Recent progress on event generators

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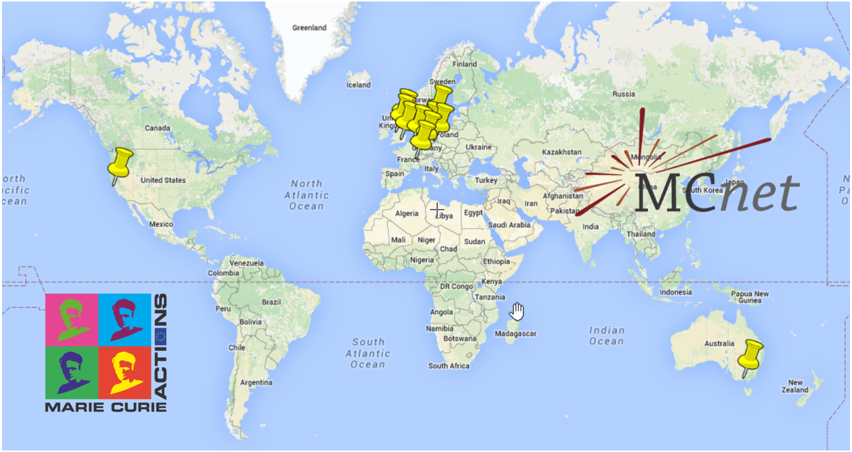
ATLAS Jamboree

SLAC, 06/13/2016

U.S. DEPARTMENT OF
ENERGY
Office of Science

SLAC NATIONAL ACCELERATOR LABORATORY

SLAC joins MCnet phase 3 as an associate partner



7 Countries, 11 Institutes, $\sim$ 100 Scientists

Where we left off last time ...

State of the art

- Parton shower Monte Carlo
- Automated 1-loop calculations
- Matching NLO to parton shower
- Merging of NLO calculations

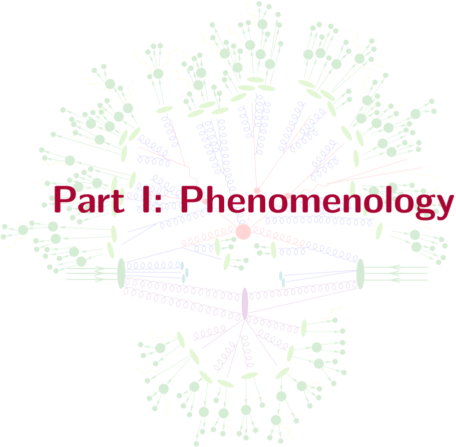
Cutting edge technology

- Differential NNLO calculations
- NNLO+parton shower

Desperately needed

- Parton showers at higher logarithmic accuracy
- Uncertainty estimates for parton showers

So what happened in the meantime?

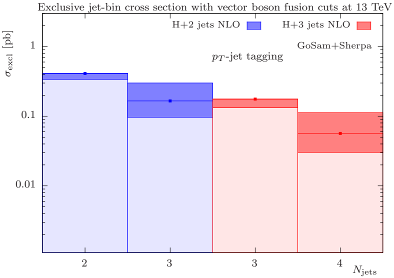


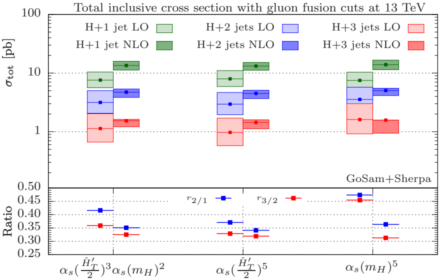
Part I: Phenomenology

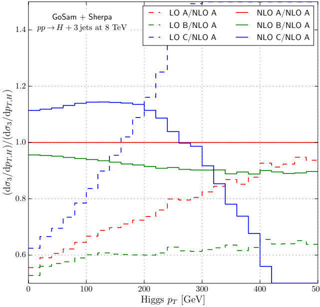
Higgs+3 jet at NLO

[Greiner,Luisoni,Schönherr,Winter,Yundin,SH] arXiv:1506.01016

- $H + 2$jets through gluon fusion is irreducible background to VBF → get handle on jet veto efficiency through $H + 3$jets at NLO
- Test jet scaling in process with topology similar to Drell-Yan lepton pair production







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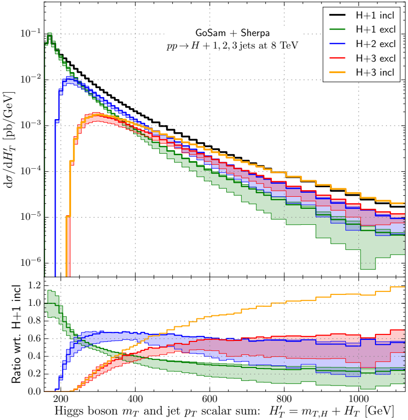

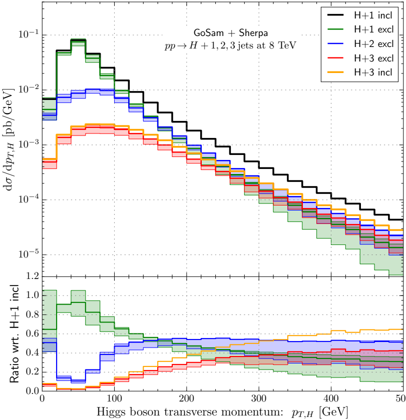

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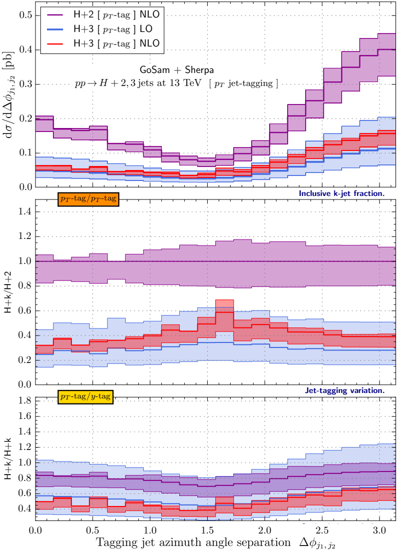
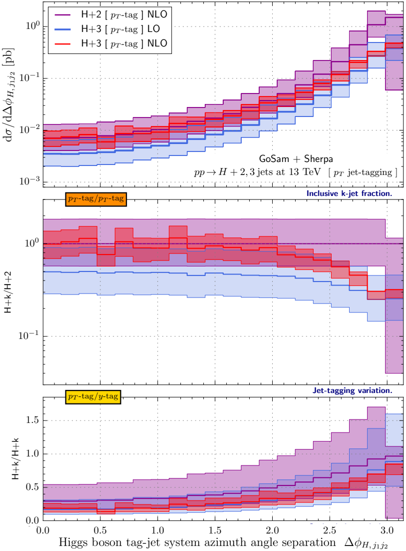
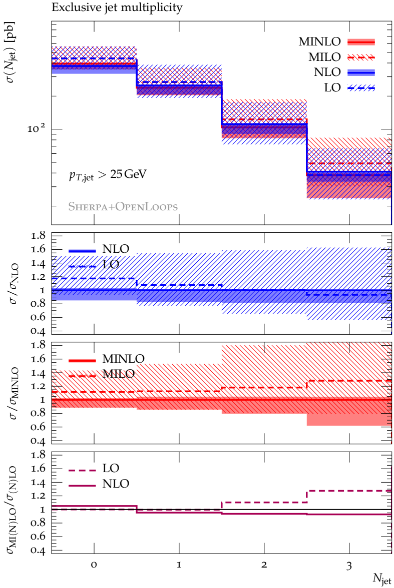




Top-quark pair production at NLO

[Maierhöfer,Moretti,Pozzorini,Siegert,SH] in preparation

- First computation of $t\bar{t}+3$ jets at NLO / MINLO accuracy
- Sherpa NLO MC framework using Comix [Gleisberg,SH] arXiv:0808.3674 combined with OpenLoops [Cascioli,Maierhöfer,Pozzorini] arXiv:1111.5206
- Public results in NTuple format [BlackHat collaboration] arXiv:1310.7439 for easy analysis & recycling will be made available at NERSC
- Scale dependence studied using $H_{T,m} = \sum m_\perp$ and MINLO [Hamilton,Nason,Zanderighi] arXiv:1206.3572 extended to massive partons



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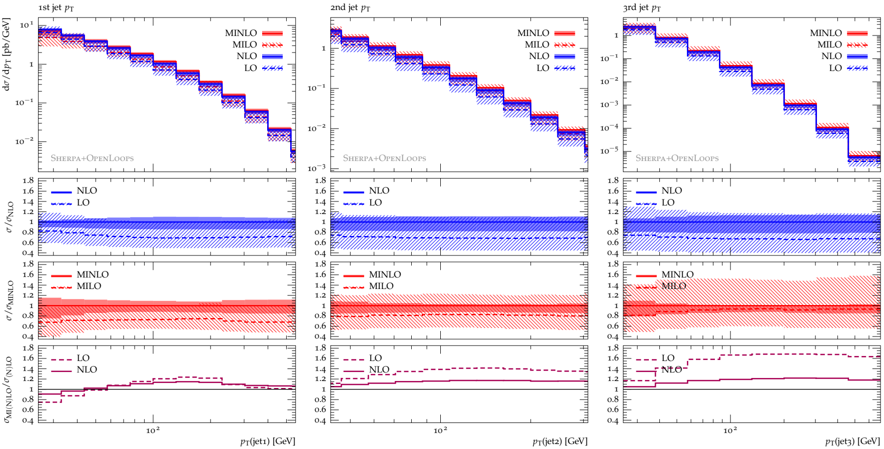
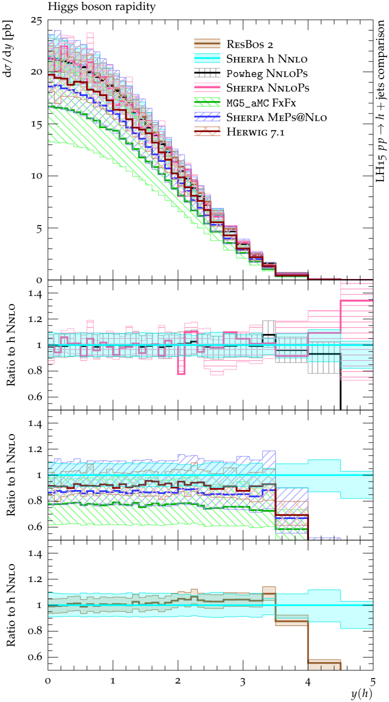
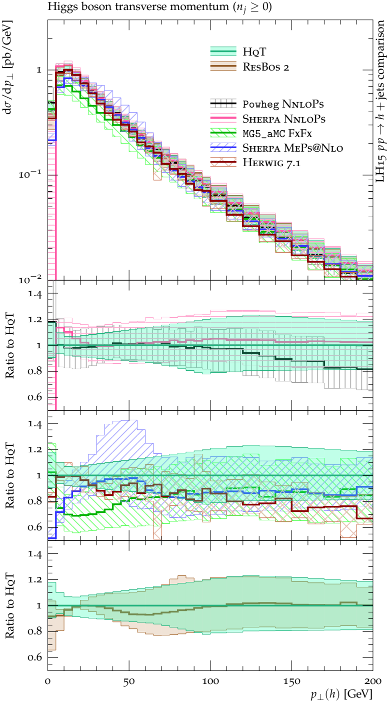


Comparison of approaches to Higgs+jets simulation

[Les Houches SM WG] arXiv:1605.04692

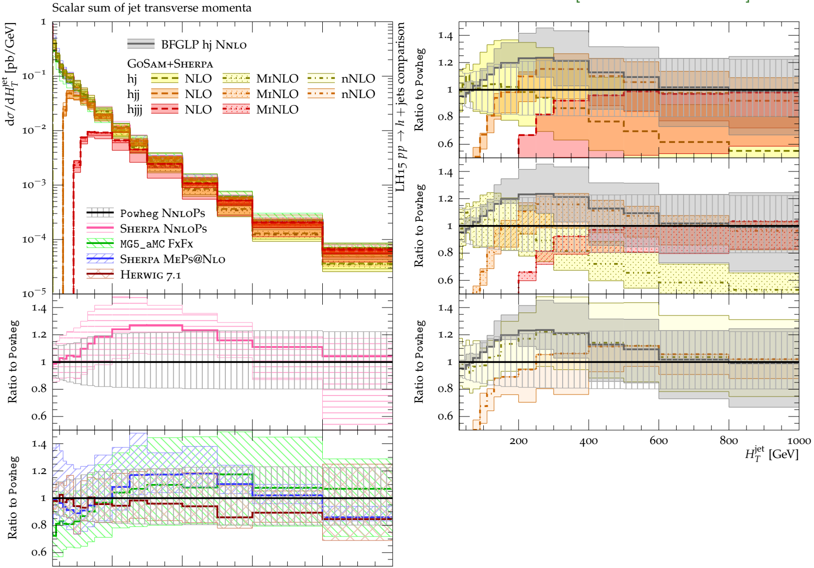
- Setup
  - Stable Higgs
  - anti-k_T jets, $R=$0.4, $p_{T,j}>$30 GeV $|\eta_j<$4.4
- Calculations & tools in the comparison
  - Fixed-order NLO for $h+\leq 3$ jets, $H_T'/2$ & MINLO
  - LoopSim
  - NNLO for $pp \to h$ (Sherpa), $pp \to h+j$ (BFGLP)
  - Resummed h-p_T (HqT & ResBos)
  - Resummed jet veto cross section (STWZ)
  - NNLO+PS (POWHEG & Sherpa)
  - Multi-jet merging up to 2 jets at NLO (Madgraph5_aMC@NLO, Herwig 7.1)
  - Multi-jet merging up to 3 jets at NLO (Sherpa)
  - High-energy resummation (HEJ)


Higgs boson rapidity
dσ/dy [pb]
25
20
15
10
5
0
ResBos 2
Sherpa h Nnlo
Powheg NnloPs
Sherpa NnloPs
MG5_aMC FxFx
Sherpa MePs@Nlo
Herwig 7.1
LH15 $pp \to h +$ jets comparison
Ratio to h Nnlo
Ratio to h Nnlo
Ratio to h Nnlo
1.4
1.2
1
0.8
0.6
0
1
2
3
4
5
$y(h)$


Higgs boson transverse momentum ($n_j \geq 0$)
dσ/d$p_\perp$ [pb/GeV]
1
10^{-1}
10^{-2}
HqT
ResBos 2
Powheg NnloPs
Sherpa NnloPs
MG5_aMC FxFx
Sherpa MePs@Nlo
Herwig 7.1
LH15 $pp \to h +$ jets comparison
Ratio to HqT
Ratio to HqT
Ratio to HqT
1.4
1.2
1
0.8
0.6
0
50
100
150
200
$p_\perp(h)$ [GeV]

Comparison of approaches

[Les Houches SM WG] arXiv:1605.04692



Comparison of approaches

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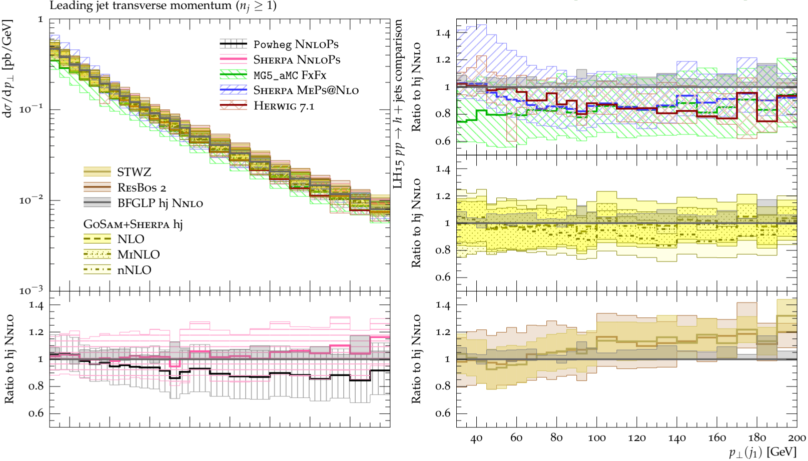

Comparison of approaches

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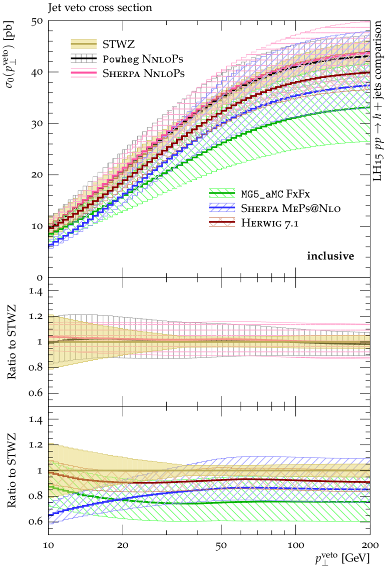
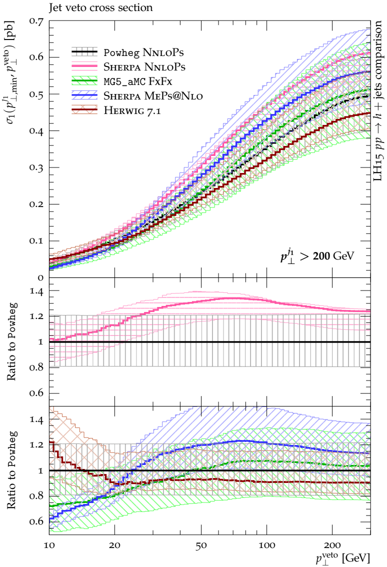
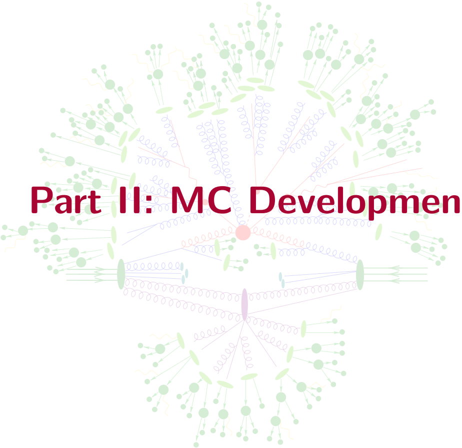




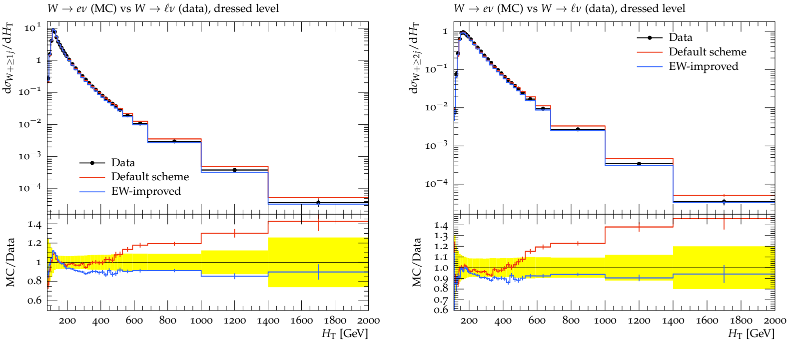


Part II: MC Development

Electroweak merging in Pythia

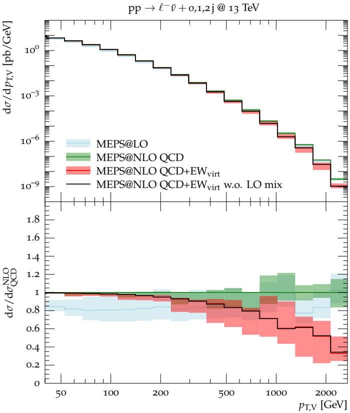
[Christiansen,Prestel] arXiv:1510.01517

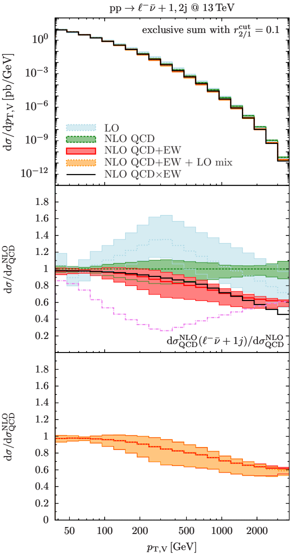
- QCD+EW parton shower implemented in Pythia 8
- Consistently merged to QCD+EW LO matrix elements



Electroweak merging in Sherpa

[Kallweit,Lindert,Maierhöfer,Pozzorini,Schönherr] arXiv:1511.08692





- QCD parton shower merged with QCD+EW matrix elements
- Implemented in Sherpa +OpenLoops framework

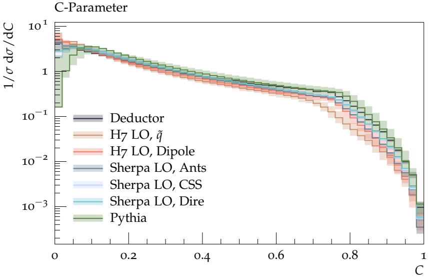
Parton-shower uncertainty estimates

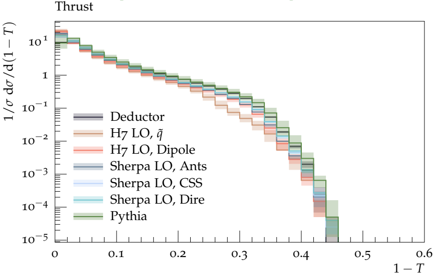
[Les Houches SM WG] arXiv:1605.04692

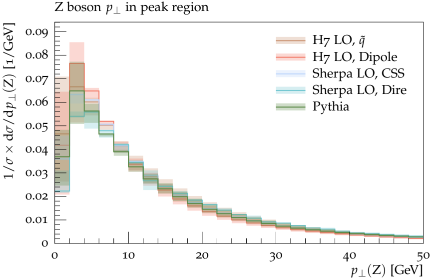
- Renormalization scale choice in parton showers
  - k_T [Amati,Bassetto,Ciafaloni,Marchesini,Veneziano] NPB173(1980)429
  - CMW rescaling [Catani,Marchesini,Webber] NPB349(1991)635
  - plus additional factor to be tuned to data (≈ 1)
- Scale variations typically not considered
  First attempt during LesHouches '15
- Participating projects
  - Deductor [Nagy,Soper] arXiv:1401.6364
  - Herwig [Bellm,Plätzer,Richardson,Siódmok,Webster] arXiv:1605.08256
    - $\tilde{q}$-shower [Gieseke,Stephens,Webber] hep-ph/0310083
    - Dipole shower [Plätzer,Gieseke] arXiv:0909.5593
  - Sherpa [Bothmann,Schönherr,Schumann] in preparation
    - Ants [Krauss,Zapp] in preparation
    - CSS [Schumann,Krauss] arXiv:0709.1027
    - Dire [Prestel,SH] arXiv:1506.05057
  - Pythia [Mrenna,Skands] arXiv:1605.08352

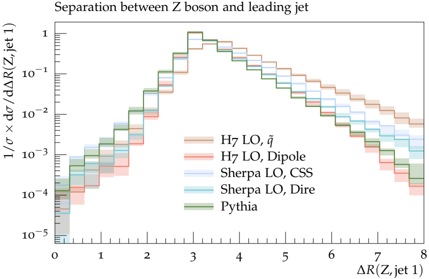
Parton-shower uncertainty estimates

[Les Houches SM WG] arXiv:1605.04692


C-Parameter


Thrust


Z boson $p_\perp$ in peak region

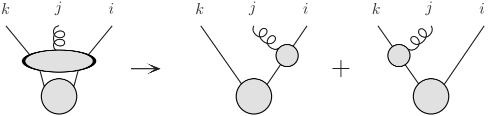

Separation between Z boson and leading jet

New parton shower formalism (DiRe)

[Prestel,SH] arXiv:1506.05057

- Soft gluon emissions correctly described if QCD evolution formulated in terms of color dipoles [Gustafsson,Pettersson] NPB306(1988)746
- Can preserve parton picture by partial fractioning soft eikonal ↔ soft enhanced part of splitting function [Catani,Seymour] hep-ph/9605323

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} \to \frac{1}{p_i p_j}\frac{p_i p_k}{(p_i+p_k)p_j} + \frac{1}{p_k p_j}\frac{p_i p_k}{(p_i+p_k)p_j}$$



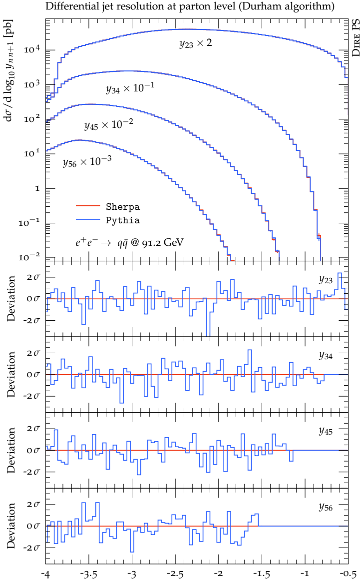
- "Spectator"-dependent kernels, singular in soft-collinear region only → capture dominant coherence effects (3-parton correlations)

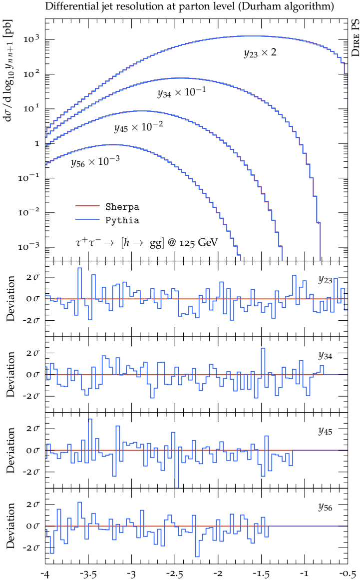
$$\frac{1}{1-z} \to \frac{1-z}{(1-z)^2+\kappa^2} \qquad \kappa^2 = \frac{k_\perp^2}{Q^2}$$

- For correct soft evolution, ordering variable must be identical at both "dipole ends" (→ recover soft eikonal at integrand level)

Validation

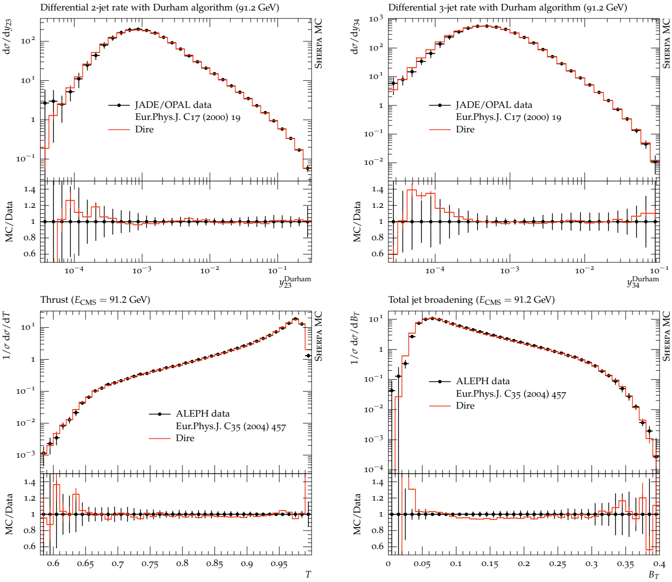
[Prestel,SH] arXiv:1506.05057


Differential jet resolution at parton level (Durham algorithm)


Differential jet resolution at parton level (Durham algorithm)

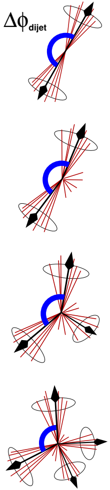
e$^+$e$^-$→hadrons as a benchmark

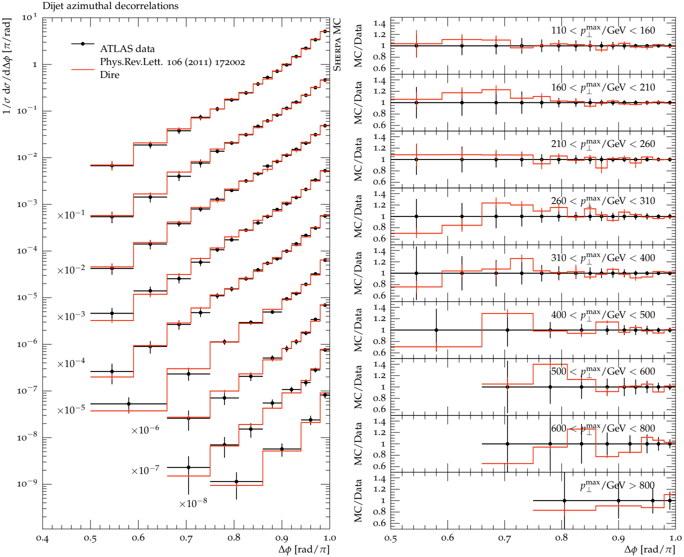
[Prestel,SH] arXiv:1506.05057



Inclusive jet production as a benchmark

[Prestel,SH] arXiv:1506.05057





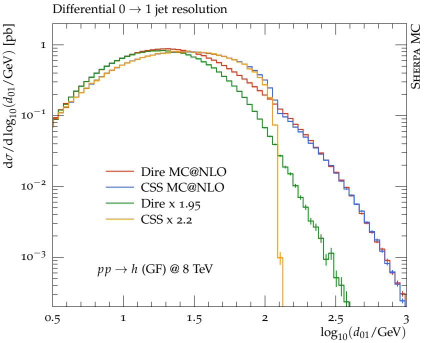
DiRe as a subtraction method and MC@NLO

[SH] in preparation

- Can view new shower as modified Catani-Seymour (CS) subtraction
- All counterterms computed and implemented in Sherpa generator

- Sherpa MC@NLO based on exponentiation of CS dipole subtraction terms

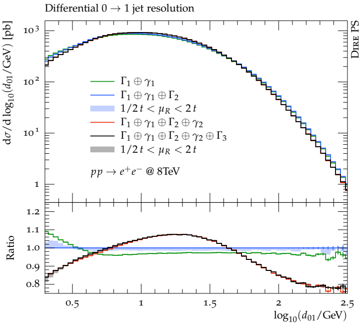
  [Krauss,Siegert,Schönherr,SH]
  arXiv:1111.1220, arXiv:1208.2815
- Modified CS subtraction automatically available for MC@NLO matching
- Important differences due to evolution variables and kernels

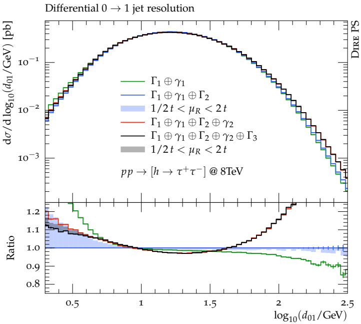


Towards higher accuracy parton showers

[Catani,Krauss,Prestel,SH] in preparation

- Parton showers do not allow to quantify theoretical uncertainties due to lack of higher-order splitting functions in the evolution
- Remedy using [Curci,Furmanski,Petronzio] NPB175(1980)27, PLB97(1980)437
- 2-loop cusp term subtracted & combined with LO soft contribution (similar to CMW rescaling [Catani,Marchesini,Webber] NPB349(1991)635)
- Implemented using weighting algorithms [Schumann,Siegert,SH] arXiv:0912.3501





Soft evolution with more color

- In soft limit real-emission amplitudes factorize as

$$|\mathcal{M}_0(1,\ldots,j,\ldots,n)|^2 \overset{j\to\text{soft}}{\longrightarrow} -\sum_{i,k\neq i}\frac{8\pi\mu^{2\varepsilon}\alpha_s}{p_ip_j}$$

$$\times\,\langle m_0(1,\ldots,i,\ldots,k,\ldots,n)|\,\frac{\mathbf{T}_i\cdot\mathbf{T}_k\;p_ip_k}{p_ip_j+p_kp_j}\,|m_0(1,\ldots,i,\ldots,k,\ldots,n)\rangle\;.$$

  $\mathbf{T}_i$ - color insertion operator for parton i
  $|m_0(1,\ldots,i,\ldots,k,\ldots,n)\rangle$ - Born amplitude

- Parton showers replace $\sum_{k\neq i}\mathbf{T}_i\cdot\mathbf{T}_k\to-\mathbf{T}_i^2$
- NLO Matched shower uses $\mathbf{T}_i\cdot\mathbf{T}_k$ in first emission
- Full matrix exponentiation in MC is work in progress
  Comparison to analytic resummation is a starting point

Soft evolution with more color

[Banfi,Salam,Zanderighi] hep-ph/0407286

- Generic NLL resummation framework exists (CAESAR)
- Observable dependence parametrized as

$$V\left(\{\tilde{p}\};k\right) = d_l \left(\frac{k_t^{(l)}}{Q}\right)^a e^{-b_l \eta^{(l)}} g_l\left(\phi^{(l)}\right)$$

- Resummed integrated spectrum for $V\left(\{\tilde{p}\};k\right) < v$ given by

$$\frac{1}{\sigma}\int_0^v \frac{d^2\sigma}{d\mathcal{B}dv'}dv' = \sum_{\delta\in\text{partonics}} \frac{d\sigma_0^{(\delta)}}{d\mathcal{B}} e^{Lg_1^{(\delta)}(\alpha_s L)+g_2^{(\delta,\mathcal{B})}(\alpha_s L)}\left[1+\mathcal{O}(\alpha_s)\right]\,, \qquad L = \log\frac{1}{v}$$

- LL / NLL coefficients g_1 and g_2 arise from 1- and 2-emission integrals
- g_2 depends on soft function $\mathcal{S}$ through

$$\log\mathcal{S}(T(L/a))\,, \qquad \text{where} \quad T(L) = \frac{1}{\pi\beta_0}\log\frac{1}{1-2\alpha_s\beta_0 L}$$

Soft evolution with more color

[Gerwick,SH,Marzani,Schumann] arXiv:1411.7325

- Soft function known analytically for low-multiplicity final states
- Generic structure in terms of anomalous dimension Γ is

$$\mathcal{S}(\xi) = \frac{\langle m_0 | e^{-\frac{\xi}{2}\mathbf{\Gamma}^\dagger} e^{-\frac{\xi}{2}\mathbf{\Gamma}} | m_0 \rangle}{\langle m_0 | m_0 \rangle} \,, \quad \mathbf{\Gamma} = -2 \sum_{i<j} \mathbf{T}_i \cdot \mathbf{T}_j \, \log \frac{Q_{ij}}{Q_{12}} + i\pi \sum_{i,j=II,FF} \mathbf{T}_i \cdot \mathbf{T}_j$$

- Insertion of color projectors $|c_\alpha\rangle\langle c^\alpha|$ leads to matrix structure

$$\mathcal{S}(\xi) = \frac{c_{\alpha\beta} H^{\gamma\sigma} \mathcal{G}^\dagger_{\gamma\rho} c^{\rho\beta} c^{\alpha\delta} \mathcal{G}_{\delta\sigma}}{c_{\alpha\beta} H^{\alpha\beta}} \,, \qquad \mathcal{G}_{\alpha\beta}(\xi) = c_{\alpha\gamma} \exp\left(-\frac{\xi}{2} \, c^{\gamma\delta} \, \Gamma_{\delta\beta} \right)$$

  where $H^{\alpha\beta} = \langle m_0 | c^\alpha \rangle \langle c^\beta | m_0 \rangle$ and $\Gamma_{\alpha\beta} = \langle c_\alpha | \mathbf{\Gamma} | c_\beta \rangle$

- $c_{\alpha\beta} = \langle c_\alpha | c_\beta \rangle$ - color "metric", $H^{\alpha\beta}$ - hard matrix
- Much effort in the literature is spent on choosing orthogonal bases
  [Sjödahl] arXiv:0906.1121, [Keppeler,Sjödahl] arXiv:1207.0609

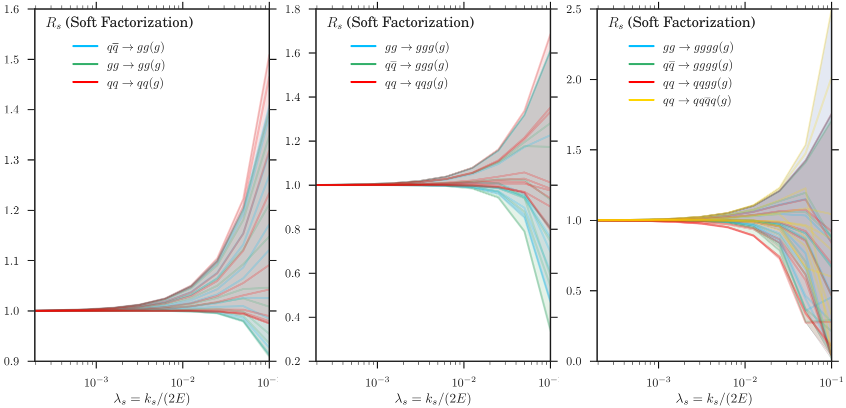
High-multiplicity NLL resummation & matching

[Gerwick,SH,Marzani,Schumann] arXiv:1411.7325

- Missing ingredients for resummation at higher multiplicity
  - Hard matrix → Matrix Element generator Comix
  - Soft anomalous dimension → Mathematica scripts
- Remaining problems
  - Non-orthogonality of color bases
    Solved by incorporation of inverse metric $c^{\alpha\beta} = (c_{\alpha\beta})^{-1}$
  - $N_c = 3$ pathologies in overcomplete color bases
    Solved by numeric matrix inversion at $N_c = 3 + \varepsilon$

High-multiplicity NLL resummation & matching

[Gerwick,SH,Marzani,Schumann] arXiv:1411.7325



- Ratio of sum-over-dipole dressed Born to exact matrix elements
- Checks correctness of soft anomalous dimension and color metric

High-multiplicity NLL resummation & matching

[Gerwick,SH,Marzani,Schumann] arXiv:1411.7325

- Expansion of resummation formula to NLO leads to LL and NLL coefficients

$$G_{12} = -\sum_{l=1}^{n} \frac{C_l}{a(a+b_l)}\,, \quad G_{11} = -\left[\sum_{l=1}^{n} C_l \left(\frac{B_l}{a+b_l} + \ldots\right) + \frac{1}{a}\frac{\mathrm{Re}[\Gamma_{\alpha\beta}]\,H^{\alpha\beta}}{c_{\alpha\beta}H^{\alpha\beta}} + \ldots\right]$$

  Combination with LO prediction → matching term

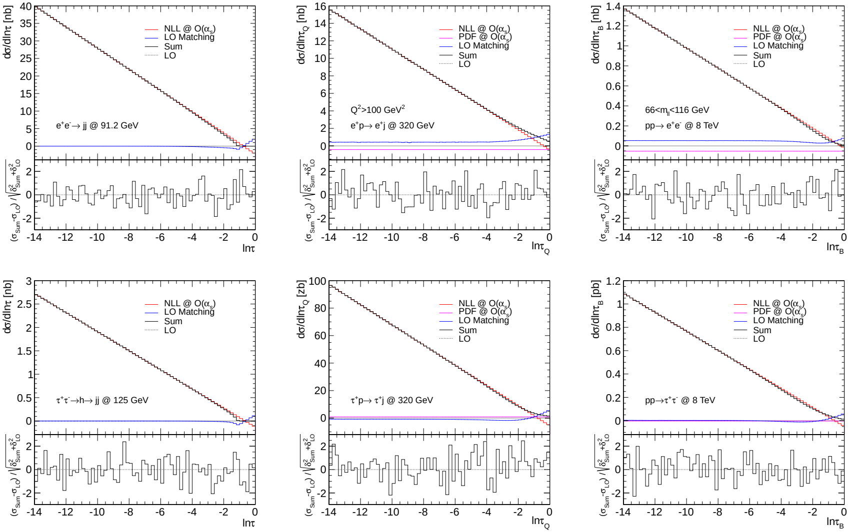
- Should be cast into fully differential procedure and automated
  Use Catani-Seymour dipole formula to generate coefficients

$$\mathcal{D}_{ij,k}(1,\ldots,n) \;=\; -\frac{1}{2p_ip_j}\,\langle m_0|\frac{\mathbf{T}_i\cdot\mathbf{T}_k}{\mathbf{T}_{ij}^2}\,\hat{V}_{ij,k}(z,k_T,\varepsilon)\,|m_0\rangle$$

- Obtain $\mathrm{Re}[\Gamma_{\alpha\beta}]\,H^{\alpha\beta}/c_{\alpha\beta}H^{\alpha\beta}$ from replacement
  $\hat{V}_{ij,k}(z,k_T,\varepsilon) \to \log Q_{(ij)k}/Q_{12}$, and rescaling by $1/a$
- Obtain G_{12} and B_l-dependent term in G_{11} from replacement
  $\hat{V}_{ij,k} \to P_{ij,i}$, restricting LL terms to $z^a > v$, and rescaling by $1/(a+b_l)$
- Rescale integration region ↔ momentum non-conservation in resummation

High-multiplicity NLL resummation & matching

[Gerwick,SH,Marzani,Schumann] arXiv:1411.7325



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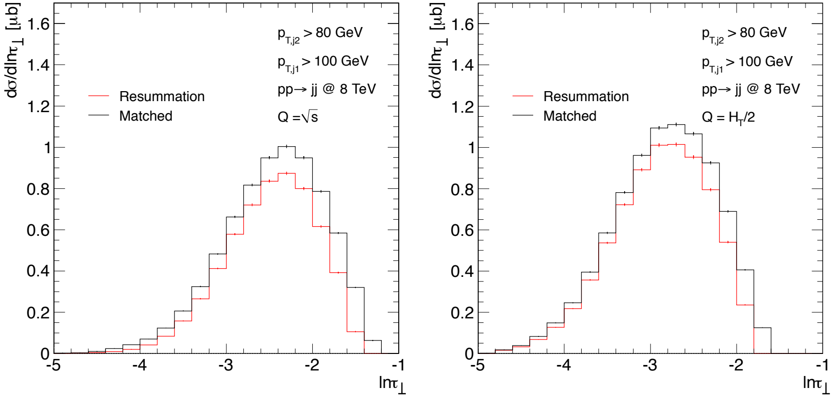
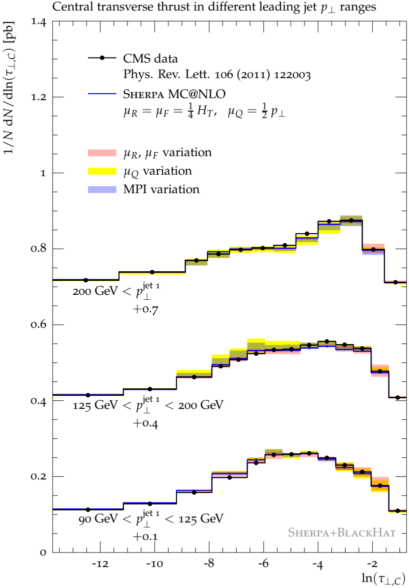
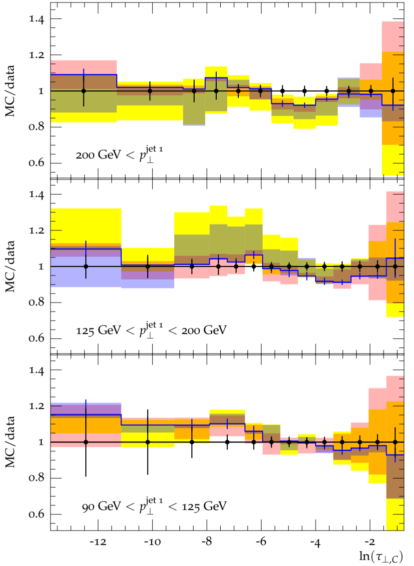


- Full result (NLL resummed and matched) for transverse thrust in $pp \to jj$

Back to the parton shower

[Schönherr,SH] arXiv:1208.2815


Central transverse thrust in different leading jet $p_\perp$ ranges

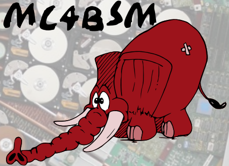


Summary & Outlook

- Decent agreement of all available matching/merging methods for Higgs
- Automated EW NLO calculations taking off & merging catching on
- Some progress on higher orders in parton evolution, but way to go

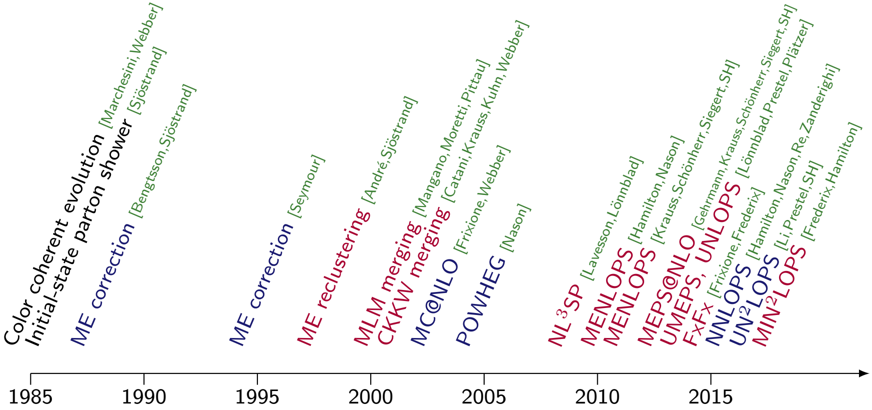
Let's see where we are at the next Jamboree ...

Join us at MC4BSM '17!



The long road to precision Monte-Carlo

Merging related
Matching related



Color coherent evolution [Marchesini,Webber]
Initial-state parton shower [Sjöstrand]
ME correction [Bengtsson,Sjöstrand]
ME correction [Seymour]
ME reclustering [André,Sjöstrand]
MLM merging [Mangano,Moretti,Pittau]
CKKW merging [Catani,Krauss,Kuhn,Webber]
MC@NLO [Frixione,Webber]
POWHEG [Nason]
NL^{3}SP [Lavesson,Lönnblad]
MENLOPS [Hamilton,Nason]
MENLOPS [Krauss,Schönherr,Siegert,SH]
MEPS@NLO [Gehrmann,Krauss,Schönherr,Siegert,SH]
UMEPS, UNLOPS [Lönnblad,Prestel,Plätzer]
F$\times$F$\times$ [Frixione,Frederix]
NNLOPS [Hamilton,Nason,Re,Zanderighi]
UN^{2}LOPS [Li,Prestel,SH]
MIN^{2}LOPS [Frederix,Hamilton]

NLO+PS matching – Basics

- Leading-order calculation for observable O

$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\,O(\Phi_B)$$

- NLO calculation for same observable

$$\langle O\rangle = \int \mathrm{d}\Phi_B\left\{\mathrm{B}(\Phi_B)+\tilde{V}(\Phi_B)\right\}O(\Phi_B)+\int \mathrm{d}\Phi_R\,\mathrm{R}(\Phi_R)\,O(\Phi_R)$$

- Parton-shower result (zero and one emission)

$$\langle O\rangle = \int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\left[\Delta^{(\mathrm{K})}(t_c)\,O(\Phi_B)+\int_{t_c}\mathrm{d}\Phi_1\,\mathrm{K}(\Phi_1)\,\Delta^{(\mathrm{K})}(t(\Phi_1))\,O(\Phi_R)\right]$$

$$\overset{\mathcal{O}(\alpha_s)}{\to}\int \mathrm{d}\Phi_B\,\mathrm{B}(\Phi_B)\left\{1-\int_{t_c}\mathrm{d}\Phi_1\mathrm{K}(\Phi_1)\right\}O(\Phi_B)+\int_{t_c}\mathrm{d}\Phi_B\mathrm{d}\Phi_1\,\mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1)\,O(\Phi_R)$$

Phase space: $\mathrm{d}\Phi_1 = \mathrm{d}t\,\mathrm{d}z\,\mathrm{d}\phi\,J(t,z,\phi)$
Splitting functions: $\mathrm{K}(t,z)\to\alpha_s/(2\pi t)\sum\mathrm{P}(z)\,\Theta(\mu_Q^2-t)$
Sudakov factors: $\Delta^{(\mathrm{K})}(t)=\exp\left\{-\int_t \mathrm{d}\Phi_1\mathrm{K}(\Phi_1)\right\}$

NLO+PS matching – MC@NLO

[Frixione,Webber] hep-ph/0204244

- Subtract $\mathcal{O}(\alpha_s)$ PS terms from subtracted NLO result ($t_c \to 0$)
  $1/N_c$ corrections faded out in soft region by smoothing function

$$\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B) = \mathrm{B}(\Phi_B) + \tilde{\mathrm{V}}(\Phi_B) + \mathrm{I}(\Phi_B) + \int \mathrm{d}\Phi_1 \Big[\mathrm{S}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1)\Big]\,f(\Phi_1)$$

$$\mathrm{H}^{(\mathrm{K})}(\Phi_R) = \Big[\mathrm{R}(\Phi_R) - \mathrm{B}(\Phi_B)\,\mathrm{K}(\Phi_1)\Big]\,f(\Phi_1)$$

- Add parton shower, described by generating functional $\mathcal{F}_{\mathrm{MC}}$

$$\langle O \rangle = \int \mathrm{d}\Phi_B\,\bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\,\mathcal{F}^{(0)}_{\mathrm{MC}}(\mu_Q^2, O) + \int \mathrm{d}\Phi_R\,\mathrm{H}^{(\mathrm{K})}(\Phi_R)\,\mathcal{F}^{(1)}_{\mathrm{MC}}(t(\Phi_R), O)$$

- Expansion of matched result up to first emission

$$\langle O \rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{K})}(\Phi_B)\Bigg[\Delta^{(\mathrm{K})}(t_c)\,O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\Delta^{(\mathrm{K})}(t(\Phi_1))\,O(\Phi_r)\Bigg] + \int \mathrm{d}\Phi_R\,\mathrm{H}^{(\mathrm{K})}(\Phi_{n+1})\,O(\Phi_R)$$

NLO+PS matching – POWHEG

[Nason] hep-ph/0409146
[Frixione,Nason,Oleari] arXiv:0709.2092

- Replace $\mathrm{BK} \to \mathrm{R} \Rightarrow \mathrm{H}^{(\mathrm{R})}$ zero, $\bar{\mathrm{B}}^{(\mathrm{R})}$ positive in physical region

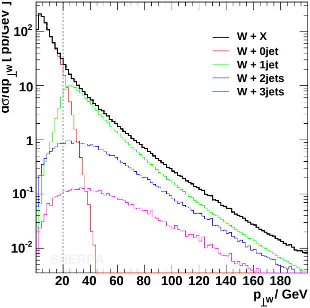
$$\langle O \rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R})}(\Phi_B) \Bigg[\Delta^{(\mathrm{R})}(t_c, s_{\mathrm{had}})\, O(\Phi_B) + \int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1 \frac{\mathrm{R}(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R})}(t(\Phi_1), s_{\mathrm{had}})\, O(\Phi_R) \Bigg]$$

- μ_Q^2 changed to hadronic centre-of-mass energy squared, s_{had}, to cover full phase space for real-emission correction
- Absence of hard events → enhanced high-p_T region $(K = \bar{\mathrm{B}}/\mathrm{B})$
  Formally beyond NLO, but often sizeable → Avoid by split $\mathrm{R} \to \mathrm{R}^s + \mathrm{R}^f$

$$\langle O \rangle = \int \mathrm{d}\Phi_B \bar{\mathrm{B}}^{(\mathrm{R^s})}(\Phi_B) \Bigg[\Delta^{(\mathrm{R^s})}(t_c, s_{\mathrm{had}})\, O(\Phi_B) + \int_{t_c}^{s_{\mathrm{had}}} \mathrm{d}\Phi_1 \frac{\mathrm{R^s}(\Phi_R)}{\mathrm{B}(\Phi_B)} \Delta^{(\mathrm{R^s})}(t(\Phi_1), s_{\mathrm{had}})\, O(\Phi_R) \Bigg] + \int \mathrm{d}\Phi_R\, \mathrm{R^f}(\Phi_R)$$

Basic idea of ME+PS merging

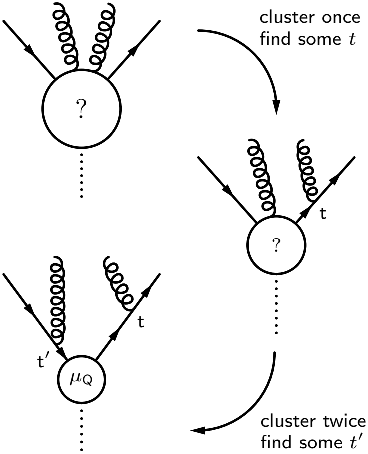
- Separate phase space into "hard" and "soft" region
- Matrix elements populate hard domain
- Parton shower populates soft domain
- Need criterion to define "hard" & "soft" $\rightarrow$ jet measure Q and corresponding cut, $Q_{\rm cut}$



Parton-shower histories

[André,Sjöstrand] hep-ph/9708390

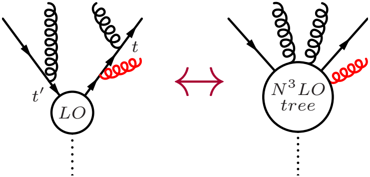
- Start with some "core" process
  for example $e^+e^- \to q\bar{q}$
- This process is considered inclusive
  It sets the resummation scale μ_Q^2
- Higher-multiplicity ME can be reduced to core by clustering
- Clustering algorithm uniquely defined by requiring exact correspondence between ME & PS
  - Identify most likely splitting according to PS emission probability
  - Combine partons into mother according to PS kinematics
  - Continue until no clustering possible



Truncated & vetoed parton showers

[Catani,Krauss,Kuhn,Webber] hep-ph/0109231
[Lönnblad] hep-ph/0112284, arXiv:1211.7204

- Higher-multiplicity MEs that can be reduced to core process are included in core's inclusive cross section (unitarity of PS)
- Sudakov suppression factors needed to make inclusive MEs exclusive
- Most efficiently computed with pseudo-showers
  - Start PS from core process
  - Evolve until predefined branching ↔ truncated parton shower
  - Emissions producing additional hard jets lead to event veto/weight



$$\Delta^{(\mathrm{K})}(t; > Q_{\mathrm{cut}}) = \exp\left\{ -\int_t \mathrm{d}\Phi_1\, \mathrm{K}(\Phi_1)\, \Theta(Q - Q_{cut}) \right\}$$

ME+PS merging

- ME⊕PS for 0+1-jet in MC@NLO notation

$$\langle O\rangle = \int \mathrm{d}\Phi_B \mathrm{B}(\Phi_B)\left[\Delta^{(\mathrm{K})}(t_c)\, O(\Phi_B) + \int_{t_c} \mathrm{d}\Phi_1 \mathrm{K}(\Phi_1)\, \Delta^{(\mathrm{K})}(t)\, \Theta(Q_{\mathrm{cut}} - Q)\, O(\Phi_R)\right]$$

$$+ \int \mathrm{d}\Phi_R\, \mathrm{R}(\Phi_R)\, \Delta^{(\mathrm{K})}(t(\Phi_R); {>}Q_{\mathrm{cut}})\, \Theta(Q - Q_{\mathrm{cut}})\, O(\Phi_R) + \ldots$$

- Reorder by parton multiplicity k, change notation $\mathrm{R}_k \to \mathrm{B}_{k+1}$
- Analyze exclusive contribution from k hard partons only $(t_0 = \mu_Q^2)$

$$\langle O\rangle_k^{\mathrm{excl}} = \int \mathrm{d}\Phi_k\, \mathrm{B}_k \prod_{i=0}^{k-1} \Delta_i^{(\mathrm{K})}(t_{i+1}, t_i; {>}Q_{\mathrm{cut}})\, \Theta(Q_k - Q_{\mathrm{cut}})$$

$$\times \left[\Delta_k^{(\mathrm{K})}(t_c, t_k)\, O_k + \int_{t_c}^{t_k} \mathrm{d}\Phi_1\; \mathrm{K}_k\, \Delta_k^{(\mathrm{K})}(t_{k+1}, t_k)\, \Theta(Q_{\mathrm{cut}} - Q_{k+1})\, O_{k+1}\right]$$

ME+PS merging – Unitarization

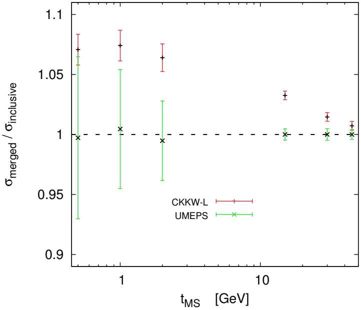
[Lönnblad,Prestel] arXiv:1211.4827, [Plätzer] arXiv:1211.5467

- Unitarity condition of PS:

$$1 = \Delta^{(\mathrm{K})}(t_c) + \int_{t_c} \mathrm{d}\Phi_1\, \mathrm{K}(\Phi_1)\, \Delta^{(\mathrm{K})}(t)$$

- ME+PS(@NLO) violates PS unitarity as ME ratio replaces splitting kernels in emission terms, but not in Sudakovs

$$\mathrm{K}(\Phi_1) \to \frac{\mathrm{R}(\Phi_1, \Phi_B)}{\mathrm{B}(\Phi_B)}$$



- Can be corrected by explicit subtraction

$$1 = \underbrace{\left\{ \Delta^{(\mathrm{K})}(t_c) + \int_{t_c} \mathrm{d}\Phi_1 \left[\mathrm{K}(\Phi_1) - \frac{\mathrm{R}(\Phi_1, \Phi_B)}{\mathrm{B}(\Phi_B)} \right] \Theta(Q - Q_{\mathrm{cut}})\, \Delta^{(\mathrm{K})}(t) \right\}}_{\text{unresolved emission / virtual correction}}$$

$$+ \underbrace{\int_{t_c} \mathrm{d}\Phi_1 \left[\mathrm{K}(\Phi_1)\Theta(Q_{\mathrm{cut}} - Q) + \frac{\mathrm{R}(\Phi_1, \Phi_B)}{\mathrm{B}(\Phi_B)} \Theta(Q - Q_{\mathrm{cut}}) \right] \Delta^{(\mathrm{K})}(t)}_{\text{resolved emission}}$$

ME+PS merging at NLO

- Analyze exclusive contribution from k hard partons

$$
\begin{aligned}
\langle O\rangle_k^{\text{excl}} \;=\; & \int \mathrm{d}\Phi_k\,\bar{\mathrm{B}}_k^{(\mathrm{K})}\,\prod_{i=0}^{k-1}\Delta_i^{(\mathrm{K})}(t_{i+1},t_i;>Q_{\text{cut}})\,\Theta(Q_k-Q_{\text{cut}})\\
& \times\left(1+\frac{\mathrm{B}_k}{\bar{\mathrm{B}}_k^{(\mathrm{K})}}\sum_{i=0}^{k-1}\int_{t_{i+1}}^{t_i}\mathrm{d}\Phi_1\mathrm{K}_i\,\Theta(Q_i-Q_{\text{cut}})\right)\\
& \times\left[\Delta_k^{(\mathrm{K})}(t_c,t_k)\,O_k\;+\int_{t_c}^{t_k}\mathrm{d}\Phi_1\;\mathrm{K}_k\,\Delta_k^{(\mathrm{K})}(t_{k+1},t_k)\,\Theta(Q_{\text{cut}}-Q_{k+1})\,O_{k+1}\right]\\
& +\int \mathrm{d}\Phi_{k+1}\,\mathrm{H}_k^{(\mathrm{K})}\,\Delta_k^{(\mathrm{K})}(t_k;>Q_{\text{cut}})\,\Theta(Q_k-Q_{\text{cut}})\,\Theta(Q_{\text{cut}}-Q_{k+1})\,O_{k+1}
\end{aligned}
$$

- Born matrix element → NLO-weighted Born
- Add hard remainder function
- Subtract $\mathcal{O}(\alpha_s)$ terms contained in truncated PS

ME+PS merging – Practical implementations

- LO schemes

Method	Shower Generator	Unitary	References
MLM	Herwig/Pythia	No	[Mangano,Moretti,Pittau] hep-ph/0108069 [Alwall et al.] arXiv:0706.2569
CKKW	Apacic	No	[Catani,Krauss,Kuhn,Webber] hep-ph/0109231
CKKW-L	Ariadne/Pythia	No	[Lönnblad] hep-ph/0112284 [Lönnblad,Prestel] arXiv:1109.4829
METS	Sherpa CSS	No	[Krauss,Schumann,Siegert,SH] arXiv:0903.1219
CKKW'	Herwig++	No	[Hamilton,Richardson,Tully] arXiv:0905.3072
UMEPS	Pythia/Herwig++	Yes	[Lönnblad,Prestel] arXiv:1211.4827 [Plätzer] arXiv:1211.5467

- NLO schemes

Method	Shower Generator	Unitary	References
NL^3	Ariadne/Pythia	No	[Lavesson,Lönnblad] arXiv:0811.2912
MEPS@NLO	Sherpa CSS	No	[Krauss,Schönherr,Siegert,SH] arXiv:1207.5030 [Gehrmann,Krauss,Schönherr,Siegert,SH] 5031
FxFx	Herwig(++)/Pythia	No	[Frederix,Frixione] arXiv:1209.6215
UNLOPS	Pythia/Herwig++	Yes	[Lönnblad,Prestel] arXiv:1211.7278

NNLO matching methods

NNLOPS

[Hamilton,Nason,Zanderighi] arXiv:1212.4504
[Hamilton,Nason,Re,Zanderighi] arXiv:1309.0017

- Based on MINLO procedure
  [Hamilton,Nason,Zanderighi] arXiv:1206.3572
- Extended to NNLL resummation and reweighted to NNLO differentially in Born phase space

UN^{2}LOPS

[Li,Prestel,SH] arXiv:1405.3607

- Based on UNLOPS merging
  [Lönnblad,Prestel] arXiv:1211.7278
- q_T-cutoff technique for NNLO, combined with subtracted MC@NLO for 1-jet contribution