

# HBT overview

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Overview on fundamentals:  
Hanbury Brown and Twiss

Positive definitene form – or not?

Two-particle symmetrization effect – or not?

Gaussian shape – or not?

Sensitive to  $U_A(1)$  symmetry restoration

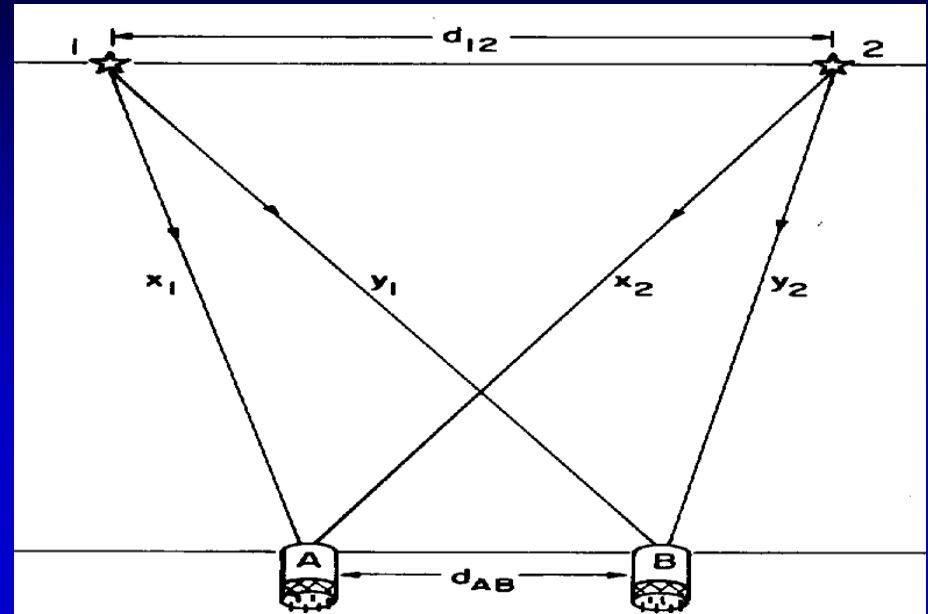
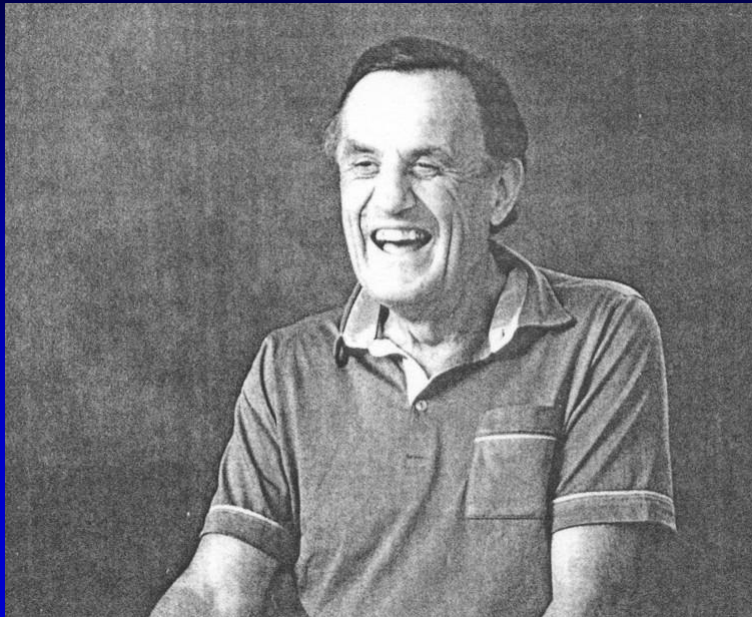
Signal of QCD phase transition

Summary, conclusions

Based on a HBT Overview talk at High Density Nuclear Matter 2017

Supported by EFOP-3.6.1-16-2016-00001 (Hungary)

# HBT: Robert HANBURY BROWN – Richard Quentin TWISS



**Two** people: Robert Hanbury Brown and Richard Quentin Twiss

– Robert, Hanbury as well as Richard and Quentin: can be **given** names, but...

– Sir Robert Hanbury Brown had a **compound family** name,

– just like Sir Christopher Llywellyn Smith, whose **compound family** name is sometimes **hyphenated**

**R. Hanbury Brown and R. Q. Twiss: Engineers**, who worked in radio and optical astronomy

„Interference between two different photons can **never** occur.”

P. A. M. Dirac, The Principles of Quantum Mechanics, Oxford, 1930

„As an **engineer** my education in physics had stopped **far short of the quantum theory**.

Perhaps just as well ... **ignorance is sometimes a bliss in science.**”

R. H. Brown: Boffin: A Personal Story ... ISBN 0-7503-0130-9

# HBT: 1 + positive-definite term

Two plane waves

**Symmetrized**, + for bosons, - for fermions

Expansion dynamics, final state interactions,  
multiparticle symmetrization effects: **negligible**

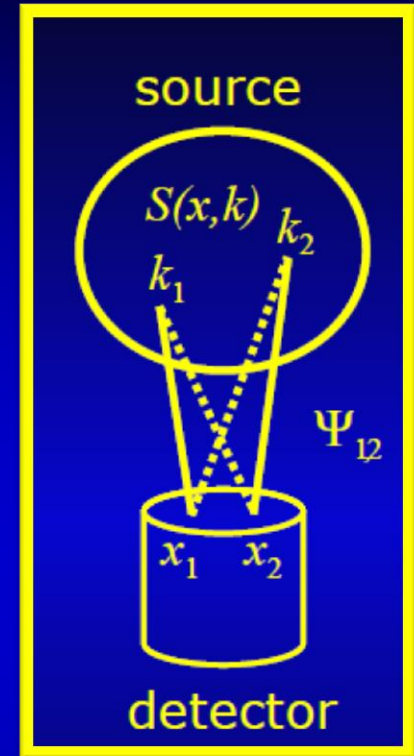
$$\Psi_1 = e^{-ik_1 x_1}$$

$$\Psi_2 = e^{-ik_2 x_2}$$

$$A_{12} \propto \frac{1}{\sqrt{2}} [e^{ik_1 x_1 + ik_2 x_2} \pm e^{ik_1 x_2 + ik_2 x_1}],$$

$$N_2(k_1, k_2) \propto \int dx_1 \rho(x_1) \int dx_2 \rho(x_2) |A_{12}|^2$$

$$C_2(k_1, k_2) = \frac{N_2(k_1, k_2)}{N_1(k_1)N_2(k_2)} = 1 \pm |\tilde{\rho}(k_1 - k_2)|^2$$



Two particle HBT correlations:

1 + positive-definite term

1+ |Fourier-transform of the source|<sup>2</sup>,

Usually evaluated in **Gaussian approximation**

**Dependence on mean momentum:**

**expansion dynamics  $\rho(x) \rightarrow S(x, k)$**

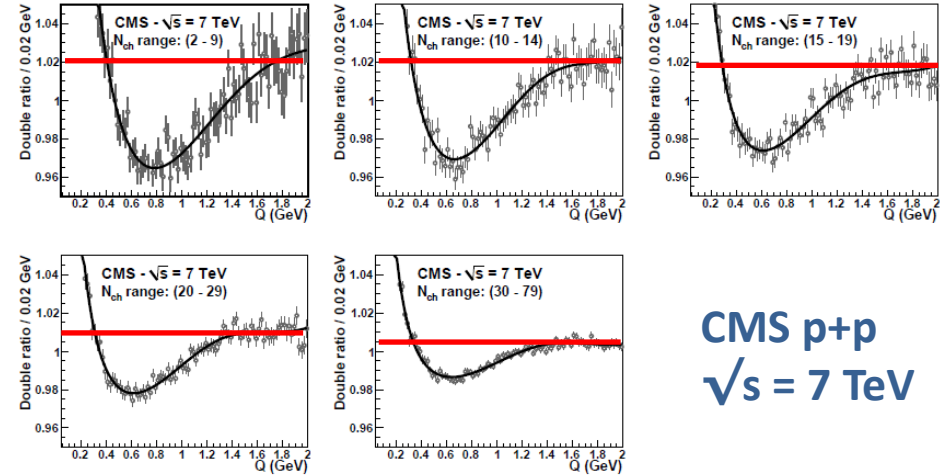
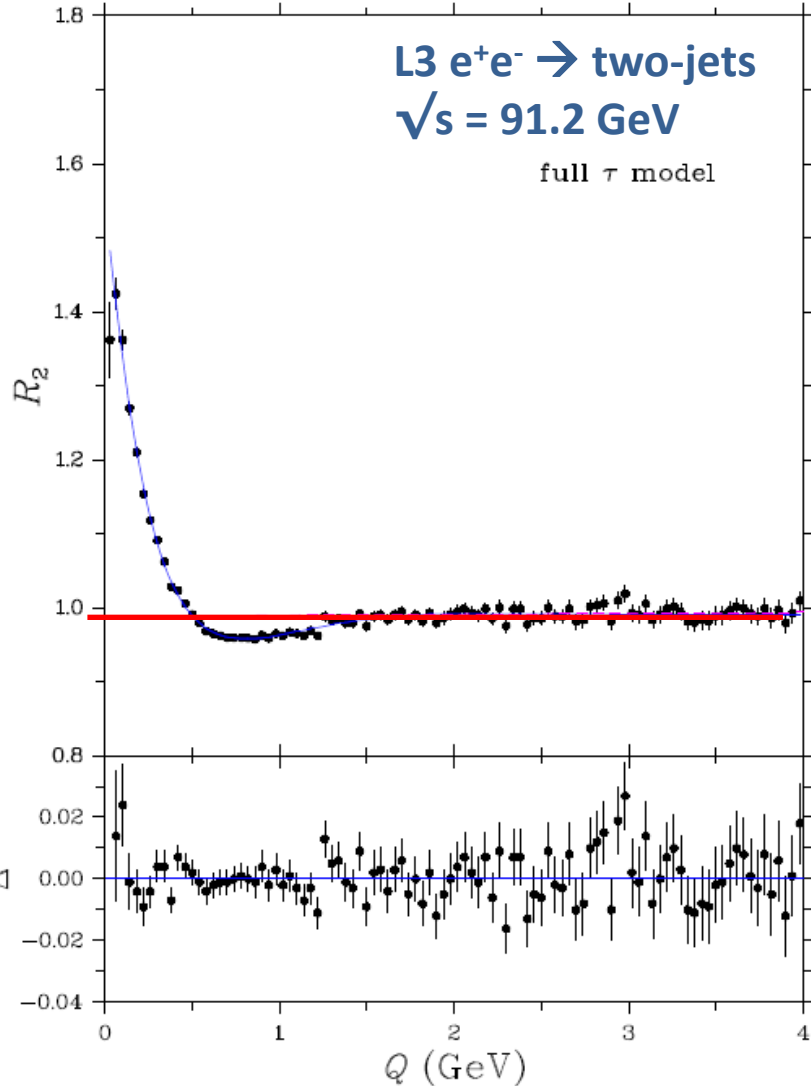
$$\tilde{\rho}(q) = \int dx e^{iqx} \rho(x)$$

**Dubna school:** use it as a tool

Kopylov and Podgoretskii:  $x \leftrightarrow k$

1+ |Fourier-transform of the source|<sup>2</sup>

# HBT: 1 + positive definite term - or not ?



Two experimental results:

L3 for Bose-Einstein in  $e^+e^-$  at LEP

[arXiv:1002.1303](https://arxiv.org/abs/1002.1303) [hep-ex]

CMS for Bose-Einstein in pp at LHC

[arXiv:1101.3518](https://arxiv.org/abs/1101.3518) [hep-ex]

Expansion dynamics: role of jets?

$\rightarrow$  strongly correlated phase-space,

$\tau$ -model:  $x \sim k \rightarrow \Delta x \Delta k \sim Q_{inv}^2$ ,

$C(Q_{inv}) \neq 1 +$  positive definite form

[arXiv:0803.3528](https://arxiv.org/abs/0803.3528) [hep-ph]

See the talk of Wes Metzger for details

# HBT: 1 + positive definite term?

## Example: Levy expansions

$$\begin{aligned} \text{1st-order Lévy polynomial} & \quad \gamma \left[ 1 + \lambda e^{-R^\alpha Q^\alpha} [1 + c_1 L_1(Q|\alpha, R)] \right] \\ \text{3rd-order Lévy polynomial} & \quad \gamma \left[ 1 + \lambda e^{-R^\alpha Q^\alpha} [1 + c_1 L_1(Q|\alpha, R) + c_3 L_3(Q|\alpha, R)] \right] \end{aligned}$$

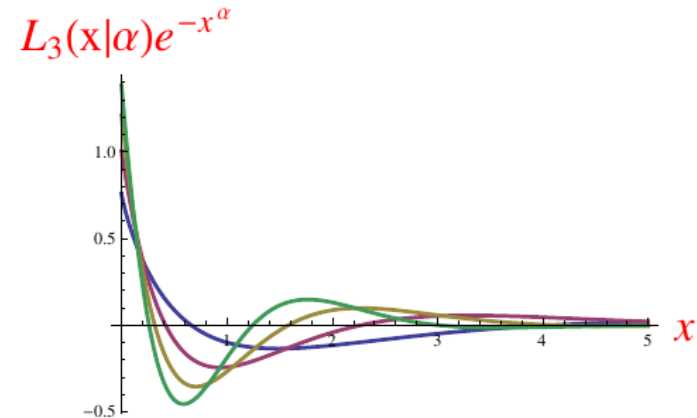
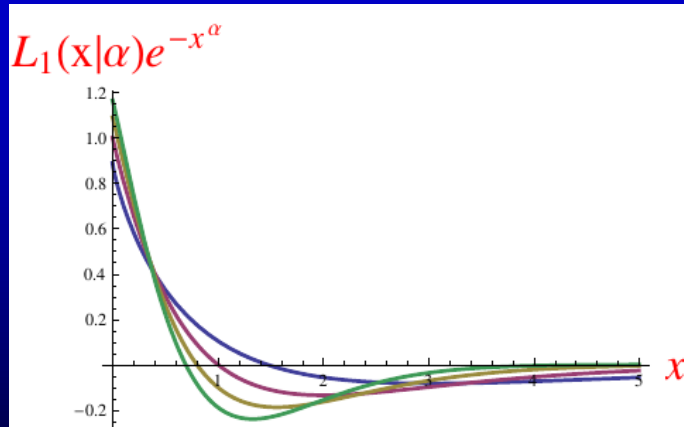
### Model-independent but:

$$L_1(x|\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ 1 & x \end{pmatrix}$$

$$L_2(x|\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ 1 & x & x^2 \end{pmatrix}$$

- Generalizes exponential ( $\alpha = 1$ ) and Gaussian ( $\alpha = 2$ )
- ubiquitous in nature
- How far from a Levy?
- Not necessarily positive definite!
  - Notation:  $x = Q R$

$$\mu_{r,\alpha} = \int_0^\infty dx x^r f(x|\alpha) = \frac{1}{\alpha} \Gamma\left(\frac{r+1}{\alpha}\right)$$



Lévy polynomials of first and third order times the weight function  $e^{-x^\alpha}$  for  $\alpha = 0.8, 1.0, 1.2, 1.4$ .

# HBT: 1 + positive definite term?

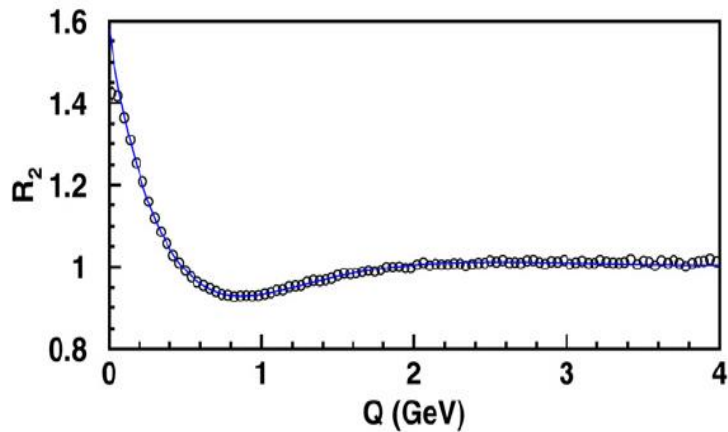


Fig. 1. The Bose-Einstein correlation function  $R_2$  for events generated by PYTHIA. The curve corresponds to a fit of the one-sided Lévy parametrization, Eq. (13).

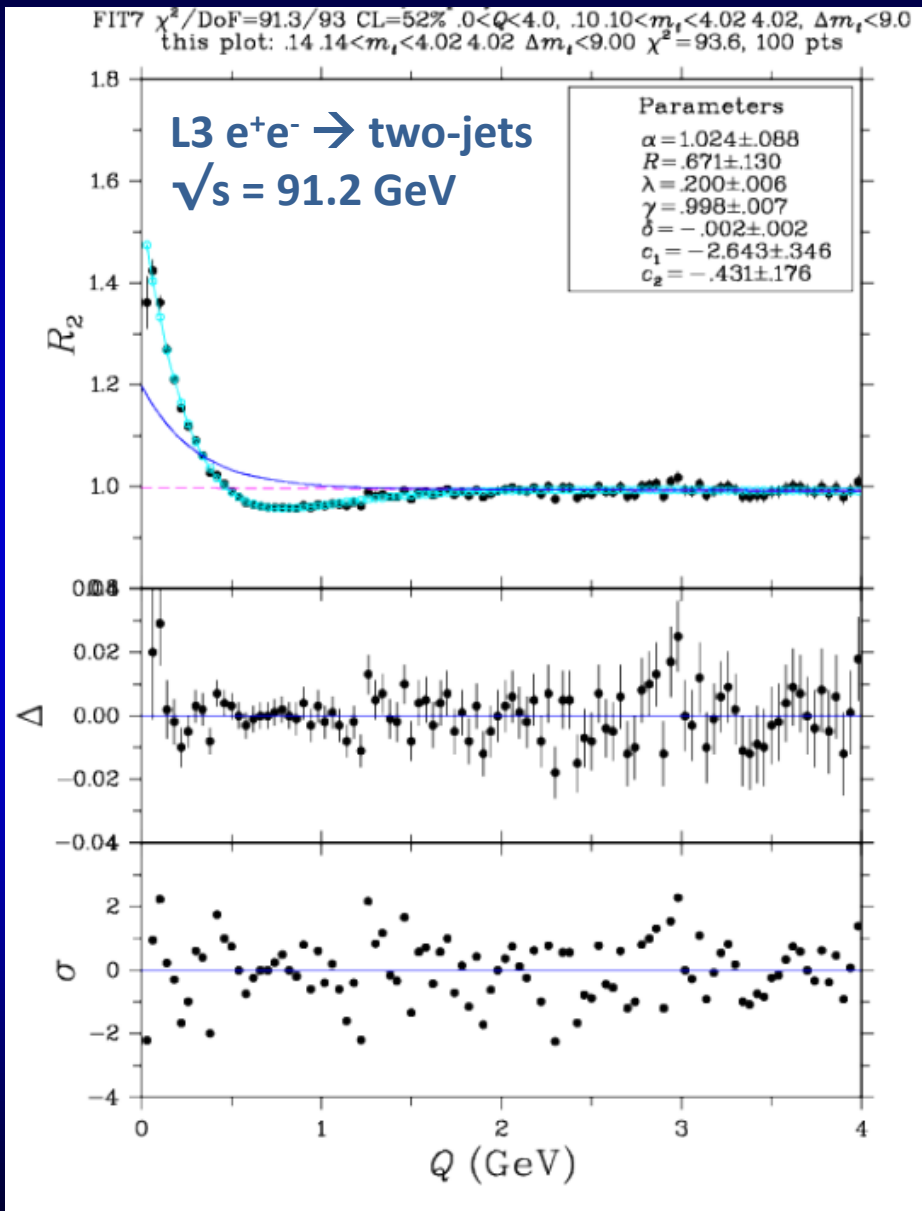
T. Csörgő et al. / Physics Letters B 663 (2008) 214–216

Check dip and background with Levy/Laguerre/Edgeworth/Gauss model independent expansions

$$t = QR$$

$$C_2(t) = N \left\{ 1 + \lambda \exp(-t^\alpha) \left[ 1 + \sum_{n=1}^{\infty} c_n L_n(t|\alpha) \right] \right\}$$

See the [talk](#) of T. Novák at Low-x 2016



# HBT: 1 + positive definite term?

## Levy expansions for 1+ positive definite forms

$$C_2(k_1, k_2) = 1 + |\tilde{f}(q_{12})|^2,$$

$$t = \left( \sum_{i,j=\text{side,out,long}} R_{i,j}^2 q_i q_j \right)^{1/2},$$

$$C_2(t) = N \left\{ 1 + \lambda \exp(-t^\alpha) \left| 1 + \sum_{n=1}^{\infty} (a_n + ib_n) L_n(t|\alpha) \right|^2 \right\}$$

where  $\{c_n = a_n + ib_n\}_{n=1}^{\infty}$  are now complex valued expansion coefficients,

### Model-independent but:

- Generalizes exponential ( $\alpha=1$ ) and Gaussian ( $\alpha=2$ )
- In this case, for 1+ positive definite forms
- ubiquitous in nature
- How far from a Levy?
- Works **also for cross-sections in elastic scattering**

$$L_1(x|\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} \\ 1 & x \end{pmatrix}$$

$$L_2(x|\alpha) = \det \begin{pmatrix} \mu_{0,\alpha} & \mu_{1,\alpha} & \mu_{2,\alpha} \\ \mu_{1,\alpha} & \mu_{2,\alpha} & \mu_{3,\alpha} \\ 1 & x & x^2 \end{pmatrix}$$

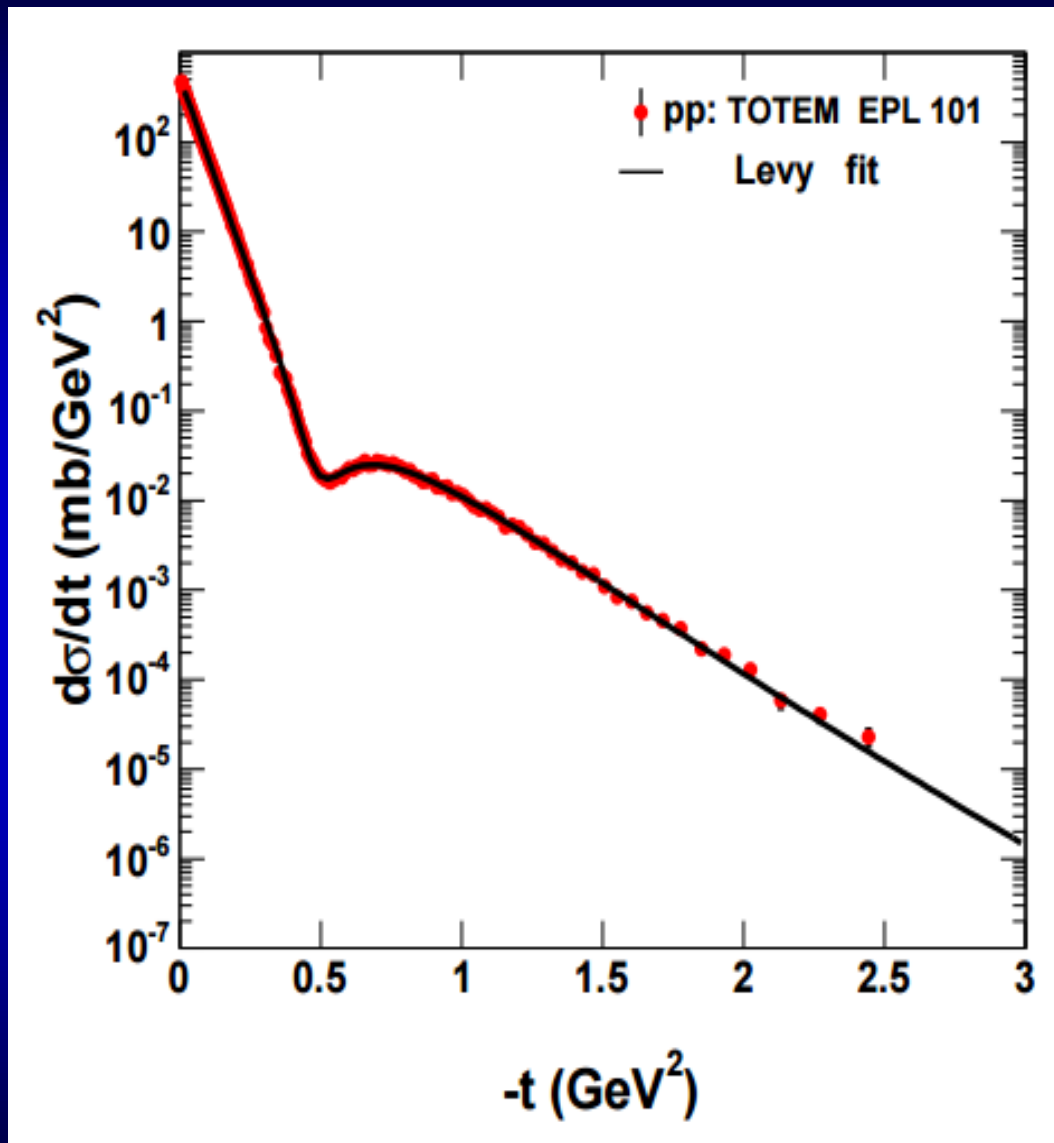
T. Novák, T. Cs., H. C. Eggers, M. de Kock:

[arXiv:1604.05513](https://arxiv.org/abs/1604.05513) [physics.data-an]

$$\mu_{r,\alpha} = \int_0^\infty dx x^r f(x|\alpha) = \frac{1}{\alpha} \Gamma\left(\frac{r+1}{\alpha}\right)$$



# Example: Levy expansion for $|f|^2$



T. Cs, W. Metzger, T. Novák, A. Ster, Proc Low-x2016 (in preparation)



# HBT: Has to be a Gaussian, IF ...

## Model-independent but Gaussian IF we assume:

- 1 + positive definite forms
- Plane wave approximation
- Two-particle symmetrization (only)
- IF  $f(q)$  is **analytic** at  $q = 0$  and
- IF means and variances are finite
- Follows an **approximate** Gaussian ( $\alpha = 2$ )

$$C_2(k_1, k_2) = 1 + |\tilde{f}(q_{12})|^2,$$

$$\tilde{f}(q_{12}) = \int dx \exp(iq_{12}x) f(x),$$

$$q_{12} = k_1 - k_2.$$

$$\tilde{f}(q) \approx 1 + iq\langle x \rangle - q^2\langle x^2 \rangle/2 + \dots,$$

$$C(q) = 1 + |\tilde{f}(q)|^2 \approx 2 - q^2(\langle x^2 \rangle - \langle x \rangle^2) \approx 1 + \exp(-q^2 R^2),$$

## Model-independent but non-Gaussian IF we assume:

- 1 + positive definite form (same as above)
- Plane wave approximation (same)
- Two-particle symmetrization only (same)
- IF  $f(q)$  is **NOT** analytic at  $q = 0$  and
- IF means and variances are **NOT** finite
- IF **Generalized Central Limit theorems** are valid
- Follows a Levy shape ( $0 < \alpha \leq 2$ )
- Earlier Gaussian recovered for  $\alpha = 2$

$$R = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}.$$

$$f(x) = \int \prod_{i=1}^n dx_i \prod_{j=1}^n f_j(x_j) \delta(x - \sum_{k=1}^n x_k).$$

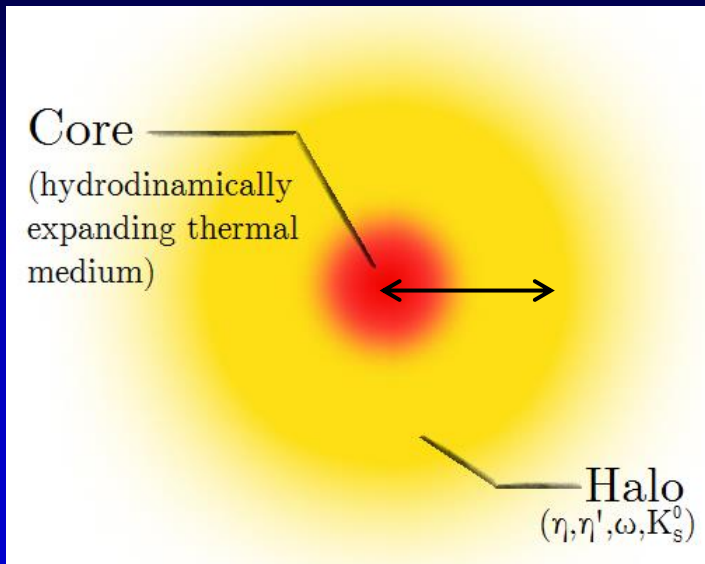
$$\tilde{f}(q) = \prod_{i=1}^n \tilde{f}_i(q)$$

$$C(q; \alpha) = 1 + \lambda \exp(-|qR|^\alpha).$$

Cs. T, S. Hegyi, W. A. Zajc, [nucl-th/0310042](https://arxiv.org/abs/nucl-th/0310042)

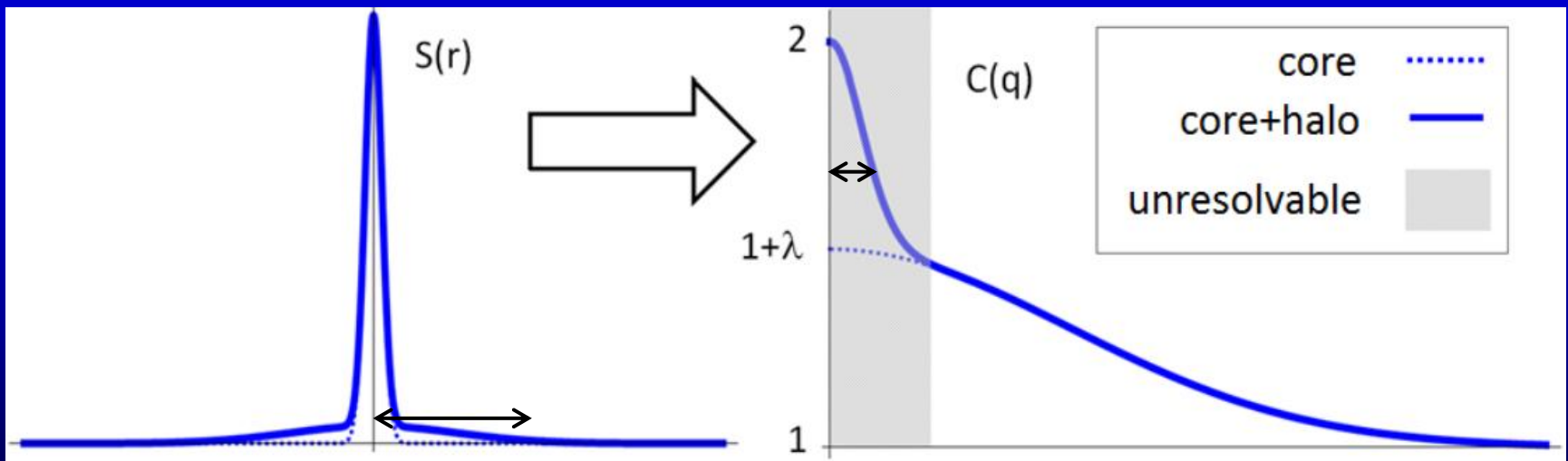
# But: core/halo model, resonances

For details: D. Kincses, [poster at QM17](#)



Resonance pions reduce the corr. strength [1, 2]  
 Core-Halo model:  $S = S_C + S_H$   
 Primordial pions - Core  $\lesssim 10$  fm  
 Resonance pions - from very far regions - Halo  
 Corr. strength  $\rightarrow$  C-H ratio:  $\lambda = \left( \frac{N_C}{N_C + N_H} \right)^2$

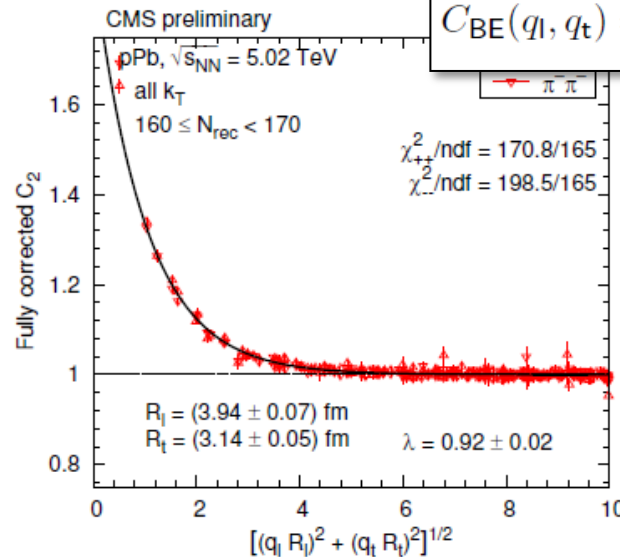
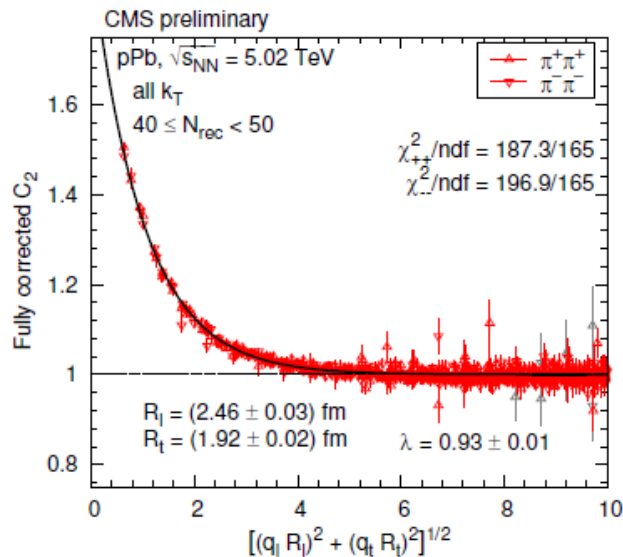
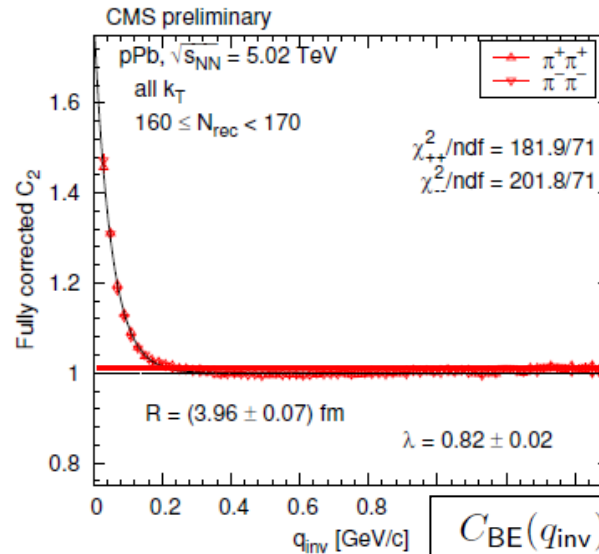
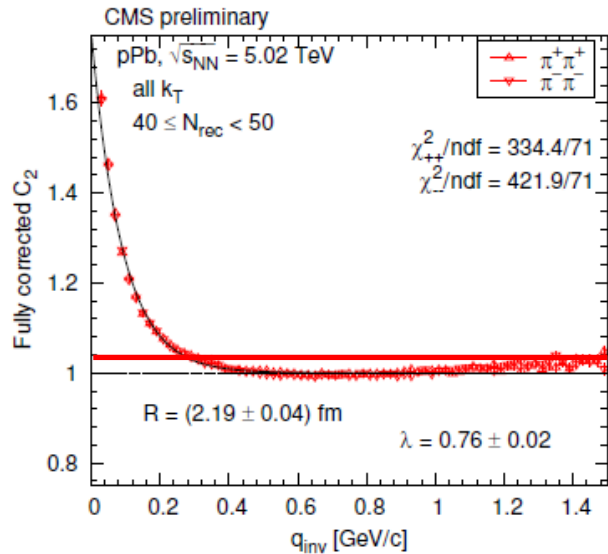
$$R = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}. \quad \text{Variance: halo dominated!}$$



[1] J. Bolz et al: Phys.Rev. D47 (1993) 3860-3870

[2] T. Cs, B. Lörstad, J. Zimányi: [hep-ph/9411307](#)

# HBT: Is C(Q) a Gaussian?



CMS Preliminary  
pPb@  $\sqrt{s} = 5.02$  TeV:

[arXiv:1411.6609](https://arxiv.org/abs/1411.6609)

- 1 + positive definite ?
- CL of the fits?
- **NOT** Gaussian !
- BUT: **Exponential** !
- IF  $\alpha \neq 2 \rightarrow \alpha = 1$  ! (?)

$$C_{BE}(q_{inv}) = 1 + \lambda \exp[-q_{inv}R],$$

$$C_{BE}(q_l, q_t) = 1 + \lambda \exp\left[-\sqrt{(q_l R_l)^2 + (q_t R_t)^2}\right],$$

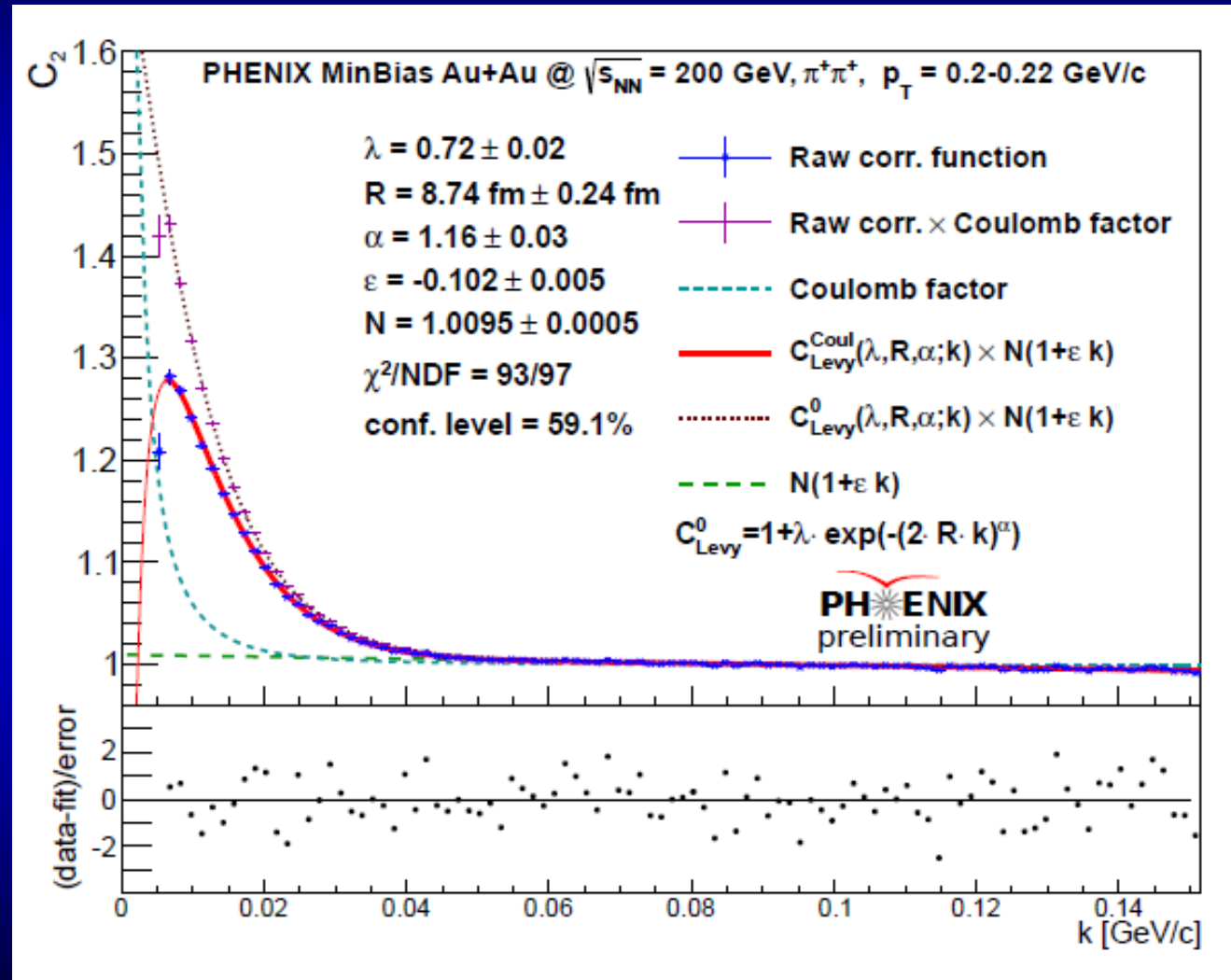
As the dimensionality  
increases from  $d=1$  to  $3$ ,  
shape analysis degrades

[arXiv:1411.6609](https://arxiv.org/abs/1411.6609)

See the talk of Sandra Padula  
for methods

# HBT: Is $C(Q)$ indeed exponential?

PHENIX Preliminary min. bias Au+Au @  $\sqrt{s_{NN}} = 200$  GeV from [arXiv:1610.05025](https://arxiv.org/abs/1610.05025)



- 1 + positive definite
- Levy expansion:  
no 1<sup>st</sup> order correction
- **CL = 59.1 %**
- **NOT Gaussian !**
- **NOT Exponential !**
- **$1 < \alpha < 2$**
- **$\alpha = 1.16 \pm 0.03$**
- **$m_t$  dependent**

What are the systematics of the source parameters,

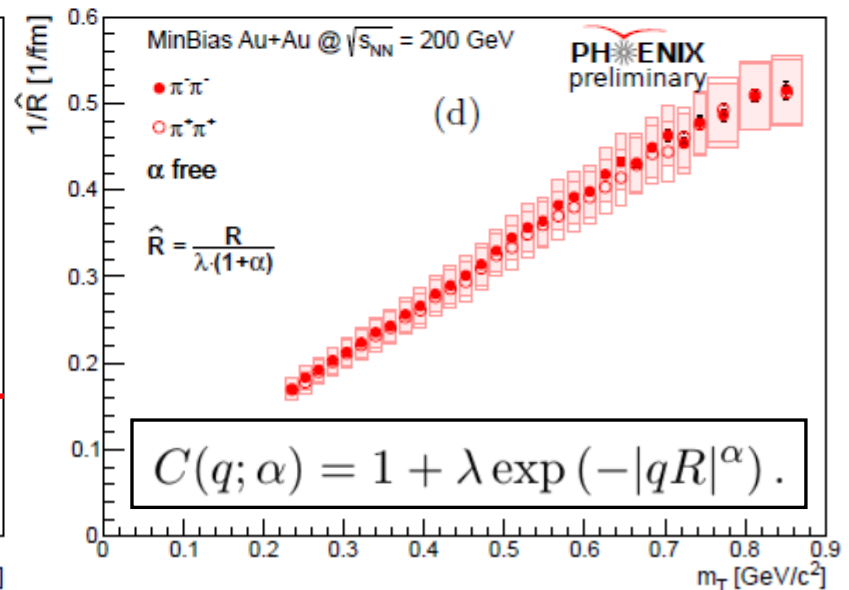
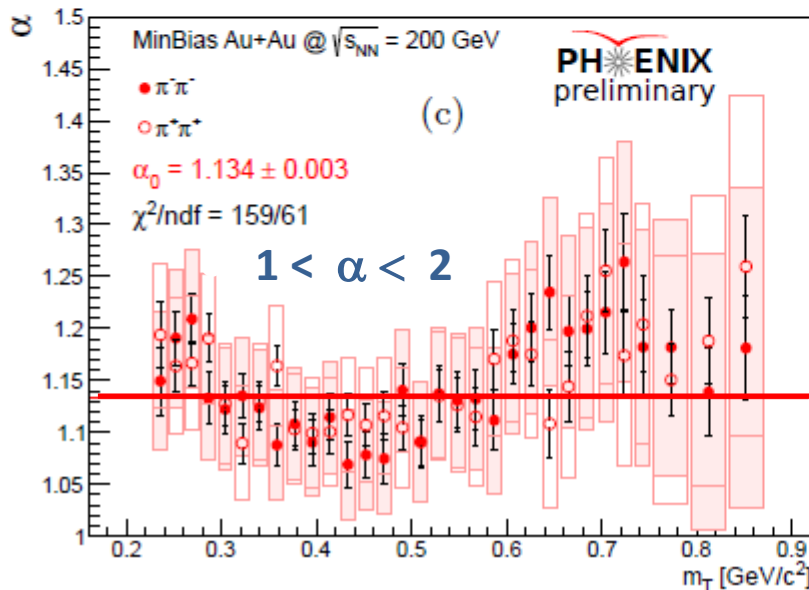
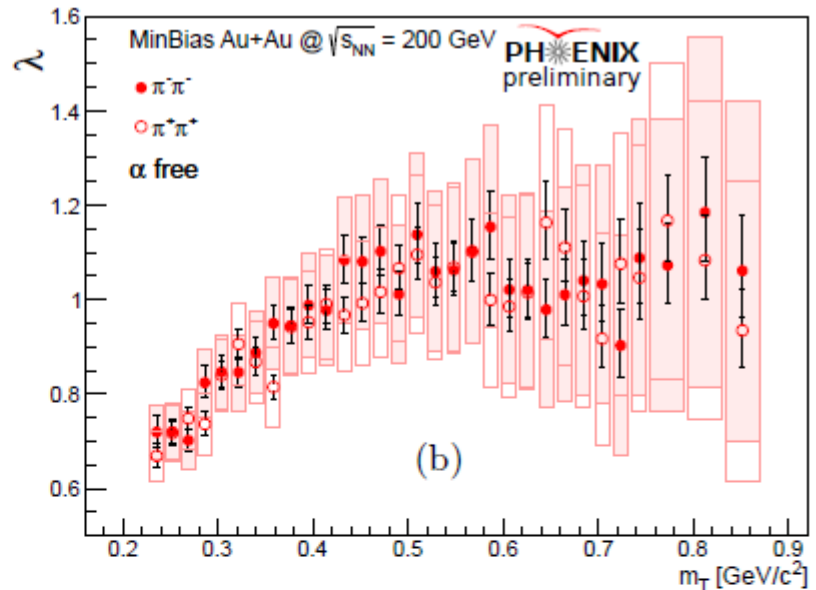
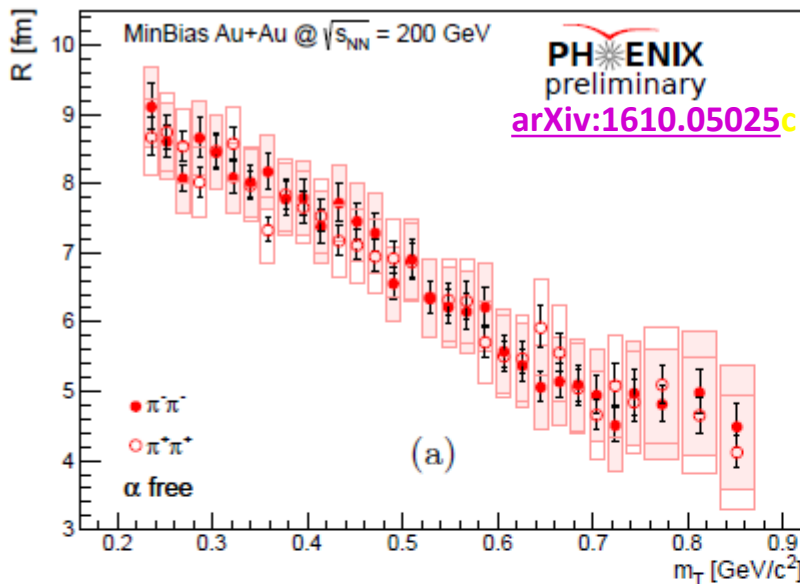
$$\lambda = \lambda(m_t),$$

$$R = R(m_t),$$

$$\alpha = \alpha(m_t) ?$$

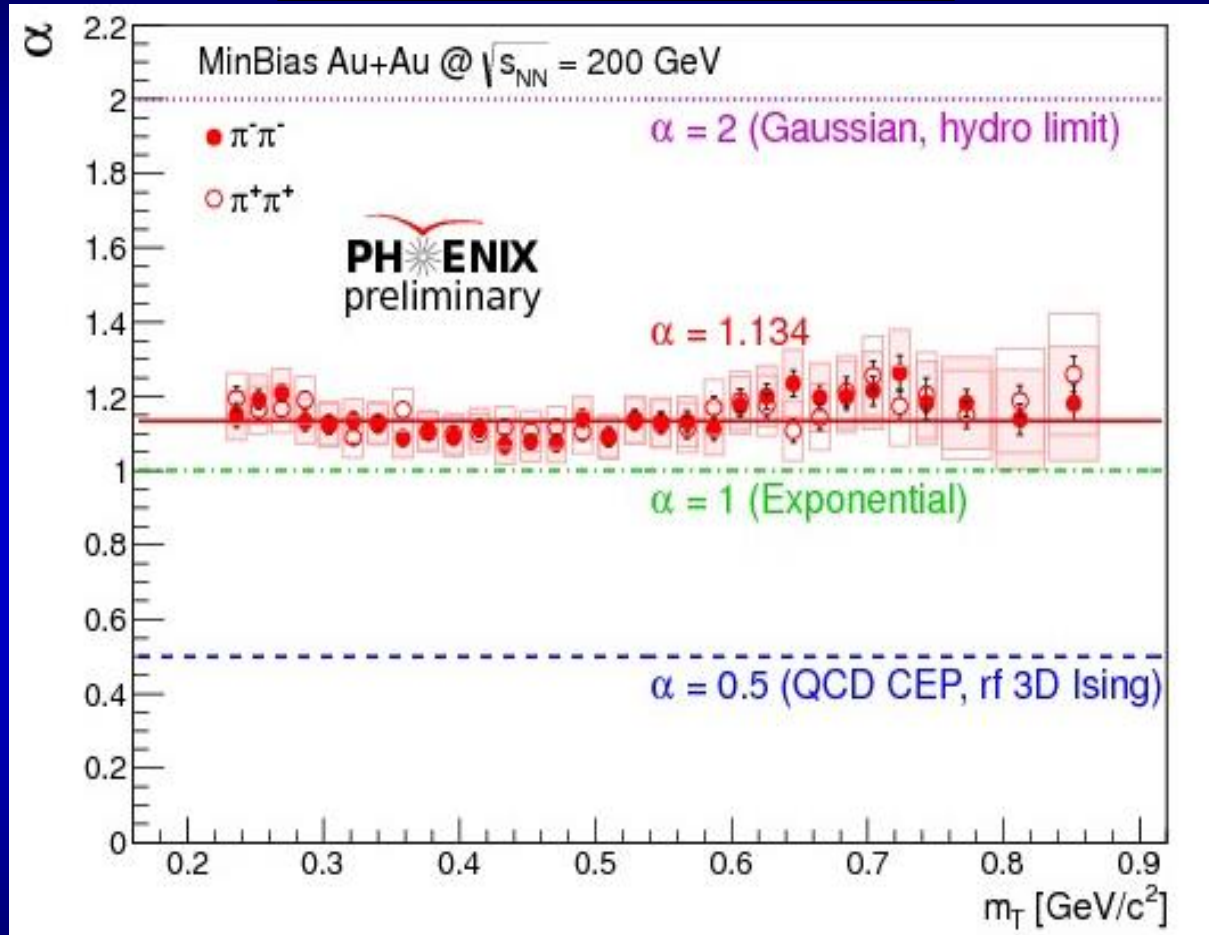
See the talk of M. Csanád for details

# HBT: Is $C(Q)$ an exponential?



# Interpretation of $\alpha$

$$C(q; \alpha) = 1 + \lambda \exp(-|qR|^\alpha).$$



Prediction: at QCD CEP,  $\alpha = \eta_c \leq 0.5$  (critical exponent of the correlation function)

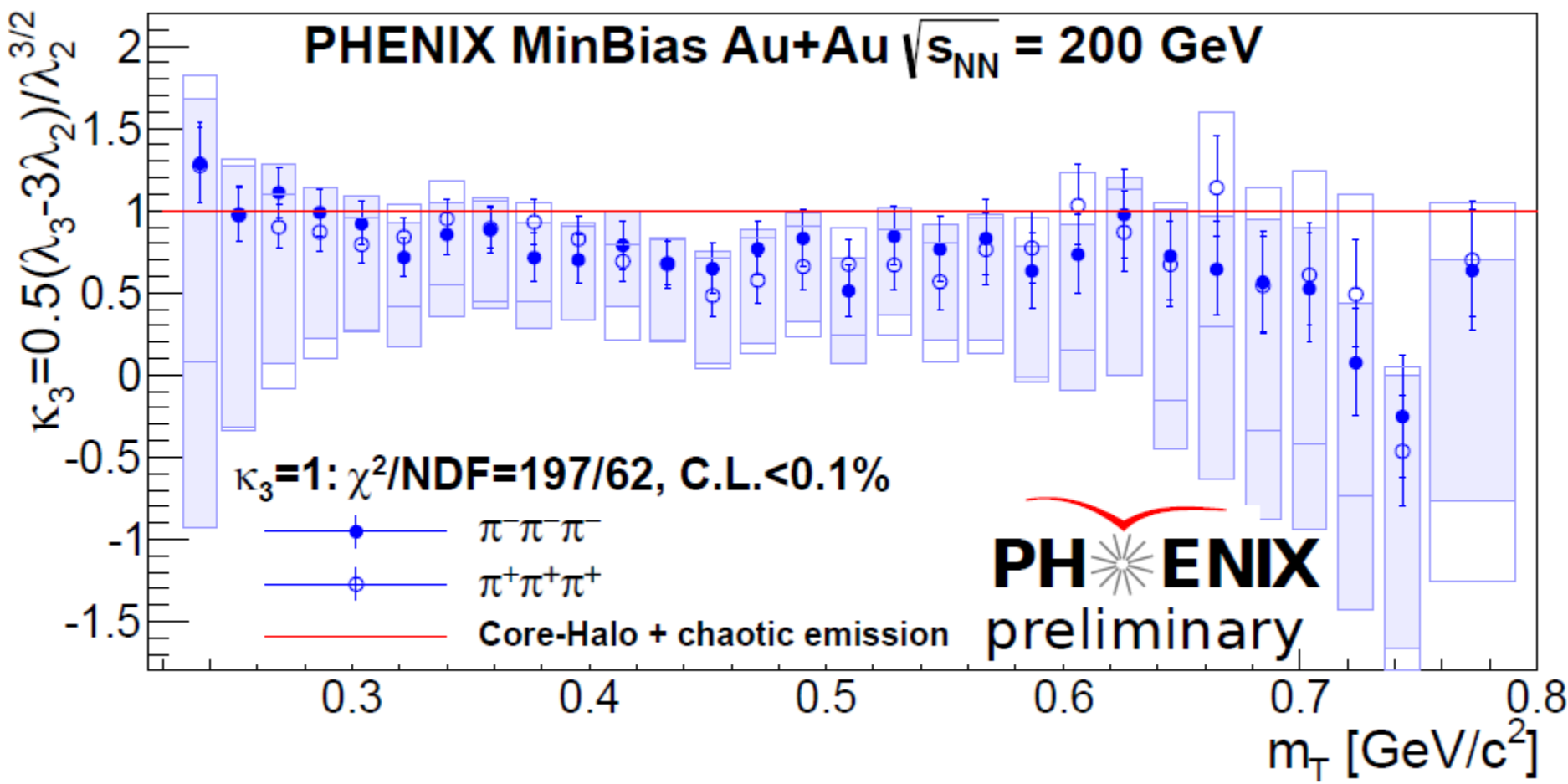
T. Cs, S.Hegy, T. Novák, W.A. Zajc, [nucl-th/0512060](https://arxiv.org/abs/nucl-th/0512060) T. Cs, [arXiv.org:0903.0669](https://arxiv.org/abs/0903.0669)

Search for the QCD critical point with  $\alpha$  ( $m_T$ ,  $\sqrt{s}$ , %, ...)



# HBT: Two-particle symmetrization - or not ?

See the talk of Tamás Novák for details

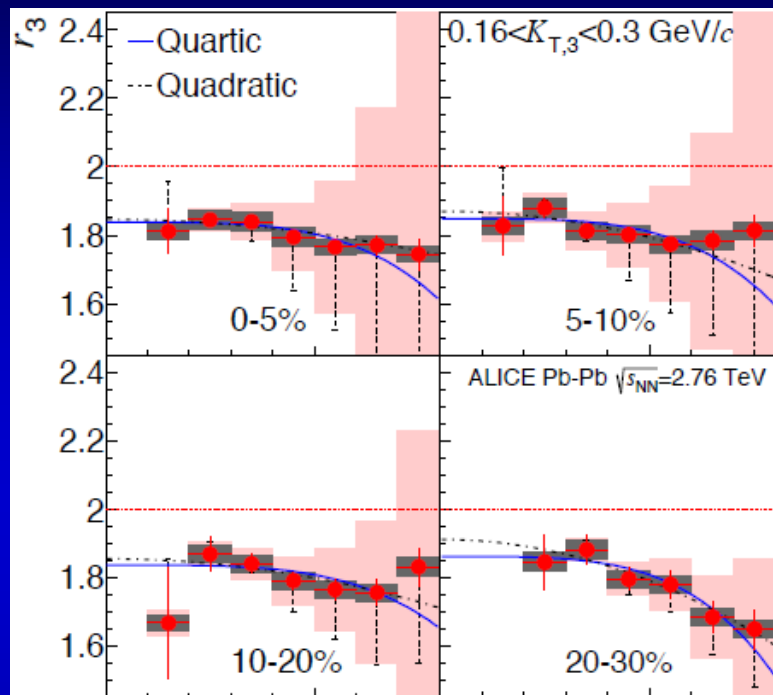


PHENIX preliminary data from A.Bagoly, [poster at QM17](#)

Centrality dependence? Excitation function? Partial coherence measurement possible!



# HBT: Two-particle symmetrization - or not ?



ALICE Pb+Pb @  $\sqrt{s_{NN}} = 2.76$  TeV

Centrality dependence!

Partial coherence if

$$r_3(Q=0) \neq 2$$

Result:

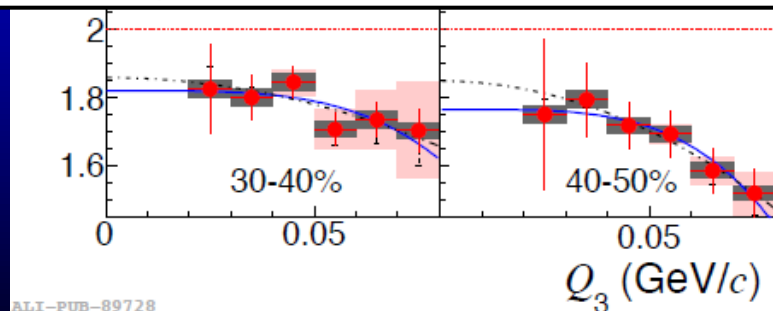
$$r_3(Q=0) < 2$$

$$p_c = 0.23 \pm 0.08$$

First  $3\sigma$  (+?) indication of

**Bose-Einstein condensation  
in a system of charged particles!**

$$r_3(Q_3) = \frac{c_3(q_{12}, q_{23}, q_{31}) - 1}{\sqrt{(C_2(q_{12}) - 1)(C_2(q_{13}) - 1)(C_2(q_{23}) - 1)}}$$

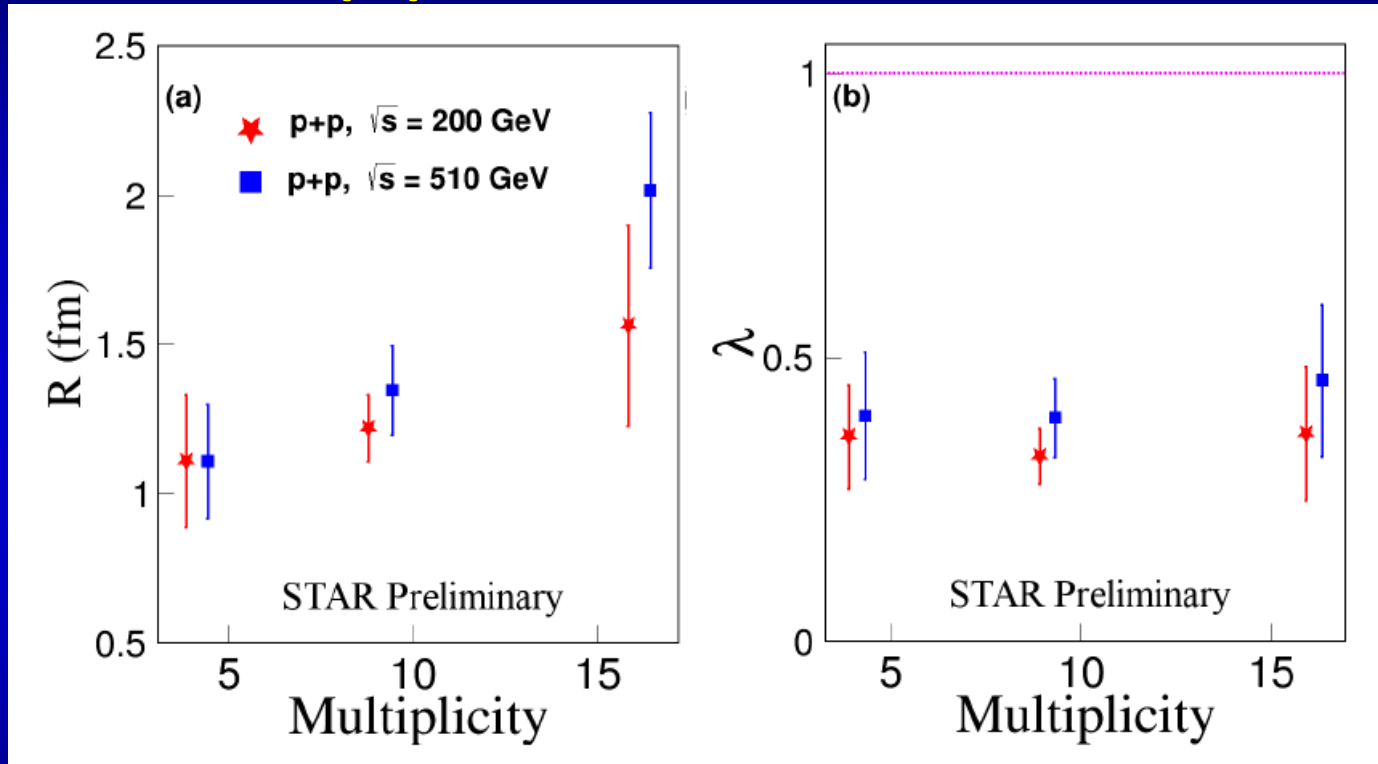


ALICE, Phys. Rev. C89 (2014) 024911

For details, see [D. Gangadharan's CERN talk](#)

# Cross-check: partial coherence for pions only - or not ?

STAR p+p  $\rightarrow$   $K^\pm K^\pm + X$   $\sqrt{s} = 200, 510$  GeV



Fewer long lived resonances expected to decay to K (but  $\phi$ )

Partial coherence not expected either: if  $\lambda_2$  (Kaons)  $< 2$  ( $f_c, p_c \neq (1,0)$ ) ?

STAR preliminary result:  $\lambda_2$ (Kaons)  $< 2$  in p+p . Halo from  $\phi$  ? Cross-checks, implications ?

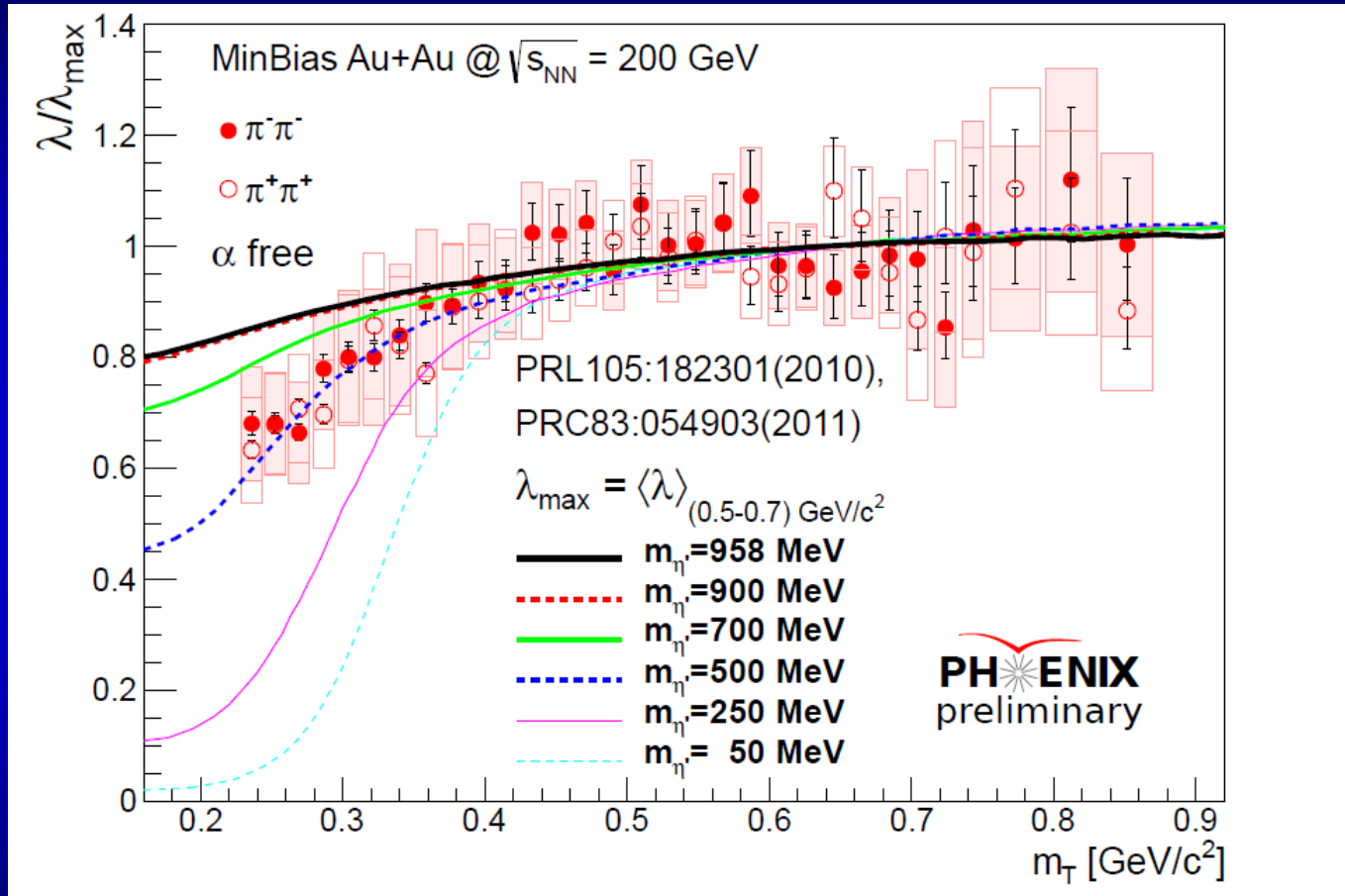
For details, see G. Nigmatkulov for STAR, [Proc. SQM 15](#)

For heavy ions: [G. Nigmatkulov's talk at HDNM17](#)

# Interpretation of $\lambda$

$$C(q; \alpha) = 1 + \lambda \exp(-|qR|^\alpha).$$

See the talk of Máté Csanád for details



PHENIX preliminary data from [arXiv:1610.05025](https://arxiv.org/abs/1610.05025)

Method: S. Vance, T. Cs., D. Kharzeev: PRL 81 (1998) 2205-2208 , [nucl-th/9802074](https://arxiv.org/abs/nucl-th/9802074)

Predictions: Cs. T., R. Vértési, J. Sziklai, [arXiv:0912.5526](https://arxiv.org/abs/0912.5526) [nucl-ex] [arXiv:0912.0258](https://arxiv.org/abs/0912.0258) [nucl-ex]

# Cross-check, pion laser model

Multi-Particle Symmetrization Effects

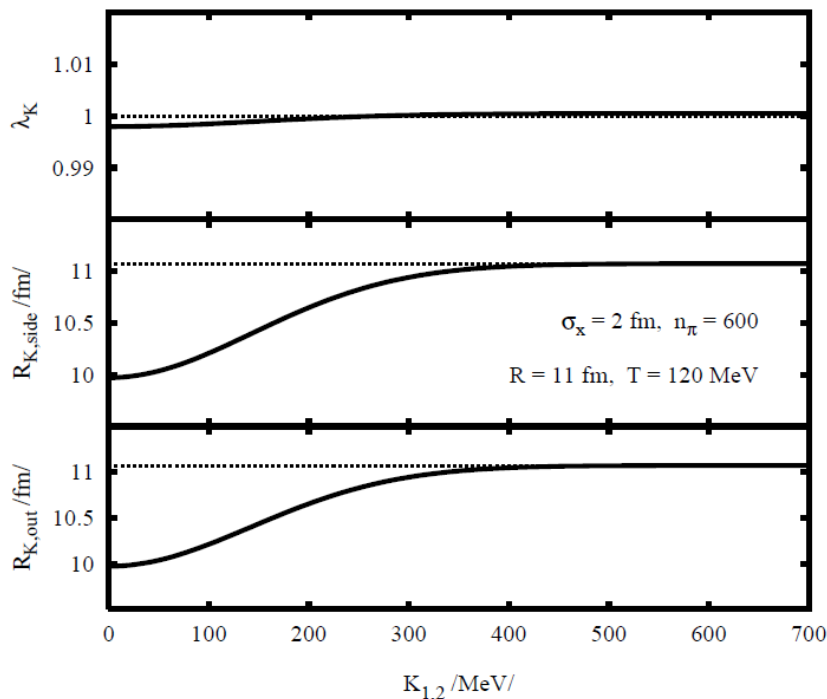


FIG. 1. Multi-particle symmetrization results at low  $K$  in a momentum-dependent reduction of the intercept parameter  $\lambda_K$ , the side-wards and the outwards radius parameters,  $R_{K,s}$  and  $R_{K,o}$  from their static values of 1 and  $R_e$ , respectively. The enhancement of these parameters at high momentum is hardly noticeable for large and hot systems.

$$\lambda(m_T)/\lambda_{\max} = 1 - H \exp(-(m_T^2 - m_\pi^2)/(2\sigma^2))$$

T. Cs, J. Zimányi, Phys.Rev.Lett. 80 (1998) 916-918,

## Pion-laser model

(Bose-Einstein condensation of wave-packets)

$$\lambda_{\mathbf{K}} = 1 + \frac{2}{(2x)^{3/2}} \left[ 1 - 2^{(5/2)} \exp\left(-\frac{\mathbf{K}^2}{\sigma_T^2}\right) \right]$$

$$x = R_e^2 \sigma_T^2$$

Suppression parameter  $H$

Controlled by HBT radii  $R_e$  and  $\sigma_T^2 = 2 \sigma^2$ , width of the depression in  $\lambda(m_T)$

What about numerical values ?

$R_e$ : limiting value of  $R(K)$  at large  $K$

$$\lambda_{\max} = 1 + 2 / (2x)^{3/2} > 1 \text{ slightly}$$

$$\rightarrow H = 2^{5/2} / (1 + 2^{1/2} x^{3/2})$$

$$\rightarrow R_e \geq 4 \text{ fm}, \sigma_T = 2 m_\pi, T \geq 170 \text{ MeV} \rightarrow x \geq 16 \rightarrow H \leq 0.06, \text{ while } H(\text{preliminary}) \sim 0.6$$

$$\rightarrow H, \text{ the size of the „hole” in } \lambda(m_T)/\lambda_{\max} \text{ or is too small for PHENIX preliminary Au+Au}$$



# Cross-check, Sinyukov-Tolstykh model

$$\rho(x_i) = (\pi R^2/2)^{-3/2} \exp(-2 \mathbf{x}_i^2/R^2).$$

The momentum spectrum

$$f(p) = \tilde{f}^2(p) = (2\pi p_0^2)^{-3/2} \exp(-\mathbf{p}^2/2 p_0^2)$$

$$C_{\pi-\pi}(p_1, p_2) = \chi \left( 1 + \lambda e^{-\frac{\Delta \mathbf{p}^2 R_{int}^2}{4}} \right).$$

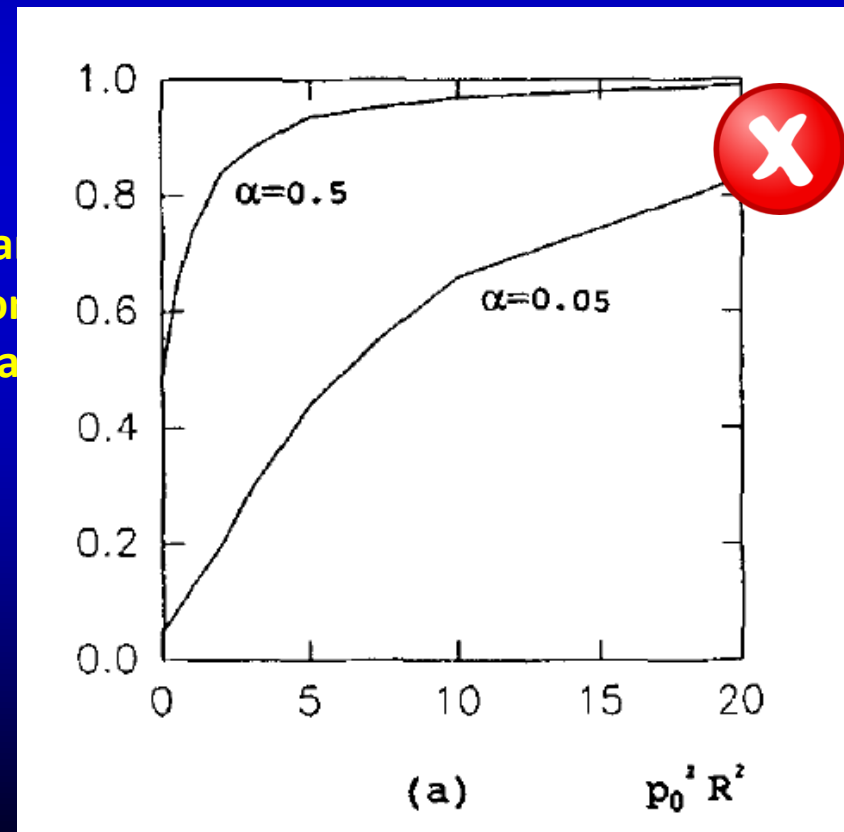
$$\lambda = \left[ 1 + \frac{1-\alpha}{\alpha (1 + p_0^2 R^2)^{3/2}} \right]^{-1}.$$

Intercept pa  
Decrease cor  
 $p_0 R \sim$  avera

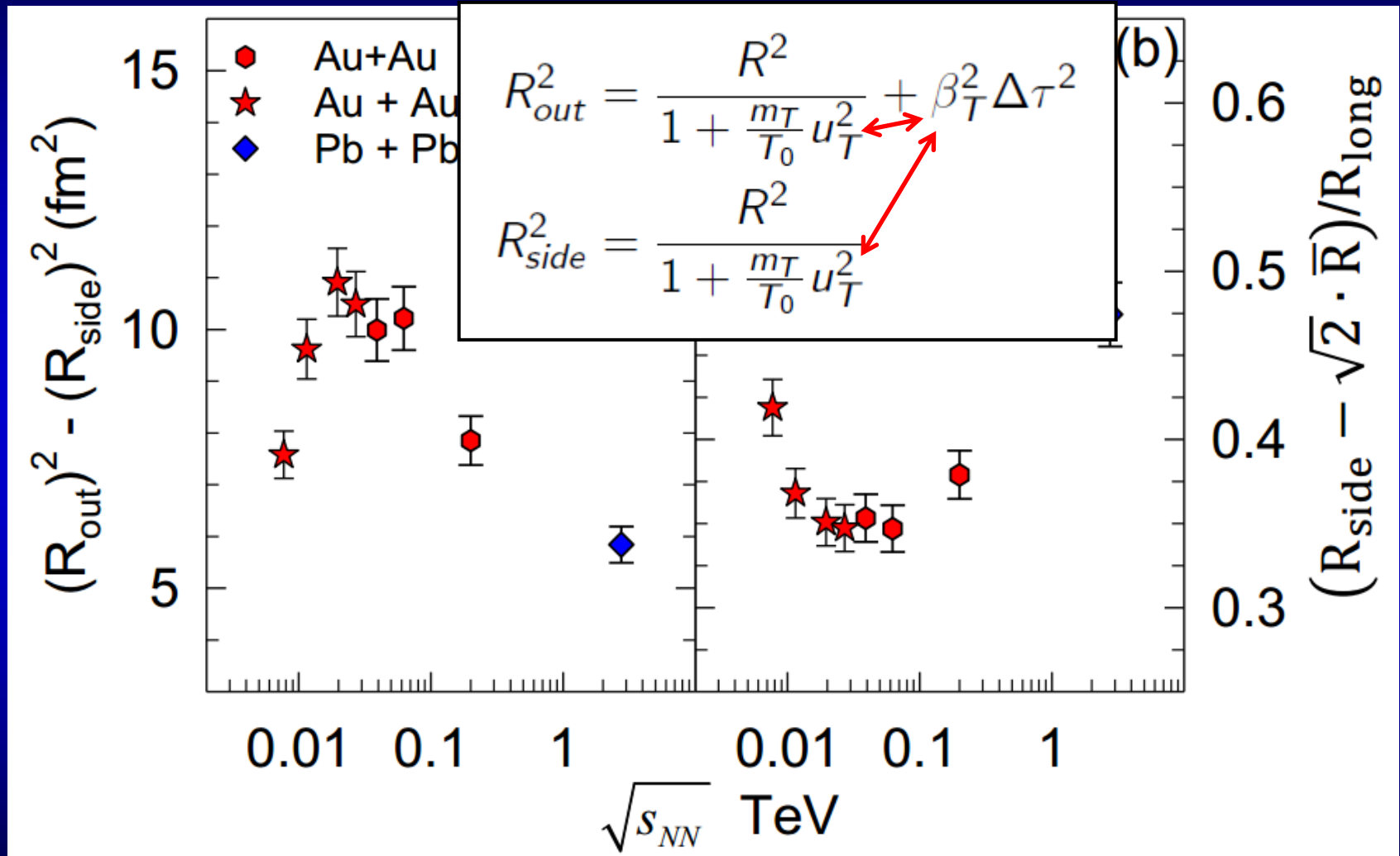
Yu, M. Sinyukov, A. Yu. Tolstykh,  
Z. Phys. C 61, 593 (1994)

Model based on partial coherence  
Static Gaussian source, size  $R$

Thermal momentum spectrum,  
Slope  $p_0^2 = m T_{eff}$



# HBT: Signal of QCD Critical Point - or not ?



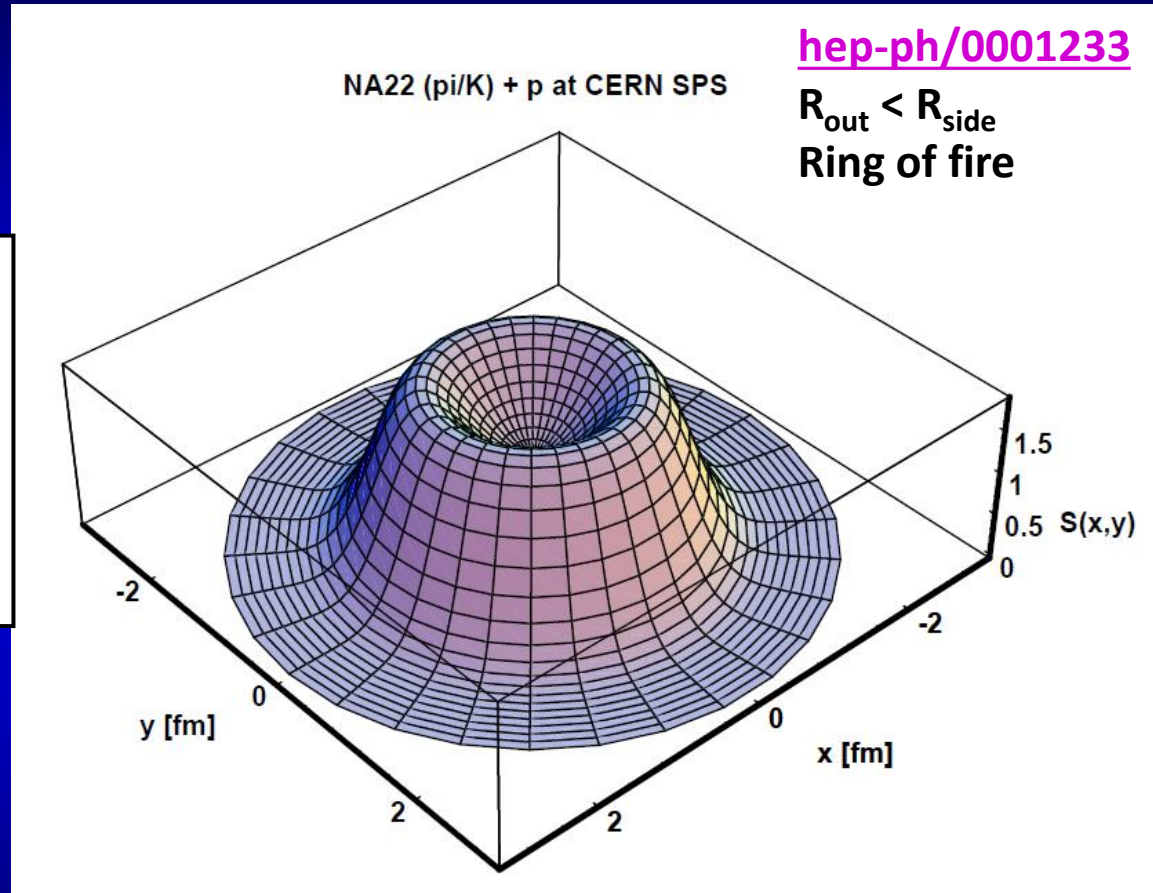
Clear indication of non-monotonic behavior in combined ALICE, STAR and PHENIX data

Roy Lacey:  $v_{s_{NN}} \sim 20\text{-}60$  GeV  $\rightarrow$  best value  $\sim 50$  GeV. Needs further study!

# HBT: Signal of QCD Critical Point - or not ?

$$R_{out}^2 = \frac{R^2}{1 + \frac{m_T}{T_0} u_T^2} + \beta_T^2 \Delta\tau^2$$

$$R_{side}^2 = \frac{R^2}{1 + \frac{m_T}{T_0} u_T^2}$$



Roy Lacey:  $v_{s_{NN}} \sim 20-60$  GeV  $\rightarrow$  best value  $\sim 50$  GeV.

Roy Lacey's 1<sup>st</sup> indication of QCD CEP **needs further study!**

Valid in a special  $m_T$  window: **only if radial flow  $u_T \sim$  velocity of the pair  $\beta_T$**

Validity can be extended: **also to  $R_{out} < R_{side}$ : use exact hydro solutions, allowing ring of fires !**



# HBT: New solutions of fireball hydro - triaxial, rotating and expanding

Coordinate-space ellipsoid at the beginning of time evolution

$$\vartheta_f$$

Final coordinate-space ellipsoid at freeze-out

$$\theta_p$$

"Momentum-space ellipsoid" (eigenframe of single-particle spectrum)

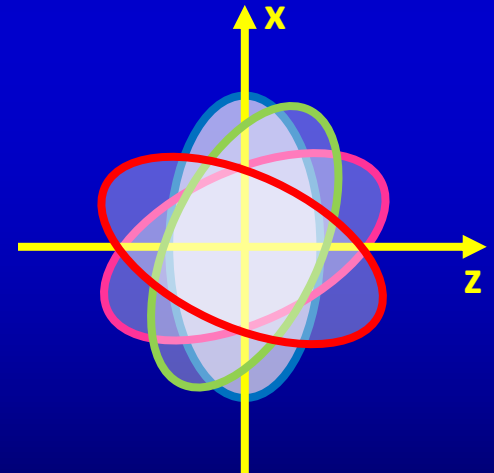
$$\theta_q$$

"HBT-space ellipsoid" (eigenframe of HBT correlations)

$$\theta'_p = \frac{1}{2} \arctan \left( \frac{2T'_{xz}}{T'_{xx} - T'_{zz}} \right) = \frac{1}{2} \arctan \left( \frac{2\omega R}{\dot{X} + \dot{Z}} \right)$$

$$\theta_p = \vartheta_f + \theta'_p$$

$$\theta'_{q,i} = \frac{1}{2} \arctan \left( \frac{2XZT'_{xz,i}}{Z^2T'_{xx,i} - X^2T'_{zz,i}} \right)$$



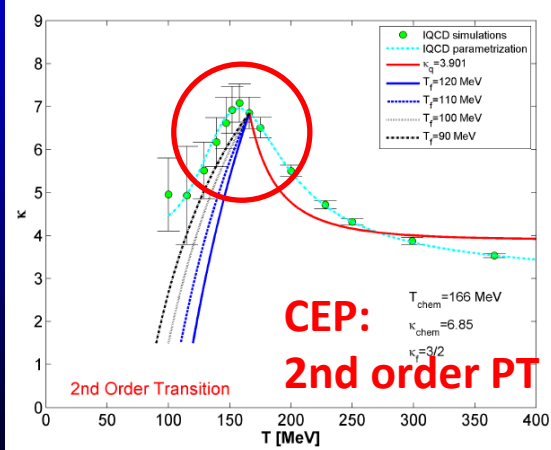
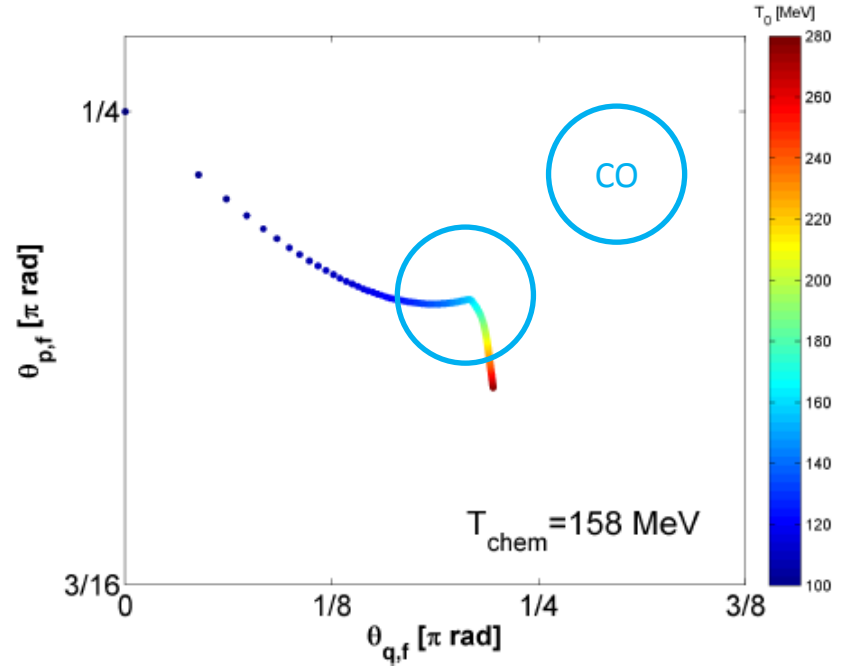
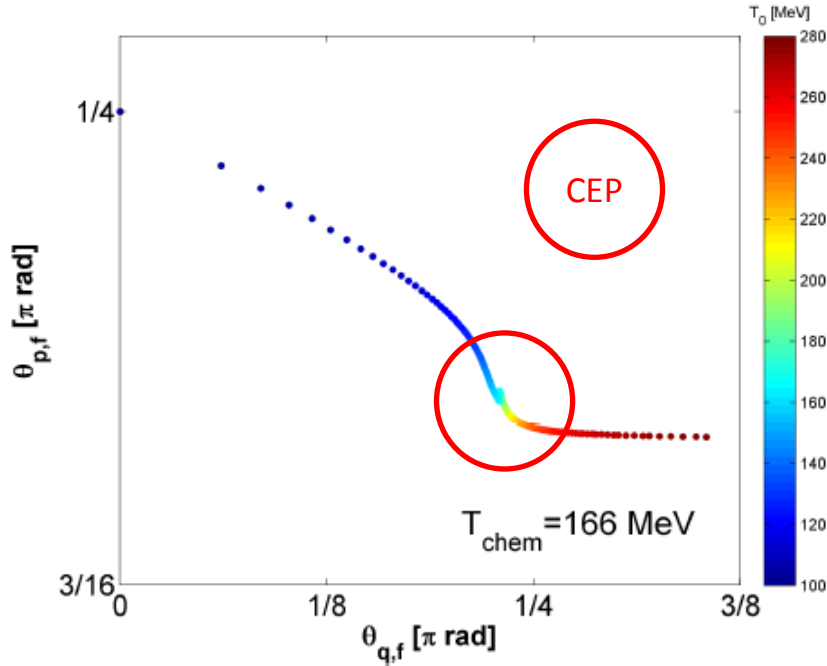
Three angles of rotation: in momentum space  $\mathbf{p}$ , in HBT's  $\mathbf{q}$ -space, and in coordinate space  $\mathbf{r}$

$$\theta_p \neq \theta_q \neq \theta_r$$

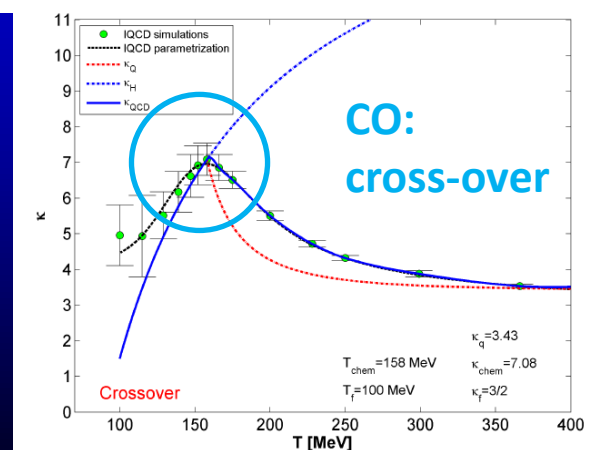
See Gábor Kasza's [talk](#)  
for more details  
[arXiv:1610.02197](#)

M.I. Nagy and T. Cs: [arXiv:1606.09160](#)

# New, triaxial and rotating solutions lattice QCD EoS for a 2<sup>nd</sup> order QCD transition



See G. Kasza's [talk](#) for more details



# Summary and conclusions

**Positive definitene form – or not?**

**Has to be checked:**

**L3 and CMS data show anticorrelated region  
in  $e^+e^-$  at LEP and in  $pp$  at LHC**

**Two-particle symmetrization effect – or not?**

**Has to be checked:**

**ALICE data indicates possible partial coherence  
→ higher order symmetrization effects**

**Gaussian shape – or not?**

**Has to be checked:**

**PHENIX preliminary data indicates non-Gaussian structure  
Levy index of stability  $\alpha < 2$  (Gaussian) significantly**

**New possibilities**

**model independent shape analysis  
to measure  $\eta'$  modification  
to identify QCD CEP from angles of rotation**

# Backup slides

## Questions?

# Hanbury Brown: a compound family name

1953MNRAS.113..123H

## A SURVEY OF 23 LOCALIZED RADIO SOURCES IN THE NORTHERN HEMISPHERE

*R. Hanbury Brown and C. Hazard*

*Mon. Not. R. astr. Soc.* (1971) **151**, 161–176.

1971MNRAS.151..161H

## A STUDY OF $\alpha$ VIRGINIS WITH AN INTENSITY INTERFEROMETER

*D. Herbison-Evans, R. Hanbury Brown, J. Davis and L. R. Allen*

(Received 28 August 1970)

*R. Hanbury Brown, J. Davis and L. R. Allen*

(Received 1973 November 5)

SERVICES ELECTRONICS RESEARCH LABORATORY, DARTMOUTH

1974MNRAS.167..121H

M



# Hanbury Brown: a family name

1971MNRAS...151..161H

*Mon. Not. R. astr. Soc.* (1971) **151**, 161–176.

## A STUDY OF $\alpha$ VIRGINIS WITH AN INTENSITY INTERFEROMETER

*D. Herbison-Evans, R. Hanbury Brown, J. Davis and L. R. Allen*

(Received 28 August 1970)

**Grandfather: Sir Robert Hanbury Brown, K.C.M.G., a notable irrigation engineer ([Wiki link](#))**

**Father: Basil Hanbury Brown**

**Twin sons:**

- Robert Hanbury Brown
- Jordan Hanbury Brown

**Daughter:**

- Marion Hanbury Brown

*„It is not all that unusual that an English last name is a compound one, with or without a hyphen.“*

*Wes Metzger*

**Thank you Wes!**

- For private communications on the family tree of Sir Robert Hanbury Brown

# R. Q. Twiss: Richard Quentin TWISS

## OBITUARIES

### Richard Quentin Twiss 1920–2005

**Fellow and Eddington Medallist of the RAS, pioneer of radio astronomy and interferometry.**

Richard Twiss was born in Simla in India in 1920. He was educated at Rugby School and completed the Mathematical Tripos at Cambridge with distinction in 1941. He spent the war years in the Admiralty working on radar, and after the war was appointed British Liaison Officer to the Research Laboratory for Electronics (RLE) at MIT in the USA, where he assisted in editing the 27-volume RLE

and the non-classical Bose–Einstein statistics of the photons must be taken into account. The debate surrounding the HBT effect led to a much deeper understanding of the nature of light and marks the beginning of modern quantum optics. In 1968 Hanbury Brown and Twiss were jointly awarded the Eddington Medal of the RAS for their work.

In 1955 Richard moved to Sydney, Australia, where he took up a research position in the CSIRO Division of Radiophysics. As well as doing more work on the HBT effect, his work on electromagnetic-wave propagation laid the theoretical foundation for both astrophysical masers

ferometer in 1972. Although the Mark II did not produce significant astronomical results, it was a major step in the development of modern optical interferometry. The Monteporzio station was closed in 1976 and Richard effectively retired from active scientific research to pursue his interests in art and music.

In 1998 Richard came to Sydney for the summer opera season and visited the Sydney University Stellar Interferometer, the modern Michelson successor to the Narrabri Stellar Intensity Interferometer, also at Narrabri. Of course, much had changed since the 1950s. He visited regularly thereafter, and shortly before his death in Sydney he applied for Australian permanent residence.  
*Bill Tango*

4.38

A&G • August 2006 • Vol. 47

#### Reference:

- **Bill Tango: Richard Quentin Twiss (1920-2005), A&G vol 47, p. 4.38 (2006)**

#### Apologies!

- **For my earlier mistaken communications on resolving „Q.” in Richard Quentin Twiss**



# HBT: 1 + positive definite term: - how to check ?

Model-independent method, to analyze Bose-Einstein correlations

IF experimental data satisfy

- The measured data **tend to a constant** for large values of the observable  $Q$ .
- There is a **non-trivial structure** at some definite value of  $Q$ , shift it to  $Q = 0$ .

Model-independent, but  
experimentally testable:

- $t = QR$
- dimensionless scaling variable
- **approximate form** of the correlations  $w(t)$
- **Identify  $w(t)$**  with a measure in an abstract Hilbert-space

$$\int dt w(t) h_n(t) h_m(t) = \delta_{n,m},$$

$$f(t) = \sum_{n=0}^{\infty} f_n h_n(t),$$

$$f_n = \int dt w(t) f(t) h_n(t).$$

e.g.  $t = Q_I R_I$



T. Csörgő and S. Hegyi, hep-ph/9912220, T. Csörgő, hep-ph/001233

# HBT: 1 + positive definite term: How to check ?

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1) N_1(\mathbf{k}_2)},$$

$$C_2(t) = \mathcal{N} \left\{ 1 + \lambda_w w(t) \sum_{n=0}^{\infty} g_n h_n(t) \right\}$$

## Model-independent AND experimentally testable:

- method for any **approximate** shape  $w(t)$
- the core-halo intercept parameter of the CF is
- coefficients by numerical integration (fits to data)
- condition for applicability: experimentally **testable**
- **Nearly Gauss** correlations,  $(-\infty, \infty) \rightarrow$  Edgeworth
- **Nearly Gauss** correlations,  $(0, \infty) \rightarrow$  **Gauss** 
- **Nearly exponential** correlations,  $(0, \infty) \rightarrow$  Laguerre
- **Nearly Levy** correlations,  $(0, \infty) \rightarrow$  **Levy expansion** 

$$\lambda_* = \lambda_w \sum_{n=0}^{\infty} g_n h_n(0)$$

$$g_n = \int dt R_2(t) h_n(t)$$

$$\int dt [R_2^2(t)/w(t)] < \infty$$

# Edgeworth expansion method

Gaussian  $w(t)$ ,  $-\infty < t < \infty$

$$t = \sqrt{2}QR_E,$$

$$w(t) = \exp(-t^2/2),$$

$$\int_{-\infty}^{\infty} dt \exp(-t^2/2) H_n(t) H_m(t) \propto \delta_{n,m},$$

$$H_n(t) = \exp(t^2/2) \left( -\frac{d}{dt} \right)^n \exp(-t^2/2).$$

$$H_1(t) = t,$$

$$H_2(t) = t^2 - 1,$$

$$H_3(t) = t^3 - 3t,$$

$$H_4(t) = t^4 - 6t^2 + 3, \dots$$

$$C_2(Q) = \mathcal{N} \left\{ 1 + \lambda_E \exp(-Q^2 R_E^2) \times \left[ 1 + \frac{\kappa_3}{3!} H_3(\sqrt{2}QR_E) + \frac{\kappa_4}{4!} H_4(\sqrt{2}QR_E) + \dots \right] \right\}.$$

**3d generalization straightforward**

- Applied by NA22, L3, STAR, PHENIX, ALICE, CMS (LHCb)

# Gauss expansion method

Gaussian  $w(t)$ ,  $0 < t < \infty$

$$\begin{aligned}L_0(t | \alpha = 2) &= \frac{\sqrt{\pi}}{2}, \\L_1(t | \alpha = 2) &= \frac{1}{2} \{ \sqrt{\pi t} - 1 \}, \\L_2(t | \alpha = 2) &= \frac{1}{32} \left\{ (\pi - 2)t^2 - \sqrt{\pi t} + 2 - \frac{\pi}{2} \right\}.\end{aligned}$$

Provides a new expansion around a Gaussian shape that is defined for the non-negative values of  $t$  only.

**Edgeworth expansion different**, its around two-sided Gaussian, includes non-negative values of  $t$  also.

# Laguerre expansion method

Model-independent but  
experimentally tested:

**w(t): Exponential**

**0 < t < ∞**

**Laguerre polynomials**

$$t = QR_L,$$
$$w(t) = \exp(-t)$$

$$\int_0^{\infty} dt \exp(-t) L_n(t) L_m(t) \propto \delta_{n,m},$$

$$L_n(t) = \exp(t) \frac{d^n}{dt^n} (-t)^n \exp(-t).$$

$$L_0(t) = 1,$$
$$L_1(t) = t - 1,$$

$$C_2(Q) = \mathcal{N} \left\{ 1 + \lambda_L \exp(-QR_L) \left[ 1 + c_1 L_1(QR_L) + \frac{c_2}{2!} L_2(QR_L) + \dots \right] \right\}$$

**First successful tests**

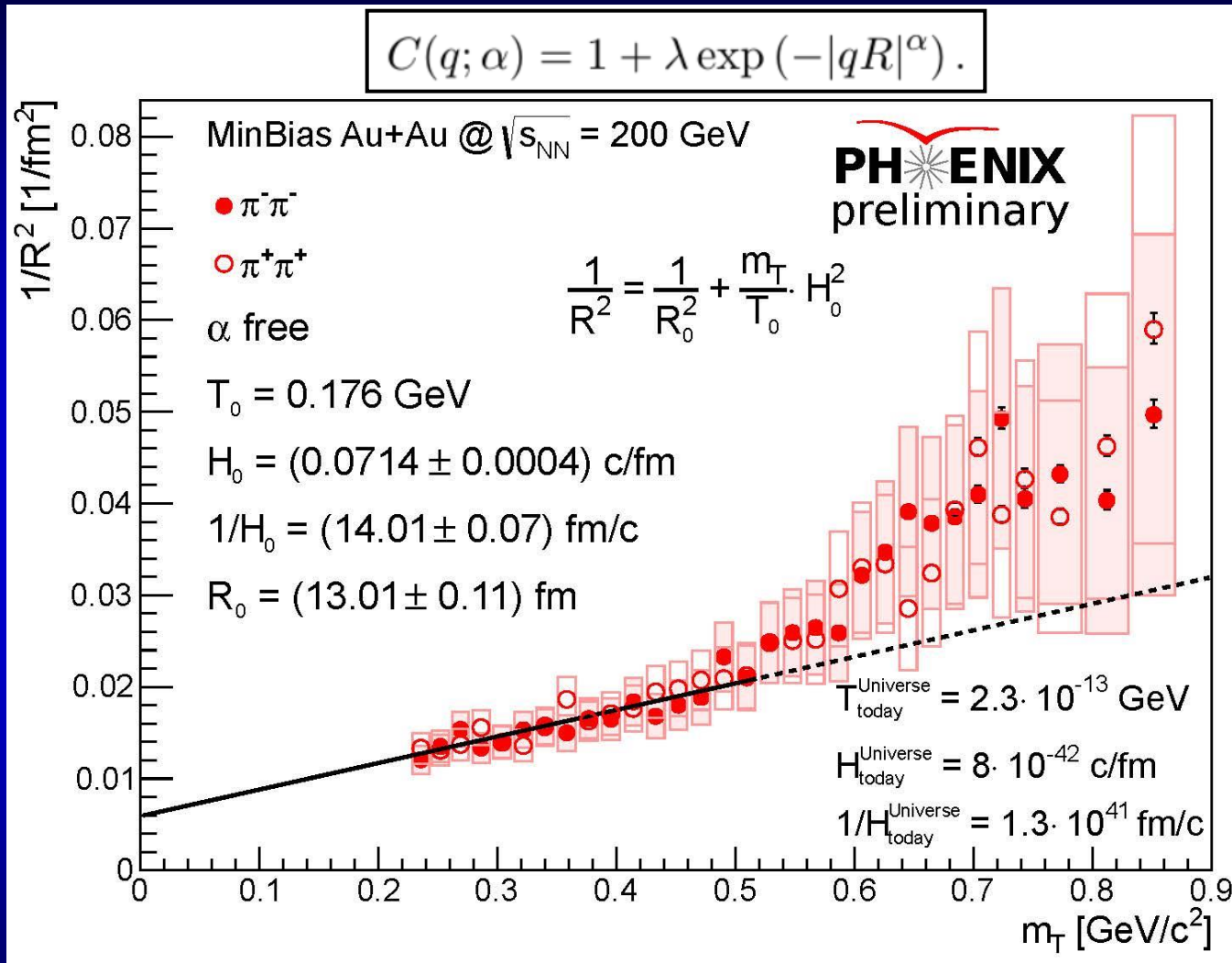
on NA22, UA1 data , convergence criteria satisfied

Intercept:  $\lambda_* \sim 1$

$$\int_0^{\infty} dt R_2^2(t) \exp(+t) < \infty,$$

$$\lambda_* = \lambda_L [1 - c_1 + c_2 - \dots],$$
$$\delta^2 \lambda_* = \delta^2 \lambda_L [1 + c_1^2 + c_2^2 + \dots] + \lambda_L^2 [\delta^2 c_1 + \delta^2 c_2 + \dots]$$

# HBT: Interpretation of R



Possibility: hydro scaling behaviour of R at low  $m_T$

Hubble ratio of Big Bang and Little Bangs  $\sim 10^{40}$  (needs centrality dependence,  $\alpha = 2 \dots$ )

M. Csanád, T. Cs, B. Lörstad, A. Ster, [nucl-th/0403074](https://arxiv.org/abs/nucl-th/0403074)

# HBT: New solutions of fireball hydro - but academic ?

QM ( $T_i \geq T \geq T_{chem}$ )

$$\partial_t \sigma + \nabla (\sigma \mathbf{v}) = 0$$

$$T \sigma (\partial_t + \mathbf{v} \nabla) \mathbf{v} = -\nabla p$$

$$\frac{1+\kappa}{T} \left[ \frac{d}{dT} \frac{\kappa T}{1+\kappa} \right] (\partial_t + \mathbf{v} \nabla) T + \nabla \mathbf{v} = 0$$

$$p = \sigma T / (1 + \kappa)$$

HM ( $T_{chem} > T \geq T_f$ )

$$\partial_t n_i + \nabla (n_i \mathbf{v}) = 0, \quad \forall i$$

$$\sum_i m_i n_i (\partial_t + \mathbf{v} \nabla) \mathbf{v} = -\nabla p$$

$$\frac{1}{T} \left[ \frac{d(\kappa T)}{dT} \right] (\partial_t + \mathbf{v} \nabla) T + \nabla \mathbf{v} = 0$$

$$p = \sum_i p_i = T \sum_i n_i$$

$$\varepsilon + p = \sum_i \mu_i n_i + T \sigma,$$

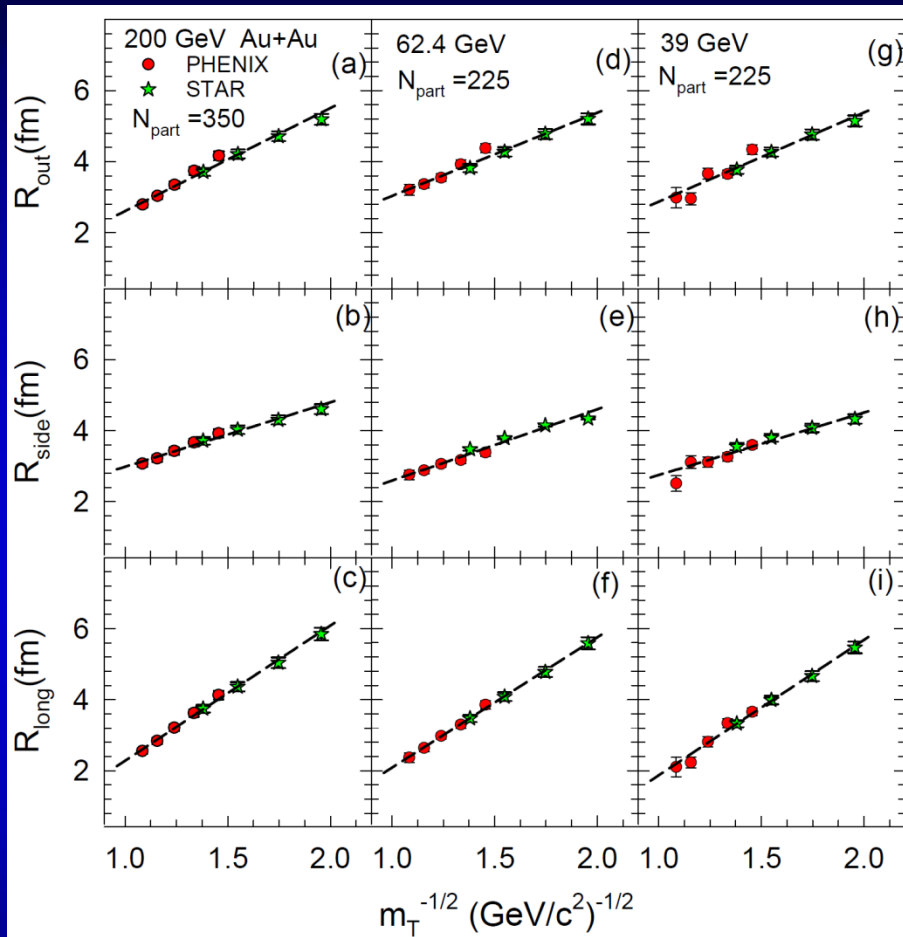
$$\varepsilon + p \approx T \sigma, \quad (T_i \geq T \geq T_{chem}),$$

$$\varepsilon + p \approx \sum_i m_i n_i \quad (T_{chem} > T \geq T_f).$$

See the talk of G. Kasza  
for details



# HBT: Signals of 3d hydro flow



$$\frac{1}{\Delta\bar{\eta}^2} = \frac{1}{\Delta\eta^2} + \frac{M_t}{T_0},$$

$$\bar{R}_\perp^2 = \frac{R_G^2}{1 + \frac{M_t}{T_0} (\langle u_t \rangle^2 + \langle \frac{\Delta T}{T} \rangle_r)},$$

$$R_i^2 = \bar{\tau}^2 \Delta\bar{\eta}^2,$$

$$R_o^2 = \bar{R}_\perp^2 + \beta_t^2 \Delta\bar{\tau}^2,$$

$$R_s^2 = \bar{R}_\perp^2$$

$$R_s^2 = R_\perp^2,$$

$$R_o^2 = R_\perp^2 + \beta_t^2 [\cosh^2(\bar{\eta}) R_\perp^2 + \sinh^2(\bar{\eta}) R_\parallel^2],$$

$$R_{ol}^2 = -\beta_t \sinh(\bar{\eta}) \cosh(\bar{\eta}) (R_\perp^2 + R_\parallel^2),$$

$$R_l^2 = \cosh^2(\bar{\eta}) R_\parallel^2 + \sinh^2(\bar{\eta}) R_\perp^2,$$

Indication of hydro scaling behaviour of  $R(\text{side,out,long})$  at low  $m_T$

$R_{\text{long}}$   $m_T$ -scaling: Yu. Sinyukov and A. Makhlin: [Z.Phys. C39 \(1988\) 69](#)

$R_{\text{side}}, R_{\text{out}}, R_{\text{long}}$   $m_T$ -scaling: T. Cs, B. Lörstad, [hep-ph/9509213](#) (shells of fire vs fireballs)

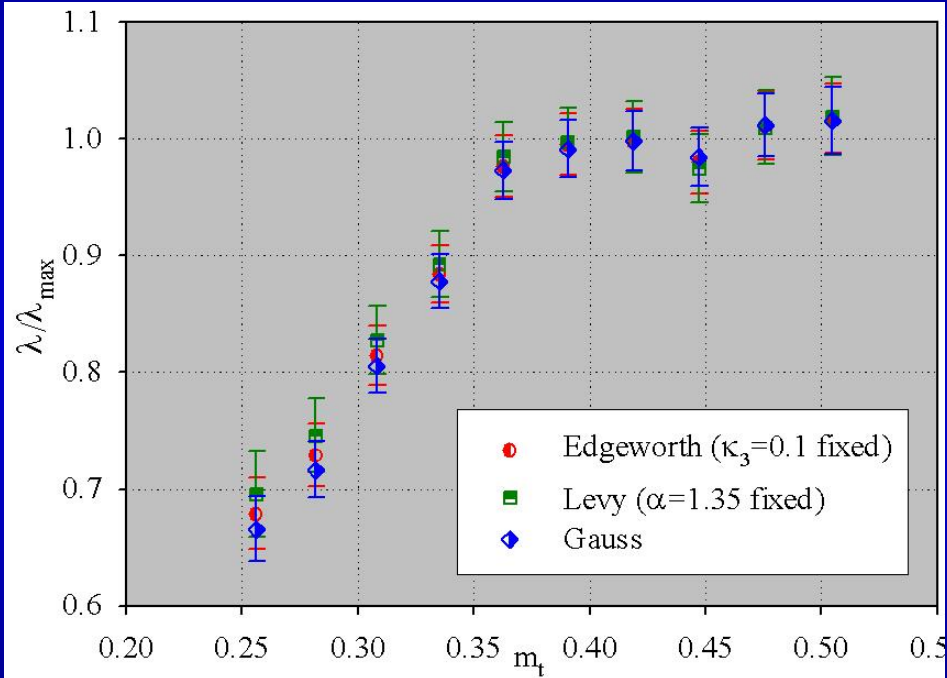
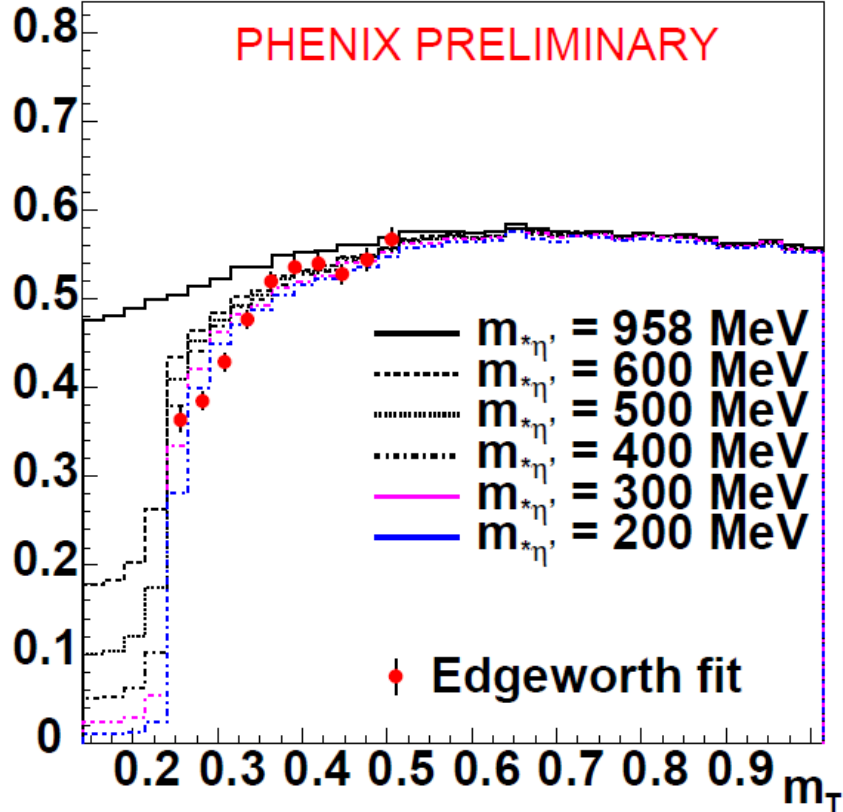
S. Chapman, P. Scotto, U. W. Heinz, [hep-ph/9408207](#)

$$\frac{1}{R^2} = \frac{1}{R_0^2} + \frac{m_T}{T_0} \cdot H_0^2$$

# HBT: Interpretation of $\lambda$ , $\alpha$ and R

$\lambda (\pi^+ \pi^+) - \text{RUN4 200GeV Au+Au}$

PHENIX PRELIMINARY



PHENIX preliminary data from  
[arXiv:nucl-ex/0509042](https://arxiv.org/abs/nucl-ex/0509042)