Kinetic theory equilibration for realistic heavy ion initial conditions

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L. Keegan, A. Kurkela, A.M., and D. Teaney, JHEP 08, 171 (2016)[1].

A. Kurkela, A.M., J.-F. Paquet, S. Schlichting, and D. Teaney, *in progress*.

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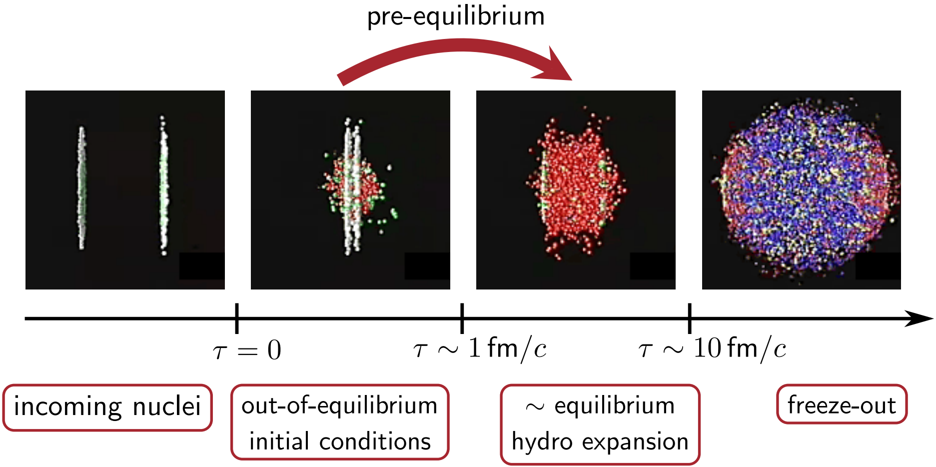
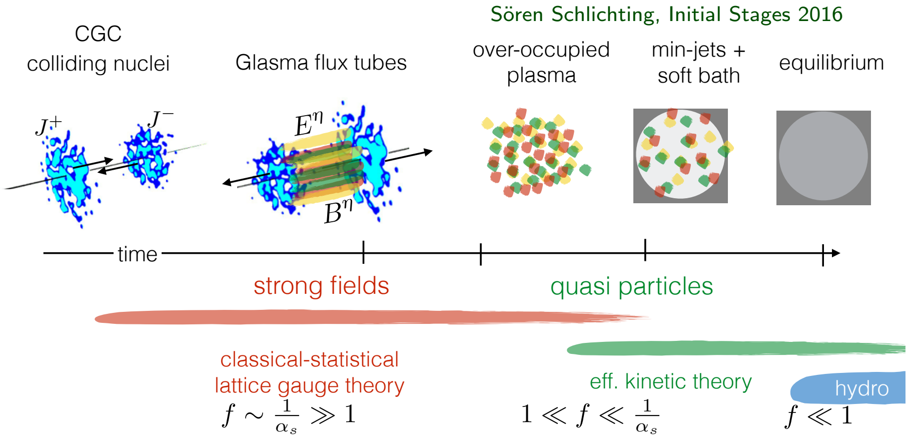
High energy heavy ion collisions in a nutshell



Goal: to understand transition from initial conditions to hydro expansion

Weak coupling picture

At high energy $\alpha_s \ll 1$



We will consistently map the IP-Glasma (CGC) initial conditions to hydrodynamics using QCD kinetic theory.

Weak coupling picture

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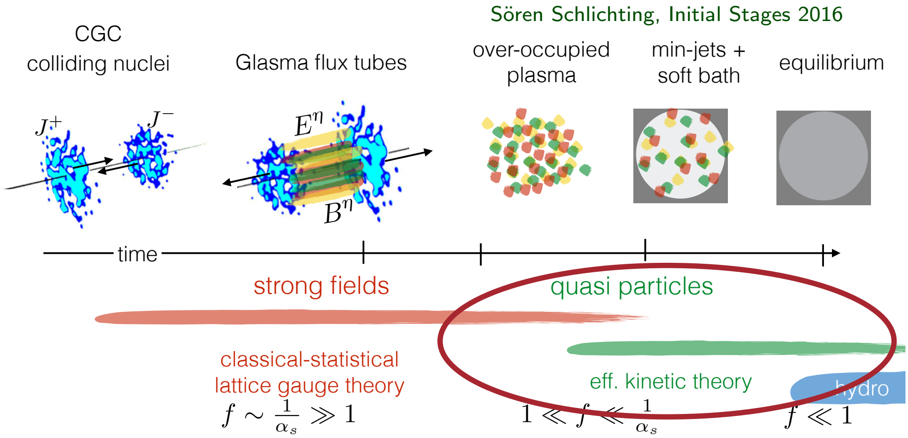


We will consistently map the IP-Glasma (CGC) initial conditions to hydrodynamics using QCD kinetic theory.

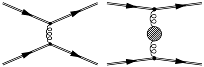
AMY effective kinetic theory

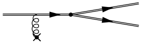
QCD effective kinetic theory

Arnold, Moore, Yaffe (2003)[3]

- Solve Boltzmann equation for $f(\mathbf{p})$—gluon distribution function

$$\partial_\tau f + \frac{\mathbf{p}}{|p|} \cdot \nabla f - \frac{p_z}{\tau} \partial_{p_z} f = - \underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]} - \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]}$$



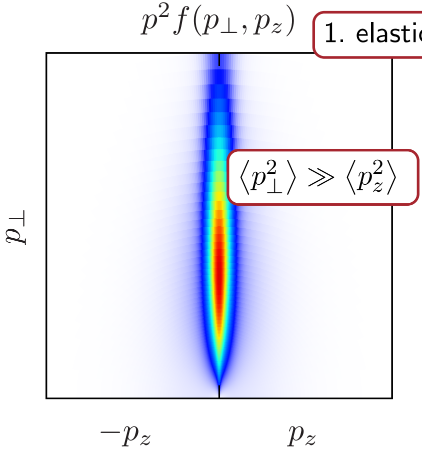


- Include transverse gradients and boost invariant expansion
- Leading order accurate QCD collision kernels
- Collision kernels $\mathcal{C}[f]$ Monte Carlo sampled at each timestep
- Extrapolate to realistic values of the coupling $\lambda = 4\pi\alpha_s N_c$

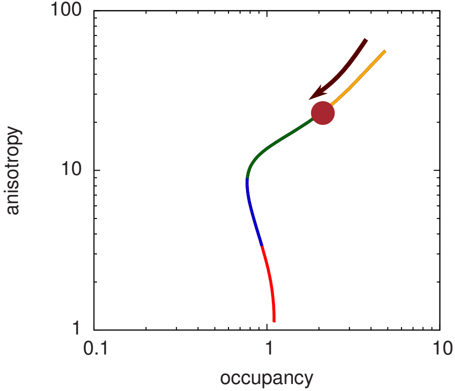
kinetic theory ⇒ hydrodynamics

Kinetic theory thermalization

"Bottom-up" thermalization scenario — Baier, Mueller, Schiff, and Son (2001)[2]
Universal-attractor — Berges, Boguslavski, Schlichting, Venugopalan (2014)[4]
Numerical realization — Kurkela and Zhu (2015)[5]



1. elastic $2 \leftrightarrow 2$ scatterings



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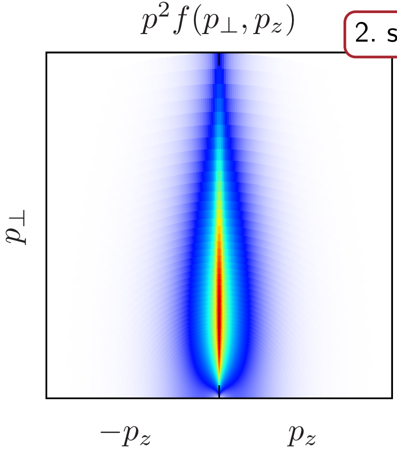
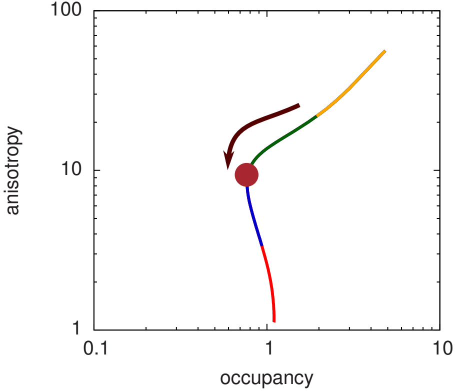


2. soft $1 \leftrightarrow 2$ splitting



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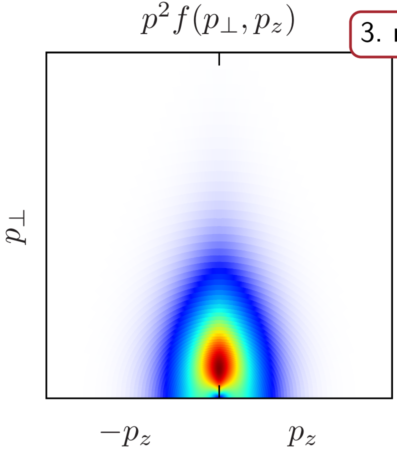
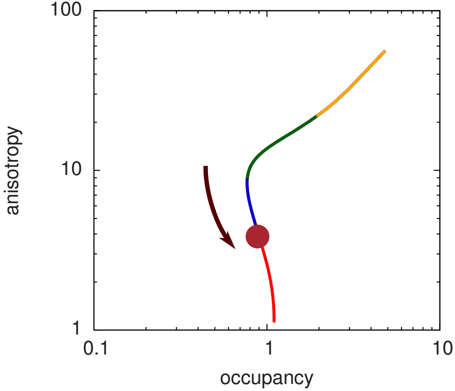


3. mini-jet quenching



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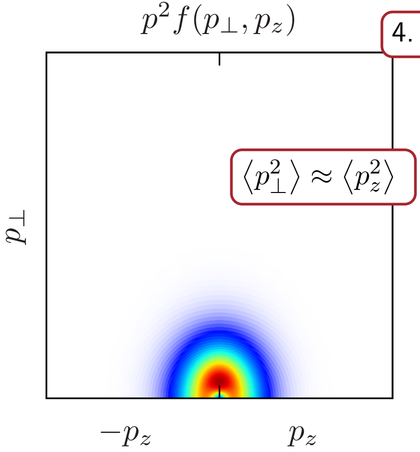
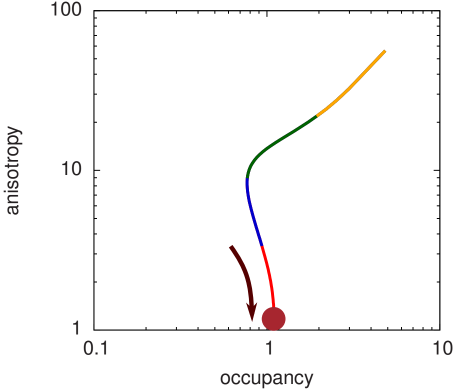


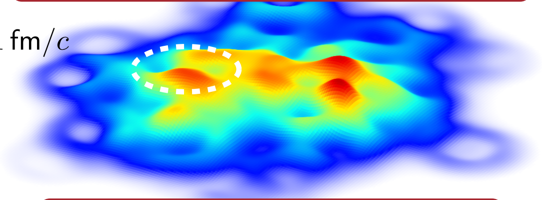
4. isotropization



Pre-equilibrium evolution with kinetic theory

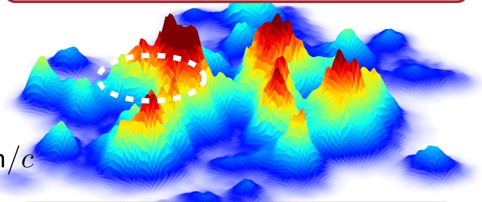
$$T^{\mu\nu}(\tau_{\text{init}}, \boldsymbol{x}) = \bar{T}^{\mu\nu}(\tau_{\text{init}}) + \delta T^{\mu\nu}(\tau_{\text{init}}, \boldsymbol{x})$$

$\tau_{\text{init}} \sim 1\,\text{fm}/c$



τ

$$T^{\mu\nu}(\boldsymbol{x}, \tau) = \nu_g \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{p} f(\boldsymbol{x}, \mathbf{p}, \tau)$$



$\tau_0 \sim 0.1\text{fm}/c$

$$|\boldsymbol{x} - \boldsymbol{x}_0| < c(\tau_{\text{init}} - \tau_0) \ll R_{\text{nucleus}}$$

Kinetic theory evolution of homogeneous background $\bar{T}^{\mu\nu}$

Hydro prediction: $$\tau^{4/3}\bar{e} = (\tau^{4/3}\bar{e})_\infty \left(\underbrace{1}_{\text{ideal}} \underbrace{- \frac{8}{3}\frac{\eta/s}{\tau T}}_{\text{viscous}} + \underbrace{C_2\left(\frac{\eta/s}{\tau T}\right)^2}_{\text{2nd order hydro}} + \ldots \right)$$

Scaling time

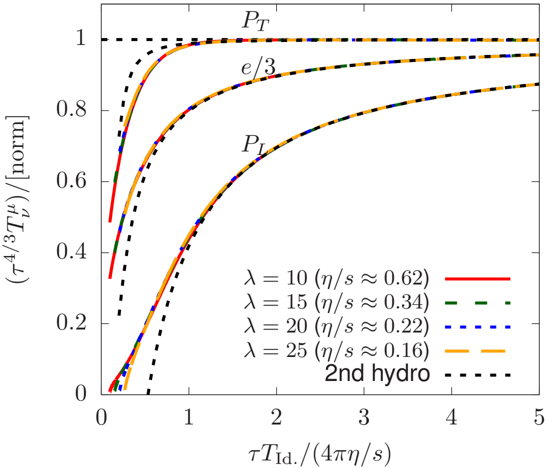
$$\underbrace{\tau \times \frac{T_{\text{Id.}}}{\eta/s} \propto \frac{\tau}{\tau_R}}$$

time in units of relaxation time

Scaling curves

$$\frac{\tau^{4/3}e}{(\tau^{4/3}e)_\infty} = \mathcal{E}\left[x = \frac{\tau T_{\text{Id.}}}{\eta/s}\right]$$

$$\frac{\tau^{4/3}P_L}{(\tau^{4/3}P_L)_\infty} = \mathcal{P}\left[x = \frac{\tau T_{\text{Id.}}}{\eta/s}\right]$$



Plane wave perturbations in transverse plane

Gluon distribution function at initial time

$$f(\tau, \mathbf{p}, \boldsymbol{x}_\perp) = \underbrace{\bar{f}_{\mathbf{p}}}_{\text{uniform background}} + \underbrace{\delta f_{\boldsymbol{k}_\perp, \mathbf{p}} e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp}}_{\text{transverse perturbations}} .$$

Linearize Boltzmann equation in perturbations

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z})\bar{f}_{\mathbf{p}} = -\mathcal{C}[\bar{f}] \qquad \text{background}$$

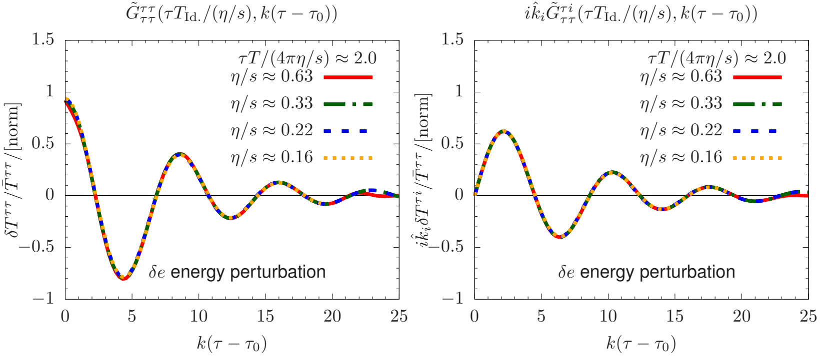
$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + \frac{i\mathbf{p}_\perp \cdot \boldsymbol{k}_\perp}{p})\delta f_{\boldsymbol{k}_\perp, \mathbf{p}} = -\delta\mathcal{C}[\bar{f}, \delta f] \qquad \boldsymbol{k}_\perp \text{ perturbation}$$

Study response functions to energy and momentum perturbations

$$\frac{\delta T^{\mu\nu}(\tau_{\text{init}}, \boldsymbol{k})}{\bar{T}^{\tau\tau}(\tau_{\text{init}})} = \tilde{G}^{\mu\nu}_{\beta\tau}\Big(\boldsymbol{k}, \tau, \tau_0\Big) \; \frac{\delta T^{\beta\tau}(\tau_0, \boldsymbol{k})}{\bar{T}^{\tau\tau}(\tau_0)}$$

Kinetic theory evolution of perturbations stress tensor $\delta T^{\mu\nu}$

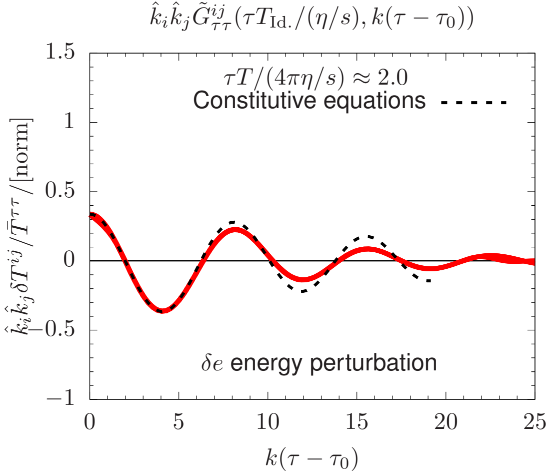
$$\tilde{G}^{\mu\nu}_{\beta\tau}\left(|\boldsymbol{k}|, \tau, \tau_0, \bar{T}^{\tau\tau}(\tau_0), \lambda\right) = \tilde{G}^{\mu\nu,\mathrm{univ}}_{\beta\tau}\left(\frac{\tau T_{\mathsf{Id.}}}{\eta/s}, |\boldsymbol{k}|(\tau - \tau_0)\right)$$



Kinetic theory evolution of perturbations stress tensor $\delta T^{\mu\nu}$

Hydrodynamic constitutive equations for k perturbations

$$\hat{k}_i\hat{k}_j\delta T^{ij}(k) = \underbrace{\delta T^{\tau\tau}}_{\text{energy } \delta e} [c_s^2 + \tfrac{2}{3}\tfrac{\eta/s}{\tau T} + \ldots] - i(k\tau) \underbrace{\hat{k}_i\delta T^{0i}}_{\text{momentum}} [\tfrac{4}{3}\tfrac{\eta/s}{\tau T} + \ldots]$$



Pre-equilibrium evolution with kinetic theory

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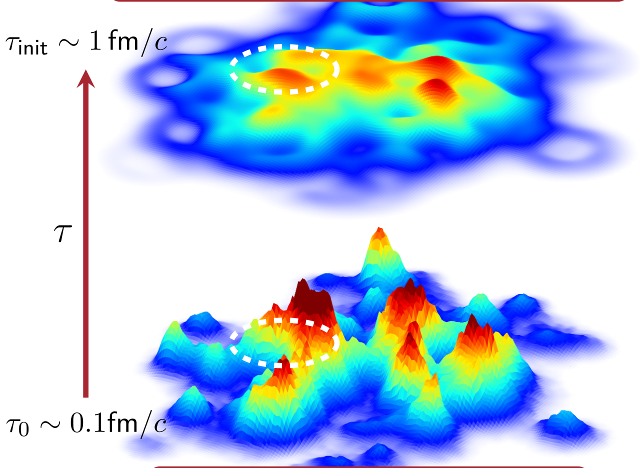
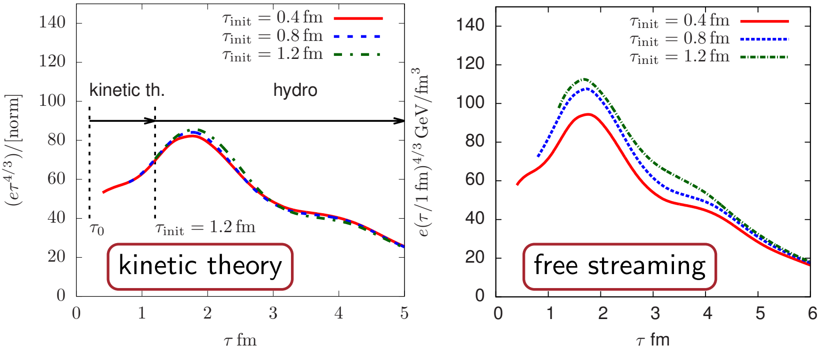


$$|\boldsymbol{x} - \boldsymbol{x}_0| < c(\tau_{\text{init}} - \tau_0) \ll R_{\text{nucleus}}$$

Matching with hydrodynamics

1. Evolve IP-Glasma initial conditions to early time $\tau_0 = 0.2\,\mathrm{fm}$
2. Kinetic theory equilibration from τ_0 to hydro initialization time τ_{init}
3. Hydrodynamic evolution from τ_{init} at $\eta/s = 2/(4\pi)$, conformal EoS

central energy density

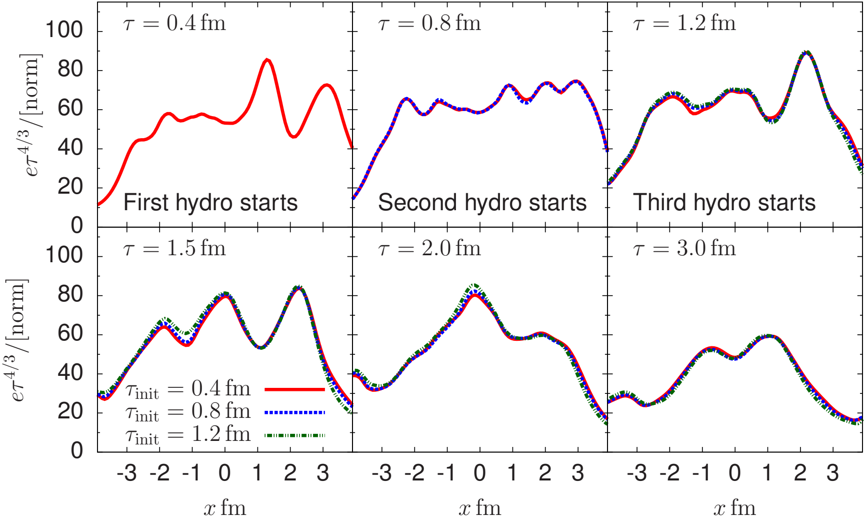


Smooth transition to hydrodynamics independent of τ_{init}!

Transverse collision cross-section

Transverse energy density $e(\tau, \boldsymbol{x})$

Kinetic theory pre-evolution

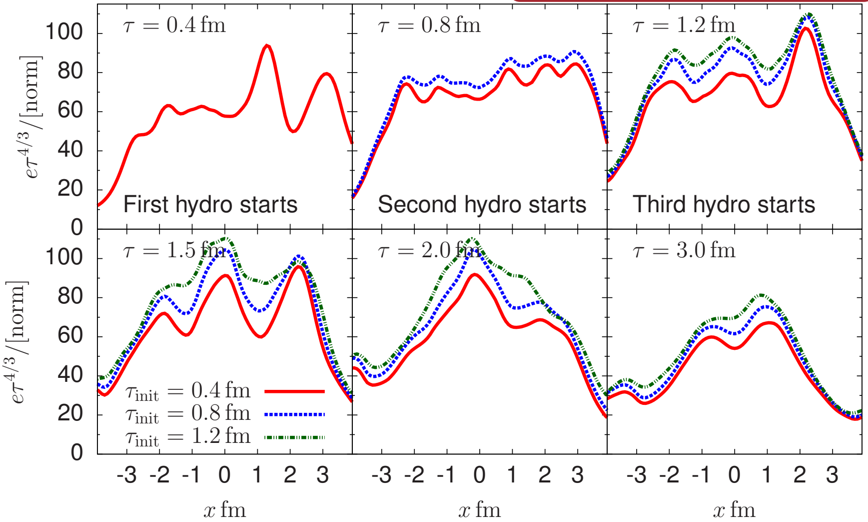


Kinetic theory pre-equilibrium eliminates dependence on initialization time τ_{init}!

Transverse collision cross-section

Transverse energy density $e(\tau, \boldsymbol{x})$

Free streaming pre-evolution

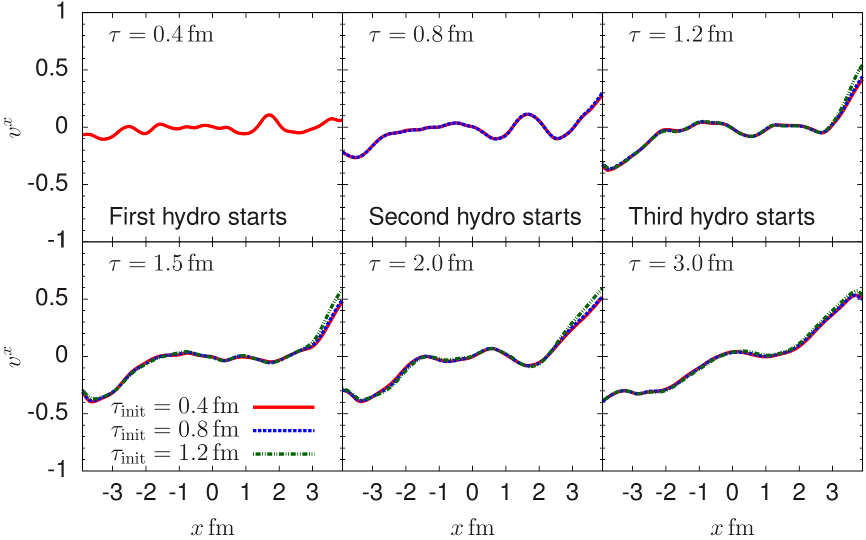


Kinetic theory pre-equilibrium eliminates dependence on initialization time τ_{init}!

Transverse collision cross-section

Transverse velocity x-direction

Kinetic theory pre-evolution



Kinetic theory pre-equilibrium eliminates dependence on initialization time τ_{init}!

Summary

Initial conditions for hydrodynamics from "bottom-up" equilibration:

- kinetic theory approach to hydrodynamics for transverse perturbations
- generated linearised response functions $G^{\mu\nu}(\tau T_{\text{Id.}}/(\eta/s), r/(\tau-\tau_0))$
- universal scaling with relaxation time $\tau T_{\text{Id.}}/(\eta/s)$
- smooth matching between IP-Glasma and hydrodynamics

Outlook

A. Kurkela, A.M., J.-F. Paquet, S. Schlichting, and D. Teaney, *in progress*.
Expect in the paper:

- Kinetic response to initial flow (response functions worked out)
- Universal equilibration in units of relaxation time $\tau T/(4\pi\eta/s)$
- Regularization for large gradients
- τ_{init} independence for physical observables ($\langle p_T \rangle$, v_2, etc.)
- Make kinetic theory response functions public

  Kinetic equilibration for your favourite initial state conditions!

Future work for kinetic theory equilibration:

- Chemical equilibration of quarks
- Pre-equilibrium photon production from "bottom-up" thermalization

Backup

Kinetic theory simulations

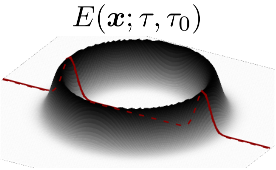
Collision kernel $\mathcal{C}[f]$ – multidimensional functional integral!

$$\mathcal{C}_{2\leftrightarrow 2}[f](\mathbf{p}) = \frac{1}{4|\mathbf{p}|\nu_g}\int \frac{d^3k}{2k(2\pi)^3}\frac{d^3p'}{2p'(2\pi)^3}\frac{d^3k'}{2k'(2\pi)^3}|\mathcal{M}(\mathbf{p},\boldsymbol{k};\mathbf{p}',\boldsymbol{k}')|^2\times$$
$$\times\,(2\pi)^4\delta^{(4)}(P+K-P'-K')\times$$
$$\times\left\{f_{\mathbf{p}}f_{\boldsymbol{k}}[1+f_{\mathbf{p}'}][1+f_{\boldsymbol{k}'}] - f_{\mathbf{p}'}f_{\boldsymbol{k}'}[1+f_{\mathbf{p}}][1+f_{\boldsymbol{k}}]\right\}$$

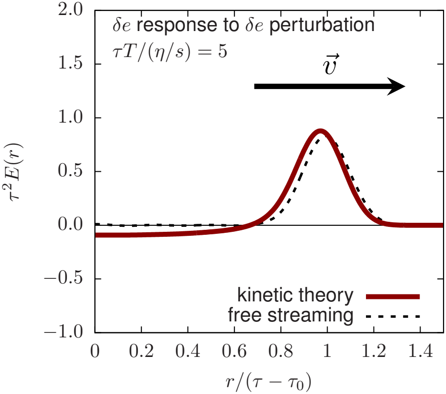
- Collisions integrals estimated with Monte Carlo sampling.
- Typical grid size for $f(p,\theta,\phi)\sim 100_p\times 100_\theta\times 20_\phi$.
- few hundreds $\boldsymbol{k}_\perp$ wavenumbers simulated for Fourier transform.

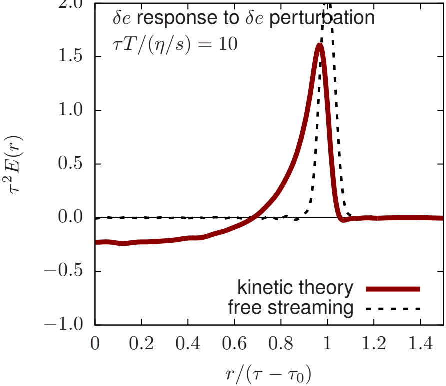
Energy Green function in coordinate space

Convolve energy perturbations with response kernel.



$$\frac{\delta e(\tau,\boldsymbol{x})}{e(\tau)} = \int d^2\boldsymbol{x}' E(|\boldsymbol{x}-\boldsymbol{x}'|;\tau,\tau_0) \times \frac{\delta e(\tau_0,\boldsymbol{x}')}{e(\tau_0)}$$





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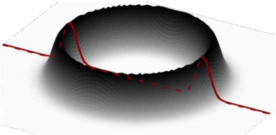
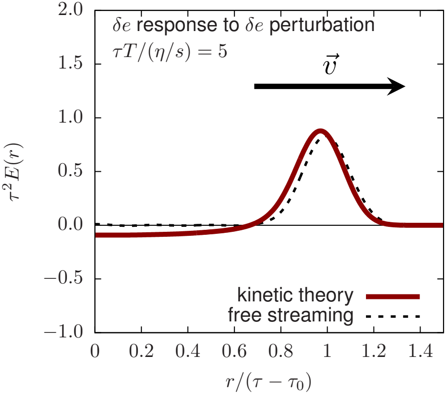
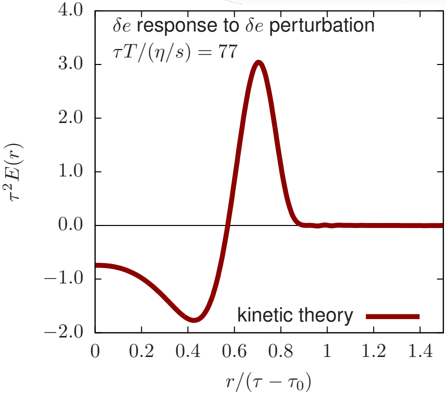
$E(\boldsymbol{x}; \tau, \tau_0)$







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