CP & Particle Correlations under Thermal Stochastic Influence

/Random Fluctuating Walk to CP/

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CP & Critical Phenomena

Collider ——— ***Fixed target***

strong interacting matter @ high T & μ_B

In the proximity of *CP*:

- **Matter becomes weakly coupled**
- **Color is no more confined**
- **Chiral symmetry is restored**

➢ **Phase transition is associated with breaking of symmetry**

Instructive: ***CP* clarified through $(\mu_B - T)$ plane**

scanning of $(\mu_B - T)$ phase diagram

CP & Critical Phenomena

A few questions arise:

- ➢ ***CP* meaning?**
- ➢ **Basic observables to be measured when *CP* achieved?**
- ➢ **New knowledge if *CP* approached?**

Answer: in terms of QCD_T @ large distances

N/Perturbative phenomena: χSB & Confinement of color

$\downarrow$ *?relations?* $\downarrow$

Phase transition of χS Restoration Deconfinement

$\downarrow$ **correlations** $\downarrow$

important issue

NO correct solution (massless quarks in the theory)

Effective models, e.g., with topological defects

Phase transitions ⇔ Topological defects (TD's)

TD's exist only in phase with SSB where $\langle \phi \rangle_{vacuum}$ emerges

Non-broken symmetry phase: ***no solutions relevant to TD's***

Minimal model: TD's (strings) arise in Abelian Higgs-like model

(Nielsen, Olesen, 1973)

$$SU(N) \xrightarrow{reduction} \left[U(1)\right]^{N-1}$$ **dual scalar thery**

gauge symmetry breaking ↓ Higgs-like mechanism

- **MA Gauge suggests special properties of QCD vacuum**
  Abelian dominance
- **Condensation of scalar d.o.f.** **(Ezawa, Iwasaki, 1982)**
  which provides
- **Dual superconductor picture of QCD vacuum** **('t Hooft 1981)**

Effective model

CP • **Fluctuation measure** • **Observables**

may be visible ↓ **through**

Fluctuations of characteristic length ξ **of chiral end mode**

Model: effective dual approach to QCD.

Fluctuations based on the order parameter $m \sim \xi^{-1}$

- **Deal with gauge-invariant quantities, TPCF as a function of** $C_{\mu}(x)$
- **Dual color string:** $U_C(x,y) \sim exp\left[ig\int_y^x dz^{\mu} C_{\mu}(z)\right]$, C_{μ}^{a} *dual to* A_{μ}^{a}
- ***Particles:*** **Bound states in terms of flux tubes**

Flux tubes

Excitations above vacuum: narrow flux tubes, $r_s \sim m^{-1}$

(in the center, $r_s \to 0$, scalar condensate vanishes)

Ensemble of a single flux tube system, $N(R)$ configurations of f.t.'s

$$Z_{flux} = \sum_{\beta} \sum_{R} N(R) \exp\left[-\beta\, E(m,R)\right] D\left(|\vec{x}|, \beta; M\right)$$

effective energy: $E(m,R) \sim m^2 R\left[a + b\ln\left(\tilde{\mu}R\right)\right]$ ***GK, 2010***

Dual gauge field C_μ - critical end mode!

$$m^2(\beta) \sim g^2(\beta)\delta^{(2)}(0)$$

$$\downarrow$$

$$c/\left(\pi r_s^2\right), \quad c \sim O(1)$$

TPCF

At large distances for any correlator (observables)

$$\langle O(\tau,\vec{x})O(\tau,0)\rangle \sim A|\vec{x}|^{c} D(|\vec{x}|,\beta;M) \text{ as } |\vec{x}|\to\infty$$

$$D(|\vec{x}|,\beta;M) = \exp[-M(\beta)|\vec{x}|], \quad D(|\vec{x}|,\beta;M) \neq 0 \text{ even at } \beta = \beta_c$$

$M^{-1}(\beta)$ is the measure of screening effect of color electric field

For $SU(N=2,3)$ **, high** T **,** N_f **massless,** $\mu = 0$

$$M(\beta) = M^{LO}(\beta) + N\alpha T \ln\left(\frac{M^{LO}(\beta)}{4\pi\alpha T}\right) + 4\pi\alpha T y_{n/p}(N) + O(\alpha^2 T)$$

$$M^{LO}(\beta) = \sqrt{4\pi\alpha\left(\frac{N}{3}+\frac{N_f}{6}\right)}\, T$$

Kajante et al. 1997

$$\langle O(\tau,\vec{x})O(\tau,0)\rangle \sim L_W^{-4} - \frac{T}{V}\sigma_0(\beta)\xi^2\left[\frac{1}{\xi}\sqrt{\frac{8\pi}{\sigma_0}} - N\ln\left(\xi\sqrt{2\pi\sigma_0}\right) + \ldots\right]!$$

GK, 2014

Result: **we find that eff. theory in terms of non-perturbative TPCF describes the fluctuations at distances** $gx / \sqrt{p} < |\vec{x}| < M^{-1}$

$$s_0(b) \sim m^2(b)a(b)$$

GK 2010

Flux-tube scheme:

- $x \sim m^{-1}$ the penetration length of color-electric field
- $x \sim r_s$ “string”-like radius
- $l \sim m_f^{-1}$ coherent length of scalar (dilaton) condensate
- $t = \sqrt{4/(3a)}\,x$ formation time of flux tube ($\rightarrow \infty$ @ CEP)

➢ For SU(3), $m » 1.95\sqrt{s_0}$ Baker et al., 1997

✓ **Lattice:** $T_c » 0.65\sqrt{s_0}$

Effective theory applicable in deconfined phase $T_c < T < 3T_c$ **!**

Dual QCD vacuum.

In $SU(3)$ gluodynamics vacuum is characterized

$$k_{GL} = \frac{\xi}{l} \sim \frac{m_\phi}{m} \begin{array}{ll} <1 & (type\ I\ vacuum,\ \text{flux tubes}\ attracted) \\ >1 & (type\ II\ vacuum,\ \text{flux tubes}\ repel) \end{array}$$

Dilatons remain massive up to the CP (1st order PT)

$k_{GL} \to \infty$ **Deconfinement!**

If $k_{GL} = 1$ parallel strings (carry the same flux) do not interact each other. $T_c \approx 172\ MeV,\ N_c = 3\ pions$

GK 2014

Singularity of $Z_{flux} \Rightarrow$ $$k_{GL} \geq \frac{3}{4}\frac{\alpha(\beta)}{\xi m_{q\bar{q}}}\left[1 + \frac{4}{3}\frac{\xi^2}{\alpha(\beta)\beta} M(\beta)\frac{L_W}{R}\right]$$

Observation of correlations between two bound states (strings) is rather useful & instructive to check the *CP* is approached!

Field theory $\Rightarrow$ RG $\Rightarrow$ Critical Behavior

Phase transitions $\Leftarrow$ presence and the properties of ***fixed points***

IR attracted Fixed Points $\left\{ \begin{array}{c} Phase\ \ Transitions\ \ of\ \ 2nd\ \ kind \\ Critical\ \ scaling \end{array} \right.$

RG Fluxes (solutions) may leave physical domain containing **IRFP** (even to ∞)

$\Downarrow$

Phase transition of the 1^{st} *kind*

Sample with production $\pi^+\pi^+,\ \pi^-\pi^-,\ \pi^0\pi^0$

$$AA\ (pp) \rightarrow high\ T\ quark\text{-}gluon\ bubble \rightarrow hadronization \rightarrow$$
$$\rightarrow chaotic\ pion's\ production\ with\ different\ directions,\ momenta,$$
$$angles$$

Bose-Einstein Correlations @ finite *T*

Def.:

BEC's are the quantum effect which enhances the probability that multiple bosons be found in the same state, same position, same momentum

What's happened once the critical *T is approached and above*

- **Shape of correlation behavior?**
- **Correlation radius size?**
- **Other characteristics to be measured?**

Size of the particle source

- Possible approach to ***CP*** study through spatial correlations of final state particles
- Size effect of space composed of “hot” particles **!** derive theoretical formulas for $2!,...,N!$ particle ***distribution-correlation functions*** (stochastic, chaotic behavior)

Stochastic scale (size) in C’s Bose-Einstein GK (2008-2010)

$$C_2(q,!)! \; !(N)\left[1+!(")e^{" q^2 L_{st}^2}\right], \qquad !(N)=\frac{\langle N(N!1)\rangle}{\langle N\rangle^2}$$

event-to-event fluctuations

Chaoticity function:

$$!(")=1/(1+")^2, \; 0<"<\# \; , \; N\sim V"d! \; m^2\frac{1}{e^{(!\,!\,m)/!}\,!\,1}$$

$$b(x)=a(x)+R(x), \; R(p_m)=\sqrt{!\;"(p,p)}, \; " =\langle a^+(p)a(p)\rangle$$ GK’98-02

Bose-Einstein Correlation

- In case of **no** ***CP*** approached, **the theory admitted** the signal is observed as an enhancement of pairs of same-sign charge particles with small relative momenta ***GK 2008***

$$C_2\left(q,\beta=T^{-1}\right)=\eta\left(N\right)\left\{1+\tilde{\lambda}\left(\beta\right)e^{-q^2L_{st}^2}\left[1+\lambda_1\left(\beta\right)e^{q^2L_{st}^2/2}\right]\right\}$$

- When ***CP*** approached:
  - **NO** signal of enhancement of pairs of same-sign charge particles is observed

! C_2 -function does not deviate from 1

$$L_{st}\to\infty\ \ as\ T\to T_c,\ \eta\left(N\right)\to 1,\ \mathrm{N}\to\infty$$

OBSERVABLES?

The scaling form C_2 is useful to predict behavior of observables @ **CP**

$L_{st} \to \infty$ *as* $T \to T_c$, $\mu \to \mu_c$ indicate the vicinity of **CP**

- Observable, e.g., $k_T^2 = \frac{1}{\nu(N)\, T^3\, L_{st}^5}$, $k_T = \left| \vec{p}_{T_1} + \vec{p}_{T_2} \right|$ GK

Experiment: **L3/CMS/ATLAS/ALICE** L **decreasing (smooth) with** k_T

- Chaoticity λ measured. *(Most important theor. study)*

$$0 < \lambda\left[\nu(N)\right] \leq 1$$

$\Downarrow$ $\Downarrow$

fully coherent phase chaotic *(critical behavior from BM to AM)*

Strength of BE correlations $\tilde{\lambda}(k_T,\beta)$ for incoherent particles emitted from independent sources

$$C_2(q,\beta) \approx \eta(N)\left\{1+\tilde{\lambda}(\beta)e^{-q^2L_{st}^2}\left[1+\lambda_1(\beta)e^{+q^2L_{st}^2/2}\right]\right\}$$

$$\tilde{\lambda}(\beta) = \frac{\gamma(\omega,\beta)}{\left[1+\nu(N)\right]^2}, \quad \nu \sim \frac{1}{n}\frac{1}{k_{GL}^2}, \text{ as } T \to T_c, \quad \gamma(\omega,\beta) \sim O(1)$$

Measure of the *CP*: fluctuation length $\xi \sim m^{-1}$ (of the "dual" gauge field)

Proposal: *GK 2009-2014*

- ✓ $\tilde{\lambda}(k_T,\beta)$ decreases with k_T far away from the *CP*, CMS (2011)

  ATLAS (2015)
- ✓ $\tilde{\lambda}(k_T,\beta) \to 0$ **as *CP* approached,** $k_{GL} \to \infty$ ***DECONFINEMENT***

Origin: infinite fluctuation length $\xi \to \infty$

Chiral restoration & Particle emission size

Theory: $L_{st} = L_{st}\left(\beta, k_T, m, \nu(N)!\right) \sim \dfrac{1}{\nu^{1/5}(N)\, m_h^{\alpha} T^{\gamma}}$ ***GK, 2009-2010***

$$\nu(N) = \frac{2-\tilde{C}_2(0)+\sqrt{2-\tilde{C}_2(0)}}{\tilde{C}_2(0)-1}, \qquad \tilde{C}_2(0) = \frac{C_2(q=0)}{\dfrac{\langle N^2\rangle}{\langle N\rangle^2} - \dfrac{1}{\langle N\rangle}}$$

$$\langle N\rangle \geq 1 + C_2(0)/2, \quad C_2(0) \leq 2$$

CMS (2011): $\sqrt{s}$=0.9 TeV; 7 TeV,- L_{st} increases with $\langle N\rangle$

ATLAS (2015): $\sqrt{s}$=0.9 TeV; 7 TeV,- L_{st} increases with $\langle N\rangle$ as well

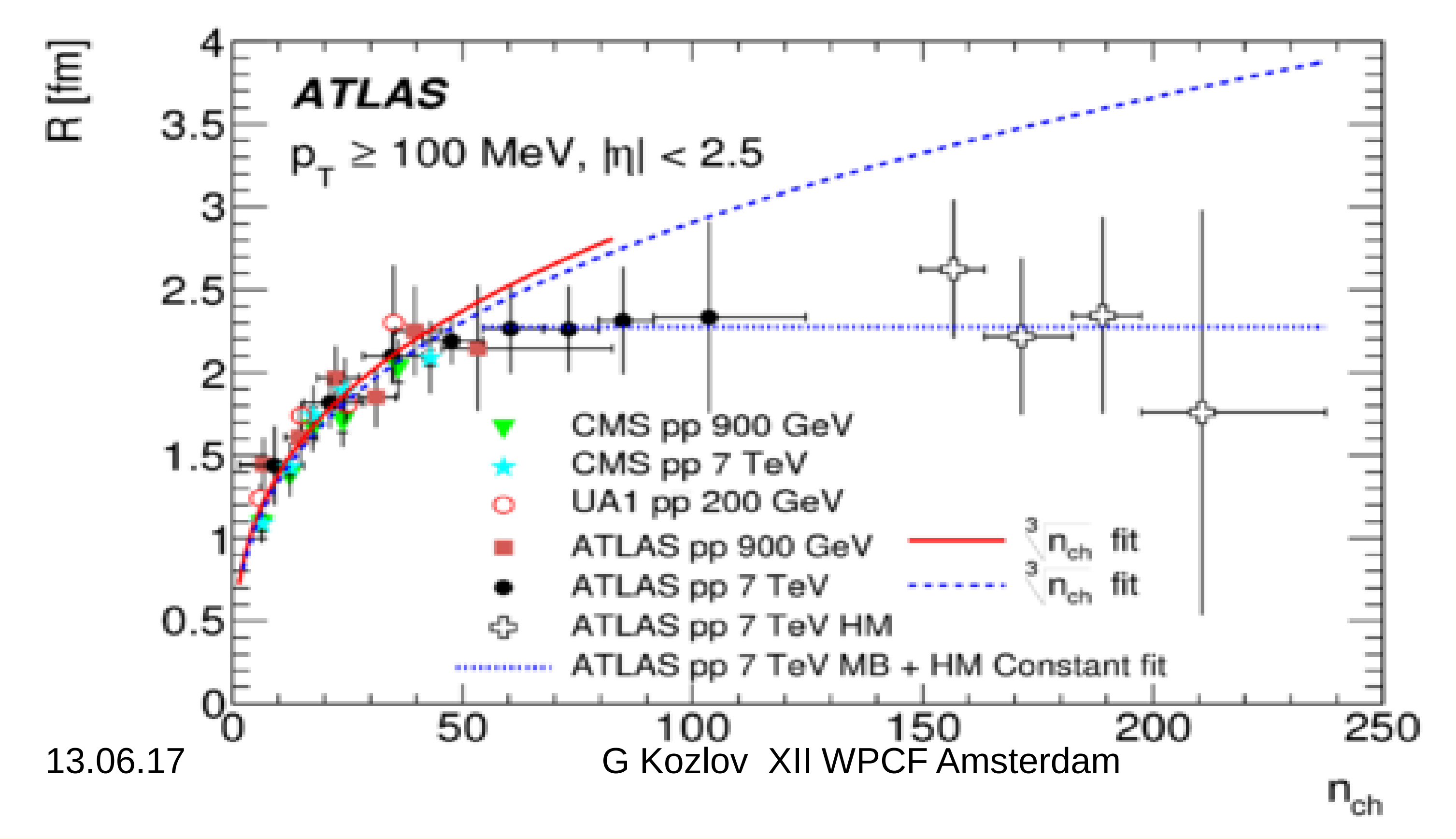
High $T \to T_c$: $\quad L_{st} \sim \left[\nu^{\delta}(N)\, m_h^{\alpha}\ T^{\gamma}\right]^{-1} \to \infty \ as\ m_h \to 0 \quad \chi SR$

Charactersitic size of the Correlation source

In terms of Ginzburg-Landau criterium k_{GL}

«Radius» increases with $n \sim n_{ch}$ $R \sim L_{st} \sim \left(\frac{n\, k_{GL}^2}{k_T^2\, T^3} \right)^{1/5}$ *GK (2009)*

ATLAS Coll., Eur. Phys. J.C75 (2015) 466



➢ Expansion of particle emission size

$$L_{st}(\beta) \sim \left[\nu(N)\, k_T^2 \; T^3\right]^{-1/5} \to \infty \;\; \textbf{\textit{as}}\; \nu(N) \to 0 \text{ at } T \to T_c$$ ***KG 2010***

The temperature at which the signal of two-particles correlations disappears is the critical temperature at *CP* : $C_2(q, T_c) = 1$

! Too rapid phase transition can include the ***explosion*** of a "hadronic fireball" just after a phase transition

➢ *Dip-effect*

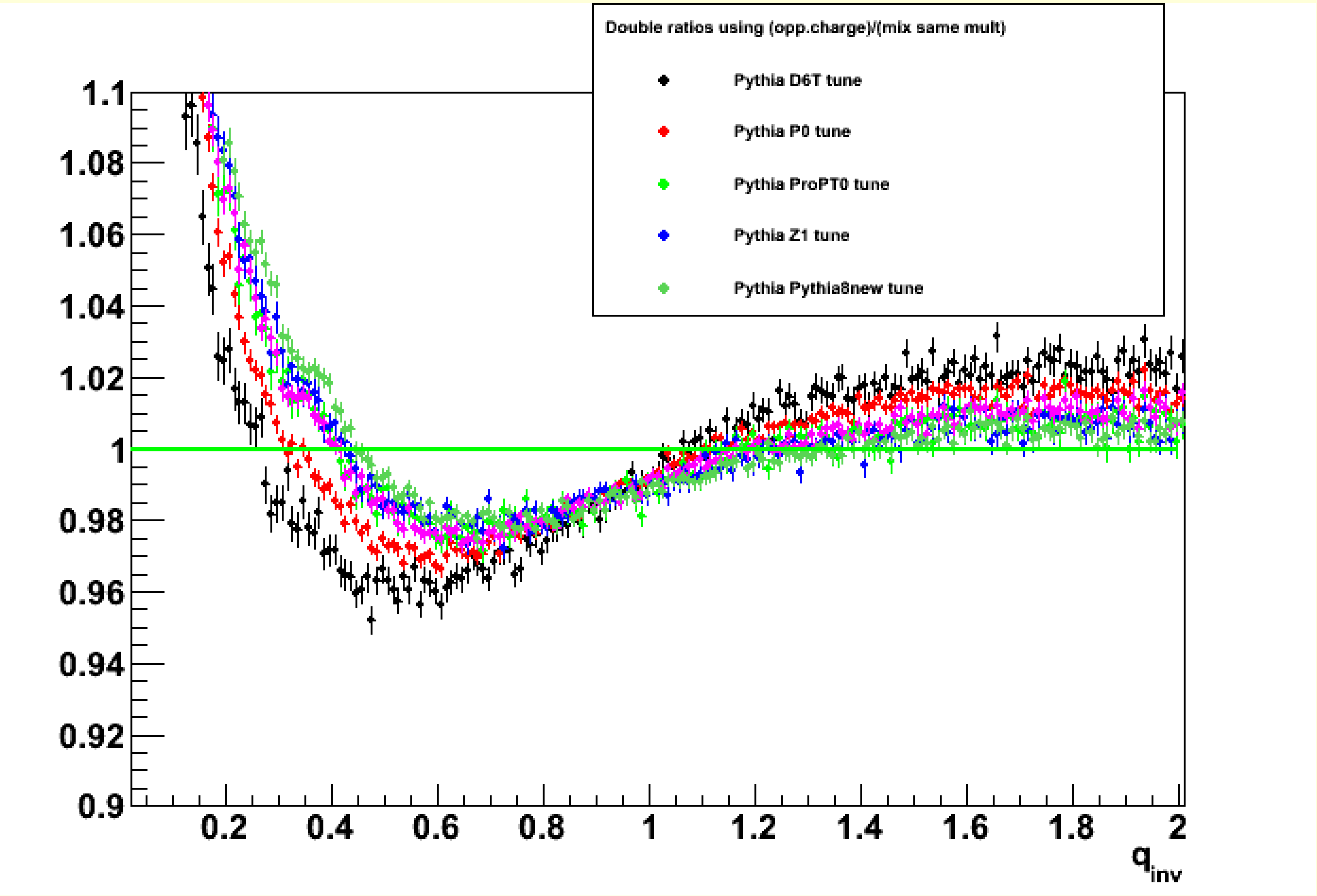
The effect of anti-correlations (the dip-effect) is predicted at low charged-particle multiplicity N **in the event:** $C_2(\{q\}, N) < 1$ **?!** ***KG 2010***

The depth of the dip in the anti-correlation region decreases as N **increases.**

Observed by CMS at LHC [CMS Coll., JHEP 5 (2011) 029]

Proposal: dip-effect disappears at *CP*

Dip-effect @ 7 TeV CMS (2011)



Random Fluctuation Walk *(BM ⇒ AM)*

Random stochastic (chaotic) walk with respect to quantum correlations of identical particles. Cross-over walk.

Model 1D x-oriented $(-\infty < x < \infty)$

$$P(x;\bar{\Gamma},\mu_c) = p\sum_{j=0}^{\infty}\bar{\Gamma}^j \frac{1}{2}\sqrt{\frac{\pi}{t}}\left[e^{-y_-^{2j}/4t} + e^{-y_+^{2j}/4t}\right]$$

$$y_{\pm}^{j} = x\mu_c \pm a^j,\ a = (\mu/\mu_c) > 1,\ t = l\,\mu_c \text{ (lattice spacing)}$$

$$\lim_{\substack{l\to 0\\ \mu_c\neq 0}} P(x;\bar{\Gamma},\mu_c) = p(\bar{\Gamma})\sum_{j=0}^{\infty}\bar{\Gamma}^j \pi\left[\delta(x\mu_c - a^j) + \delta(x\mu_c + a^j)\right]$$

$$p(\bar{\Gamma}) = \frac{1}{2\pi}(1-\bar{\Gamma}),\ 0 < \bar{\Gamma} \le 1,$$ NC: $2\left(p + \Gamma p + \ldots + \Gamma^j p + \ldots\right) = 1$

The limit $\Gamma \to 1 \Rightarrow$ broad behavior of *P: vicinity of **CP** is approached*

$\bar{\Gamma} \to 0 \ \Rightarrow\ P(x;\bar{\Gamma}\to 0,\mu_c) \to 1/(2\pi)$ trivial

Random fluctuation weight $\bar{!}$

Vicinity of **CP**: theory conformal, scalar dilaton field !

$$\bar{!}\;''\;\;!\left(\#\right)=\left[1+\#\left(N\right)\right]^{2}$$ stochastic (external) influence strength

$$/\left(N\right)\;!\;\frac{1}{n\,k_{GL}^{2}}\,O\left(\frac{m_{/}^{2}}{\# m^{2}}\right)$$

Dual Higgs-Abelian gauge model (A_{m} ! B_{m})

dual superconductor QCD vacuum

$$k_{GL}\sim\frac{m_{!}}{m_{B}}\begin{cases}<1 & \textit{vacuum type" I, two flux tubes attracted}\\ >1 & \textit{vacuum type" II, two flux tubes repel}\end{cases}$$

CP: k_{GL} ! " as $/\sim m_{B}^{!1}$ " # fluctuation length/ penetration depth of color-electric field/radius of the flux tube

Analyticity of probability $P(x;!,m_c)$

Large x (sharp increasing of L_{st}) / or k ! 0 (IR analogue)

To smooth the particularity (speciality) of $P(x;!,m_c)$

$$P(x;!,m_c)'' \quad G(k;!,m_c) = p(!)\overset{\infty}{\underset{j=0}{+}} !^{j}\, \mathbf{cos}\, \#\%\frac{k}{m_c} a^{j} \&(,\quad G(0;!) = \frac{1}{2)}$$

Fluctuation length through the even moments of the order $2s$:

$$!^{2}_{(2s)}('') \sim m_{(2s)}('') = \frac{\#^{2s} G(k;'',m_c)}{\#k^{2s}}\Big|_{k=0}$$

- ➢ Finite ! will provide analytical form of G, however large ! '' non-analytical behavior of G @ k ! 0

The dual QCD vacuum will influence (through k_{GL}) ! up to cross-over: unified process of phase transition between BM and AM

Analyticity of probability $P(x;!,\mathrm{m}_c)$ cont'd I

$G(k;!,\mathrm{m}_c)$ in terms of CBE function $C_2(q,!)$

$$G(k;!,m_c)\sim\frac{const}{\bar{C}_2(q;!)!\ 1},\quad \bar{C}_2(q;!)''\ \frac{C_2(q;!)}{!(N)},\quad 0<!\ \#1$$

CP: $\lim_{!''1} l.h.s.''\ \#$ and $\lim_{!''1} r.h.s.''\ \#$

Fluctuation length result: $\left|!^{2}_{(2s)}('')\right| = p('')\sum_{j=0}^{)}\#\frac{a^j}{\mathrm{m}_c}\&^{2s}\ ''^{j},\ a=\frac{\mathrm{m}}{\mathrm{m}_c}>1$

Converged @ $(a/\mathrm{m}_c)^{2s}\ !\ <1,\ \left|!^{2}_{(2s)}('')\right|\#\ p('')\mathrm{m}_c^{-2s}(1+a^{2s}'')$ finite

If $\mathrm{m}!\ \mathrm{m}_c$ and $!''\ 1\ \ !''\ \#$ divergence of $!''$ **CP**

Analyticity of probability $P\left(x;!,\mathrm{m}_c\right)$ cont'd II

- Infinite # of divergent (singular) terms in $G\left(k;!,\mathrm{m}_c\right)$

  Why?

Because wide range of $!,\ \mathrm{m}$; singularity @ $k \ll \mathrm{m}_c \ \left(k\ !\ \ 0\right)$

  - To find non-analytical part @ $k\ !\ 0$

$G\left(k;!,\mathrm{m}_c\right)=G_{BM}\left(k;!,\mathrm{m}_c\right)+G_{AM}\left(ak;!,\mathrm{m}_c\right)$ linear non-homog. eq.

BM $G_{BM}\left(k;!,\mathrm{m}_c\right)=p\left(!\right)\boldsymbol{cos}\left(k/\mathrm{m}_c\right)$ regular if $k\ !\ 0$, for all $!$

AM $G_{AM}\left(k;!,\mathrm{m}_c\right)=!\,G\left(ak;!,\mathrm{m}_c\right)$, for $a^{!2}<''<1$

$G_{AM}\left(k;!,\mathrm{m}_c\right)''\ \ 0,\ if\ \ !\ ''\ \ 0$ The phase with **BM** does exist only

While $G\left(ak;!\ ''\ 0,\mathrm{m}_c\right)''\left(1/2\#\right)\boldsymbol{cos}\left(k\mathrm{m}/\mathrm{m}_c^2\right)$ Finite

Special solution for AM

$$G_{AM}\left(k;!\,(S),m_c\right)=C\left(!\,(S)\right)\left(k/m_c\right)^{!\,(''(S))}Q(k)$$

$$!\,\left(''(S)\right)=\ln\left|\frac{Q(k)}{\#Q(ak)!\,!\,(S)}\right|/\ln a,\quad a>1$$

! associated function of the 1st order to the measure ! with the proper function of the operator of dilatation transformation $u: uQ(k)=Q(ak)$

For any $a=(\mathrm{m}/\mathrm{m}_c)>1$: $Q(ak)=a^{!}Q(k)+a^{!}\,\boldsymbol{ln}(a)''Q_0(k)$

Vicinity of CP (a close to 1 from above) $Q(ak)!\;Q(k)$

Finally, $!\,\left(''(S)\right)!\;''\frac{\ln !\,(S)}{\ln a}+!\,(,\;2(S+1)!\;!\ln a''\;''(S)>2S!\;!\ln a$

Fluctuation length (cont'd) @ CP

The solution of $Q(|k|)$ through the functional dilatation equation $u\, Q(|k|) \approx Q(a \cdot |k|)$

Solution with smoothly changing function, e.g., the log-periodic one with the period $\log(a)$

Method: $|k| \rightarrow \log(|k|),\ a \cdot |k| \rightarrow \log(a) = \log(|k|)$

$$Q(|k|) \sim \frac{1}{\log a} \sum_{m=0}^{\infty} b_m \cos\left(2\pi m \frac{\log|k|}{\log a} + \vartheta_m\right)$$

If $b_m = 1$, $\vartheta_m = 0 \Rightarrow Q(|k|) \sim \frac{1}{2\log a}\left[1 + \sum_{m=-\infty}^{+\infty} \delta\left(\frac{\log(|k|)}{\log a} - m\right)\right]$

Solution for AM phase

$$G_{AM}\left(k;!,m_c\right) \sim \frac{i^{''\#2}}{\Gamma\left(''\#1\right)} \%_{(''\#2)}^{2}\left(!\right)\left|\frac{k}{m_c}\right|^{''} \frac{1}{\log a} \sum_{m=0} b_m \cos\left(2\&m\frac{\log\left(|k|\right)}{\log a} + '_m\right)$$

Divergent if a) $! \,'' \, 1$ because of $!\left(''\right)$

b) $a = \dfrac{m}{m_c} ! \; 1$ from above

➢ $C_{!,!}\left(S\right) = \dfrac{i^{!\,\%2}}{\&\left(!\,\%1\right)} !^{2}_{(''\%2)}\left(!\right)$ *used within calculation*

The **AM** disappears if $\dfrac{!\left(S\right)}{S} '' \; 2$ as $S \,!\, ''$ at $!\left(S\right) '' \; 0$

Results:

a). $C_2^{\exp}$ is the monotonous function with the Dip-effect @ small $\langle N \rangle$, far away from *CP*.

b). Hot emission volume: *CP* signature: $C_2^{\exp} = 1$.
Dip-effect disappears as $\langle N \rangle >> 1$,

c). Source size L_{st} increases (smoothly) with N at low T.

d). L_{st} blows up as T ! T_c due to ! $\left(N\right)$″ 0, m_h ! 0;
L_{st} singular @ transition point, *CP*.

e). / decreases with k_T; / ! 0 as T ! T_c where / ! ″ .

Too rapid phase transition can include the ***explosion*** of a "hadronic fireball" just after a phase transition

Results to RFW

1. Theoretical search for **CP,** cross-over between **BM** and **AM** in 1D random fluctuation walk accompanied by the **CBE.**

2. Main points: $\bar{\Gamma}$, $!\left(''\right)$, $m_{(2s)}\left(!\right) \sim ''^{2}_{(2s)}\left(!\right)$, $a = \left(m / m_{c}\right) > 1$, k_{GL}.

3. Solution: **BM** (regular) + **AM** (singular).

4. Smooth $!\left(''\right)\#$ **BM**

$!\left(''\right)\#\ 1$ (strong particle chaoticity) $!$ **AM**

asymp. behavior at $k\ !\ 0$ (**IR**), large x is approached

$!$

infinite size of particle source.

5. Close to **AM:** $k_{GL}\ !\ \dfrac{1}{n}\dfrac{2}{\left(m / m_{c}\right)^{2} '' 1}$

CP (cross-over) $k_{GL}\ !\ \ ''$ at $m = m_{c}$.