Implications of HBT analyses

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Wigner function and interaction region

M. Hillery, R.F. O'Connel, M. Scully and E.P. Wigner, *Phys. Rep.* **106**(1984)121. R.L. Hudson, *Rep. Math. Phys.* **6**(1974)249. K.Zalewski, *Acta Phys. Pol.* **B39**(2008)181.

- Phase space density $p_{\mathbf{K}}(\mathbf{x})$.
- Wigner function

$$W(\mathbf{K},\mathbf{X}) = \int \frac{d^3q}{(2\pi)^3}\, \rho(\mathbf{K},\mathbf{q})e^{i\mathbf{q}\mathbf{X}}, \quad (t, S(K,X))$$

where

$$\mathbf{K} = \frac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2), \qquad \mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2.$$

- Smoothness assumption (and more).

$$p_{\mathbf{K}}(\mathbf{X}) \doteq W(\mathbf{K},\mathbf{X}).$$

Density matrix and momentum measurements

G. Goldhaber, S. Goldhaber, W. Lee and A. Pais, *Phys. Rev.* **120**(1960)414. J. Karczmarczuk, *Nucl. Phys.* **B78**(1974)370.

$$P_1(\mathbf{p}_1) \sim \rho(\mathbf{p}_1;\mathbf{p}_1).$$

GGLP equation for the out-of-diagonal elements

$$P_2(\mathbf{p}_1,\mathbf{p}_2) = P_1(\mathbf{p}_1)P_1(\mathbf{p}_2) + N_2|\rho(\mathbf{p}_1;\mathbf{p}_2)|^2.$$

Remaining equations

$$P(\mathbf{p}_1,\ldots,\mathbf{p}_n) \sim \sum_Q \prod_{i=1}^{n} \rho(\mathbf{p}_i;\mathbf{p}_{Qi}); \qquad n = 3,\ldots.$$

Consistency and "technical" problems

Invariance group

A. Bialas and K.Zalewski, Phys. Rev. D72(2005)036009. K. Zalewski, Phys. Rev. D74(2006)114022. S. Pratt, *Phys. Rev. Lett.* **53**(1984)1219.

The predicted momentum distributions do not change under the following transformations of the density matrix:

$$\rho(\mathbf{p}_1;\mathbf{p}_2) \to \rho(\mathbf{p}_2;\mathbf{p}_1) \qquad \text{and/or}$$
$$\rho(\mathbf{p}_1;\mathbf{p}_2) \to e^{if(\mathbf{p}_1)}\rho(\mathbf{p}_1;\mathbf{p}_2)e^{-if(\mathbf{p}_2)},$$

where $f(\mathbf{p})$ is an arbitrary, real function of $\mathbf{p}$.
These changes, however, can strongly modify the deduced interaction region. E.g. $f(\mathbf{p}) = \frac{1}{2}\sum_{i=x,y,z} a_i p_i^2$ implies

$$W(\mathbf{K},\mathbf{X}) \to W(\mathbf{K},\mathbf{X}+\mathbf{a}(\mathbf{K})) \quad \text{where} \quad a_i(\mathbf{K}) = a_i K_i.$$

Limitations of the pure HBT method

K. Zalewski, *Phys. Rev.* **D77**(2008)074006.

Definition of the cumulants $\mathcal{K}(n_x, n_y, n_z)$:

$$\mathcal{K}(\mathbf{t}) = log\langle e^{i\mathbf{t}\cdot\mathbf{X}}\rangle,$$

$$\mathcal{K}(\mathbf{t}) = \sum_{n_x,n_y,n_z}^{\infty} \mathcal{K}(n_x, n_y, n_z)\frac{(it_x)^{n_x}}{n_x!}\frac{(it_y)^{n_y}}{n_y!}\frac{(it_z)^{n_z}}{n_z!}.$$

Theorem

- Every even cumulant of $p_{\mathbf{K}}(\mathbf{x})$ can be measured.
- No odd cumulant of $p_{\mathbf{K}}(\mathbf{x})$ can be measured.
- The first order cumulants, i.e. $\langle\mathbf{X}\rangle_{\mathbf{K}}$, fix the phase $f(\mathbf{p})$, up to an irrelevant constant, and thus the full $p_{\mathbf{K}}(\mathbf{x})$.

Implications

P. Danielewicz and S. Pratt, *Phys. Rev.* **C75**(2007)034907.

- The first order cumulants $\langle \mathbf{X} \rangle_{\mathbf{K}}$ cannot be measured; therefore; the best one can hope for is to find the profiles of the individual homogeneity regions.
- The second order cumulats are $\langle (x_i - \overline{x}_i)(x_j - \overline{x}_j) \rangle_{\mathbf{K}}$ for $i, j = x, y, z$; therefore; the HBT radii for individual homogeneity regions are measurable.
- The distribution of distances $\overline{p}_{\mathbf{K}}(\mathbf{x}_1 - \mathbf{x}_2)$ depends only on the even cumulants and on all of them. Therefore, the imaging method, applied to single homogeneity regions, gives reliable information, which goes far beyond the information from the determination of the HBT radii.

Emission function and Wigner function

K. Zalewski, *Acta Phys. Pol.* **B34**(2003)3379. G.I. Kopylov and M.I. Podgoretskii, *Yad. Phys.* **18**(1973)656.

$$\rho(\mathbf{K},\mathbf{q}) \sim \int d^4X \, S(K,X) e^{iqx}.$$

- Instant source at $t = 0$

$$S(K,X) = \delta(t) W(\mathbf{K},\mathbf{X}).$$

- No coherence in time

$$S(K,X) = \frac{W_{dt}(\mathbf{K},X)}{dt}.$$ normalization, sum rule

- For $S(K,X)$ independent of $\mathbf{q}^2$, e.g. assuming $K_0 = \sqrt{m^2 + \mathbf{K}^2}$ (mass shell approximation), in the frame $\mathbf{K} = \mathbf{0}$:

$$W(\mathbf{K},\mathbf{X}) = \int dt \, S(K,X).$$