Energy dependence of interferometry scales in ultrarelativistic heavy-ion collisions

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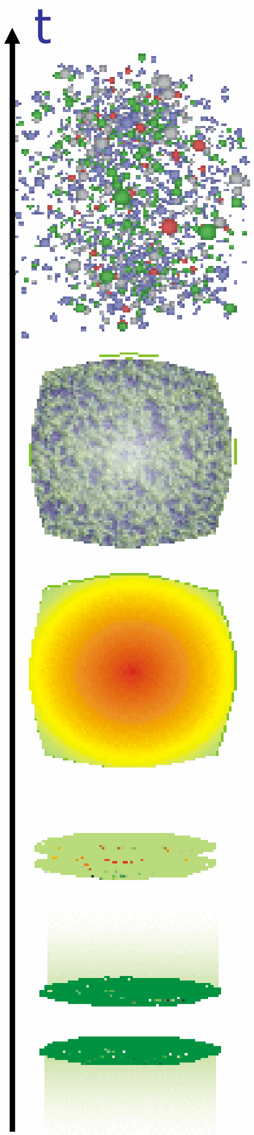
OUTLOOK

- **I. Thermalization (is early thermalization really needed?), IC, HBT-puzzle and all that…**
- **II. Transition from very initial and very non-equilibrium state in HIC to thermal one. Phenomenological approach.**
- **III. Matter evolution at chemically equilibrated stage.**
- **IV. Matter evolution at non-equilibrated stage. HydroKinetic Model (HKM).**
- **V. System's decay and spectra formation. Whether it possible to apply Cooper-Frye prescription for continuously emitting and not equilibrated system? If possible, how?**
- **VI. The HKM results for RHIC energies. Pion, kaon, proton spectra; pion and kaon HBT radii.**
- **VII. Energy dependence of the interferometry scales.**

Expecting Stages of Evolution in Ultrarelativistic A+A collisions

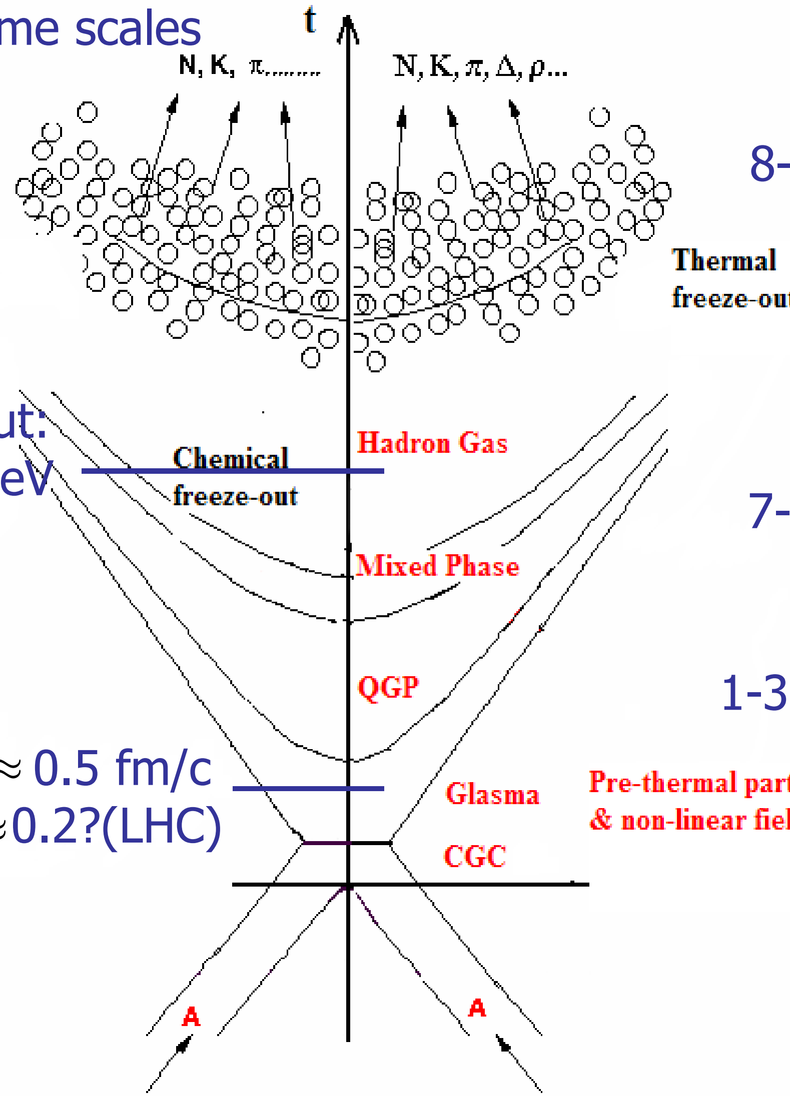


Relatively small space-time scales
(HBT puzzle)

Early thermal freeze-out:
T_th $\approx$ Tch $\approx$ 150 MeV

Elliptic flows

Early thermalization at $\approx$ 0.5 fm/c
$\approx$0.2?(LHC)



8-20 fm/c

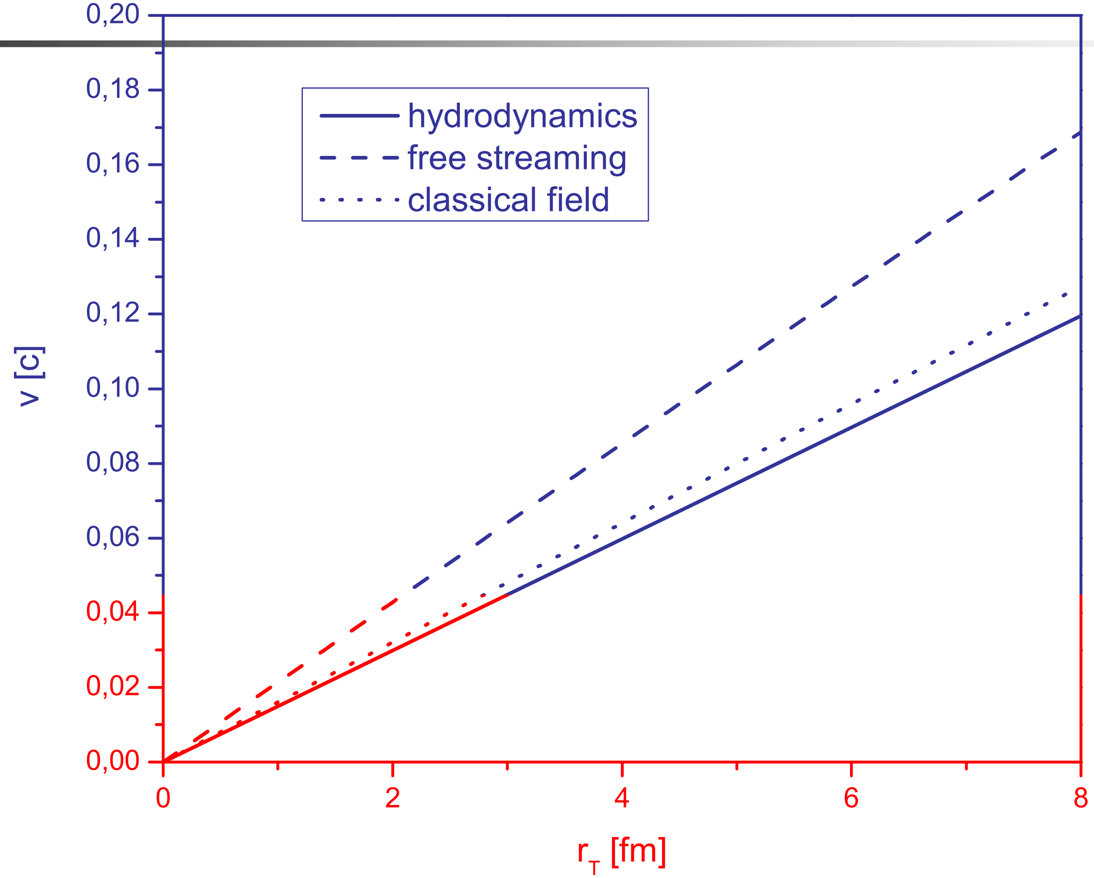
7-8 fm/c

1-3 fm/c

or strings

Pre-thermal transverse flow

Collective velocities developed between τ_0 =0.3 and τ =1.0 fm/c

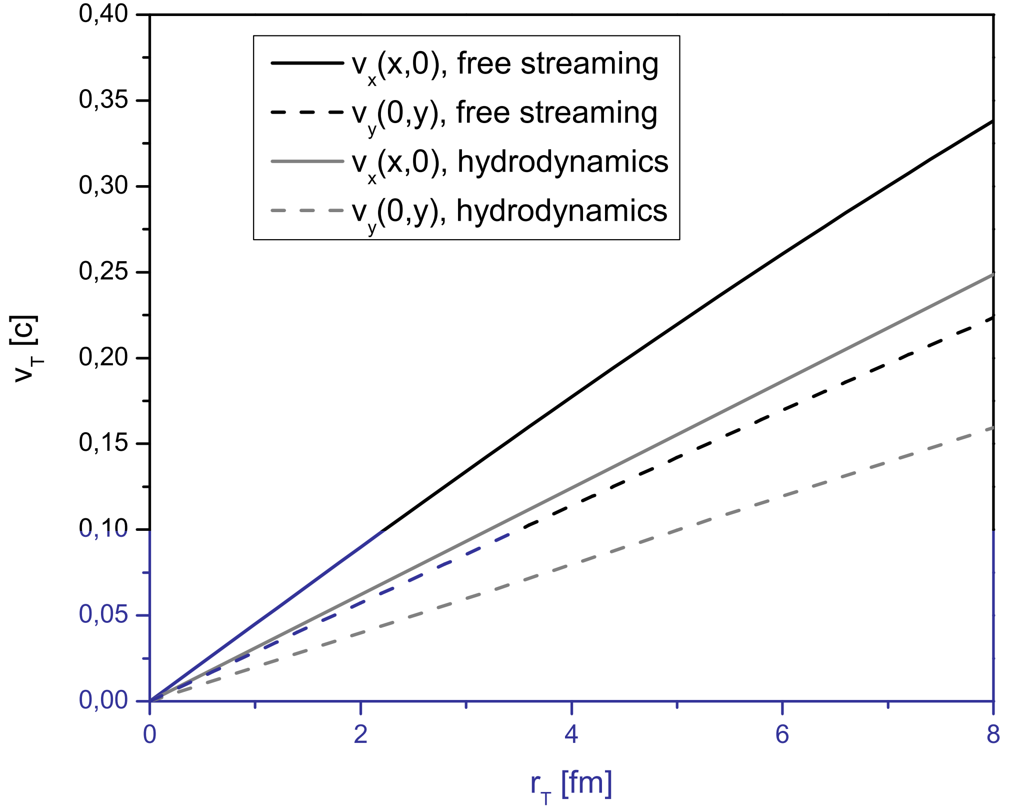


Central collisions

Collective velocity developed at pre-thermal stage from proper time tau_0 =0.3 fm/c by supposed thermalization time tau_th = 1 fm/c for scenarios of partonic free streaming and free expansion of classical field. The results are compared with the hydrodynamic evolution of perfect fluid with hard equation of state p = 1/3 epsilon started at τ_0 . Impact parameter b=0.

Yu.S. Acta Phys.Polon. B37 (2006) 3343**;** Gyulassy, Yu.S., Karpenko, Nazarenko Braz.J.Phys. 37 (2007) 1031. Yu.S., Nazarenko, Karpenko: Acta Phys.Polon. B40 1109 (2009) .

Collective velocities and their anisotropy developed between τ_0 =0.3 and τ=1.0 fm/c



Non-central collisions b=6.3 fm

Collective velocity developed at pre-thermal stage from proper time τ_0 =0.3 fm/c by supposed thermalization time tau_i = 1 fm/c for scenarios of partonic free streaming. The results are compared with the hydrodynamic evolution of perfect fluid with hard equation of state p = 1/3 epsilon started at τ_0 . Impact parameter b=6.3 fm.

Summary-1

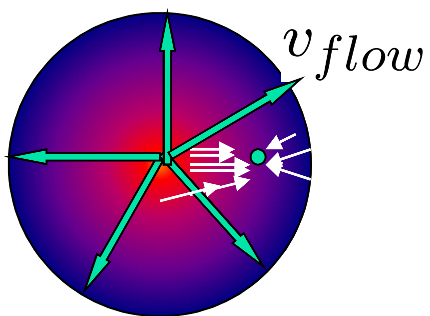
Yu.S., Nazarenko, Karpenko: Acta Phys.Polon. B40 1109 (2009)

- **The initial transverse flow in thermal matter as well as its anisotropy are developed at pre-thermal, either partonic or classical field — Glasma, stage with even more efficiency than in the case of very early perfect hydrodynamics.**

- **Such radial and elliptic flows develop no matter whether a pressure already established. The general reason for them is an essential finiteness of the system in transverse direction.**

- **The anisotropy of the flows transforms into asymmetry of the transverse momentum spectra only of (partial) thermalization happens.**

- **So, the results, first published in 2006, show that whereas the assumption of (partial) thermalization in relativistic A + A collisions is really crucial to explain soft physics observables, the hypotheses of early thermalization at times less than 1 fm/c is not necessary.**



$$v_{flow,i} \sim r_i t/\lambda^2_{homog,i}$$

Phenomenological model of pre-thermal evolution

Yu.S. ECT Workshop on flows and dissipation in ultrarelativistic A+A collisions September 14-18, 2009

Akkelin, Yu.S. (2009), in preparation

- **If some model (effective QCD theory) gives us the energy-momentum tensor at time τ_0 , one can estimate the flows and energy densities at expected time of thermalization τ_{th}, using equations for ideal fluid with (known) source terms.**

- **This phenomenological approach is motivated by Boltzmann equations, accounts for the energy and momentum conservation laws and contains two parameters: supposed time of thermalization τ_{th} and "initial" relaxation time.**

$$\partial_\mu \tilde{T}^{\mu\nu}_{hyd}(x) = -T^{\mu\nu}_{free}(x)\partial_\mu \mathcal{P}_{\tau_0\to\tau}(\tau)$$

$$\mathcal{P}(\tau) = \left(\frac{\tau_f - \tau}{t_f - \tau_0}\right)^{\frac{\tau_f - \tau_0}{\tau_{\mathtt{rel}}(\tau_0)}}$$

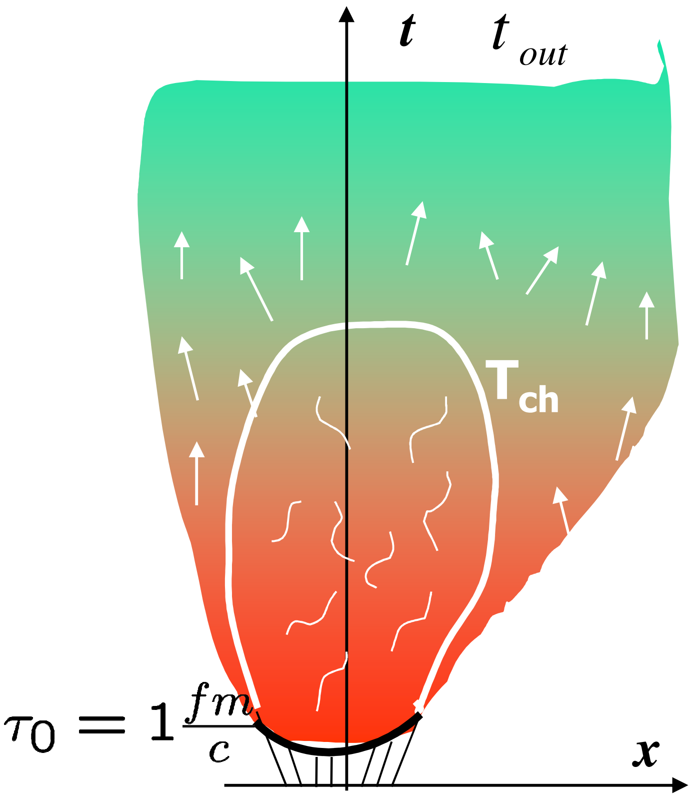
where

$$\tilde{T}^{\mu\nu}_{hyd} = T^{\mu\nu}_{hyd}(\epsilon \to (1-\mathcal{P}(\tau))\epsilon, p \to (1-\mathcal{P}(\tau))p)$$

HydroKinetic Model (HKM) of A+A collisions

I. Matter evolution in chemically equilibrated space-time zone

Locally (thermally & chemically) equilibrated evolution and initial conditions (IC)



$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p \cdot g^{\mu\nu}$$

$$p = p(\epsilon, \{Q_\alpha\})$$

$$\begin{cases} \partial_\nu T^{\mu\nu} = 0 \\ \partial_\nu (Q_\alpha u^\nu) = 0 \qquad \alpha = B, S, E \end{cases}$$

IC for central Au+Au collisions

The "effective" initial distribution is the one which being used in the capacity of initial condition bring the average hydrodynamic results for fluctuating initial conditions:

I. $\epsilon = \epsilon_0 e^{-\mathbf{r}^2/R^2}$ **Gaussian-like IC**

$R = 5.4 fm$

II. $\epsilon = \epsilon_0 F_G(\mathbf{r})$, $F_G(\mathbf{r})$ **is Glauber-like profile**

III. $\epsilon = \epsilon_0 F_{CGC}(\mathbf{r})$, $F_{CGC}(\mathbf{r})$ **is CGC-like profile**

Initial rapidity profiles:

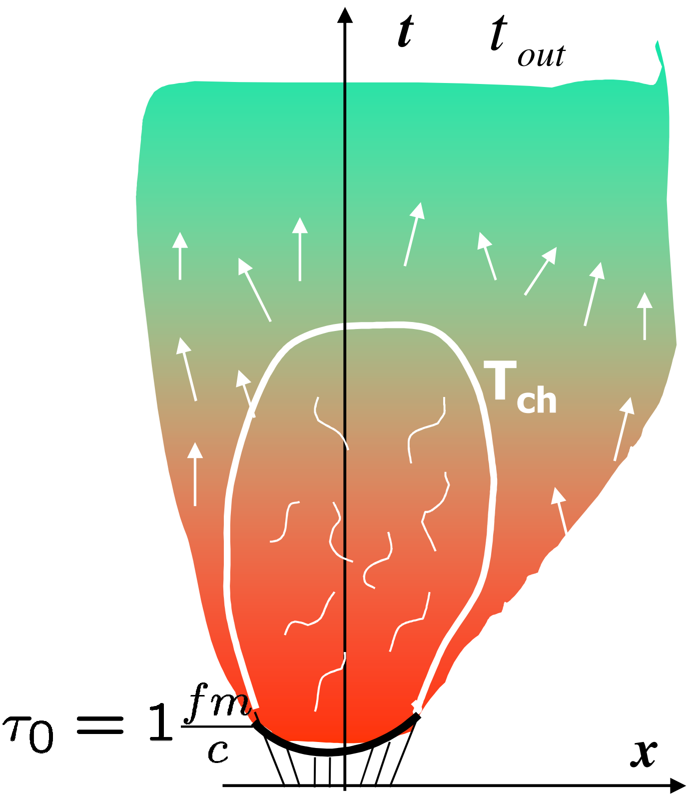
$$y_T = \alpha \frac{r}{R}$$

$$y_L = \eta = \frac{1}{2} \ln[(t+z)/(t-z)]$$

ϵ_0 and α are only fitting parameters in HKM

HKM

II. Evolution of the hadronic matter in non-equlibrated zone.



Decay of the system and spectra formation

Boltzmann equations and probabilities of particle free propagation

Boltzmann eqs (differential form)

$$\frac{p^\mu}{p^0}\frac{\partial f_i(x,p)}{\partial x^\mu} = G_i(x,p) - L_i(x,p)$$

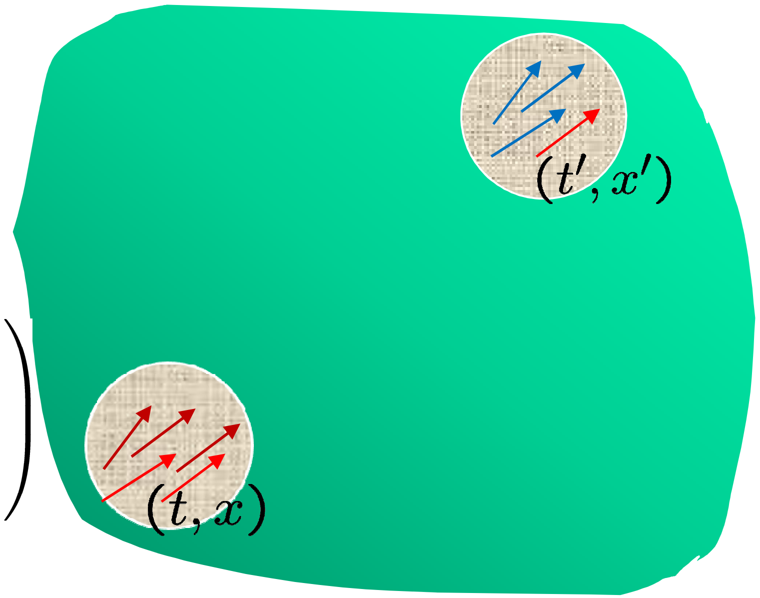
$G_i(x,p)$ and $L_i(x,p) = J_i(x,p) f_i(x,p)$ are G(ain), L(oss) terms for i p. species

Probability of particle free propagation (for each component i)

$$x = (t, \mathbf{x})$$

$$\overline{x}_{t\to s} = (s, \mathbf{x} + \frac{\mathbf{p}}{p^0}(s-t))$$

$$\mathcal{P}_{t\to t'}(x,p) = \exp\left(-\int_t^{t'} ds J(\overline{x}_{t\to s}, p)\right)$$



Spectra and Emission function

Boltzmann eqs (integral form)

$$f(t,\mathbf{x},p) = f(\overline{x}_{t\to t_0},p)\mathcal{P}_{t_0\to t}(\overline{x}_{t\to t_0},p) + \int_{t_0}^{t} G(\overline{x}_{t\to s},p)\mathcal{P}_{s\to t}(\overline{x}_{t\to s},p)ds$$

$\mathcal{P}_{s\to t}[J]$

$(s, \mathbf{x} - \frac{\mathbf{p}}{p^0}(t-s))$

$(t,\mathbf{x})$

$\mathbf{p}$

Index i is omitted everywhere

$$\frac{d^3N}{d^3p}(t) \equiv n(t,p) = \int d^3x f(t,\mathbf{x},p)$$

Spectrum

$$n(t\to\infty,p) = \int d^3x f(t_0,\mathbf{x},p)\mathcal{P}_{t_0\to t}(t_0,\mathbf{x},p) + \int d^3x \int_{t_0}^{t} dt' G(t',\mathbf{x},p)\mathcal{P}_{t'\to t}(t',\mathbf{x},p)$$

$S(t',x,p) \neq f_{l.eq}(x,p) * \exp(-(\tau-\tau_0)^2/\Delta\tau^2)$

Relax. time approximation for emission function (Yu.S. , Akkelin, Hama PRL, 2002)

For (quasi-) stable particles

$$J(x,p) \approx R_{l.eq.}(x,p) + J^{decay}(x,p),$$

$$G(x,p) \approx R_{l.eq.}(x,p) f_{l.eq.}(x,p) + G^{decay}(x,p)$$

Kinetics and hydrodynamics below T_{ch} =165 MeV

For hadronic resonances

$$m_j/T \ll 1 \Longrightarrow f_j(x,p_i) \approx \frac{p_j^0}{m_j} n_j(x)\delta^3(\mathbf{p}_j - m_j\mathbf{u}(x)),$$

$$G_i^{decay}(x,p_i) = \sum_j \sum_k \Gamma_{j\to ik} \frac{n_j(x)}{p_i^0 p_k^0 F_{j\to ik}} \delta(m_j u^0(x) - p_k^0 - p_i^0)$$

$$F_{i\to kl} = \frac{2\pi}{m_i^2}((m_i^2 - m_k^2 - m_l^2)^2 - 4m_i^2 m_k^2)^{1/2} \qquad p_k^0 = \sqrt{m_k^2 + (m_j\mathbf{u}(x) - \mathbf{p}_i)^2}$$

$$L_i^{decay}(x,p_i) = \sum_k \sum_l \int \frac{d^3p_k}{p_k^0} \int \frac{d^3p_l}{p_l^0} \Gamma_{i\to kl} f_i(x,p_i) \frac{m_i}{F_{i\to kl}} \delta^{(4)}(p_i - p_k - p_l) = \frac{m_i}{p_i^0} \Gamma_i f_i(x,p_i)$$

$$f_i(x,p) \approx f_{i,l.eq.}(x,p), \qquad \int d^3p \otimes, \quad \sum_i \int d^3p \frac{p^\nu}{p^0} \otimes$$

$$\frac{p^\mu}{p^0}\partial_\mu f_i(x,p) = (f_i^{l.eq.}(x,p) - f_i(x,p))R_i(x,p) + G_i^{decay}(x,p) - L_i^{decay}(x,p)$$

$$\partial_\mu(n_i(x)u^\mu(x)) = -\Gamma_i n_i + \sum_j b_{ij}\Gamma_j n_j(x) \qquad \partial_\nu T^{\mu\nu} = 0$$

Representations of non-loc.eq. distribution function (for quasi- stable particles)

$$f_i(x,p) = f_i^{l.eq.}(T(x),\mu_i(x),u(x);p) + g_i[T,\mu_i,u;p](x,p)$$

If $f_i(t_0,\mathbf{r},p) = f_i^{l.eq}(t_0,\mathbf{r},p)$ **at the initial (hadronisation) time**

$$f_i(x,p) = f_i^{l.eq.}(x,p) + \underbrace{\frac{G_i^{decay}(x,p)}{R_i(x,p)} - \frac{G_i^{decay}(\overline{x}_{t\to t_0},p)}{R_i(\overline{x}_{t\to t_0},p)}\mathcal{P}_{t_0\to t}(\overline{x}_{t\to t_0},p) - \int_{t_0}^{t}\frac{d}{ds}[f_i^{l.eq}(\overline{x}_{t\to s},p) + \frac{G_i^{decay}(\overline{x}_{t\to s},p)}{R_i(\overline{x}_{t\to s},p)}]\mathcal{P}_{s\to t}(\overline{x}_{t\to s},p)ds}_{g_i(x,p)}$$

$$\overline{x}_{t\to s} = (s, \mathbf{x} + \frac{\mathbf{p}}{p^0}(s-t))$$

Iteration procedure:

I. Solution of perfect hydro equations with given initial l.eq. conditions

$$\partial_\nu T^{\nu\mu}[f^{\mathtt{leq}}] = 0,\ \partial_\nu n_i^\nu[f^{\mathtt{leq}}] = -\Gamma_i n_i[f^{\mathtt{leq}}] + \sum_j b_{ij}\Gamma_j n_j[f^{\mathtt{leq}}]$$

$$p = p(\epsilon, n)$$

$$\Downarrow$$

$$u^\mu_{id}(x),\ T_{id}(x), \mu_{id}(x)$$

II. Decomposition of energy-momentum tensor

$$f = f^{l.eq}(T, u, \{\mu_i\}; p) + g[T_{id}, u_{id}, \{\mu_{id,i}\}; p]$$

$$\Downarrow$$

$$T^{\mu\nu}(x) = T^{\mu\nu}[f^{l.eq.}] + T^{\mu\nu}[g]$$

III. Ideal hydro with "source" instead of non-ideal hydro

$$\partial_\mu T^{\mu\nu}(x) = 0 \qquad \partial_\nu n_i^\nu(x) = -\Gamma_i n_i(x) + \sum_j b_{ij}\Gamma_j n_j(x)$$

$$\Downarrow$$

$$\partial_\nu T^{\nu\beta}[f^{l.eq}(T, u, \{\mu_i\}, p)] = F^\beta(x)$$

where

$$F^\beta(x) = -\partial_\nu T^{\nu\beta}[g[T_{id}, u_{id}, \{\mu_{id,i}\}; p]$$

(known function)

Full procedure for HKM

$$\partial_\mu T_{id}^{\mu\nu} = F^\nu(x)$$

↓

$$T(x),\ u^\nu(x), \mu(x)$$

↓

$$f_i(x,p) = f^{l.eq.}(T,u,\mu_i,p) + g_i[T_{id}, u_{id}, \mu_{id,i}, p]$$

This approach accounts for

- **conservation laws**
- **deviations from loc. eq.**
- **viscosity effects in hadron gas:** $\eta \propto \tau_{\mathtt{rel}} n T$

Equation of state in non-equilibrated zone

$T < T_{ch} = 165$ **MeV** → **EoS** $p = p(\epsilon, \{n_i\})$

$i = 1, ..., N = 359 \ (m_i < 2.6 \text{ GeV})$

Pressure and energy density of multi-component Boltzmann gas

Below T_{ch} we account for the evolution of all N densities of hadron species in hydro calculation, and compute EoS dynamically for each chemical composition of N sorts of hadrons in every hydrodynamic cell in the system during the evolution. Using this method, we do not limit ourselves by chemically frozen or chemically equilibrated evolution, keeping nevertheless thermodynamically consistent scheme.

System's decoupling and spectra formation

- Emission function

$$S(x,p) = G(x,p)\mathcal{P}(x,p)$$

For pion emission

$$G \approx f_{\pi}^{l.eq.}(x,p)R_{\pi+h}(x,p) + G_{H\to\pi}(x,p)$$

$R_{\pi+h}(x,p)$ **is the total collision rate of the pion, carrying momentum *p* with all the hadrons *h* in the system in a vicinity of point *x*.**

$G_{H\to\pi}(x,p)$ **is the space-time density of pion production caused by gradual decays during hydrodynamic evolution of all the suitable resonances *H* including cascade decays**

The cross-sections in the hadronic gas are calculated in accordance with UrQMD .

PARAMETERS the RHIC TOP ENERGY

Fitting parameter at $\tau_i = 1 fm/c$

	Max initial energy density $\epsilon_0 = \epsilon(\tau_i, \mathbf{r} = 0)$	**Initial transverse flows** $\langle v_T \rangle$
Glauber IC	15.5 GeV/fm^3	0.23
CGC IC	20 GeV/fm^3	0.19
Gaussian IC	17 GeV/fm^3	0.26

$$\langle \epsilon \rangle_{\tau_i} \approx 8 \text{ GeV/fm}^3.$$

In CGC approach at RHIC energies this energy density corresponds to the value

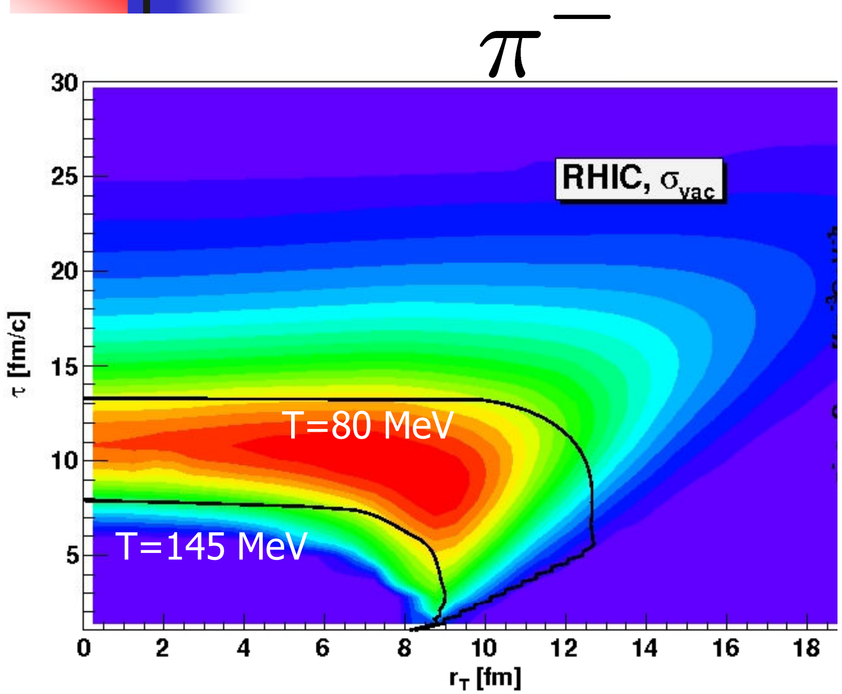
$$\Lambda_s^4/g^2 \approx 6 \text{ GeV}^4 \text{ at } \tau_0 \approx 3/\Lambda_s = 0.3 \text{ fm/c (RHIC)}$$
$$(\Lambda_s = g^2\mu \equiv 4\pi\alpha_s\mu = 2 \text{ GeV (RHIC)}).$$

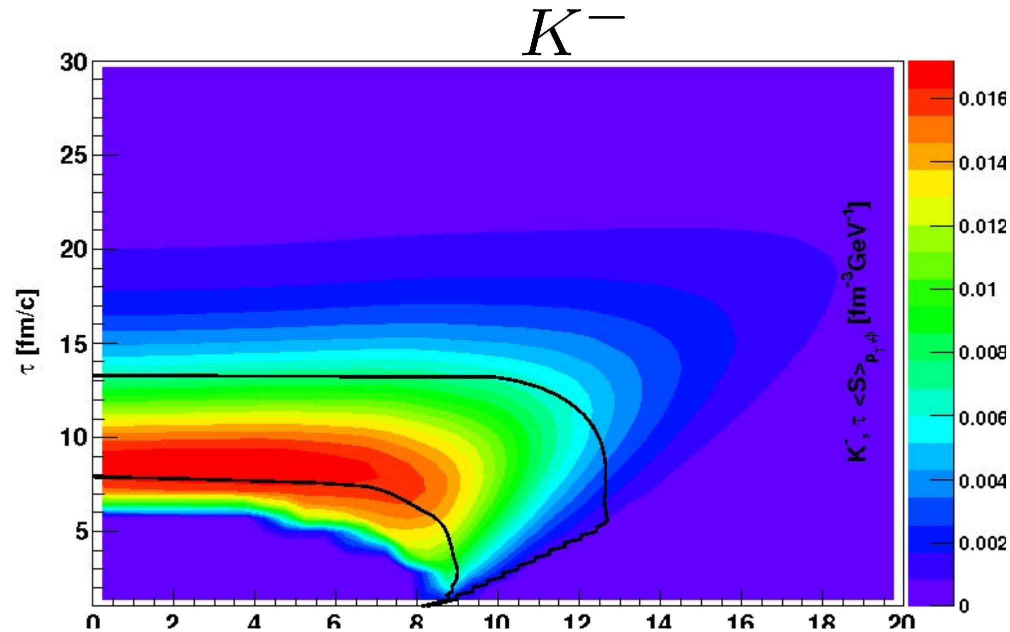
$$g \approx 1.61 \text{ if } \Lambda_s = 2 \text{ GeV}$$

In CGC approach at RHIC energies the value $g=2$ **is used (T. Lappi, J.Phys. G, 2008)**

The RESULTS for RHIC TOP ENERGY

Pion, kaon and proton emission densities (Gaussian IC, vacuum c.s.)

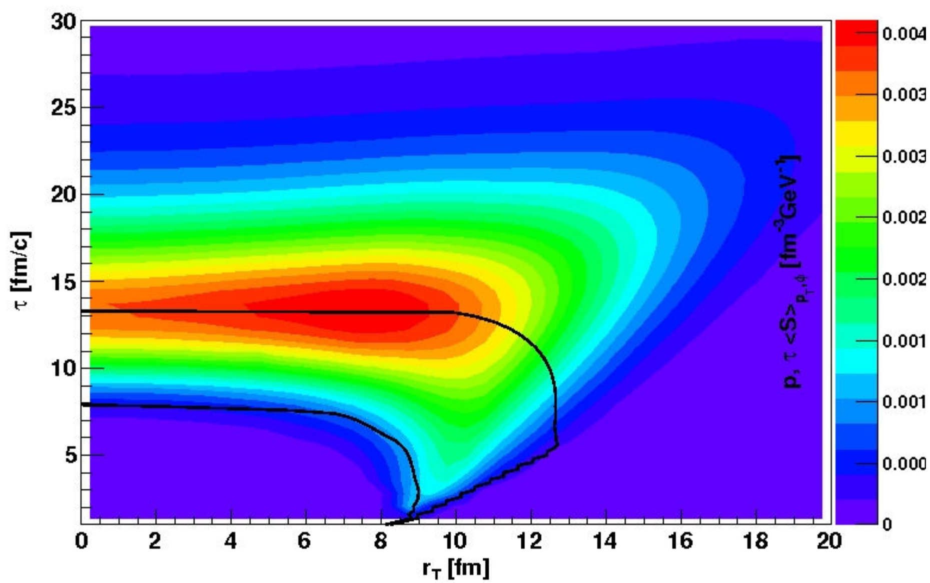




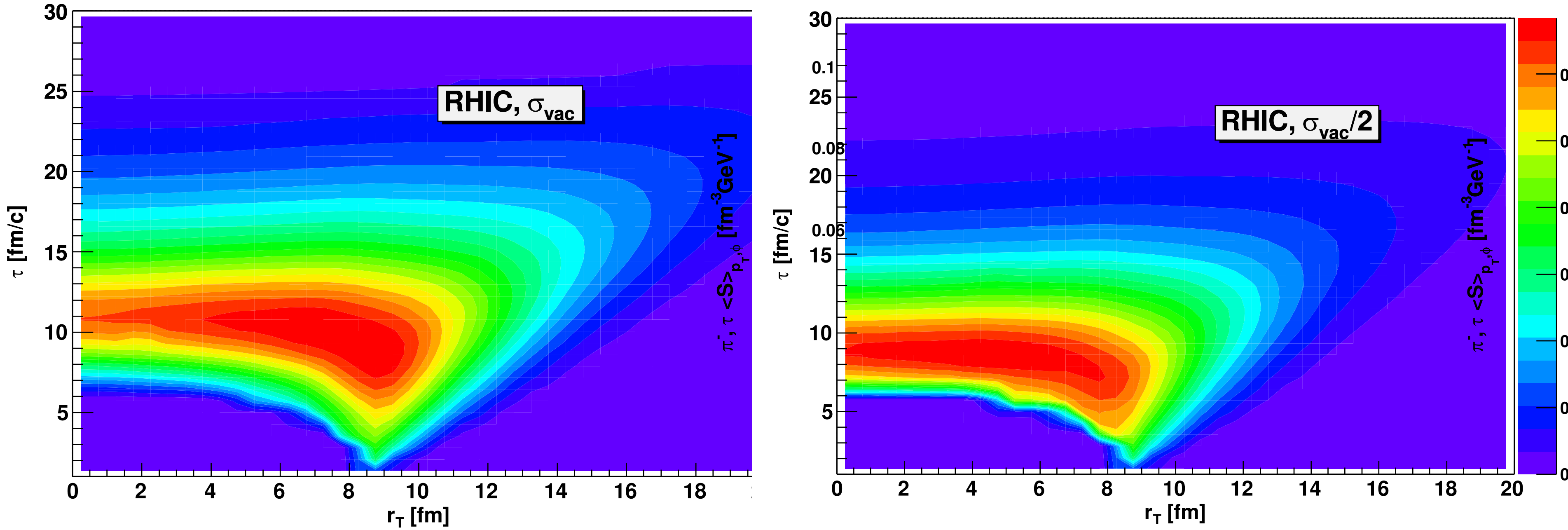
At the point of maximal pion emissi

$$\frac{\eta}{s} \approx 3\frac{1}{4\pi} \qquad \frac{s_\pi}{s_{tot}} \approx 0.6$$

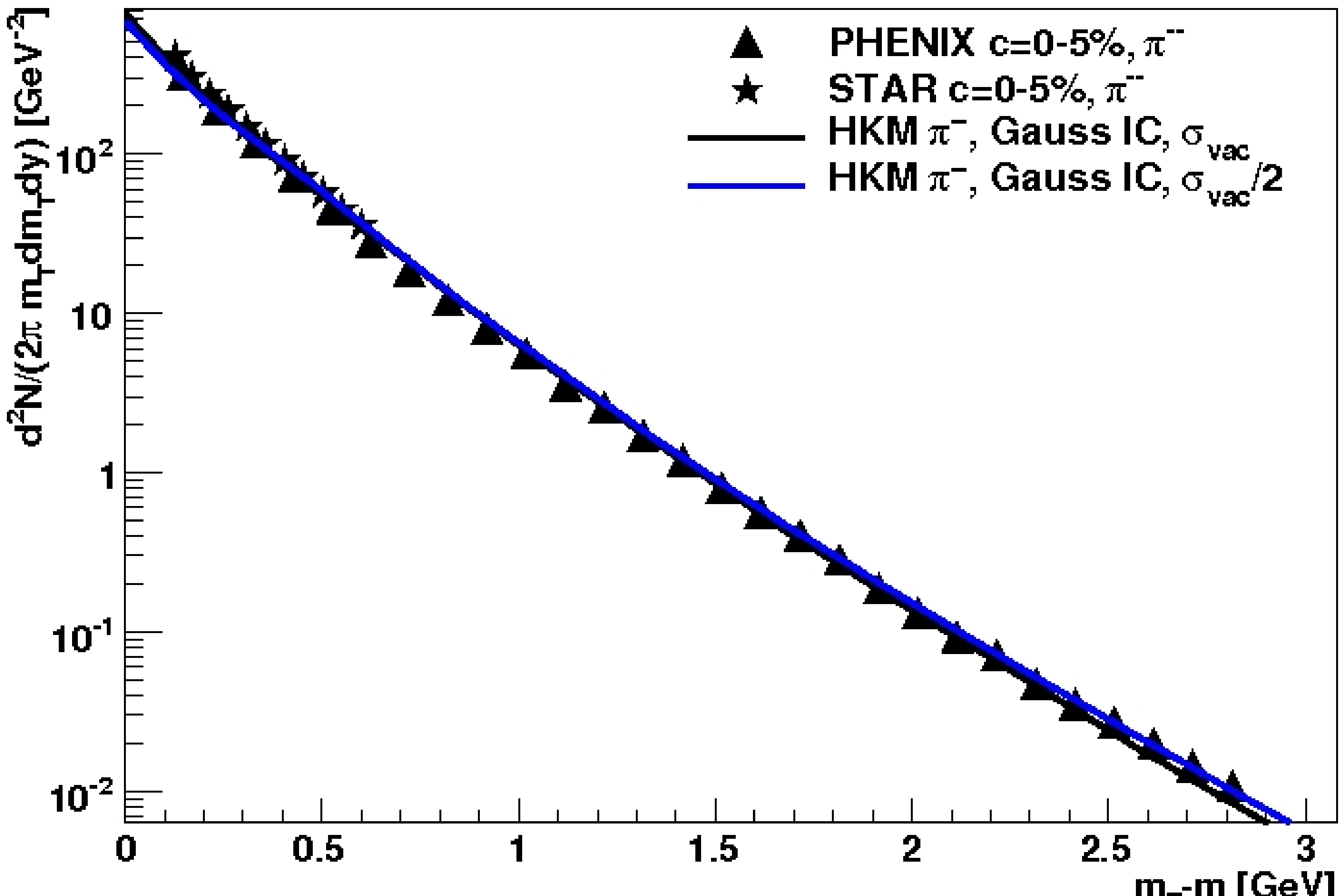
$$K_n^\pi = \frac{\lambda_{m.f.p.}}{\lambda_{hydr}} \approx 0.46; \qquad K_n^p \approx 0.25$$



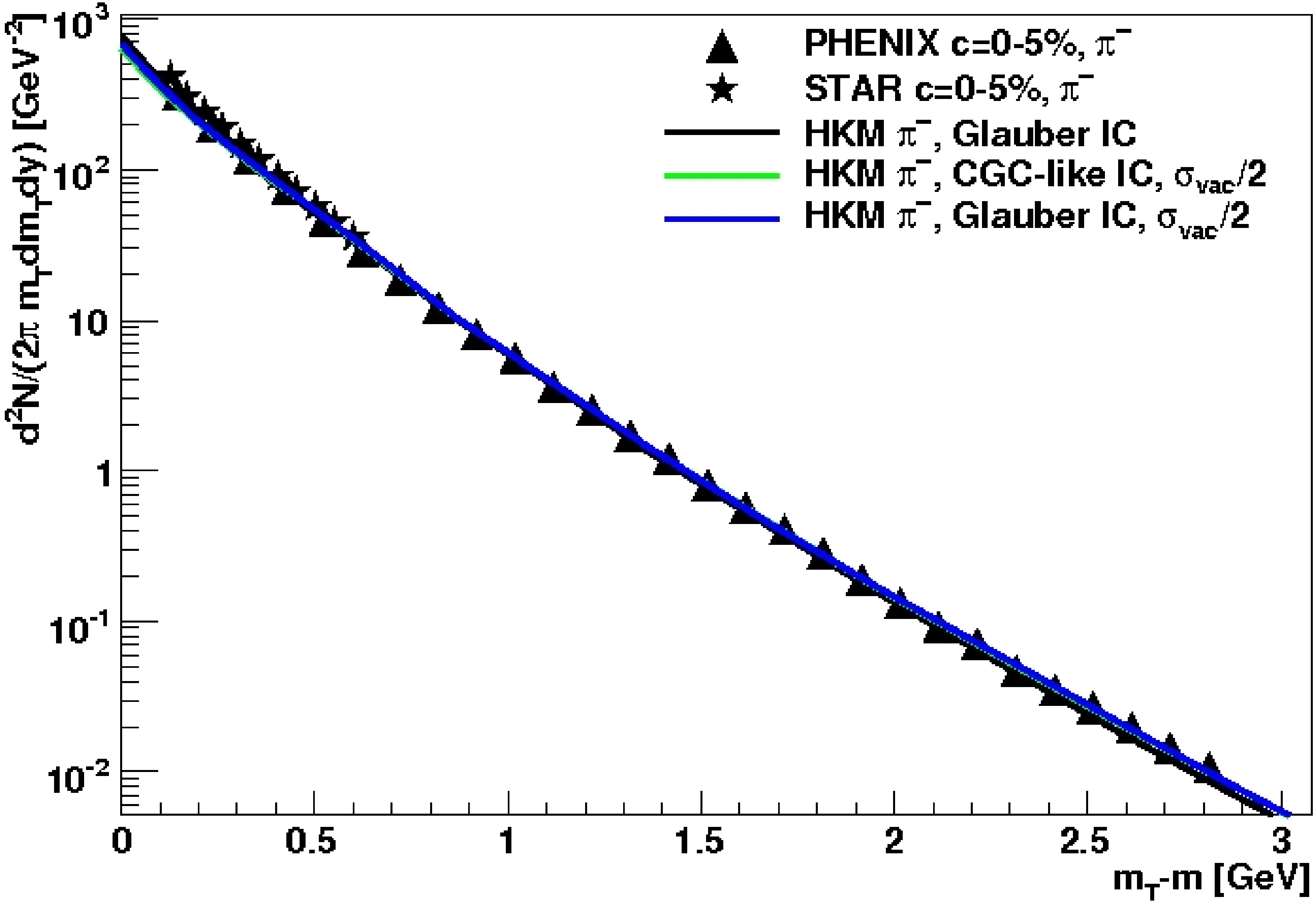
Pion emission densities (Gaussian IC) for total and "transport" cross sections



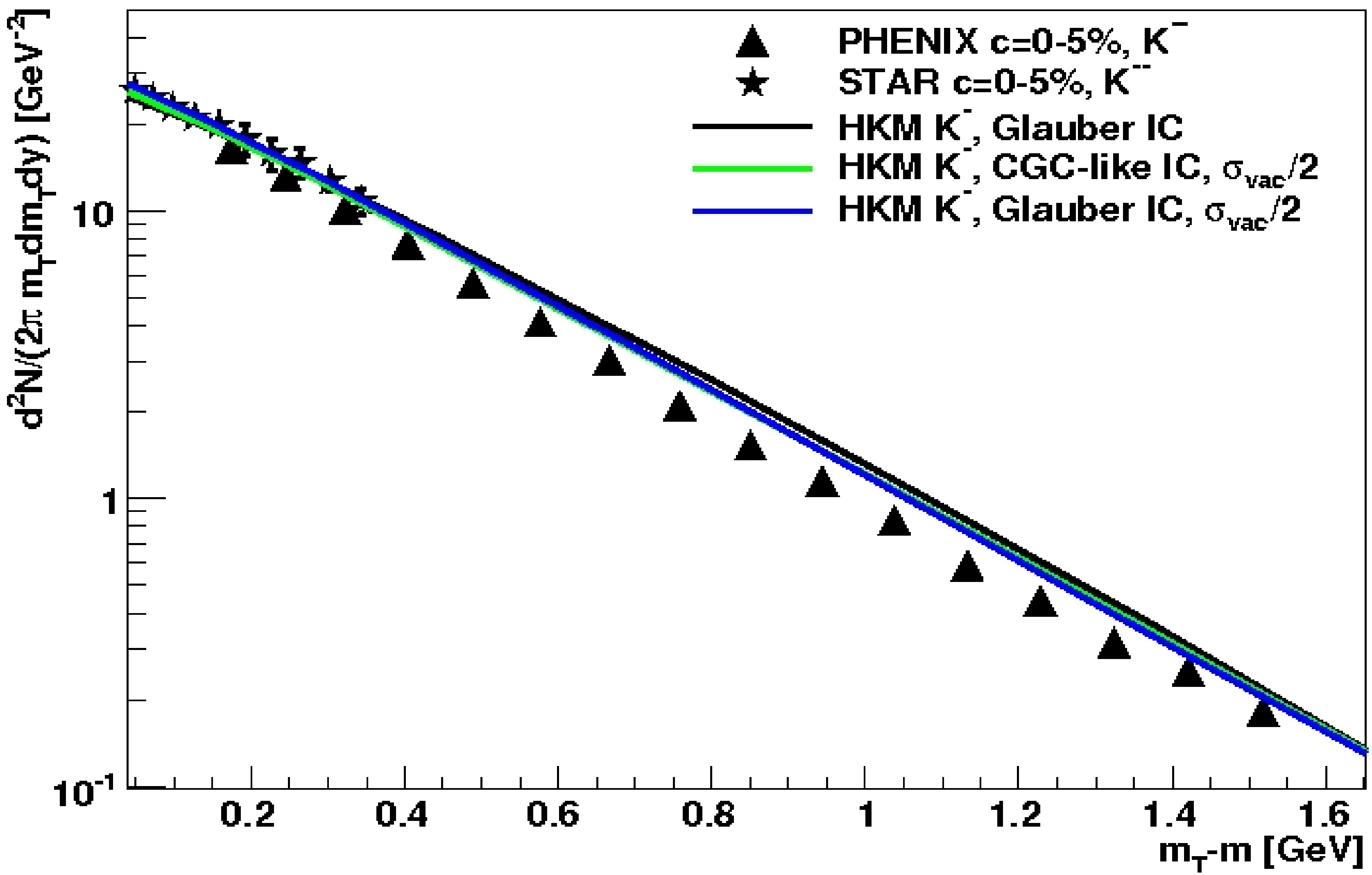
Momentum transverse spectra of pions in HKM for top RHIC energy and different cross-sections between hadrons (Gaussain profile)



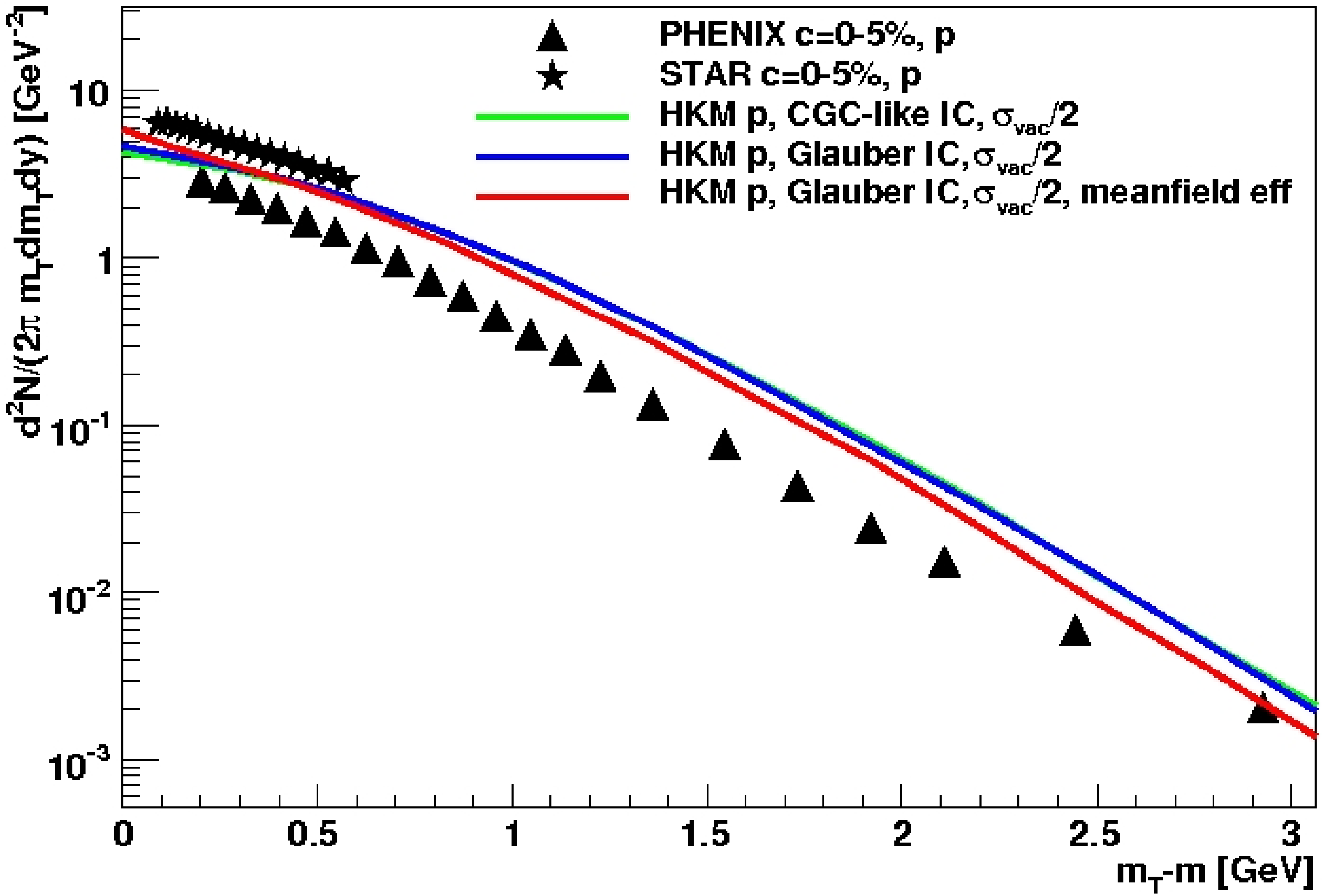
Momentum transverse spectra of pions in HKM for top RHIC energy and different cross-sections between hadrons (Glauber profile) different types of profiles (CGC and Glauber) of initial energy density



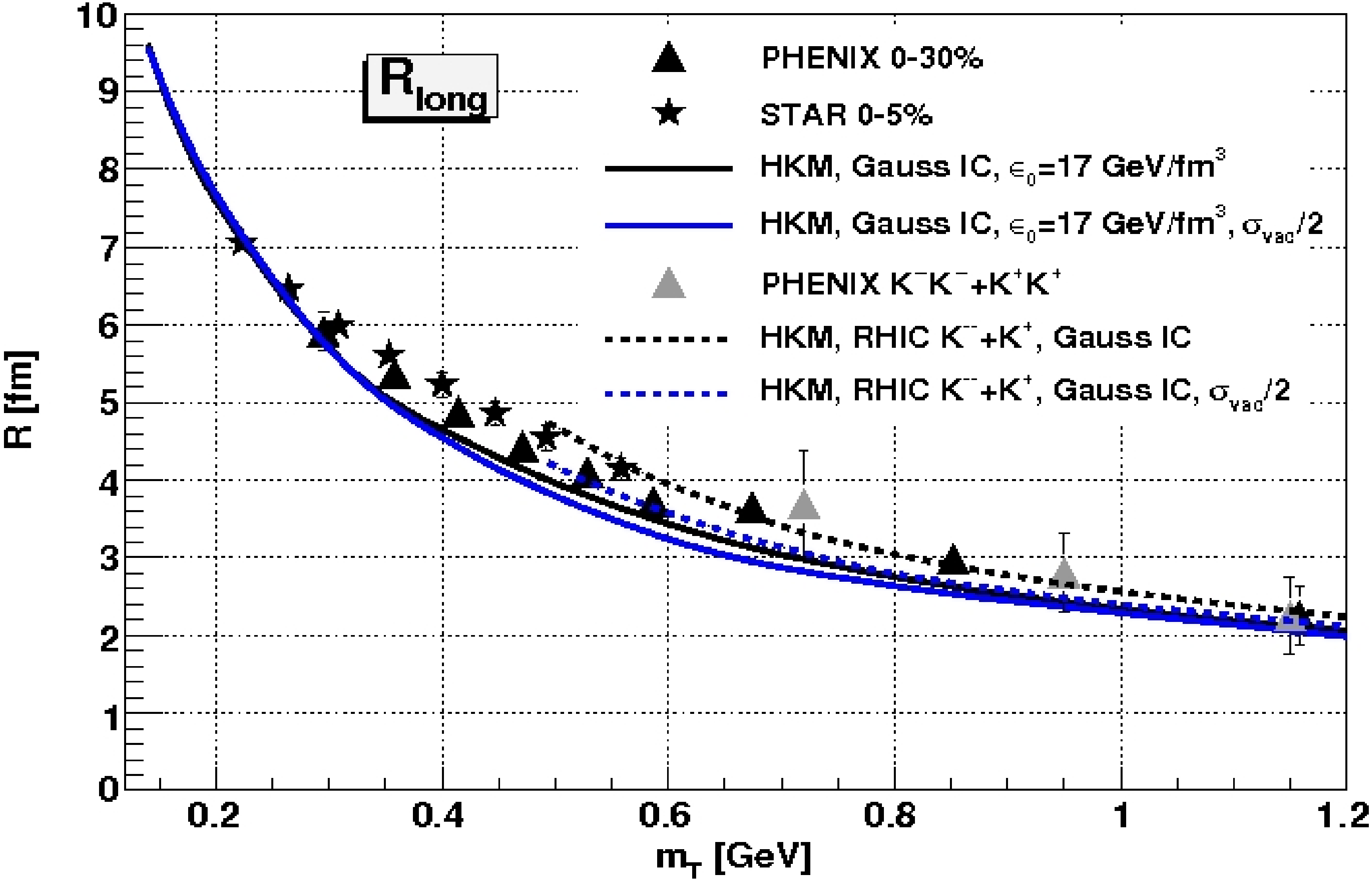
Momentum transverse spectra of kaons in HKM for top RHIC energy and different cross-sections between hadrons (Glauber profile) different types of profiles (CGC and Glauber) of initial energy density



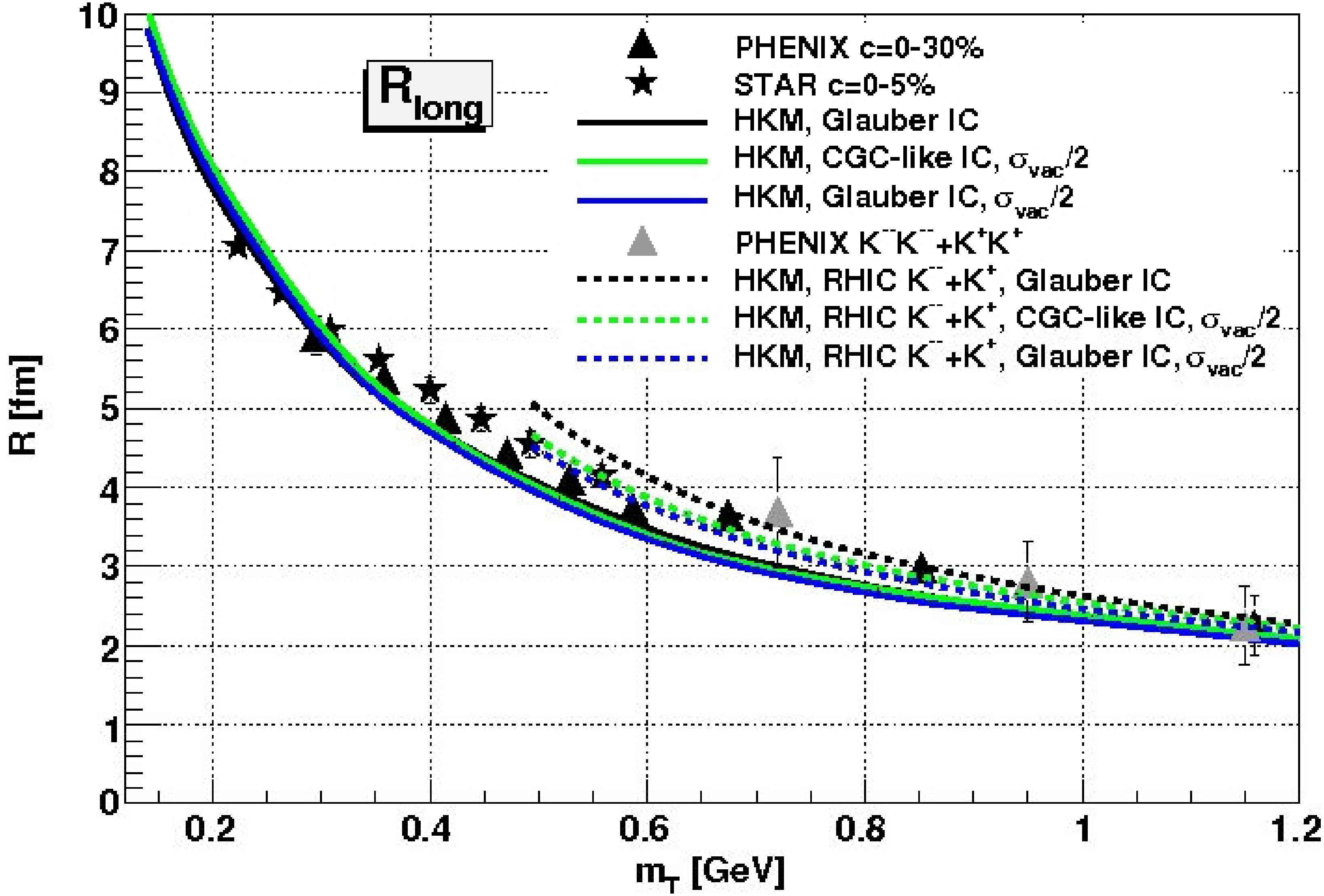
Momentum transverse spectra of protons in HKM for top RHIC energy and different types of profiles (CGC and Glauber) of initial energy density without and with including of the mean field effect for protons (12% of the proton transverse rapidity field off in the interval (0-1))



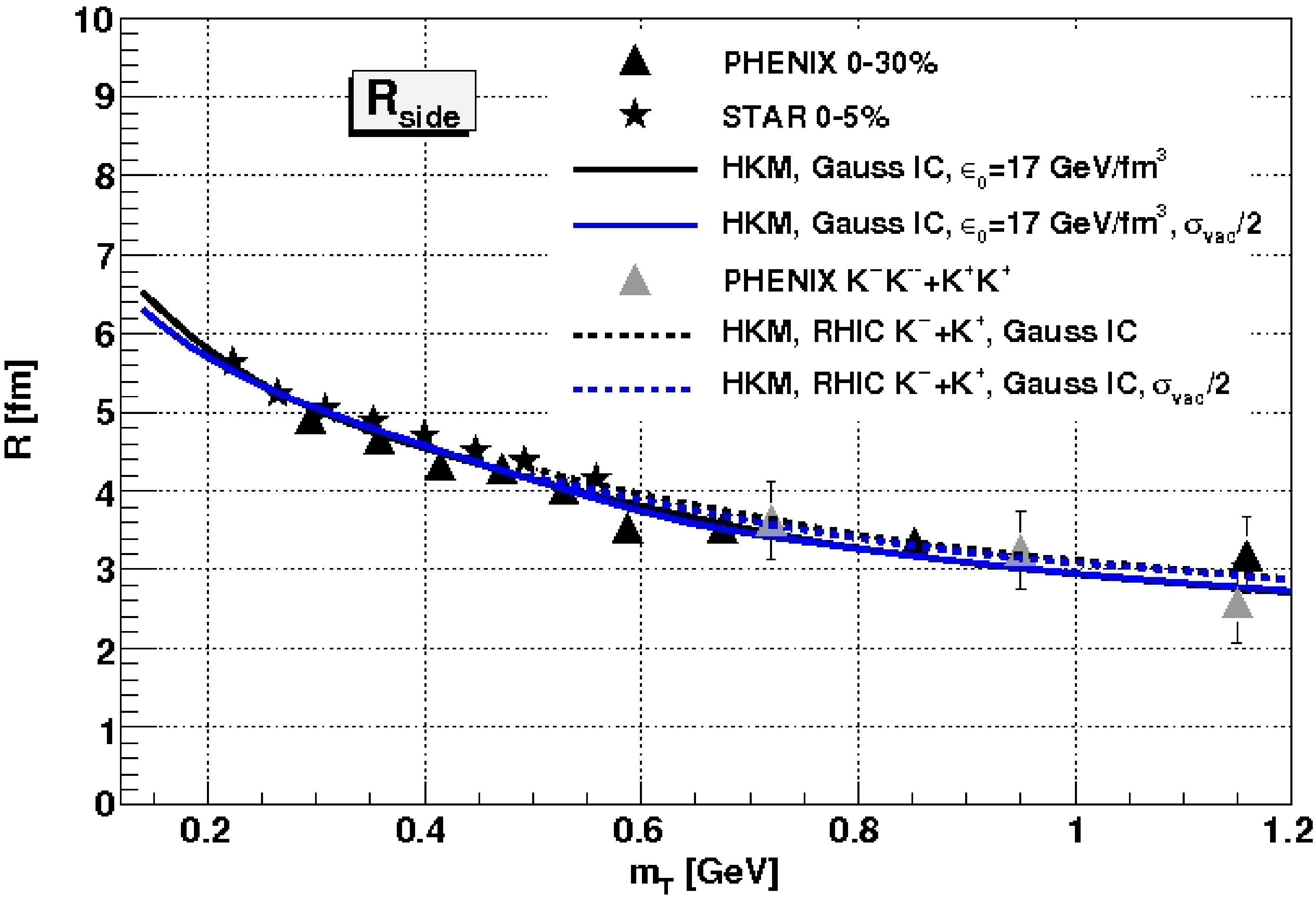
Long- interferometry radii of pions and kaons at RHIC in HKM for Gauss IC with total and “transport” cross-sections.



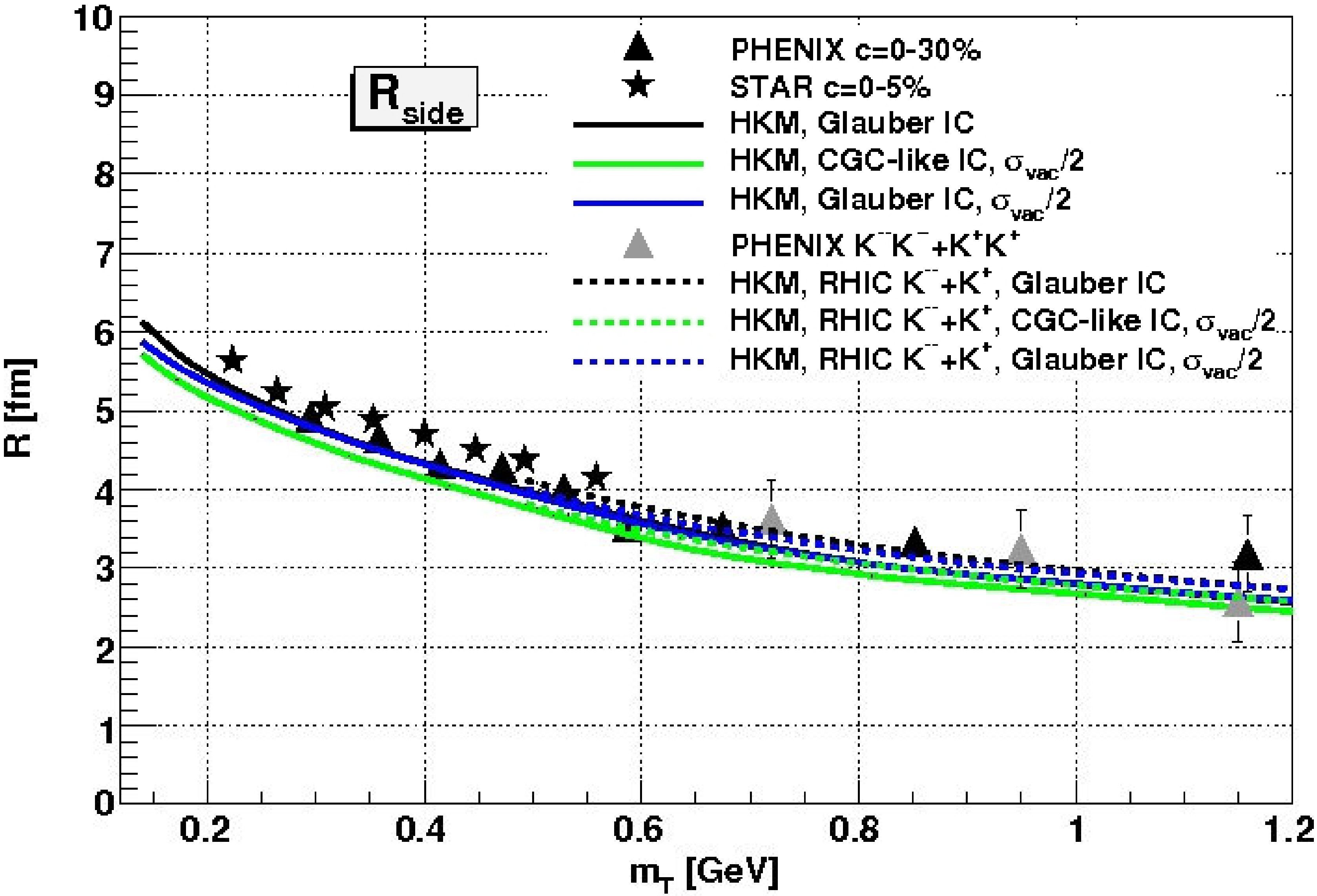
Long- interferometry radii of pions and kaons at RHIC in HKM for Glauber and CGC IC with total and “transport” cross-sections.



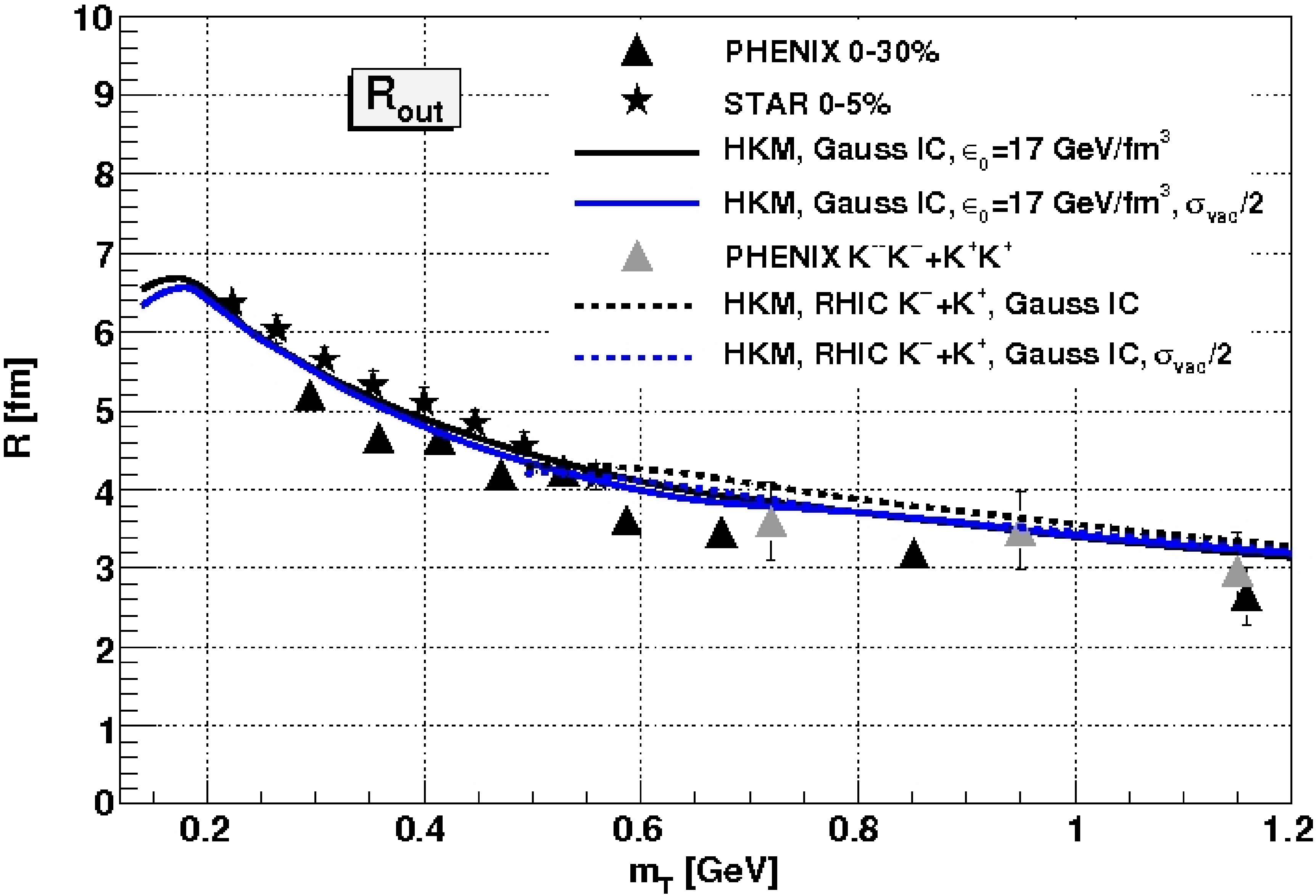
Side- interferometry radii of pions and kaons at RHIC in HKM for Gaussian IC with total and "transport" cross-sections.



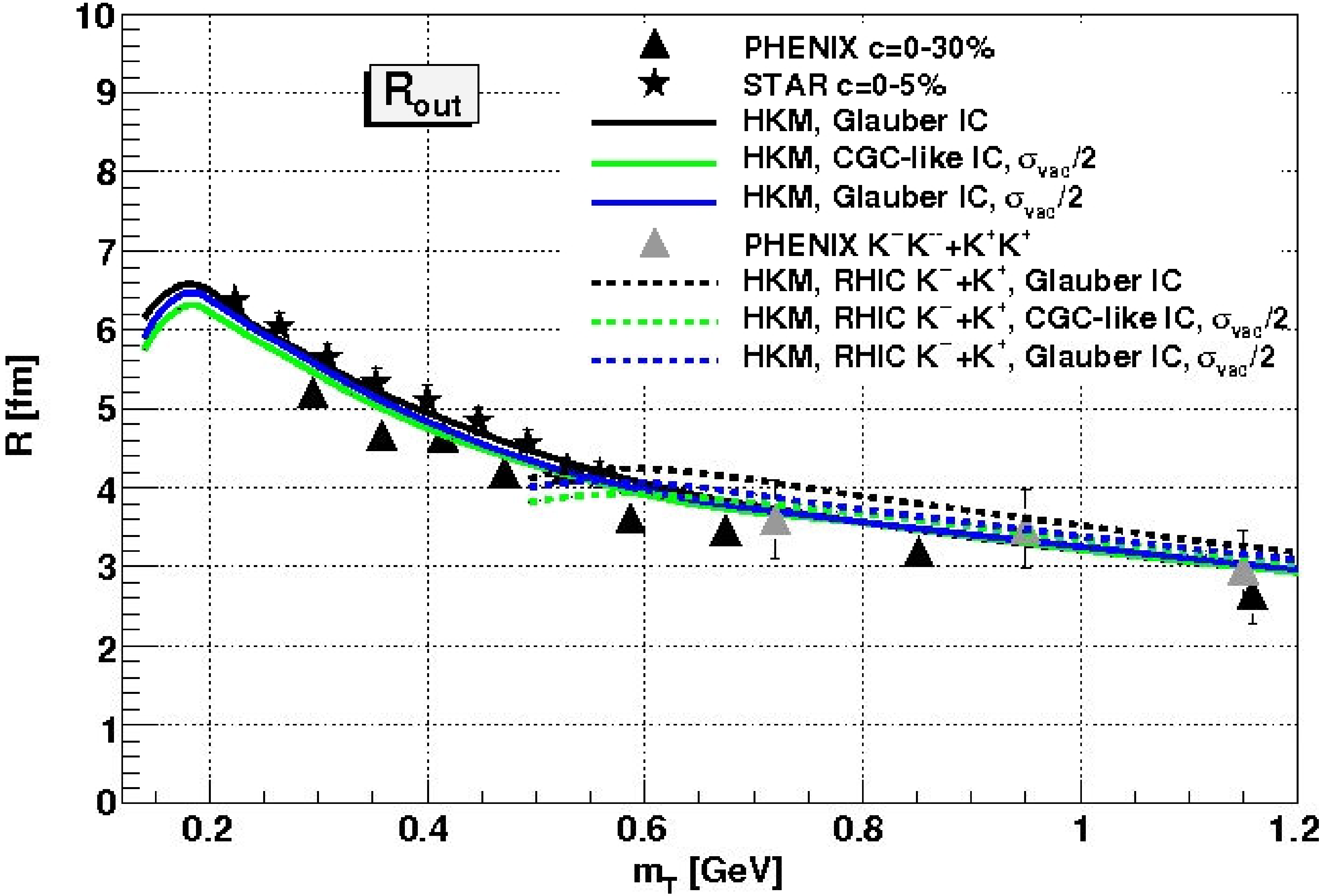
Side- interferometry radii of pions and kaons at RHIC in HKM for Glauber and CGC IC with total and “transport” cross-sections.



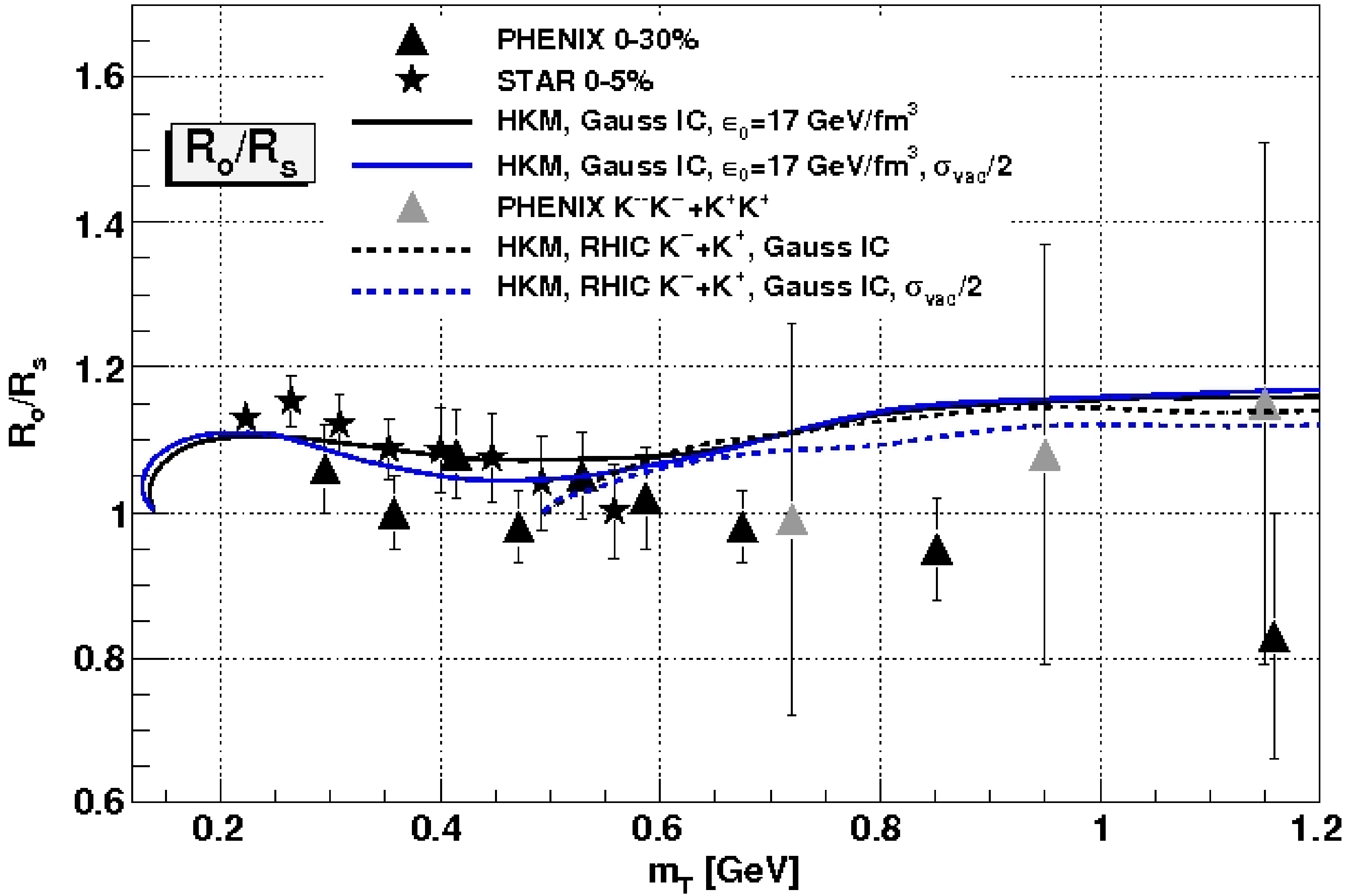
Out- interferometry radii of pions and kaons at RHIC in HKM for Gaussian IC with total and “transport” cross-sections.



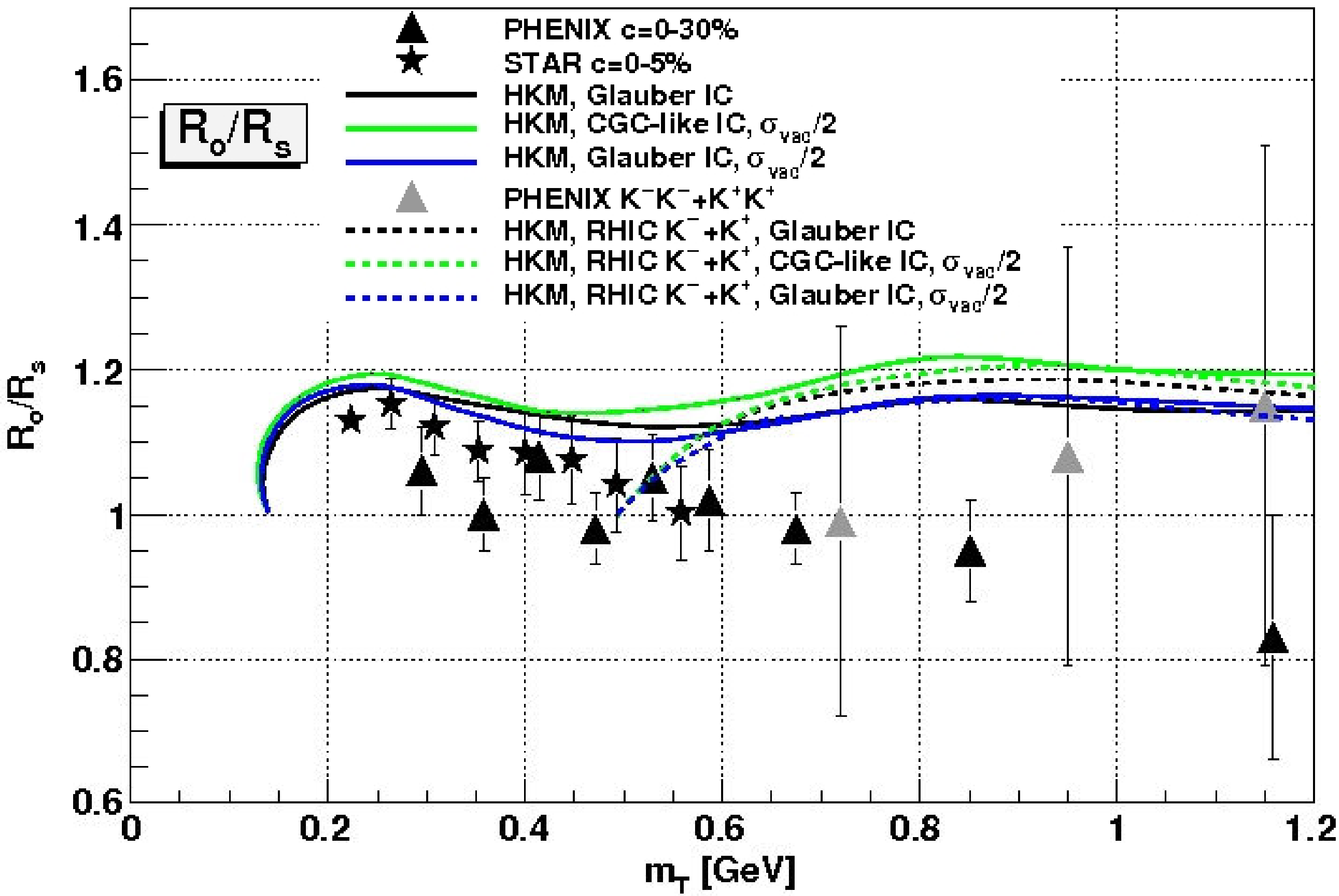
Out- interferometry radii of pions and kaons at RHIC in HKM for Glauber and CGC IC with total and “transport” cross-sections.



Out- to side- ratios of pions and kaons at RHIC in HKM for Gaussian IC with total and “transport” cross-sections.



Out- to side- ratios of pions and kaons at RHIC in HKM for Glauber and CGC IC with total and "transport" cross-sections.



Conclusions-2

A good description of the pion and kaon spectra and HBT (including out-to-side ratio of the interferometry radii) in cental Au+Au collisions at the RHIC energies is reached within the hydrokinetic model. The latter allows one to describe the continuous particle emission from a finite multicomponent hadron-resonance system, expanding hydrodynamically into vacuum, in the way which is consistent with Boltzmann equations.

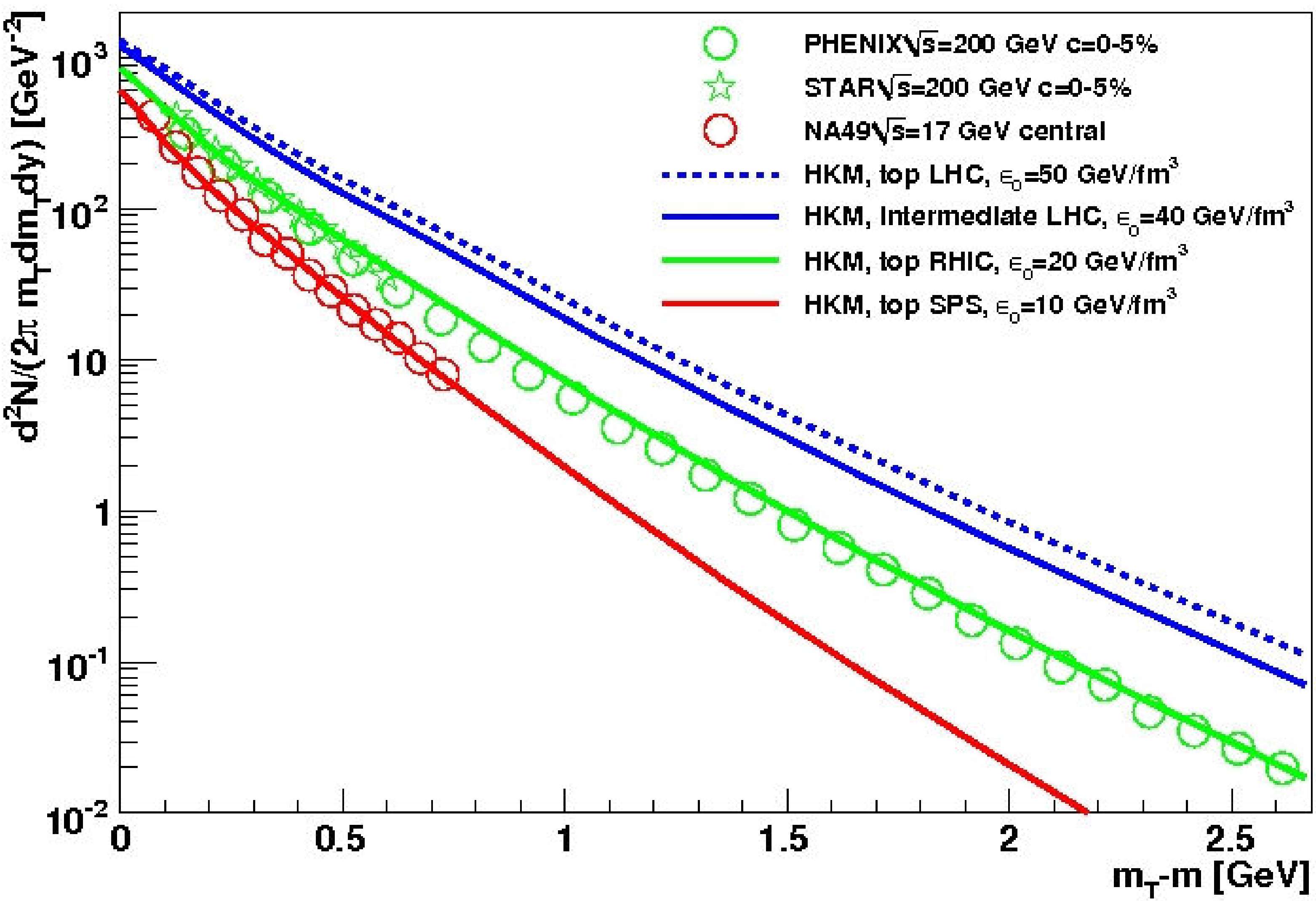
- **The following factors reduces space-time scales of the emission and Rout/Rside ratio.**
- **essentially non-flat initial energy density distributions (Gaussian, Glauber, CGC);**
- **developing of initial transv. flows at early pre-thermal stage;**
- **+ additional developing of transv. flow due to shear viscosity;**
- **more hard transition EoS, corresponding to cross-over.**

+

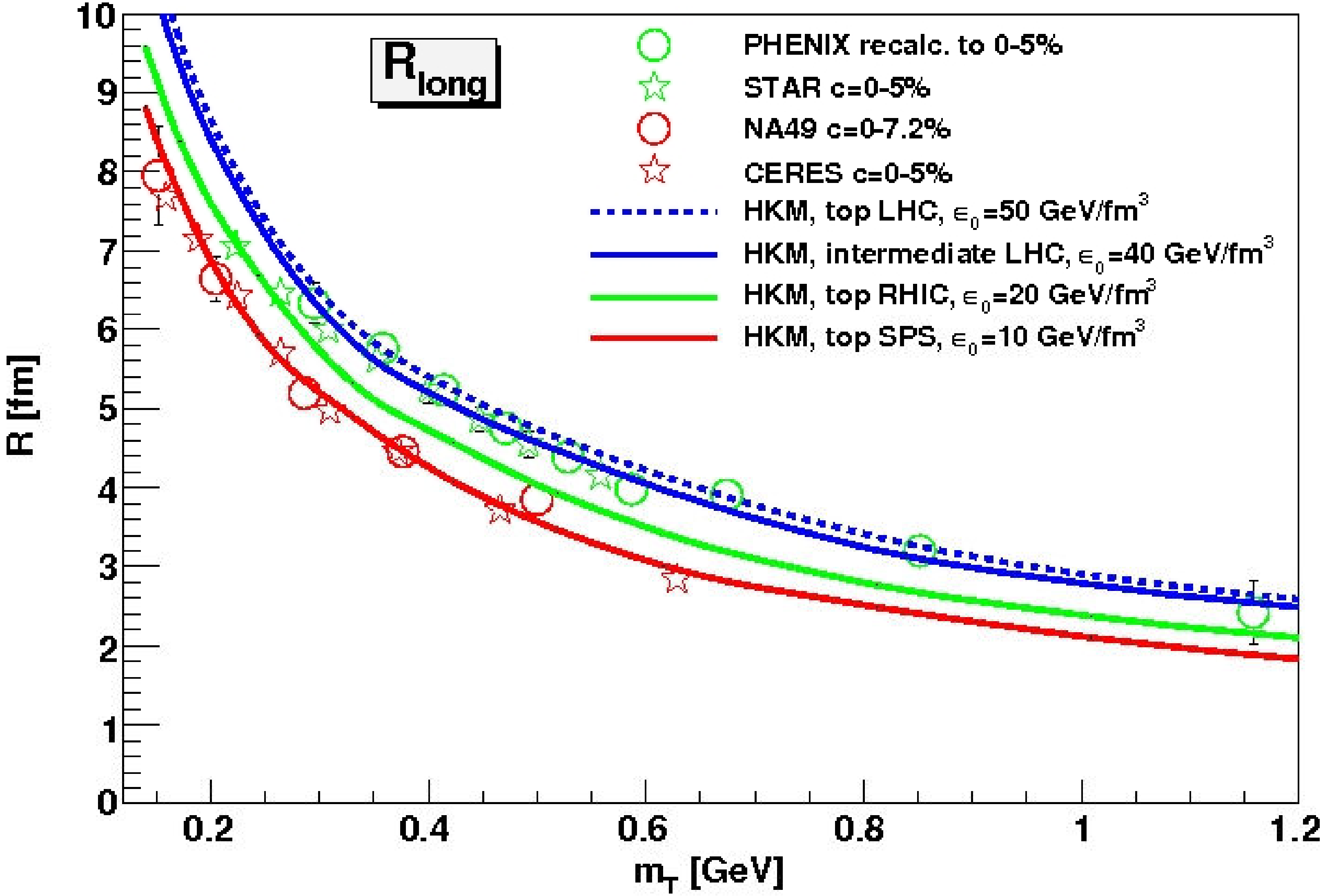
- **Correct description of evolution and decay of strongly interacting and chemically/thermally non-equilibrated system after hadronisation!**

Energy dependence of space-time scales

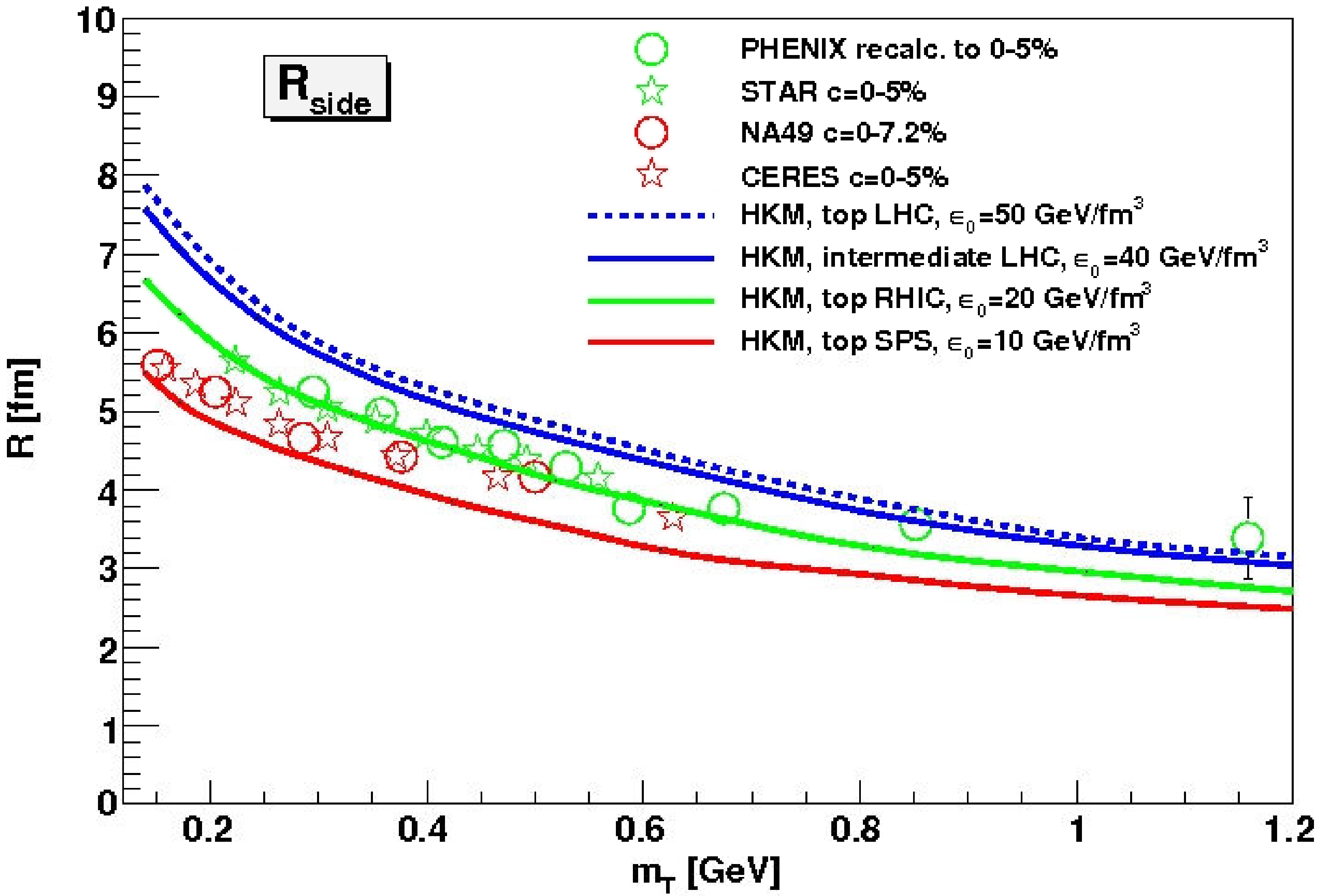
Pion spectra at top SPS, RHIC and two LHC energies in HKM



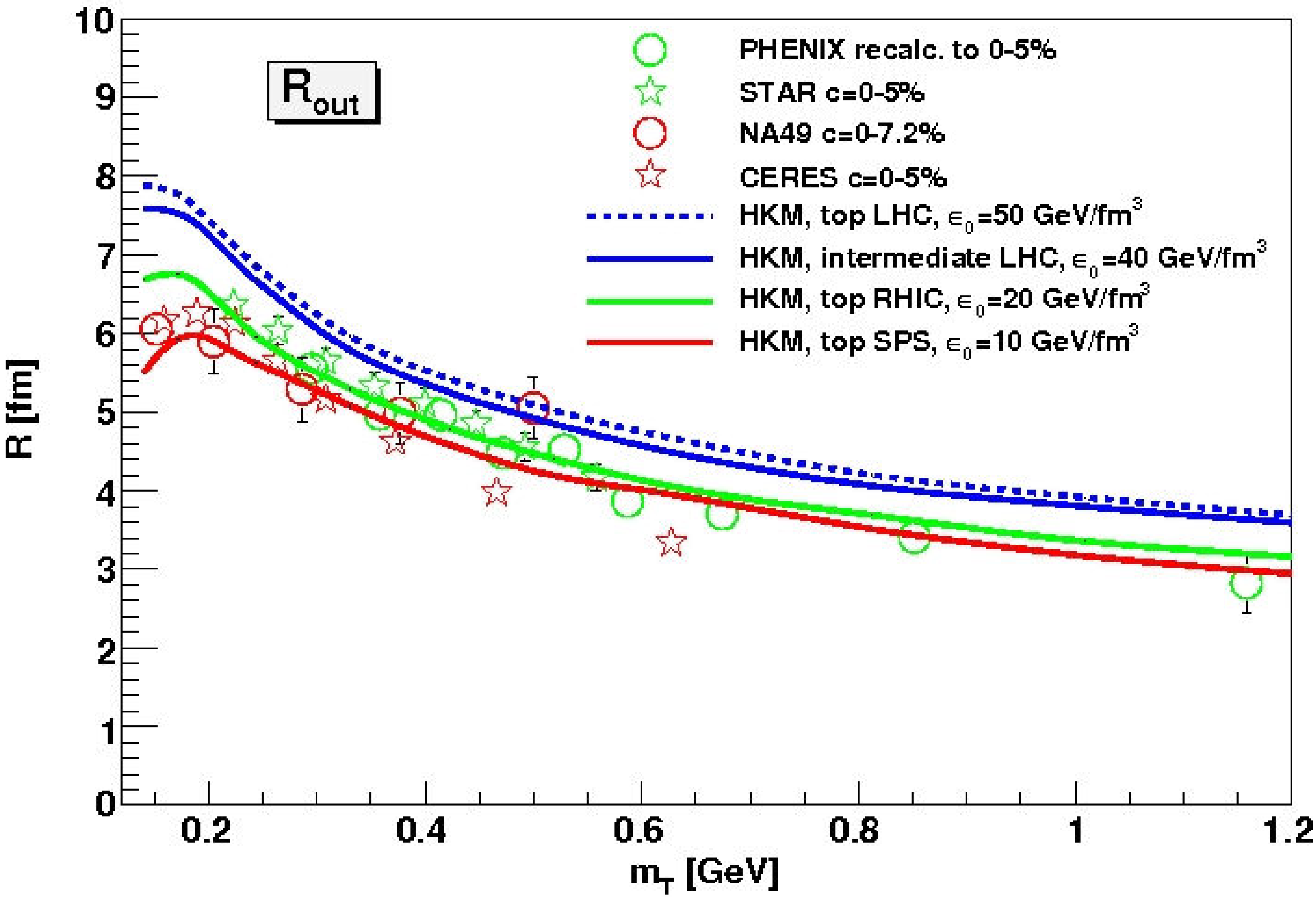
Long- radii at top SPS, RHIC and two LHC energies in HKM



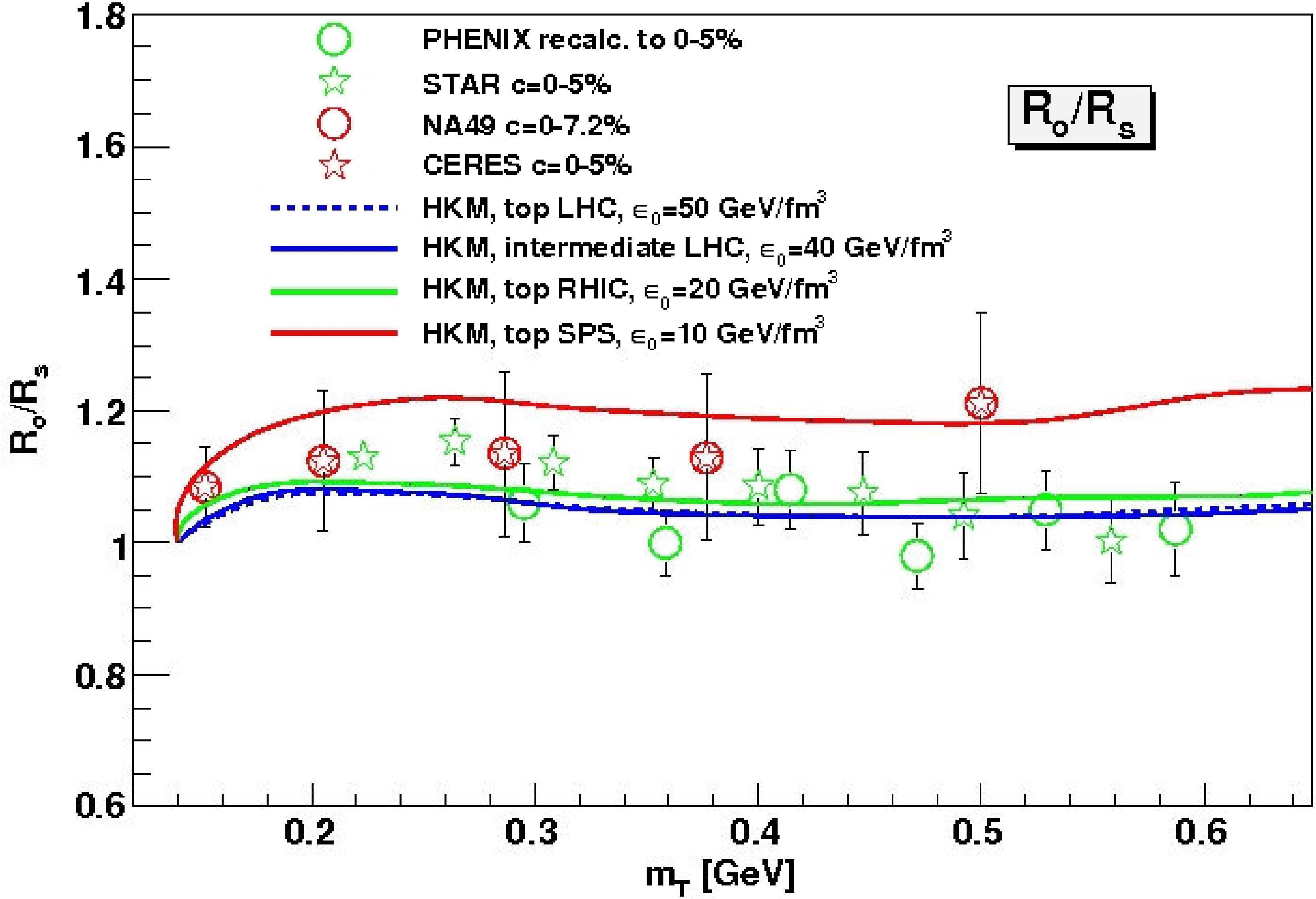
Side- radii at top SPS, RHIC and two LHC energies in HKM



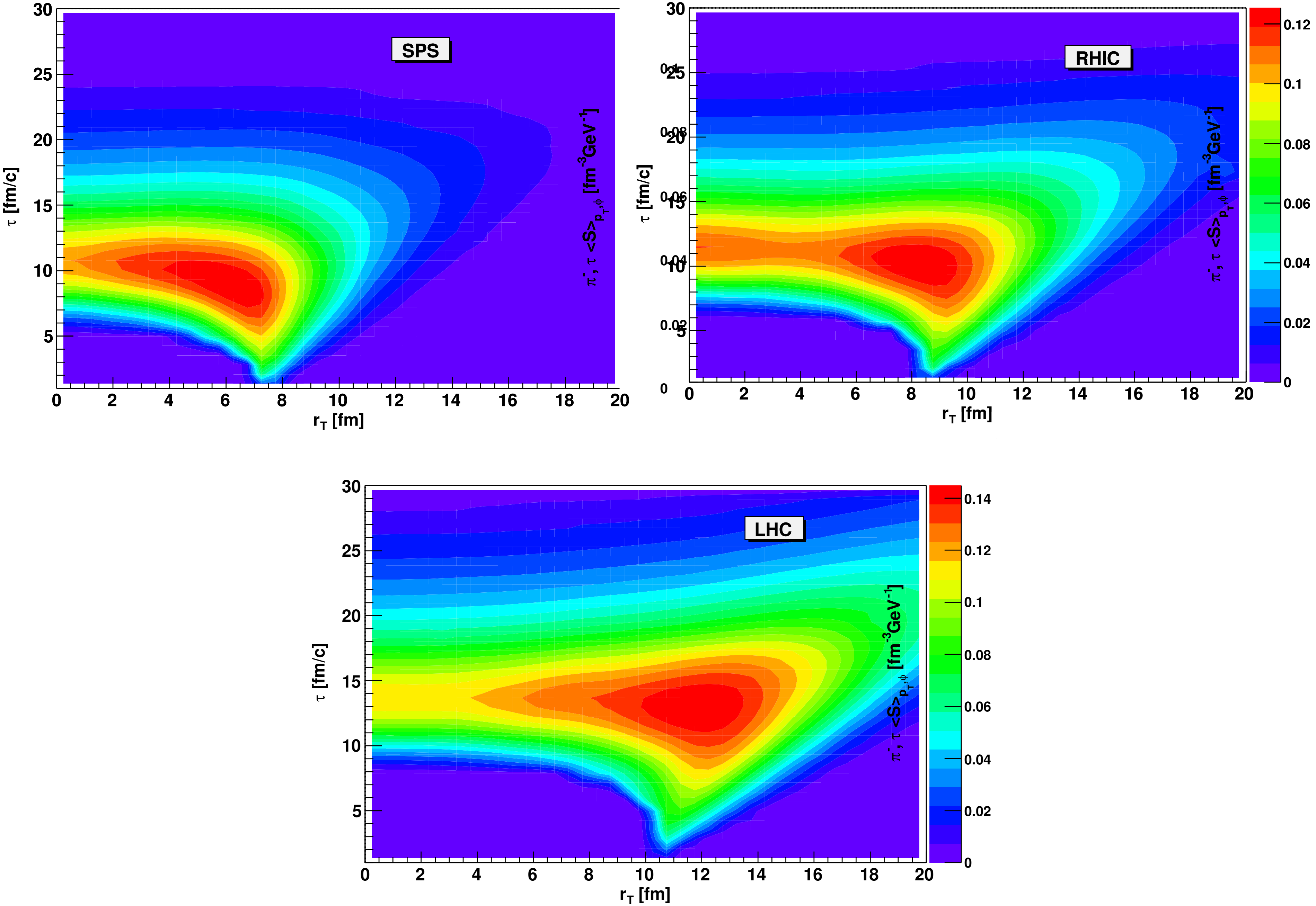
Out- radii at top SPS, RHIC and two LHC energies in HKM



Out- to side- ratio at top SPS, RHIC and two LHC energies in HKM

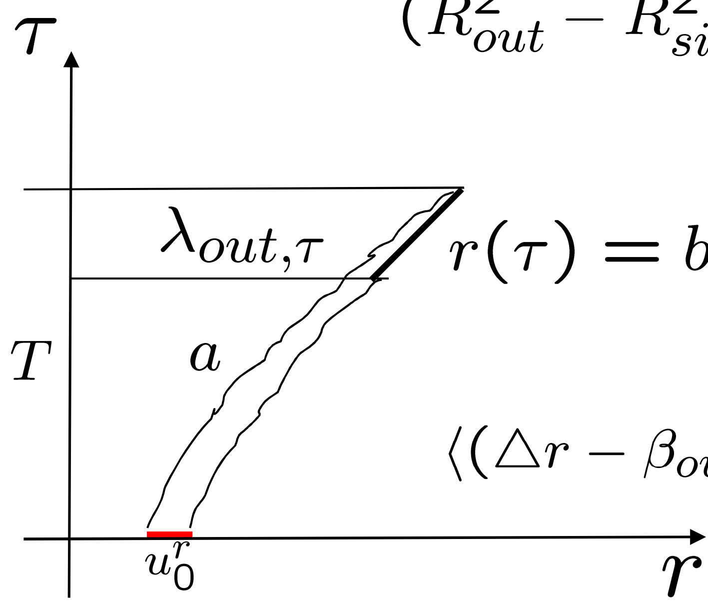


Emission functions for top SPS, RHIC and LHC energies



The R_{out}/R_{side} ratio as function on in-flow and energy

$$(R^2_{out} - R^2_{side})_{vol} \approx const \cdot \langle(\triangle r - \beta_{out}\triangle\tau)^2\rangle_{surf}$$



$$r(\tau) = b + v\tau \ \ (v > 0)$$

$$\langle(\triangle r - \beta_{out}\triangle\tau)^2\rangle_s \approx const \cdot \lambda^2_{out,\tau}(1-v)^2$$

$$v \approx 1 - \frac{1-u^r_0}{aT(\epsilon_0)} \ \text{ at } aT(\epsilon_0) \to \infty$$

$$\frac{R_{out}}{R_{side}} \approx 1 + const \cdot \frac{\lambda^2_{out,T}(1-u^r_0)^2}{a^2T^2(\epsilon_0)} \to 1.$$

Conclusion 3

- The main mechanisms that lead to the paradoxical behavior of the interferometry scales, are exposed.

- In particular, decrease of R_{out}/R_{side} ratio with growing energy and saturation of the ratio at large energies happens due to a magnification of $r-\tau$ positive correlations between space and time positions of emitted pions and a developing of pre-thermal collective transverse flows.

Saddle point approximation

Spectrum

$$n(t \to \infty, p) = \int d^3r f(t_0, \mathbf{r}, p) \mathcal{P}_{t_0 \to t}(t_0, \mathbf{r}, p) + \int d^3r \int_{t_0}^{t} dt' S(t', \mathbf{r}', p)$$

Emission density

$$S(t', \mathbf{r}', p) = f^{\mathrm{leq}}(t', \mathbf{r}' + \frac{\mathbf{p}}{p_0}(t' - t_0), p) Q(t', \mathbf{r}', p)$$

where

$$Q(t', \mathbf{r}', p) = \frac{d}{dt'} \mathcal{P}_{t' \to t}(t', \mathbf{r}' + \frac{\mathbf{p}}{p_0}(t' - t_0), p)$$

Normalization condition

$$\int_{t_0}^{\infty} dt' Q(t', \mathbf{r}', p) = 1 - \mathcal{P}(t_0, \mathbf{r}', p)$$

Eqs for saddle point $t_\sigma(\mathbf{r}, p)$**:**

$$\frac{dQ(t', \mathbf{r}', p)}{dt'}\Big|_{t' = t'_\sigma} = 0,$$

$$\frac{d^2 Q(t', \mathbf{r}', p)}{dt'^2}\Big|_{t' = t'_\sigma} < 0,$$

Physical conditions at $t_\sigma(\mathbf{r}, p)$

$$\frac{1}{\langle v\sigma \rangle (x) n(x)} = \tau_{\mathrm{scat}} \approx \tau_{\mathrm{exp}} = -\left(\frac{1}{n(x)} u^\mu \partial_\mu n\right)^{-1}$$

Cooper-Frye prescription

$$\mathbf{r} = \mathbf{r}' + \frac{\mathbf{p}}{p_0}(t'_\sigma(\mathbf{r}', p) - t_0)$$

Spectrum in new variables ⇒

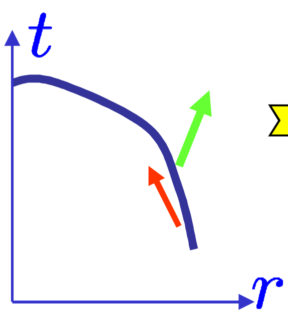
$$n(t \to \infty, p) \approx \int d^3r \left| 1 - \frac{\mathbf{p}}{p_0}\frac{\partial t_\sigma}{\partial \mathbf{r}} \right| \int_{t_0}^{t} dt' S(t', \mathbf{r}, p).$$

Emission density in saddle point representation ⇒

$$S(t', \mathbf{r}, p) = f^{\mathtt{leq}}(t', \mathbf{r} + \frac{\mathbf{p}}{p_0}(t' - t_\sigma), p)$$
$$\times \frac{\mathcal{P}_{t_\sigma \to \infty}(t_\sigma, \mathbf{r}, p)}{\tau_{\mathtt{rel}}(t_\sigma(\mathbf{r}, p), \mathbf{r}, p)} \exp(-(t' - t_\sigma(\mathbf{r}, p))^2 / 2D^2(\mathbf{r}, p))$$

Temporal width of emission ⇒

$$D(\mathbf{r}, p) = \frac{1}{\sqrt{2\pi}} \tau_{\mathtt{rel}}(t_\sigma, \mathbf{r}, p) \mathcal{P}^{-1}_{t_\sigma \to \infty}(t_\sigma, \mathbf{r}, p) \approx \frac{e}{\sqrt{2\pi}} \tau_{\mathtt{rel}}(t_\sigma, \mathbf{r}, p)$$



⇒

$$\mathcal{P}^{-1}_{t_\sigma \to \infty} = \exp\left(\int_{t_\sigma}^{\infty} ds \, \tau^{-1}_{\mathtt{rel}}(s, \mathbf{r} + \frac{\mathbf{p}}{p^0}(s - t_\sigma(\mathbf{r}, p), p) \right) \approx e$$

R_{coll}

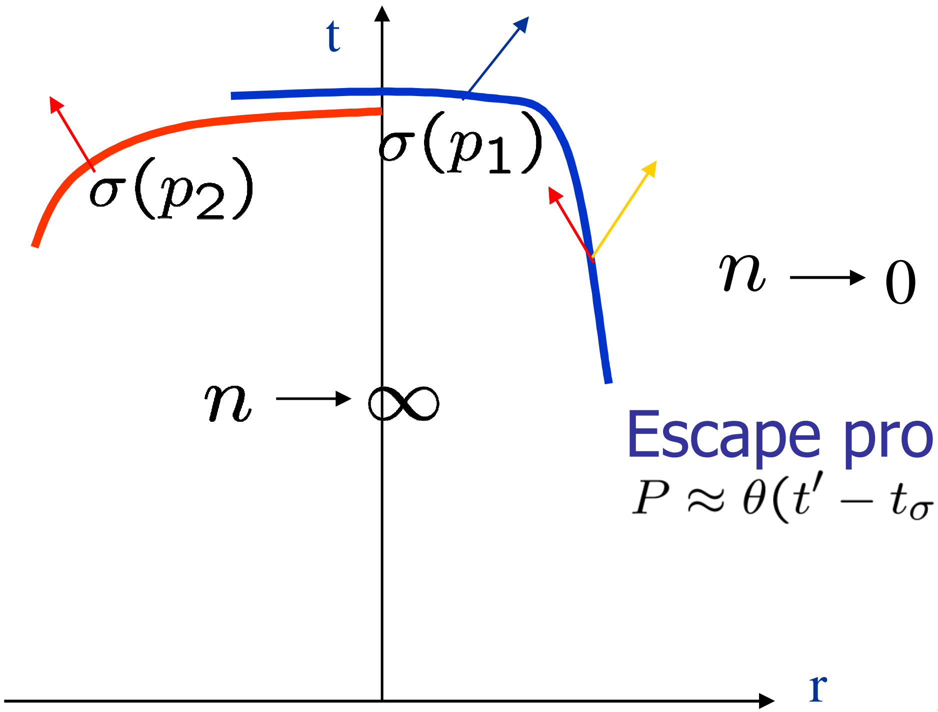
≈ 1

Generalized Cooper-Frye f-la ⇒

$$p^0 n(t \to \infty, p) = \int_{\sigma(p)} d\sigma_\mu p^\mu f^{\mathtt{leq}}(x, p)$$

Generalized Cooper-Frye prescription:

$$p^0\frac{d^3N}{d^3p} = \int_{\sigma(p)} d\sigma_\mu p^\mu f^{l.eq.}(x,p) \qquad d\sigma_\mu p^\mu > 0$$



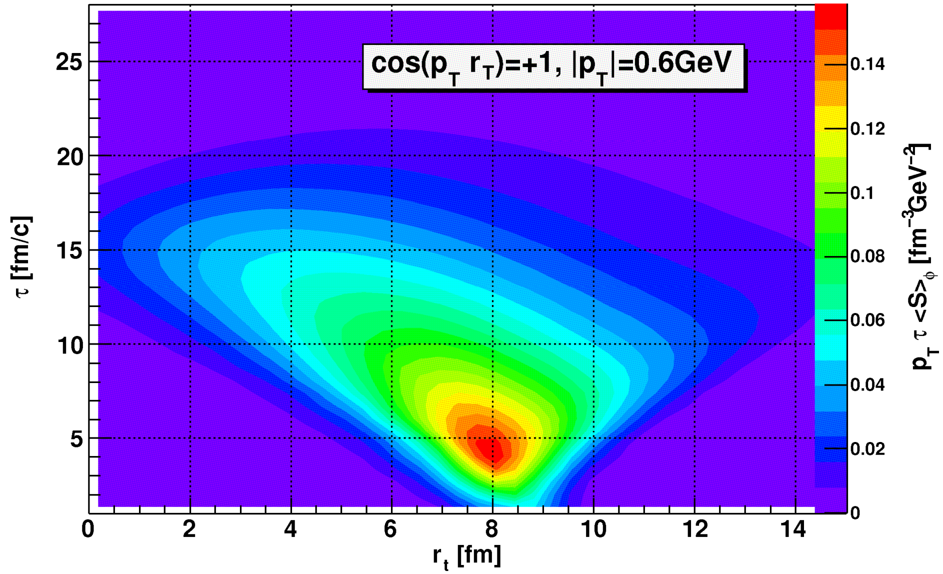
Escape probability

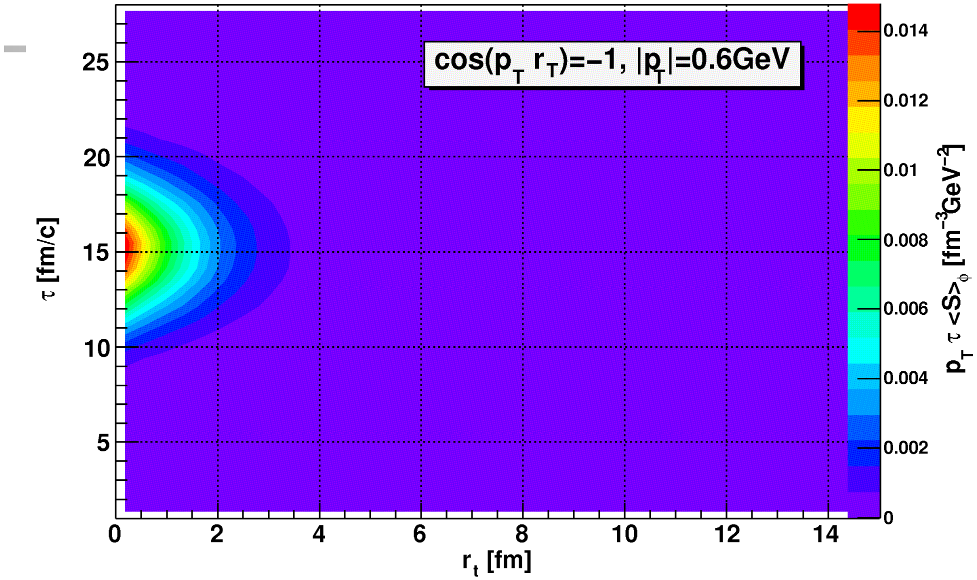
$$P \approx \theta(t' - t_\sigma(\mathbf{r},p))\theta(d\sigma_\mu(\mathbf{r},p)p^\mu)$$

$$p^0\frac{d^3N}{d^3p} = \int_{\sigma(p)} d\sigma_\mu p^\mu f^{l.eq.}(r,p)\theta(d\sigma_\mu p^\mu)$$

Yu.S. (1987)-particle flow conservation; K.A. Bugaev (1996) (current form)

Momentum dependence of freeze-out



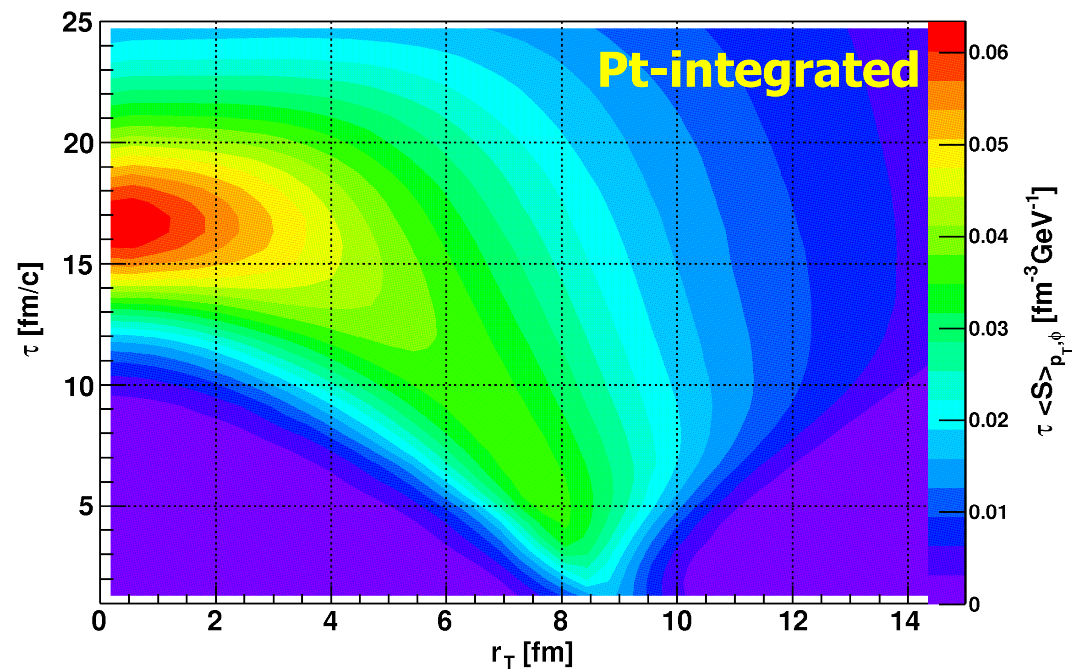


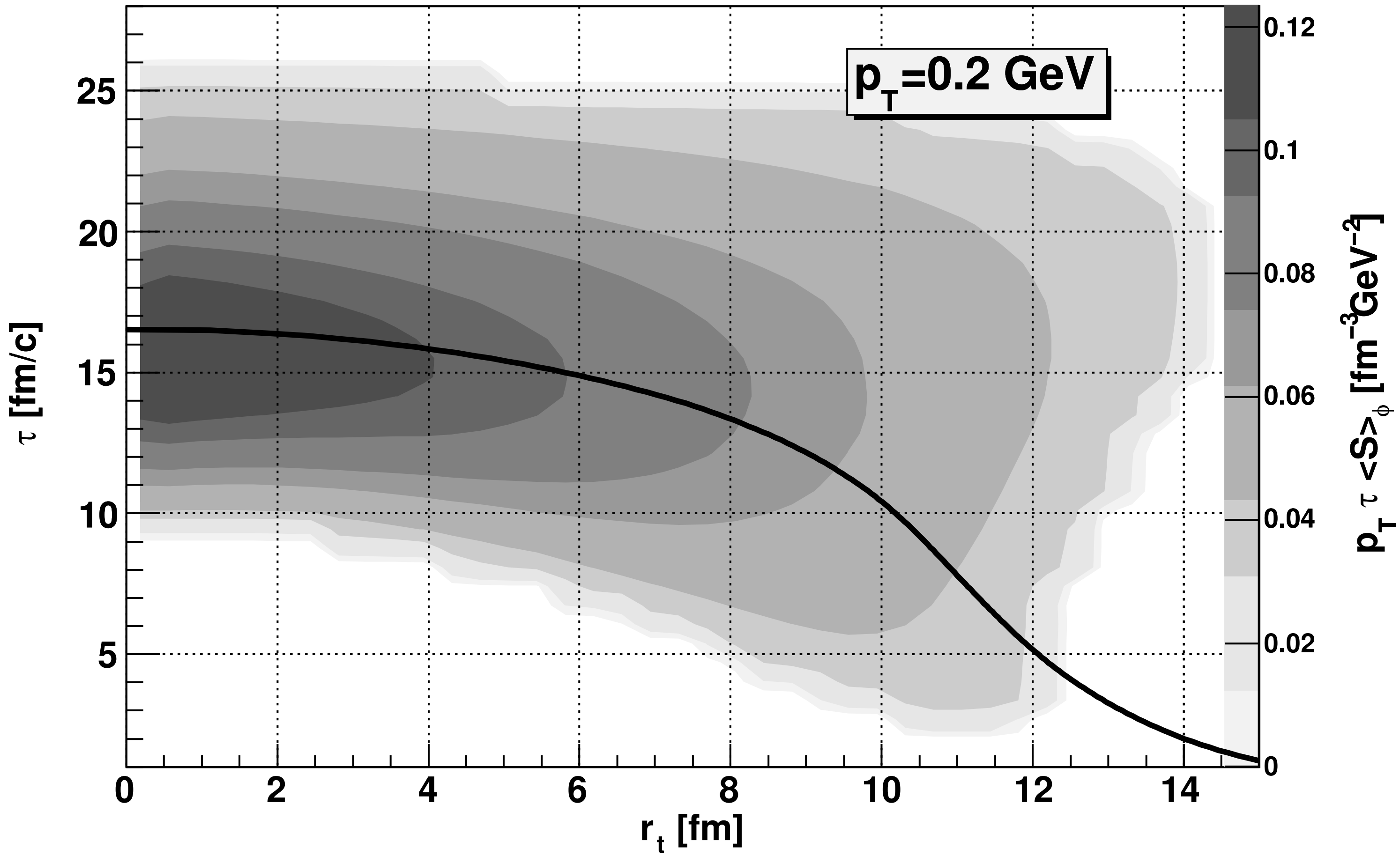
Here and further for Pb+Pb collisions we use:

initial energy density

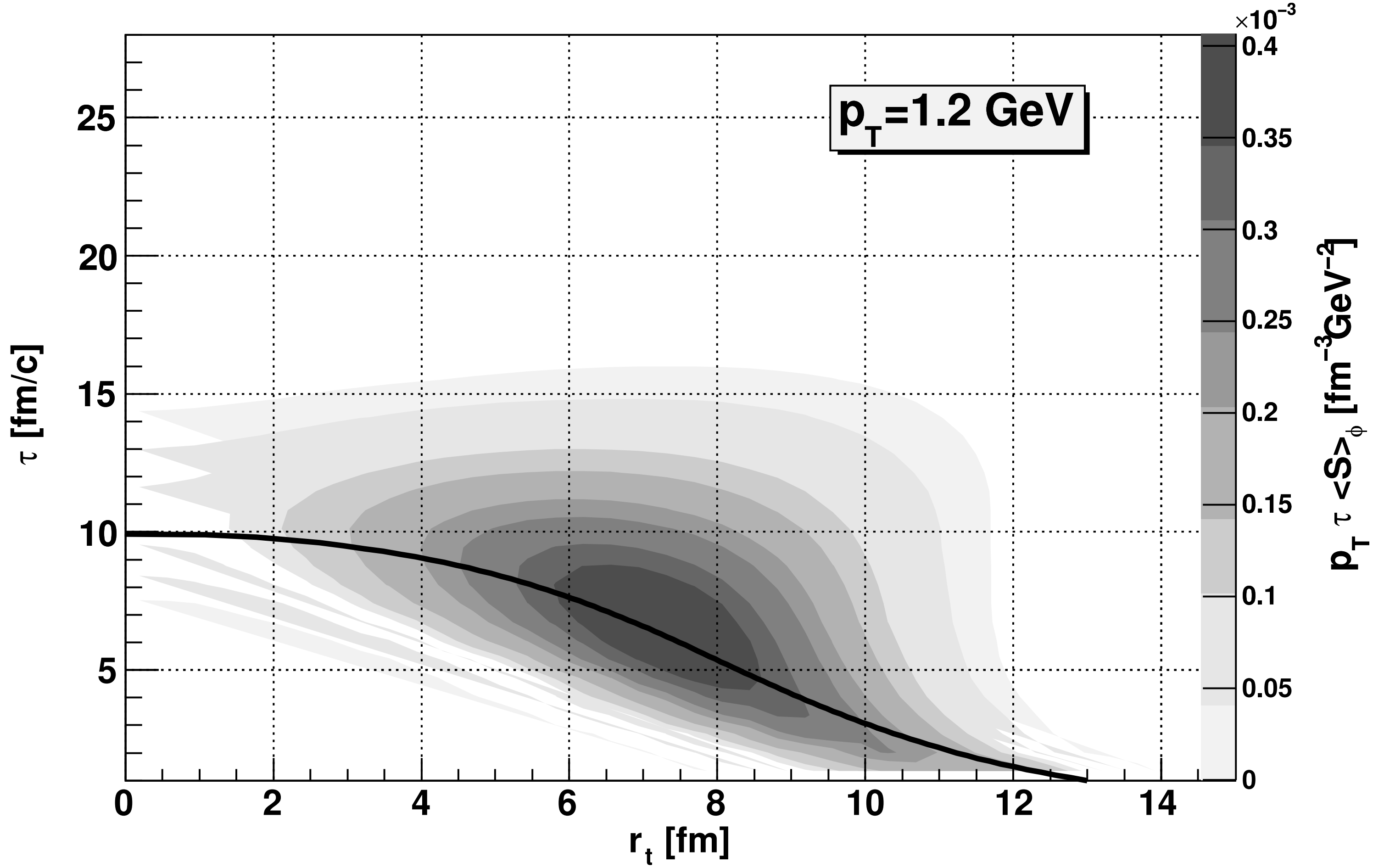
ϵ_i=6 GeV/fm^3 (T_i= 247 MeV)

EoS from Lattice QCD when T< 160 MeV, and EoS of chemically frozen hadron gas with 359 particle species at T< 160 MeV.

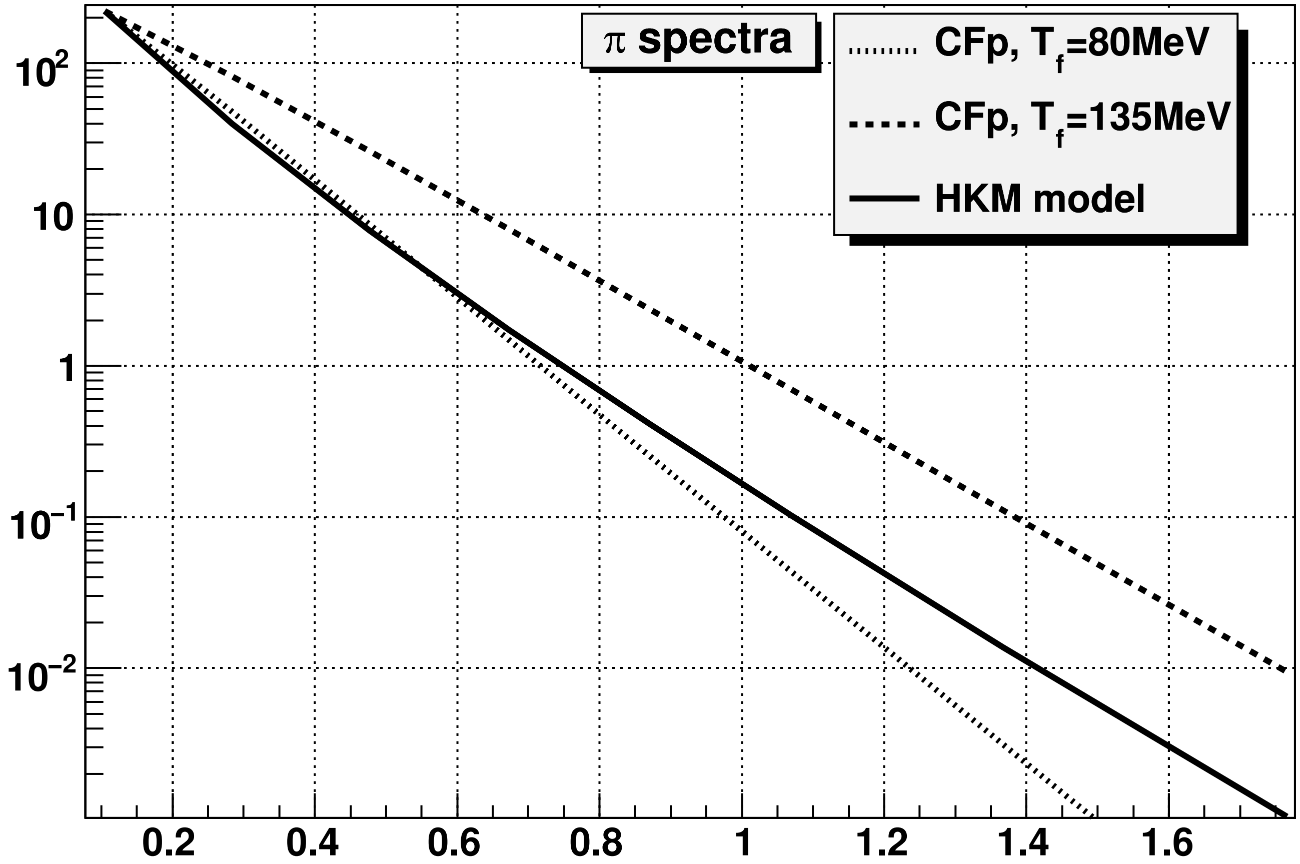




The pion emission function for different pT in hydro-kinetic model (HKM)
The isotherms of 80 MeV is superimposed.



The pion emission function for different pT in hydro-kinetic model (HKM). The isotherms of 135 MeV (bottom) is superimposed.



Transverse momentum spectrum of pi– in HKM, compared with the sudden freeze-out ones at temperatures of 80 and 160 MeV with arbitrary normalizations.

Conditions for the utilization of the generalized Cooper-Frye prescription

i) For each momentum p, there is a region of r where the emission function has a sharp maximum with temporal width $D(\mathbf{r},p)$.

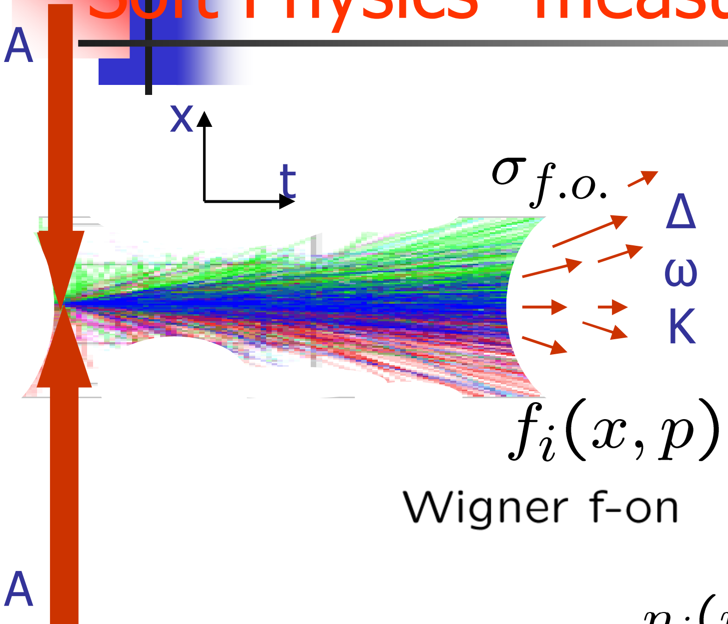
ii) The width of the maximum, which is just the relaxation time (inverse of collision rate), should be smaller than the corresponding temporal homogeneitylength of the distribution function: $\lambda(\mathbf{r},p) \gg D(\mathbf{r},p) \simeq \frac{e}{2\pi}\tau_{\mathtt{rel}}(\mathbf{r},p)$. 1% accuracy!!!

iii) The contribution to the spectra from the residual region of r where the saddle point method is violated does not affect essentially the particle momentum spectrum.

iiii) The escape probabilities $\mathcal{P}_{t_0\to\infty}(t_0,\mathbf{r},p)$ for particles to be liberated just from the initial hyper-surface $t0$ are small almost in the whole spacial region (except peripheral points)

Then the momentum spectra can be presented in Cooper-Frye form *despite it is, in fact, not sadden freeze-out and the decaying region has a finite temporal width*. Also, what is very important, such a generalized Cooper-Frye representation is related to *freeze-out hypersurface that depends on momentum* p and *does not necessarily encloses the initial dense matter*.

"Soft Physics" measurements



$$N_i = \int \frac{d^3p}{p^0} d\sigma_\mu p^\mu f_i(x,p)$$

$T_{f.o.}$
Landau, 1953
$\sigma_{f.o.}$

$$n_i(p) \equiv p^0 \frac{d^3N_i}{d^3p} = \int d\sigma_\mu p^\mu f_i(x,p)$$

Cooper-Frye prescription (1974)

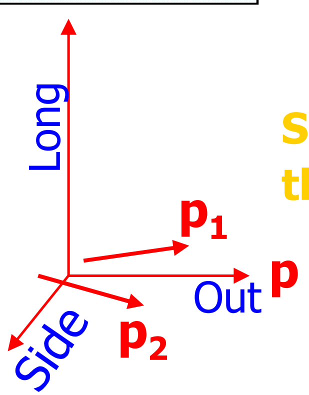
$$n_i(p_1,p_2) \equiv p_1^0 p_2^0 \frac{d^6N_i}{d^3p_1 d^3p_2} = C(p,q)\, n(p_1)\, n(p_2)$$

$p=(p_1+p_2)/2$

$q= p_1- p_2$

QS correlation function

$$C(p,q) = \left|\int d\sigma_\mu p^\mu \exp{(iqx)} f(x,p)\right|^2 / n(p)^2$$
$$= 1 + \exp{(R_L^2 q_L^2 + R_S^2 q_S^2 + R_O^2 q_O^2)}$$



Space-time structure of the matter evolution:

$$R_L(p_T) \approx \tau \sqrt{\frac{T_{f.o.}}{m_T}}$$

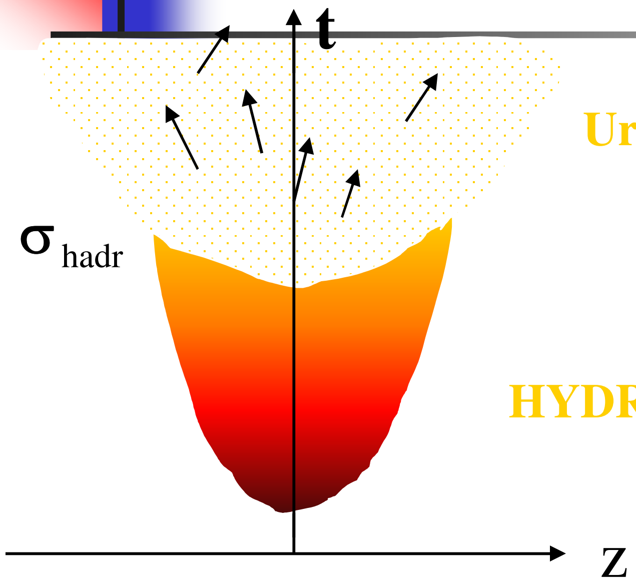
$$R_S^2 \approx R_T^2/(1 + I^2 m_T/T_{f.o.}),\ I^2 \propto \ < v_T^2 >$$

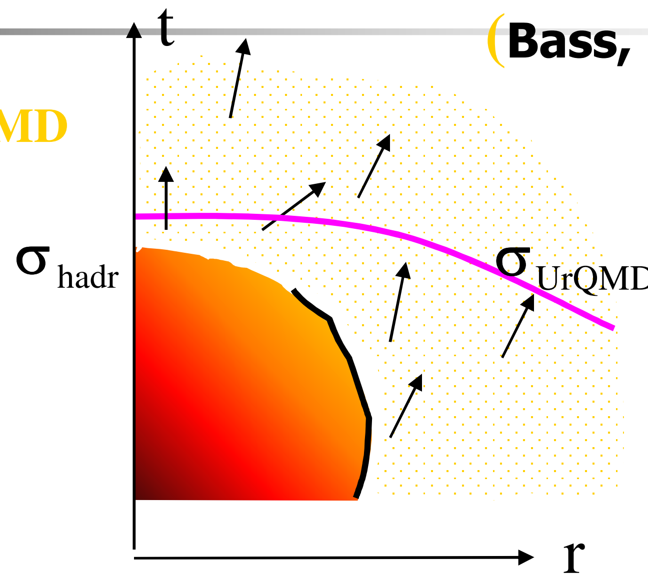
BW $R_{out}/R_{side} >> 1$ Exp: $R_{out}/R_{side} \approx 1$

$$R_O^2 \approx R^2 + v^2 \langle \triangle\tau^2 \rangle_p - 2v \langle \triangle x_O \triangle\tau \rangle_p,\ v = \frac{p_{out}}{p_0}$$

Sudden freeze-out ⟹ Hybrid models: HYDRO + UrQMD

(Bass, Dumitru (2000))





$\sigma_{hadr}: \tau \equiv \sqrt{t^2 - z^2} = const$

at $r = const$

$\sigma_{hadr}: \tau(r)$ at $z = 0$

The problems:

- the sudden freeze-out/decay of the system does contradict simulations in microscopic/cascade models
- the system just after hadronization is not so dilute to apply hadronic cascade models;
- hadronization hypersurface $\tau(r)$ contains non-space-like sectors (causality problem: Bugaev, PRL 90, 252301, 2003);

⟹ **The initial conditions for hadronic cascade models should be based on non-local equilibrium distributions**

Conclusions-1

- **The following factors reduces space-time scales of the emission and Rout/Rside ratio.**
  - **developing of initial flows at early pre-thermal stage;**
  - **more hard transition EoS, corresponding to cross-over;**
  - **non-flat initial (energy) density distributions, similar to Gaussian;**
  - **early (as compare to standard CF-prescription) emission of hadrons, because escape probability account for whole particle trajectory in rapidly expanding surrounding (no mean-free pass criterion for freeze-out)**
  - **Viscosity [Heinz, Pratt]**

- **The hydrokinetic approach to A+A collisions is proposed. It allows one to describe the continuous particle emission from a hot and dense finite system, expanding hydrodynamically into vacuum, in the way which is consistent with Boltzmann equations and conservation laws, and accounts also for the opacity effects.**

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Conclusions-2

The CFp might be applied only in a generalized form, accounting for the direct momentum dependence of the freeze-out hypersurface corresponding to the maximum of the emission function at fixed momentum **p** in an appropriate region of **r**.

Conclusions-3

- **A reasonable description of the pion spectra and HBT (except some an overestimate for R_{out}) in cental Au+Au collisions at the RHIC energies is reached with the value of the fitting parameter $\Lambda_s^4/g^2 \approx 6\ \mathrm{GeV}^4$ or the average energy density $\langle\epsilon\rangle = 33\ \mathrm{GeV/fm}^3$ at the initial time τ_0.**

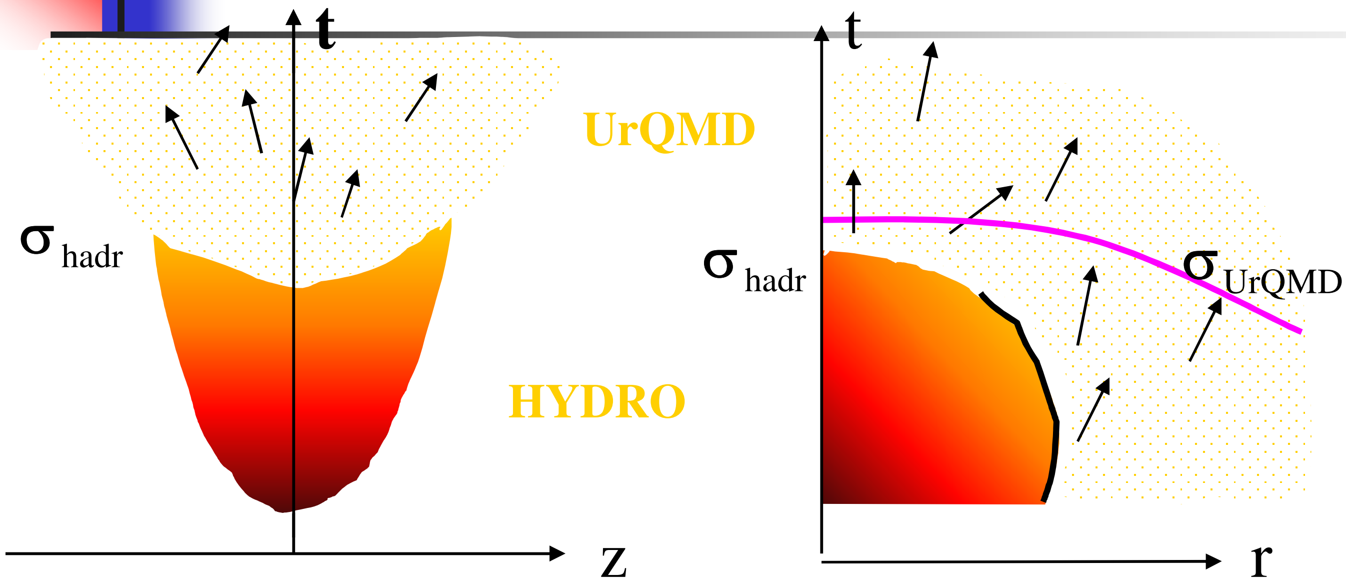
- **The initial time $\tau_0 = 0.3$ fm/c and transverse width $R_\epsilon =$ 5.3 fm (in the Gaussian approximation) of the energy density distribution are obtained from the CGC estimates.**

- **The EoS at the temperatures $T > T_c \approx T_{ch}$ corresponds to the lattice QCD calculations at $\mu_B = 0$.**

- **The used temperature of the chemical freeze-out $T_{ch} = 165$ MeV is taken from the latest results of particle number ratios analysis (F. Becattini, J.Phys. G, 2008).**

- **The anisotropy of pre-thermal transverse flows in non-central collisions, bring us a hope for a successful description of the elliptic flows with thermalization reached at a relatively late time:1-2 fm/c.**

Hybrid models: HYDRO + UrQMD (Bass, Dumitru (2000))



$$\sigma_{hadr} : \tau \equiv \sqrt{t^2 - z^2} = const \quad \text{at} \quad r = const$$

$$\sigma_{hadr} : \tau(r) \quad \text{at } z = 0$$

The problems:

- the system just after hadronization is not so dilute to apply hadronic cascade models;
- hadronization hypersurface $\tau(r)$ contains non-space-like sectors (causality problem: Bugaev, PRL 90, 252301, 2003);
- hadronization happens in fairly wide 4D-region, not just at hypersurface σ_{hadr}, especially in crossover scenario.

⇒ **The initial conditions for hadronic cascade models should be based on non-local equilibrium distributions**