model

$$\langle n_F \rangle = \langle \mu_F \rangle \langle N \rangle, \qquad \omega_{n_F} = \omega_{\mu_F} + \langle \mu_F \rangle \omega_N \ ,$$

where n_F is a number of particles produced in the interval $\delta\eta_F$ and the same for n_B.

Consider two-particle correlation functions, the observed one:

$$C_2(\eta_1, \eta_2) \equiv \frac{\rho_2(\eta_1, \eta_2)}{\rho(\eta_1)\rho(\eta_2)} - 1 \ , \tag{1}$$

where

$$\rho(\eta) = \frac{dN_{ch}}{d\eta} \ , \qquad \rho_2(\eta_1, \eta_2) = \frac{d^2 N_{ch}}{d\eta_1 \, d\eta_2} \tag{2}$$

and the two-particle correlation function $\Lambda(\eta_1, \eta_2)$, characterizing correlation between particles, produced from the same string:

$$\Lambda(\eta_1, \eta_2) \equiv \frac{\lambda_2(\eta_1, \eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 \ . \tag{3}$$

Connection between the two-particle correlation functions

In this model we have the following connection:

$$C_2(\eta_1,\eta_2) = \frac{\omega_N+\Lambda(\eta_1,\eta_2)}{\langle N\rangle}$$

[*V.V.,Nucl.Phys.A939(2015)21*]. (Note that one often looses the constant part $\omega_N/\langle N\rangle$ of C_2, obtaining C_2 by di-hadron correlation approach.)

At midrapidities, implying uniform rapidity distribution:

$$\rho(\eta) = \frac{dN_{ch}}{d\eta} = \rho_0 = \frac{\langle n_F\rangle}{\delta y_F} = \frac{\langle n_B\rangle}{\delta y_B} = \langle N\rangle\mu_0 \ , \qquad \mu_0 = \frac{\langle\mu_F\rangle}{\delta y_F} = \frac{\langle\mu_B\rangle}{\delta y_B}$$

the correlation functions depends only on a difference of rapidities:

$$\eta_{sep} = \eta_1 - \eta_2$$

Note that we use the two-particle correlation functions integrated over azimuth:

$$C_2(\eta_{sep}) = \frac{1}{\pi}\int_0^\pi C_2(\eta_{sep},\phi_{sep})\, d\phi_{sep} \ , \qquad \Lambda(\eta_{sep}) = \frac{1}{\pi}\int_0^\pi \Lambda(\eta_{sep},\phi_{sep})\, d\phi_{sep} \ .$$

C_2 through multiplicities in two small windows

For two small windows $\delta\eta_1$ and $\delta\eta_2$ around η_1 and η_2 we have

$$\rho(\eta) = \frac{\langle n \rangle}{\delta\eta} , \qquad \rho_2(\eta_1, \eta_2) = \frac{\langle n_1 n_2 \rangle}{\delta\eta_1 \, \delta\eta_2} , \tag{4}$$

$$C_2(\eta_1, \eta_2) = \frac{\langle n_1 n_2 \rangle}{\langle n_1 \rangle \langle n_2 \rangle} - 1 , \tag{5}$$

where n_1 and n_2 are the event multiplisities in these windows $\delta\eta_1$ and $\delta\eta_2$. Note that when $\eta_1 = \eta_2 = \eta$, $\eta_{sep} = 0$, we have to use

$$\rho_2(\eta, \eta) = \frac{\langle n(n-1) \rangle}{\delta\eta^2} , \qquad C_2(0) = \frac{\langle n(n-1) \rangle}{\langle n \rangle^2} - 1 = \frac{\omega_n - 1}{\langle n \rangle} , \tag{6}$$

where n is the number of particles in small window $\delta\eta$ around the point η. (see e.g. [C.Pruneau,S.Gavin,S.Voloshin,Phys.Rev.C66(2002)044904] or [V.V.,Nucl.Phys.A939(2015)21]).

Observables in small windows

For small symmetric observation windows, $\delta\eta_F = \delta\eta_B \equiv \delta\eta \ll 1$, we have [*V.V.,Nucl.Phys.A939(2015)21*] by the model independent way:

$$\omega_{\mu_F} = \omega_{\mu_B} \equiv \omega_\mu = 1 + \langle\mu\rangle\Lambda(0)\ , \qquad \omega_{n_F} = \omega_{n_B} \equiv \omega_n = 1 + \langle n\rangle C_2(0) \tag{7}$$

and

$$b_{corr} \equiv \frac{\mathrm{cov}(n_F, n_B)}{\sqrt{D_{n_F} D_{n_B}}} = \frac{\langle n\rangle}{\omega_n} C_2(\eta_{sep}) \tag{8}$$

where $\langle n\rangle = \rho_0\delta\eta$ and $\langle\mu\rangle = \mu_0\delta\eta$.

In the model: $\langle n\rangle = \langle N\rangle\langle\mu\rangle$, $\rho_0 = \langle N\rangle\mu_0$ and

$$\omega_n = 1 + \langle\mu\rangle[\omega_N + \Lambda(0)] = 1 + \mu_0\delta\eta[\omega_N + \Lambda(0)] \tag{9}$$

$$b_{corr} = \frac{\langle\mu\rangle}{\omega_n}[\omega_N + \Lambda(\eta_{sep})] = \frac{\mu_0\delta\eta}{\omega_n}[\omega_N + \Lambda(\eta_{sep})] \tag{10}$$

We see that the both observables are intensive, but not strongly intensive.

Strongly intensive observable Σ

The strongly intensive observable Σ between multiplicities in forward and backward windows is defined as
[*M.I.Gorenstein,M.Gazdzicki,Phys.Rev.C84(2011)014904*]

$$\Sigma(n_F, n_B) \equiv \frac{1}{\langle n_F\rangle + \langle n_B\rangle}[\langle n_F\rangle\omega_{n_B} + \langle n_B\rangle\omega_{n_F} - 2\,\mathrm{cov}(n_F, n_B)] \qquad (11)$$

For symmetric reaction and symmetric observation windows $\delta\eta_F{=}\delta\eta_B{=}\delta\eta$:

$$\Sigma(n_F, n_B) = \omega_n\,(1 - b_{corr}) = \omega_n - \omega_n\,b_{corr} \qquad (12)$$

Substituting the ω_n and b_{corr}, calculated in the model [V.V.,Nucl.Phys.A939(2015)21], we find

$$\boxed{\Sigma(\eta_{sep}) = 1 + \mu_0\delta\eta[\Lambda(0) - \Lambda(\eta_{sep})]}$$

We see that really the $\Sigma(\eta_{sep})$ is strongly intensive quantity.
It does not depend on $\langle N\rangle$ and ω_N.

Properties of the Σ in independent identical string model

$$\Sigma(\eta_{sep}) = 1 + \mu_0 \delta\eta[\Lambda(0) - \Lambda(\eta_{sep})]$$

The $\Sigma(0) = 1$ and increases with the gap between windows, η_{sep},
because the $\Lambda(\eta_{sep})$ decrease with η_{sep},
as the correlations in string go off with increase of η_{sep}.

The rate of the $\Sigma(\eta_{sep})$ growth with η_{sep} is proportional
to the width of the observation window $\delta\eta$ and μ_0 -
- the multiplicity produced from one string.

The model predicts saturation of the $\Sigma(\eta_{sep})$ on the level $1 + \mu_0 \delta\eta\, \Lambda(0)$
at large η_{sep}, as $\Lambda(\eta_{sep}) \to 0$ at the $\eta_{sep} \gg \eta_0$,
where the λ_0 is a correlation length in a string.

Arbitrary rapidity width of observation windows

The simple substitution:

$$\Lambda(\eta_{sep}) \to J_{FB} = \frac{1}{\delta\eta_F \delta\eta_B} \int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_B} d\eta_2 \; \Lambda(\eta_1 - \eta_2) \; ,$$

$$\Lambda(0) \to J_{FF} = \frac{1}{\delta\eta_F^2} \int_{\delta\eta_F} d\eta_1 \int_{\delta\eta_F} d\eta_2 \; \Lambda(\eta_1 - \eta_2) \; .$$

Then for $\delta\eta_F = \delta\eta_B = \delta\eta$ we have:

$$\omega_n = 1 + \mu_0 \delta\eta [\omega_N + J_{FF}] \; , \tag{13}$$

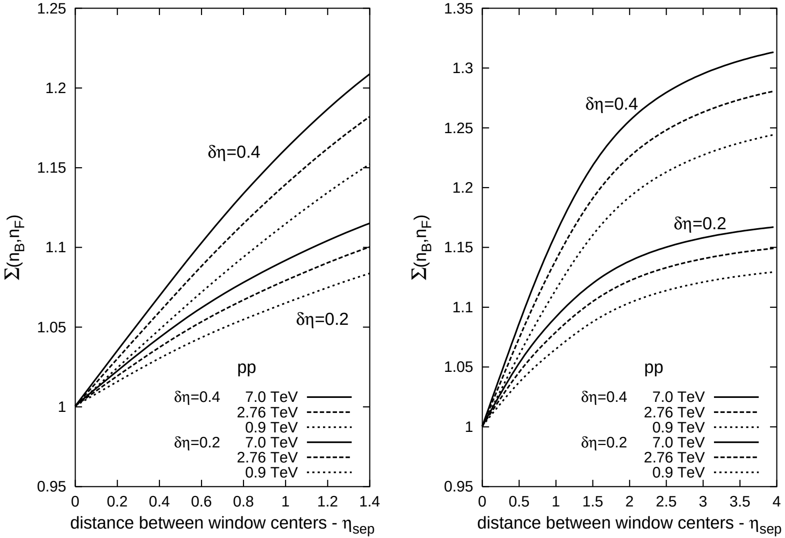
$$b_{corr} = \frac{\mu_0 \delta\eta}{\omega_n} [\omega_N + J_{FB}] \; . \tag{14}$$

$$\Sigma(\eta_{sep}) = 1 + \mu_0 \delta\eta [J_{FF} - J_{FB}] \; . \tag{15}$$

See calculation of the integrals in [V.V.,Nucl.Phys.A939(2015)21] (Appendix A).

$\Lambda(\eta_{sep}, \phi_{sep})$ was fitted by the ALICE b_{corr} pp data with FB windows of small acceptance, $\delta\eta = 0.2, \delta\phi = \pi/4$ *[ALICE collab.,JHEP 05(2015)097]*.

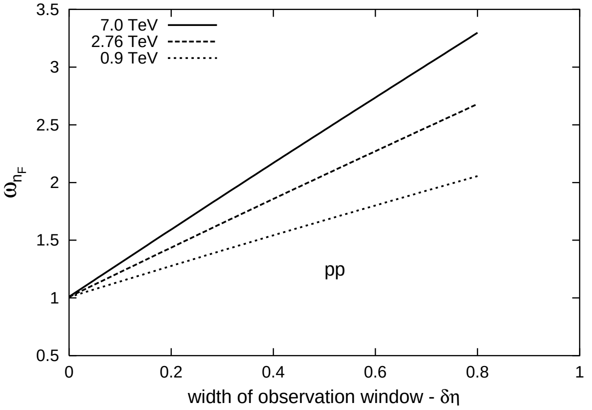
Σ for 2π azimuth windows



in ALICE TPC acceptance

in wider pseudorapidity range

The scaled variance of multiplicity in the observation window



Stronger dependence on collision energy due to dependence on the scaled variance in the number of strings - ω_N

The pair correlation function of a single string

The parametrization for the pair correlation function $\Lambda(\eta,\phi)$ of a single string (reflecting the Schwinger mechanism of a string decay, V.V.,Nucl.Phys.A939(2015)21) was used:

$$\Lambda(\eta,\phi) = \Lambda_1 e^{-\frac{|\eta|}{\eta_1}} e^{-\frac{\varphi^2}{\varphi_1^2}} + \Lambda_2 \left(e^{-\frac{|\eta-\eta_0|}{\eta_2}} + e^{-\frac{|\eta+\eta_0|}{\eta_2}} \right) e^{-\frac{(|\varphi|-\pi)^2}{\varphi_2^2}} \ . \qquad (16)$$

This formula has the nearside peak, characterizing by parameters Λ_1, η_1 and φ_1, and the awayside ridge-like structure, characterizing by parameters Λ_2, η_2, η_0 and φ_2 (two wide overlapping hills shifted by $\pm\eta_0$ in rapidity, η_0 - the mean length of a string decay segment). We imply that in formula (16)

$$|\varphi| \leq \pi \ . \qquad (17)$$

If $|\varphi| > \pi$, then we use the replacement $\varphi \to \varphi + 2\pi k$, so that (17) was fulfilled. With such completions the $\Lambda(\eta,\phi)$ meets the following properties

$$\Lambda(-\eta,\phi) = \Lambda(\eta,\phi) \ , \quad \Lambda(\eta;-\phi) = \Lambda(\eta,\phi) \ , \quad \Lambda(\eta,\phi+2\pi k) = \Lambda(\eta,\phi) \qquad (18)$$

Fitting the model parameters by FBC in small windows

Results of the fitting of the model parameters by FB correlations between two small windows, separated in azimuth and rapidity:

$\sqrt{s}$, TeV		0.9	2.76	7.0
LRC	$\mu_0\omega_N$	0.7	1.4	2.1
SRC	$\mu_0\Lambda_1$	1.5	1.9	2.3
	η_1	0.75	0.75	0.75
	ϕ_1	1.2	1.15	1.1
	$\mu_0\Lambda_2$	0.4	0.4	0.4
	η_2	2.0	2.0	2.0
	ϕ_2	1.7	1.7	1.7
	η_0	0.9	0.9	0.9

$\omega_N = \frac{\langle N^2\rangle - \langle N\rangle^2}{\langle N\rangle}$ is the e-by-e scaled variance of the number of strings,
μ_0 is the average rapidity density of the charged particles from one string,
$i=1$ corresponds to the nearside and $i=2$ to the awayside contributions,
η_0 is the mean length of a string decay segment.
V.V.,Nucl.Phys.A939(2015)21,
ALICE collab.,JHEP 05(2015)097

Windows separated in azimuth and rapidity

The two-particle correlation function C_2 is defined through the inclusive ρ_1 and double inclusive ρ_2 distributions:

$$C_2(\eta_F,\phi_F;\eta_B,\phi_B) = \frac{\rho_2(\eta_F,\phi_F;\eta_B,\phi_B)}{\rho_1(\eta_F,\phi_F)\rho_1(\eta_B,\phi_B)} - 1 \tag{19}$$

$$\rho_1(\eta,\phi) = \frac{d^2N}{d\eta\, d\phi}\ , \qquad \rho_2(\eta_F,\phi_F;\eta_B,\phi_B) = \frac{d^4N}{d\eta_F\, d\phi_F\, d\eta_B\, d\phi_B} \tag{20}$$

For a small window $\delta\eta\,\delta\phi$ around η, ϕ we have

$$\rho_1(\eta,\phi) \equiv \frac{\langle n\rangle}{\delta\eta\,\delta\phi}\ , \tag{21}$$

here $\langle n\rangle$ is the mean multiplicity in the acceptance $\delta\eta\,\delta\phi$.

Connection of the FB correlation coefficient with two-particle correlation function -1

For two small windows: $\delta\eta_F\,\delta\phi_F$ around η_F, ϕ_F and $\delta\eta_B\,\delta\phi_B$ around η_B, ϕ_B we have

$$\rho_2(\eta_F,\phi_F;\eta_B,\phi_B) \equiv \frac{\langle n_F n_B\rangle}{\delta\eta_F\,\delta\phi_F\,\delta\eta_B\,\delta\phi_B}\ . \qquad (22)$$

The formulae (21) and (22) are the base for the experimental measurement of the one- and two-particle densities of charge particles ρ_1 and ρ_2, and hence of the two-particle correlation function C_2 (19), for which by (21) and (22) we have:

$$C_2(\eta_F,\phi_F;\eta_B,\phi_B) \equiv \frac{\langle n_F n_B\rangle - \langle n_F\rangle\langle n_B\rangle}{\langle n_F\rangle\langle n_B\rangle}\ . \qquad (23)$$

where n_F and n_B are the event multiplicities in these two small windows.

Connection of the FB correlation coefficient with two-particle correlation function - 2

Again for small FB windows we have

$$b_{corr} = \frac{\langle n_F \rangle}{\omega_{n_F}} C_2(\eta_F, \phi_F; \eta_B, \phi_B) \qquad (24)$$

and

$$\omega_{n_F} = 1 + \langle n_F \rangle C_2(\eta_F, \phi_F; \eta_F, \phi_F) \ , \qquad (25)$$

so

$$b_{corr} = \frac{\langle n_F \rangle}{1 + \langle n_F \rangle C_2(\eta_F, \phi_F; \eta_F, \phi_F)} C_2(\eta_F, \phi_F; \eta_B, \phi_B) \qquad (26)$$

We see that the traditional definition of the FB correlation coefficient in the case of small observation windows coincides with the standard definition of two-particle correlation function C_2 upto some common factor, which depends on the width of the windows.

Connection of the FB correlation coefficient with two-particle correlation function - 3

In the central rapidity region due to translation invariance:

$$C_2(\eta_F, \phi_F; \eta_B, \phi_B) = C_2(\eta_{sep}, \phi_{sep}) \tag{27}$$

and for small windows we have

$$\omega_{n_F} = 1 + \langle n_F \rangle C_2(0,0) \ , \tag{28}$$

$$b_{rel} = \frac{\langle n_F \rangle}{1 + \langle n_F \rangle C_2(0,0)} C_2(\eta_{sep}, \phi_{sep}) \tag{29}$$

Up to this point, a model independent consideration.

Model with independent identical strings

In the model with independent identical strings we have

$$C_2(\eta_F, \eta_B; \phi_F - \phi_B) = \frac{\omega_N + \Lambda(\eta_F, \eta_B; \phi_F - \phi_B)}{\langle N \rangle} ,$$

where ω_N is the event-by-event scaled variance $\omega_N = D_N / \langle N \rangle$ of the number of emitters and

$$\Lambda(\eta_F, \eta_B; \phi_F - \phi_B) = \frac{\lambda_2(\eta_F, \eta_B; \phi_F - \phi_B)}{\lambda_1(\eta_F)\lambda_1(\eta_B)} - 1 \qquad (30)$$

is the two-particle correlation function for charged particles produced from a decay of a single emitter (string).
In the central rapidity region it takes the following simple form:

$$C_2(\eta_{sep}, \phi_{sep}) = \frac{\omega_N + \Lambda(\eta_{sep}, \phi_{sep})}{\langle N \rangle}$$

Σ for $\delta\eta\ \delta\phi$ windows

For small windows:

$$\Sigma(\eta_{sep},\phi_{sep}) = 1 + \frac{\delta\eta\,\delta\phi}{2\pi}\mu_0\,[\Lambda(0,0) - \Lambda(\eta_{sep},\phi_{sep})]$$

For arbitrary width of $\delta\eta\ \delta\phi$ windows, again the simple substitution:

$$\Lambda(\eta_{sep},\phi_{sep}) \to J_{FB}(\eta_{sep},\phi_{sep})\ , \qquad \Lambda(0,0) \to J_{FF}\ ,$$

$$J_{FB}(\eta_{sep},\phi_{sep}) = \frac{1}{\delta y_F \delta\varphi_F \delta y_B \delta\varphi_B}\int_{\delta y_F \delta\varphi_F} dy_1 d\varphi_1 \int_{\delta y_B \delta\varphi_B} dy_2 d\varphi_2\, \Lambda(\eta_1-\eta_2;\phi_1-\phi_2)\ , \tag{31}$$

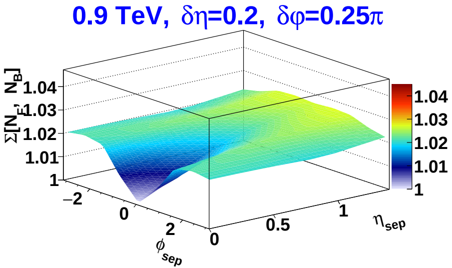
$$J_{FF} = \frac{1}{(\delta y_F \delta\varphi_F)^2}\int_{\delta y_F \delta\varphi_F} dy_1 d\varphi_1 \int_{\delta y_F \delta\varphi_F} dy_2 d\varphi_2\, \Lambda(\eta_1-\eta_2;\phi_1-\phi_2)\ , \tag{32}$$

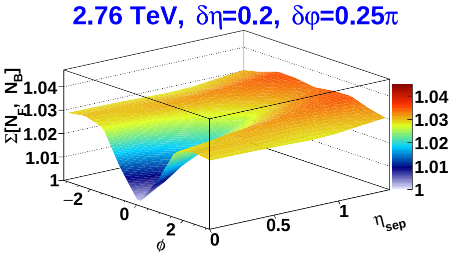
$\Lambda(\eta;\phi)$ is the pair correlation function for a single string, Then

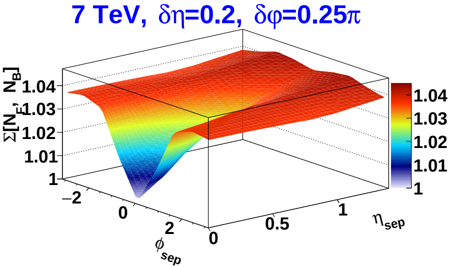
$$\Sigma(\eta_{sep},\phi_{sep}) = 1 + \frac{\delta\eta\,\delta\phi}{2\pi}\mu_0\,[J_{FF} - J_{FB}(\eta_{sep},\phi_{sep})]\ . \tag{33}$$

See calculation of the integrals in [V.V.,Nucl.Phys.A939(2015)21] (Appendix A).

Σ for $\delta\eta\ \delta\phi$ windows separated in azimuth and rapidity - 1

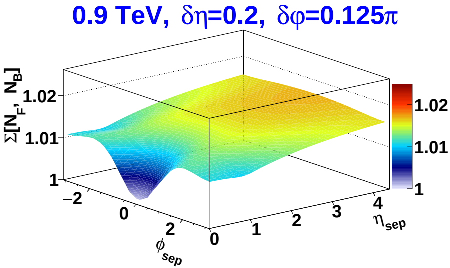


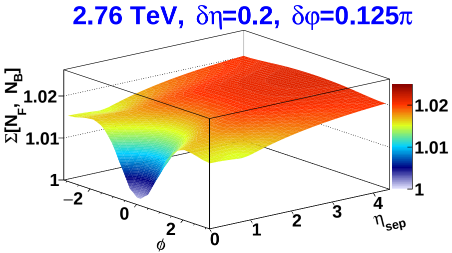


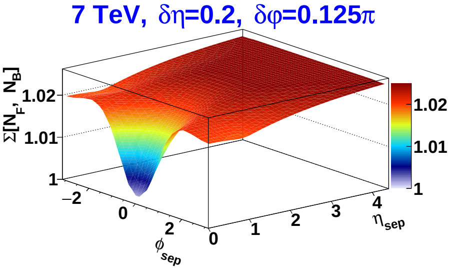


in ALICE TPC acceptance (ALICE experimental pp data analysis is in progress)

Σ for $\delta\eta$ $\delta\phi$ windows separated in azimuth and rapidity - 2







with $\pi/8$ azimuth windows and for wider pseudorapidity range

AA vs pp interactions - 1

The same value of $\Sigma(n_F, n_B)$ in AA collisions, as in pp,
if we suppose the formation of the same strings in AA and pp collisions.
Because the $\Sigma(n_F, n_B)$ does not depends on the mean value, $\langle N \rangle$,
and the event-by-event fluctuations, ω_N, in the number of strings.
It depends only on string properties.

If we suppose the formation of new strings in AA collisions
(and may be in central pp collisions at high energy)
with some new characteristics, compared to pp collisions,
due to e.g. string fusion processes, then

$$\Sigma_{AA}(\eta_{sep}) = 1 + \mu_0^{AA} \delta\eta [\Lambda_{AA}(0) - \Lambda_{AA}(\eta_{sep})]$$

For the fused strings, forming in AA collisions, we will have
1) larger multiplicity from one string, $\mu_0^{AA} > \mu_0$,
2) smaller correlation length, $\lambda_0^{AA} < \lambda_0$.

AA vs pp interactions - 2

This corresponds to the analysis of the net-charge fluctuations in the framework of the string model for pp and AA collisions *[A.Titov, V.V., PoS(Baldin ISHEPP XXI)047(2012)]*.

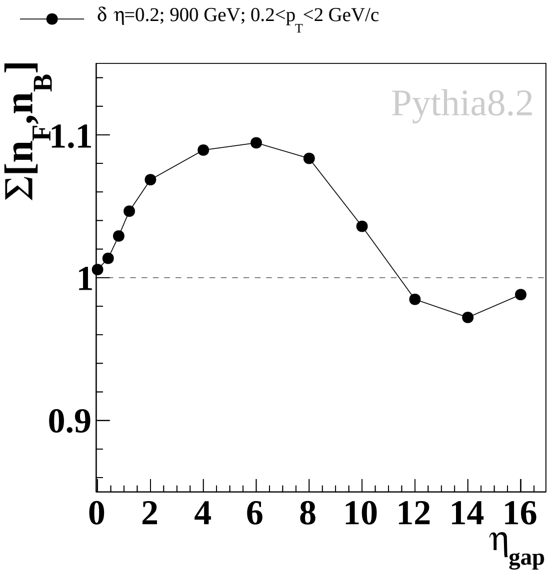
$$\Sigma_{AA}(\eta_{sep}) = 1 + \mu_0^{AA}\delta\eta[\Lambda_{AA}(0) - \Lambda_{AA}(\eta_{sep})]$$

Both factors lead to the steeper increase of $\Sigma_{AA}(\eta_{sep})$ with η_{sep} in the case of AA collisions, compared to pp.

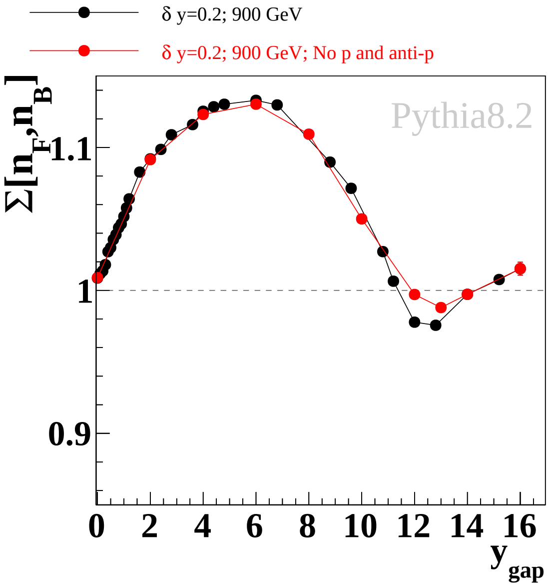
Qualitatively, this is in accordance with the experimental data (see *I. Sputowska, Correlations in Particle Production in Nuclear Collisions at LHC Energies, Ph.D. thesis, Institute of Nuclear Physics PAN, Cracow, Poland, Cern Document Server (2016)*).

In reality - a mixture of fused and single strings.
(See e.g. *[E.V.Andronov,Theor.Math.Phys.185(2015)1383]*)
It requires further theoretical studies (todo).

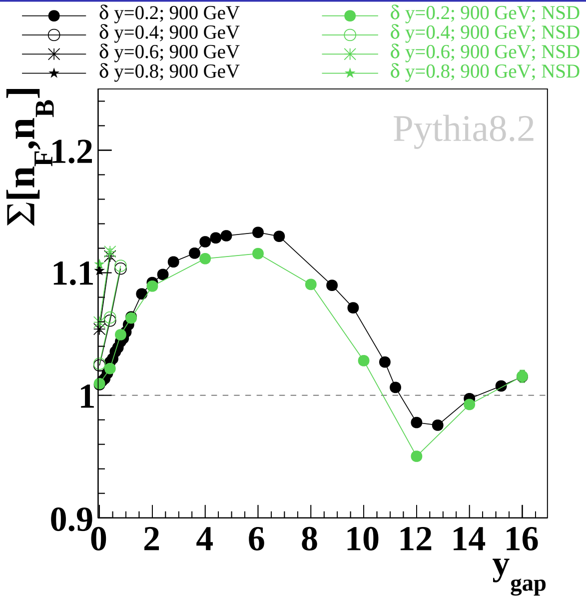
PYTHIA 8.2 simulations



PYTHIA 8.2 simulations - without $p\bar{p}$



PYTHIA 8.2 simulations - NSD



Conclusions

The model with independent identical strings [V.V.,Nucl.Phys.A939(2015)21] explains well the dependence of the strongly intensive observable $\Sigma(n_F, n_B)$ on the distance between forward and backward windows, η_{sep}, and the width of windows, $\delta\eta$, for pp collisions.

It predicts the same value of $\Sigma(n_F, n_B)$ for AA collisions, as in pp, if we suppose the formation of the same strings in AA and pp collisions.

The deviation of $\Sigma(n_F, n_B)$ in AA collisions from the pp case, is the signature of formation of strings with new characteristics, e.g. due to string fusion processes.

Backup slides

Backup slides

Connection of the FB correlation analysis with two-particle correlation function C_2

A. Capella and A. Krzywicki, Phys.Rev.D**18**, 4120 (1978):

$$b_{nn} = \frac{\mathrm{cov}(n_F, n_B)}{D_{n_F}} = \frac{\frac{\langle n_B \rangle}{\delta y_F{}^2} \int_{\delta y_F} d\eta_1 \int_{\delta y_B} d\eta_2 \, C_2(\eta_1 - \eta_2)}{1 + \frac{\langle n_F \rangle}{\delta y_F{}^2} \int_{\delta y_F} d\eta_1 \int_{\delta y_F} d\eta_2 \, C_2(\eta_1 - \eta_2)} \qquad (34)$$

$$\omega_{n_F} = \frac{D_{n_F}}{\langle n_F \rangle} = 1 + \frac{\langle n_F \rangle}{\delta y_F{}^2} \int_{\delta y_F} d\eta_1 \int_{\delta y_F} d\eta_2 \, C_2(\eta_1 - \eta_2) \qquad (35)$$