Azimuthally Sensitive Buda-Lund Hydrodynamic Model and Fits to Spectra, Elliptic Flow and asHBT

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- **Buda-Lund hydro model**
- **Observables**
- **Review of old central hydro results**
- **New non-central hydro results**
- **Conclusion**

The Buda-Lund hydro model

- **General form of a particle emission function**

$$S(x,p)\,d^4x = f(x,p)\;\; p_\mu\; d\sigma^\mu(x)$$

- **With (Boltzmann) probability distribution for fluids**

$$S(x,p)\,d^4x = \frac{g}{(2\pi)^3}\frac{p_\mu\, d\sigma^\mu(x)}{\exp\left(\frac{p_\mu u^\mu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right) + s_q}$$

The Buda-Lund hydro model

$\sigma(x)$ **hypersurface**

$u(x)$ **flow field**

$\mu(x)$ **chemical potential**

$T(x)$ **temperature**

must comply with the 5 differential equations of fluid dynamics:

- **continuity**
- **momentum conservation**
- **energy conservation**

The Buda-Lund hydro model

- **Equations of non-relativistic inviscid fluid dynamics**

$$\partial_t\, n + \nabla \cdot (n\mathbf{v}) = 0\,,$$
$$\partial_t\, \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -(\nabla p)/(mn)\,,$$
$$\partial_t\, \epsilon + \nabla \cdot (\epsilon\mathbf{v}) = -p\nabla \cdot \mathbf{v}\,,$$

- **Not closed, EoS needed, for example**

$$p = nT$$

$$\epsilon = \kappa p$$

The Buda-Lund hydro model

5 model principles

- **3D expansion with axial or ellipsoidal symmetry**
- **Local thermal equilibrium**
- **Analytic expressions for the observables**
- **Reproduction known exact hydro solutions (nonrelativistic, hubble and bjorken limits)**
- **Core-halo picture (long lived resonances)**

M. Csanád, T.Csörgő, B. Lörstad: Nucl.Phys.A742:80-94,2004; nucl-th/0310040

The Buda-Lund Hydro Model

Buda-Lund form of hydro fields:

in several cases parametric solutions of hydrodynamics

see M. Csanád's talk

$$d^4\sigma(x) = u(x) H(\tau) d^4 x$$

$$u(x) = (\gamma, \sinh\eta_x, \sinh\eta_y, \sinh\eta_z)$$

$$\frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - s$$

$$\frac{1}{T(x)} = \frac{1}{T_0}\left(1 + \frac{T_0 - T_s}{T_s} s\right)\left(1 + \frac{(T_0 - T_e)}{T_e}\frac{(\tau - \tau_0)^2}{2\Delta\tau^2}\right)$$

The Buda-Lund Hydro Model (2)

- **Where (in case of axial symmetry):**

$$H(\tau) = \frac{1}{(2\pi\Delta\tau^2)^{1/2}} \exp\left(-\frac{(\tau-\tau_0)^2}{2\Delta\tau^2} \right)$$

$$s = \frac{r_t^2}{(2R_G^2)} + \frac{(\eta - y_0)^2}{2\Delta\eta^2}$$

$$\sinh(\eta_t) = \frac{\langle u_t \rangle r_t}{R_G} \quad = \boldsymbol{H_t\, r_t}$$

H_t :transverse Hubble constant

The Buda-Lund hydro model (3)

- **Observables**

$$\mathbf{N_1}(p) = \int d^4x\, S(x,p)$$

$$C_2(Q,p) = 1 + \lambda_* \exp(-Q_o^2 R_o^2 - Q_s^2 R_s^2 - Q_l^2 R_l^2)$$

$$S(x,p) = S_c(x,p) + S_h(x,p)$$

$$\mathbf{N_1}(p) = \frac{1}{\sqrt{\lambda_*}} \int d^4x\, S_c(x,p)$$

Buda-Lund Hydro: Observables

- **Final form of the Invariant Momentum Distribution:**

$$N(\mathbf{p}) = \frac{g}{(2\pi)^3} \overline{E}\,\overline{V}\,\overline{C} \exp\left(-\frac{p \cdot u(\overline{x}) - \mu(\overline{x})}{T(\overline{x})} + s_q \right)$$

$$\overline{E} = m_t \cos h(\overline{\eta})$$

$$\overline{V} = 2\pi^{(3/2)} \overline{R}_{par} \overline{R}_t^{\,2} \frac{\overline{\Delta\tau}}{\Delta\tau}$$

$$\overline{C} = \frac{1}{\sqrt{\lambda_*}} \exp\left(\frac{\overline{\Delta\eta}^2}{2}\right)$$

Buda-Lund Hydro Model: HBT radii

- **Final form of the HBT radii:**

$$\overline{R_t}^2 = \overline{R}_G^2 / [1 + (< u_t >^2 + (T_0 - T_s) / T_s) \overline{E} / T_0]$$

$$\overline{R}_{par}^2 = \tau_0^2 / \Delta\overline{\eta}^2$$

$$\Delta\overline{\eta}^2 = \Delta\eta^2 / (1 + \Delta\eta^2 \overline{E} / T_0)$$

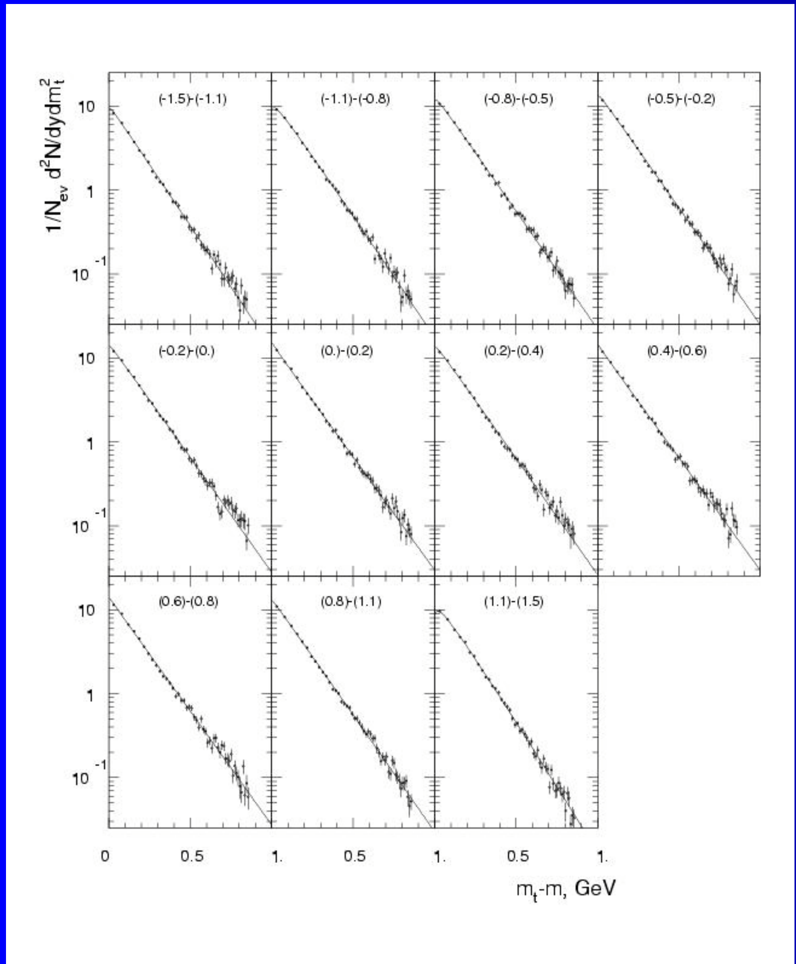
$$\Delta\overline{\tau}^2 = \Delta\tau^2 / [1 + ((T_0 - T_e) / T_e) \overline{E} / T_0]$$

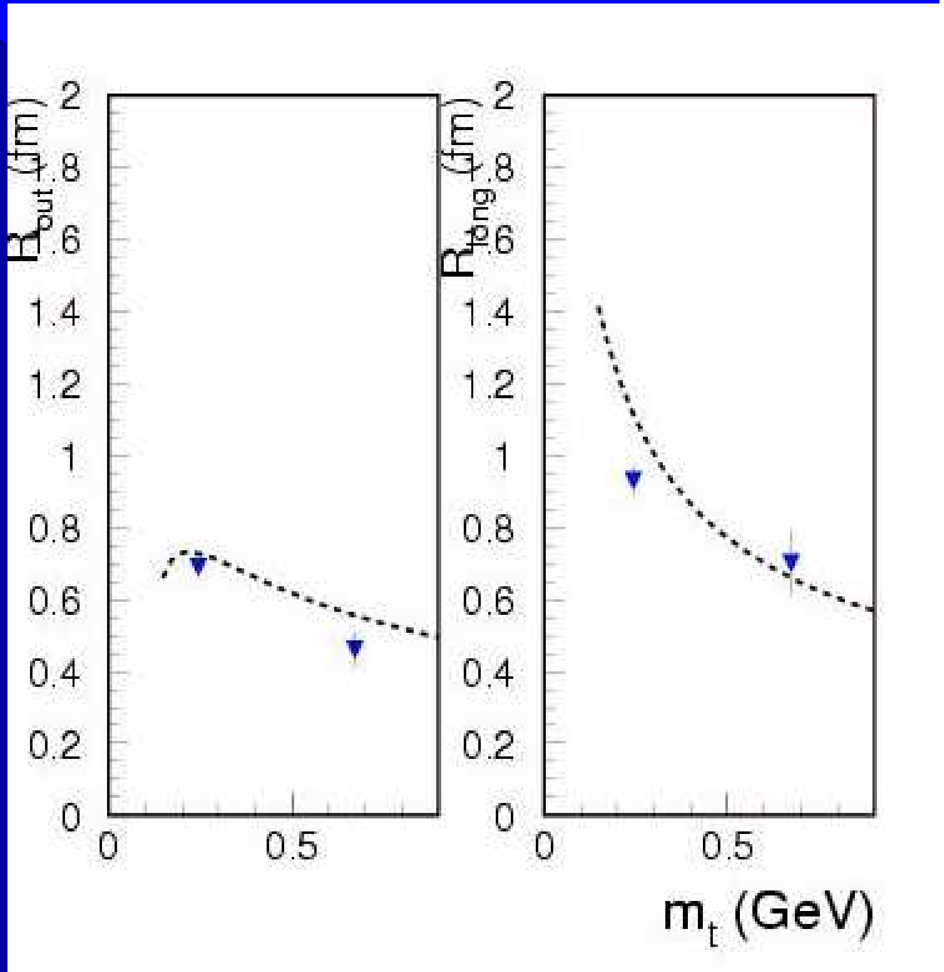
$$\overline{\tau} = \tau_0, \qquad \overline{r}_x = u_t R_G p_t / [T_0 + \overline{E}(u_t)(T_0 - T_s) / T_s]$$

$$\overline{r}_y = 0$$

$$\overline{\eta} = (y_0 - y) / [1 + \Delta\eta^2 m_t / T_0]$$

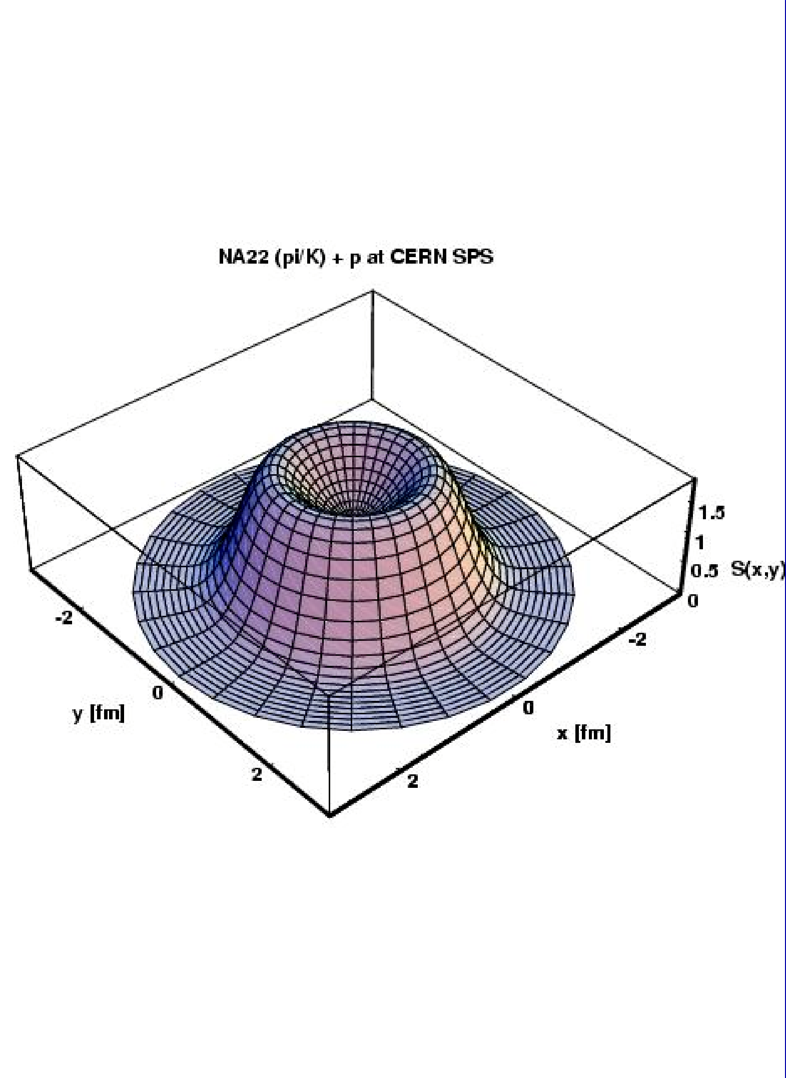
Buda-Lund fits to NA22 h+p

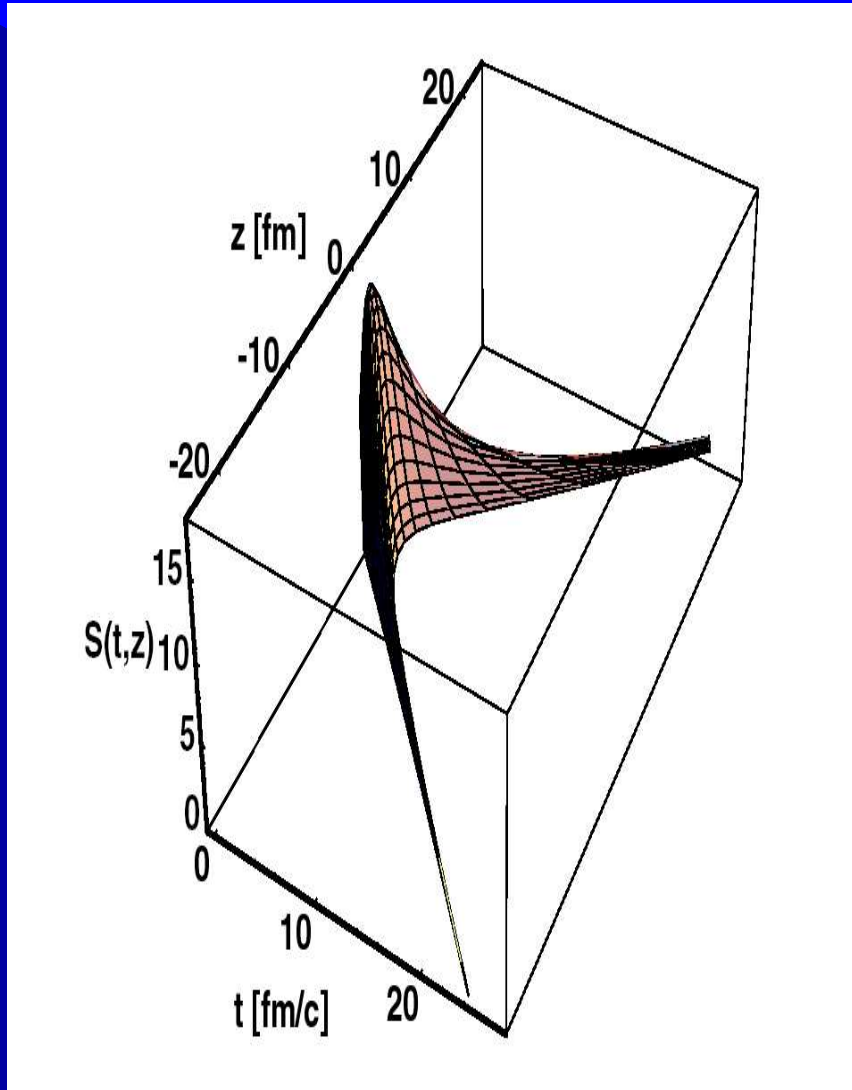




N. M. Agababyan et al, EHS/NA22 , PLB 422 (1998) 395
T. Csörgő, hep-ph/001233, Heavy Ion Phys. 15 (2002) 1-80

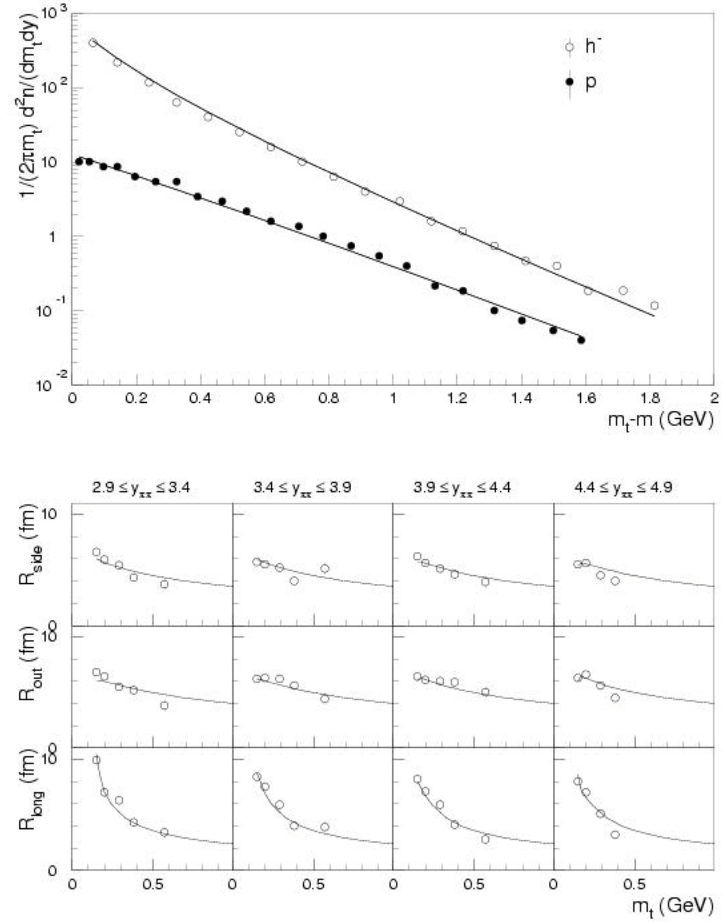
Emission function from NA22 h+p



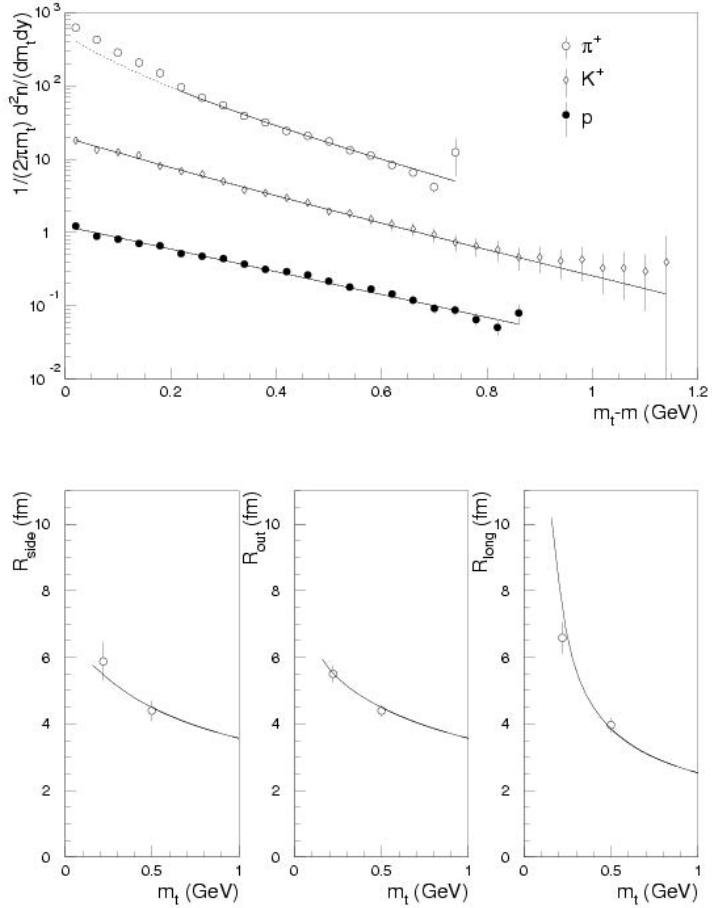


BudaLund fits to SPS Pb+Pb

NA49

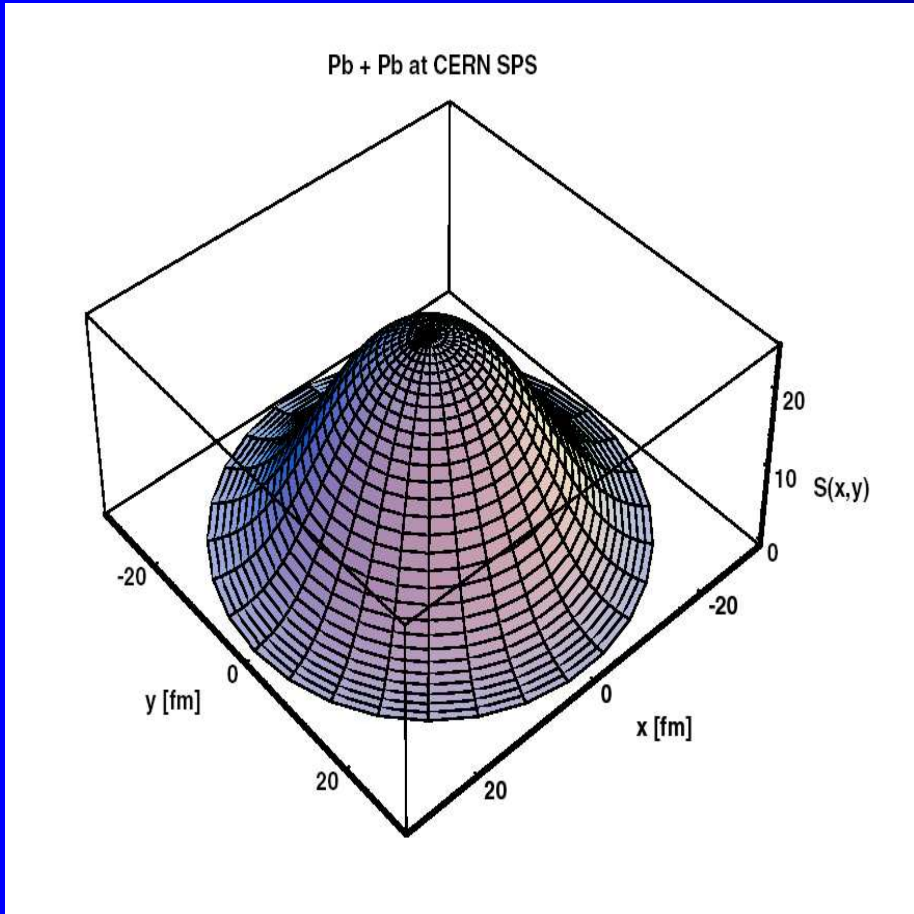


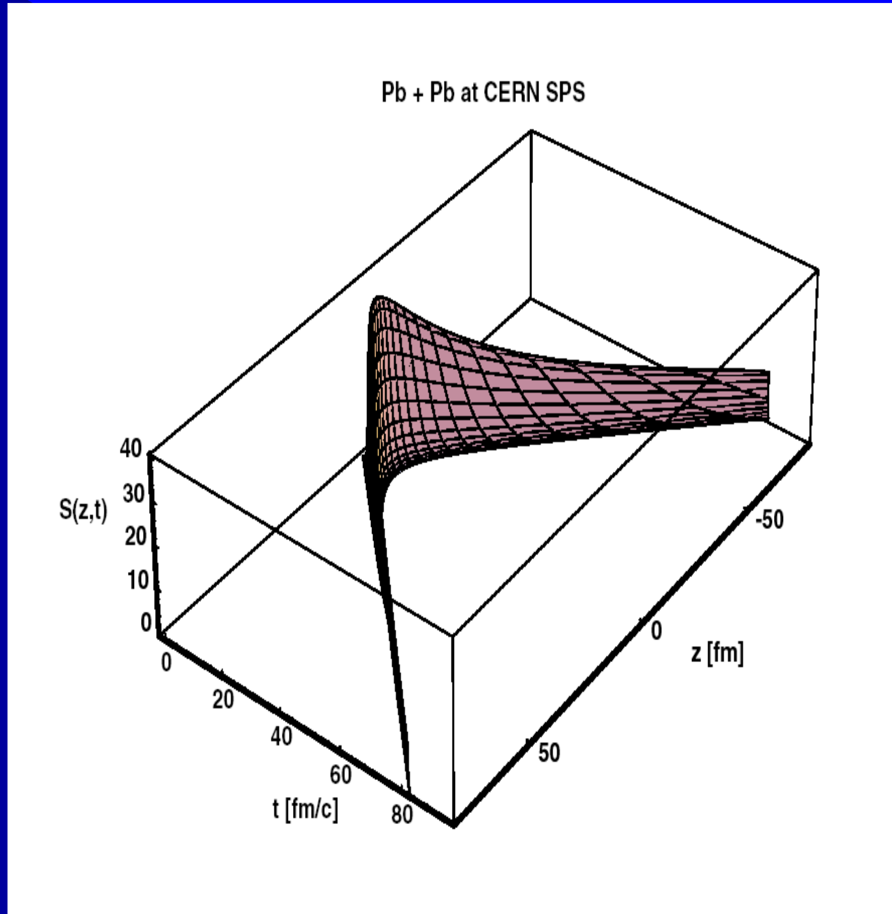
NA44



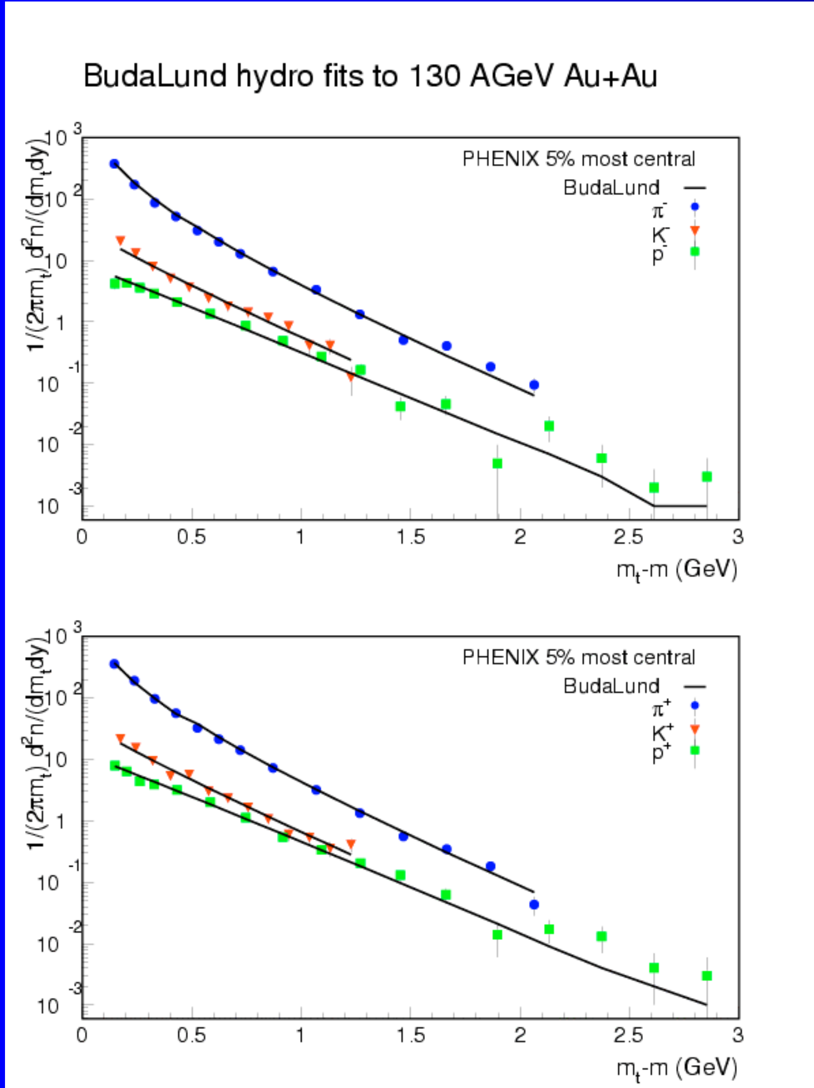
A. Ster, T.Csörgő, B. Lörstad , Nucl.Phys. A661 (1999) 419-422, nucl-th/9907338

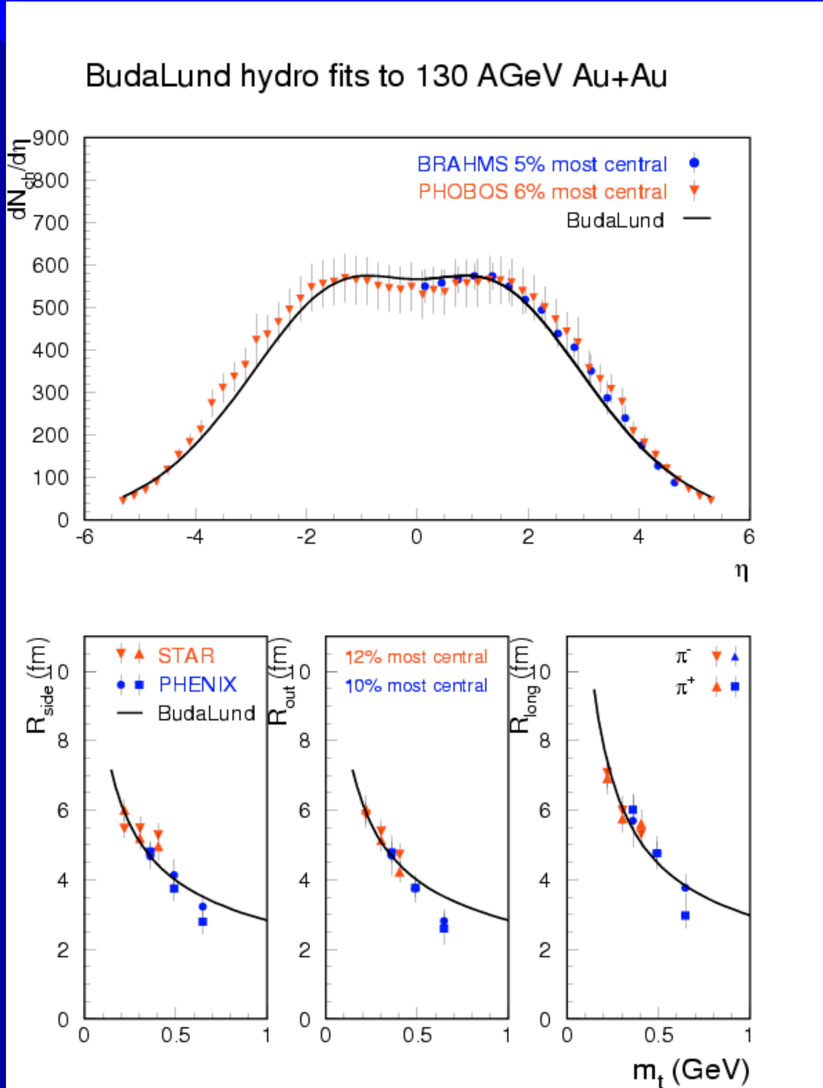
Emission function from SPS Pb+Pb





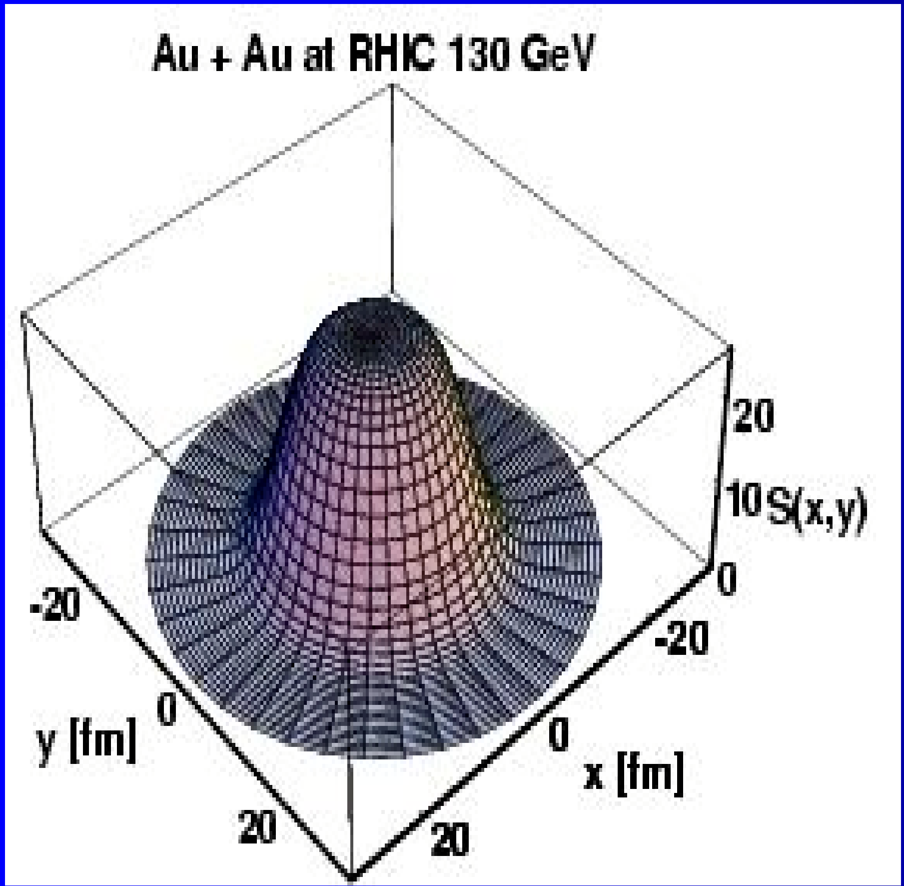
BudaLund fits to RHIC Au+Au

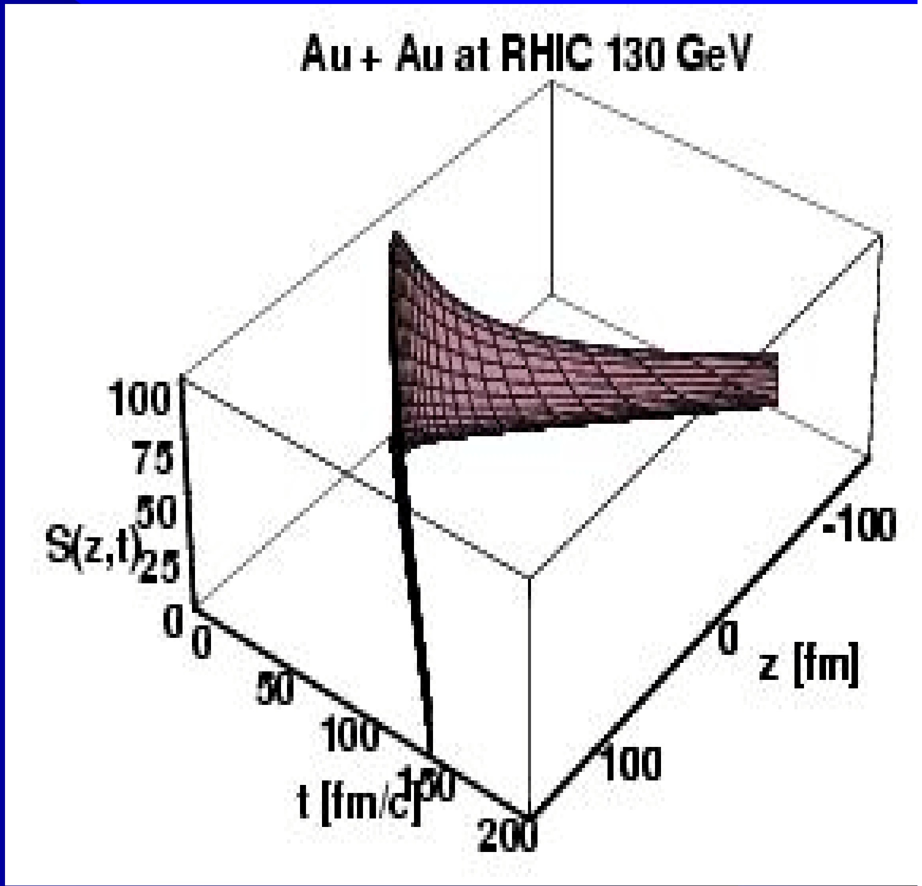




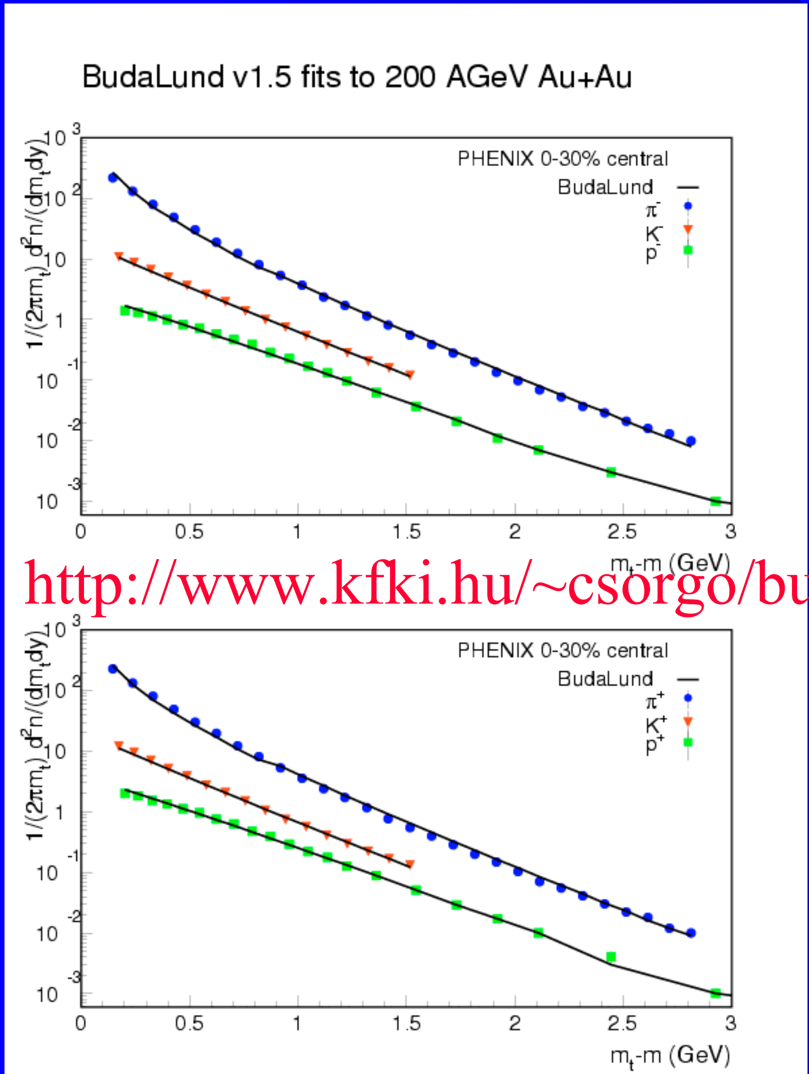
A. Ster, et al., Acta Phys.Polon. B35 (2004) 191-196, nucl-th/0311102

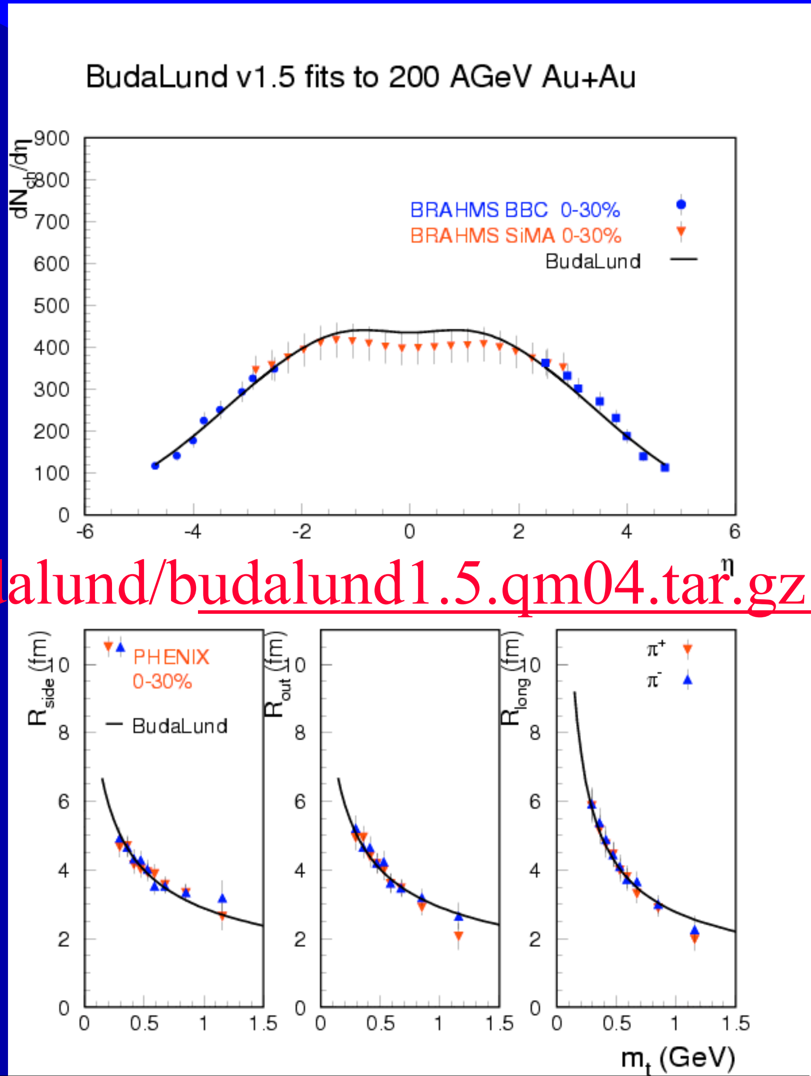
Emission function from RHIC Au+Au





BudaLund fits to RHIC Au+Au

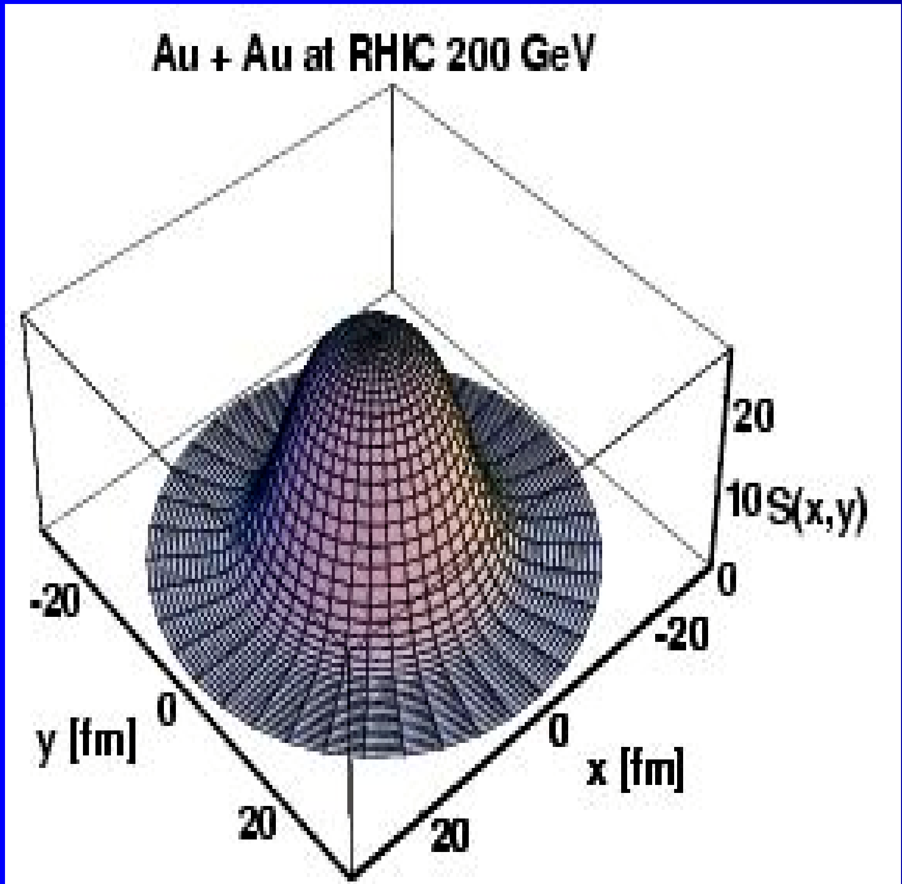


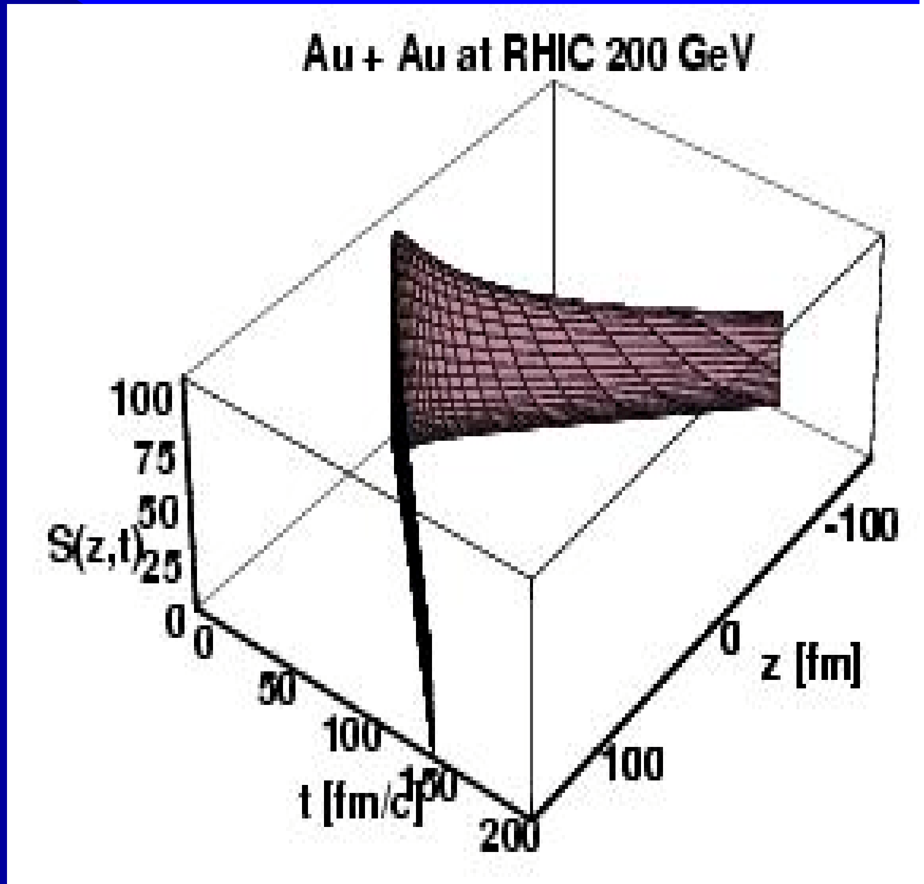


http://www.kfki.hu/~csorgo/budalund/budalund1.5.qm04.tar.gz

M. Csanád, et al., J.Phys.G30: S1079-S1082, 2004, nucl-th/0403074

Emission function from RHIC Au+Au



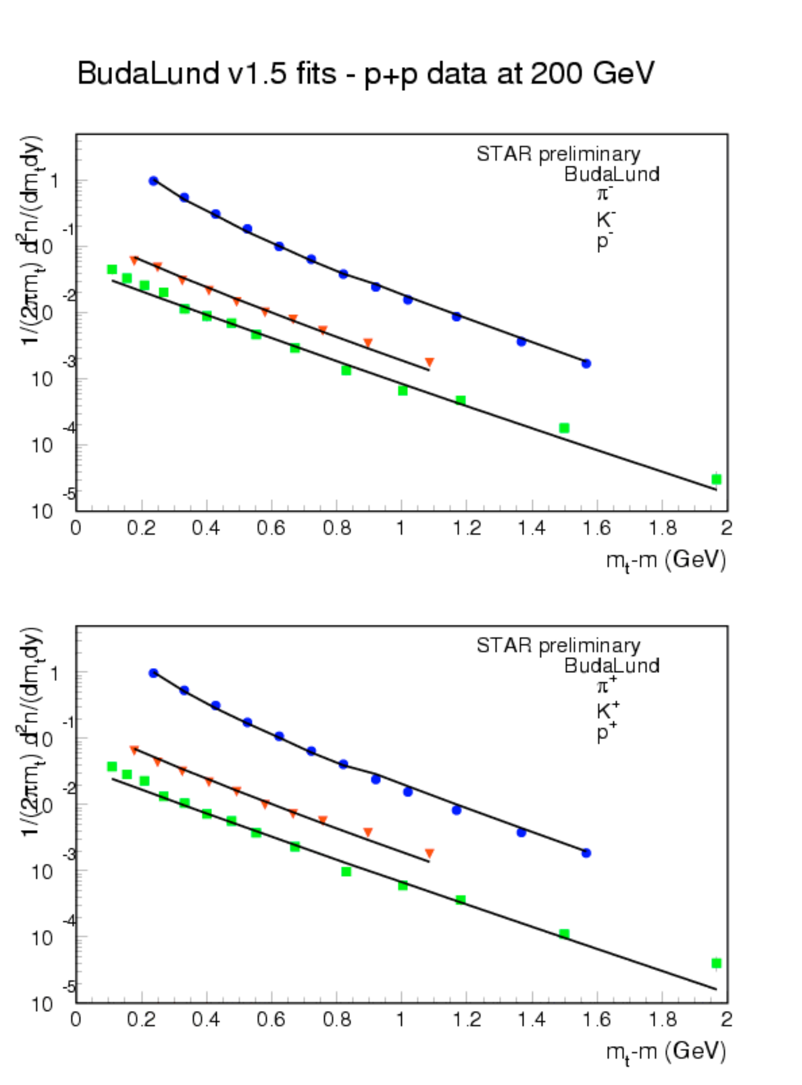


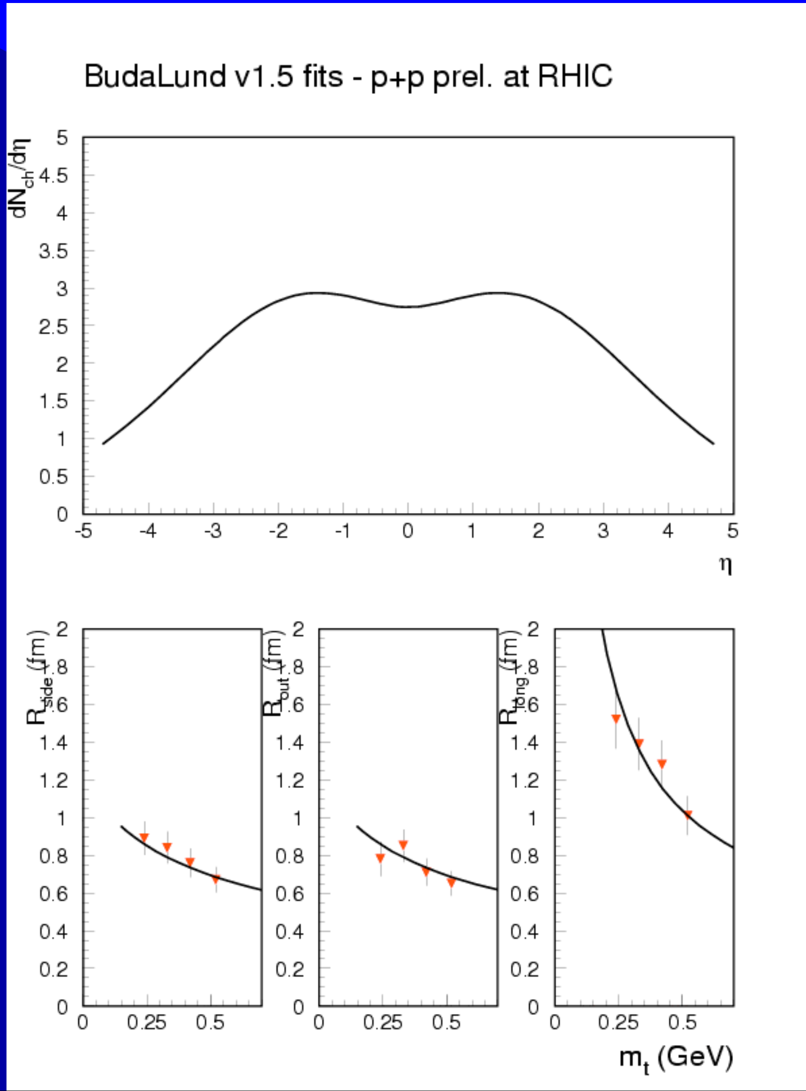
Buda-Lund fit results

BL v1.5 parameters	RHIC 200 GeV Au+Au	RHIC 130 GeV Au+Au	Pb+Pb SPS	h+p SPS
T_0 [MeV]	196 ± 13	214 ±7	139 ± 6	140 ± 3
$<u_t>$	1.6 ± 0.2	1.0 ± 0.1	0.55 ± 0.06	0.20 ± 0.07
R_G [fm]	13.5 ± 1.7	28.0 ± 5.5	7.1 ± 0.2	0.88 ± 0.13
R_s [fm]	12.4 ± 1.6	8.6 ± 0.4	28 ± 21	1.4 ± 0.3
T_{surf} [MeV]	0.5 T_0	0.5 T_0	0.5T_0	0.5T_0
τ_0 [fm/c]	5.8 ± 0.3	6.0 ± 0.2	5.9 ± 0.6	1.4 ± 0.1
$\Delta\tau$ [fm/c]	0.9 ± 1.2	0.3 ± 1.2	1.6 ± 1.5	1.3 ± 0.3
$\Delta\eta$	3.1 ± 0.1	2.4 ± 0.1	2.1 ± 0.4	1.36 ± 0.02
T_{evap} [MeV]	117 ± 12	102 ± 11	87 ± 24	-
μ_0^π [MeV]	-2 ± 14	63 ± 11		
μ_0^K [MeV]	16 ± 19	98 ± 19		
μ_0^{p-}[MeV]	97 ± 28	315 ± 27		
μ_B [MeV]	61 ± 52	77 ± 38	0 fixed	0 fixed
χ^2/NDF	114/208=0.55	158/180=0.9	342/277=1.2	642/683=0.9
CL	100 %	88 %		

$$\beta_t = \frac{\langle u_t \rangle}{\sqrt{1+\langle u_t \rangle^2}}$$

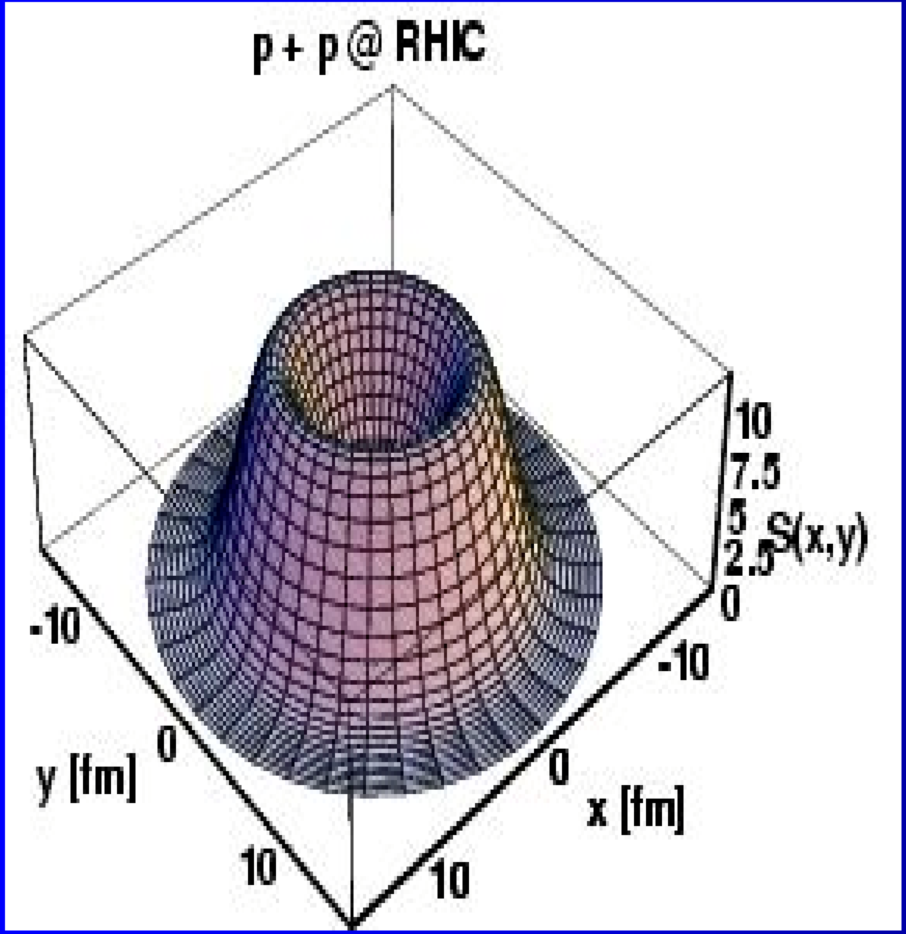
BudaLund fits to RHIC preliminary p+p

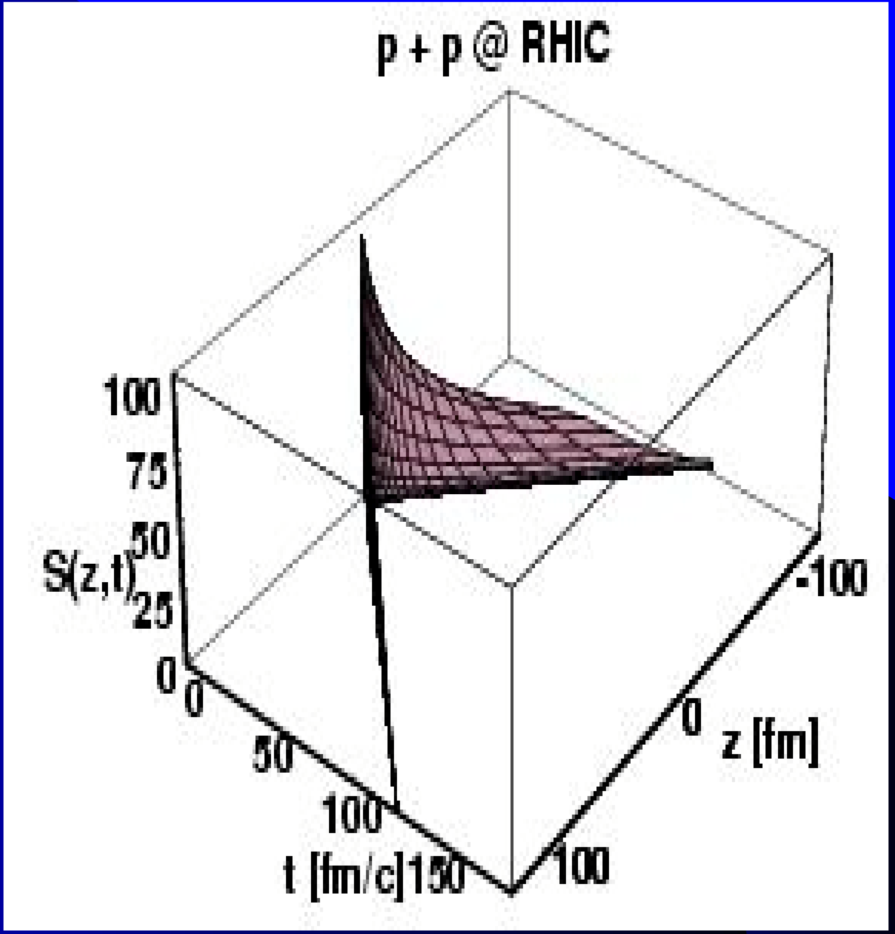




T. Csörgő, et al., Heavy Ion Physics, hep-ph/0406042

Emission function from RHIC p+p

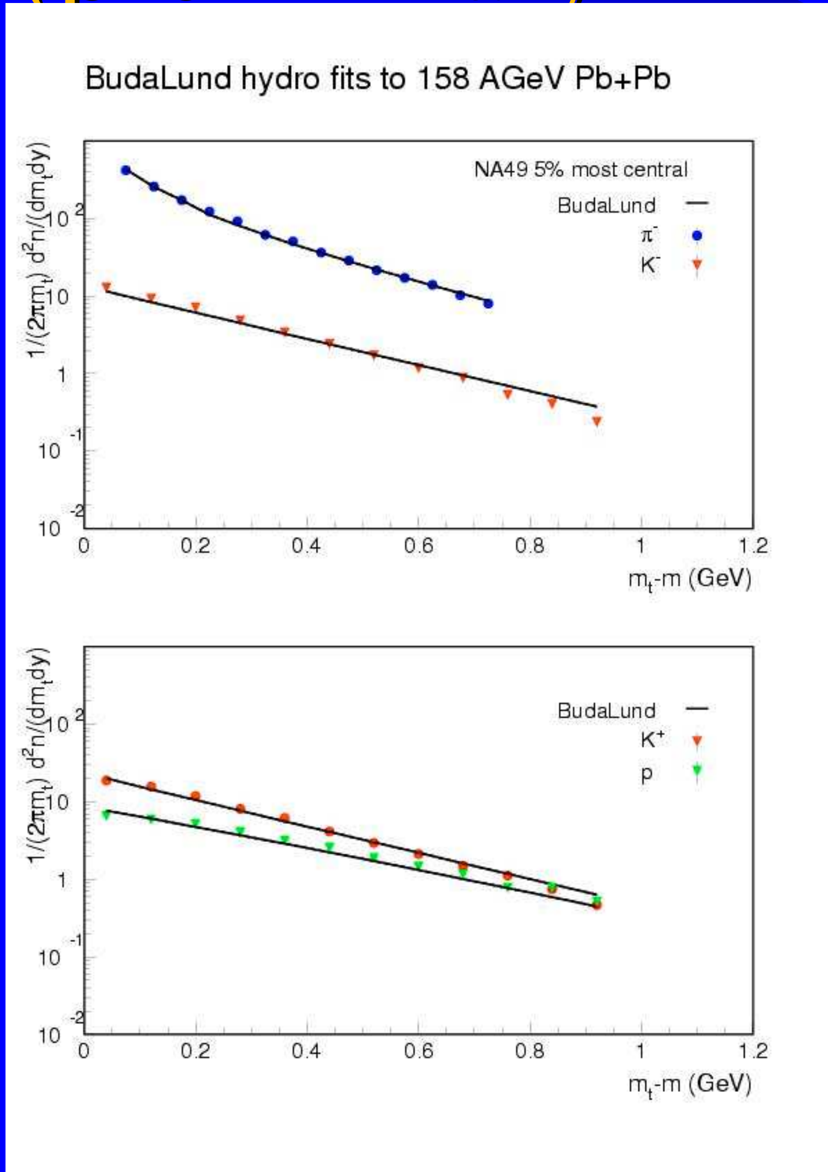


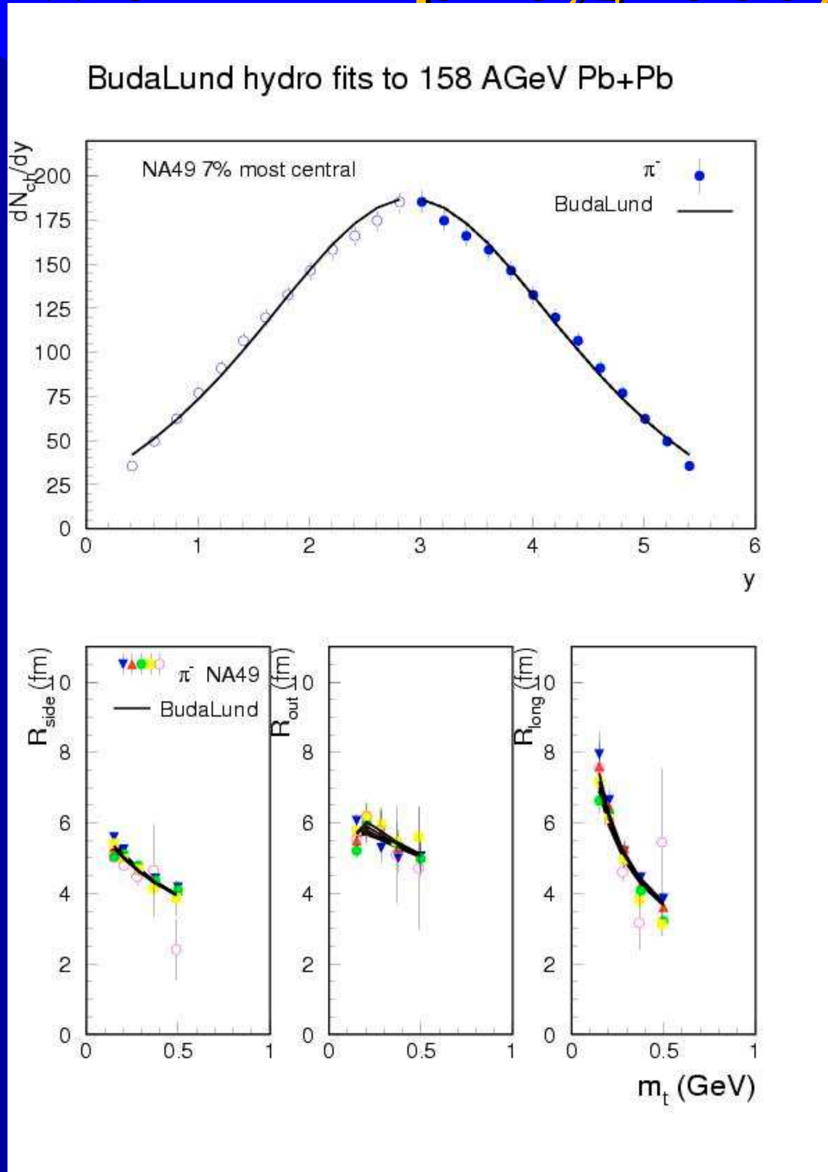


BudaLund fit results

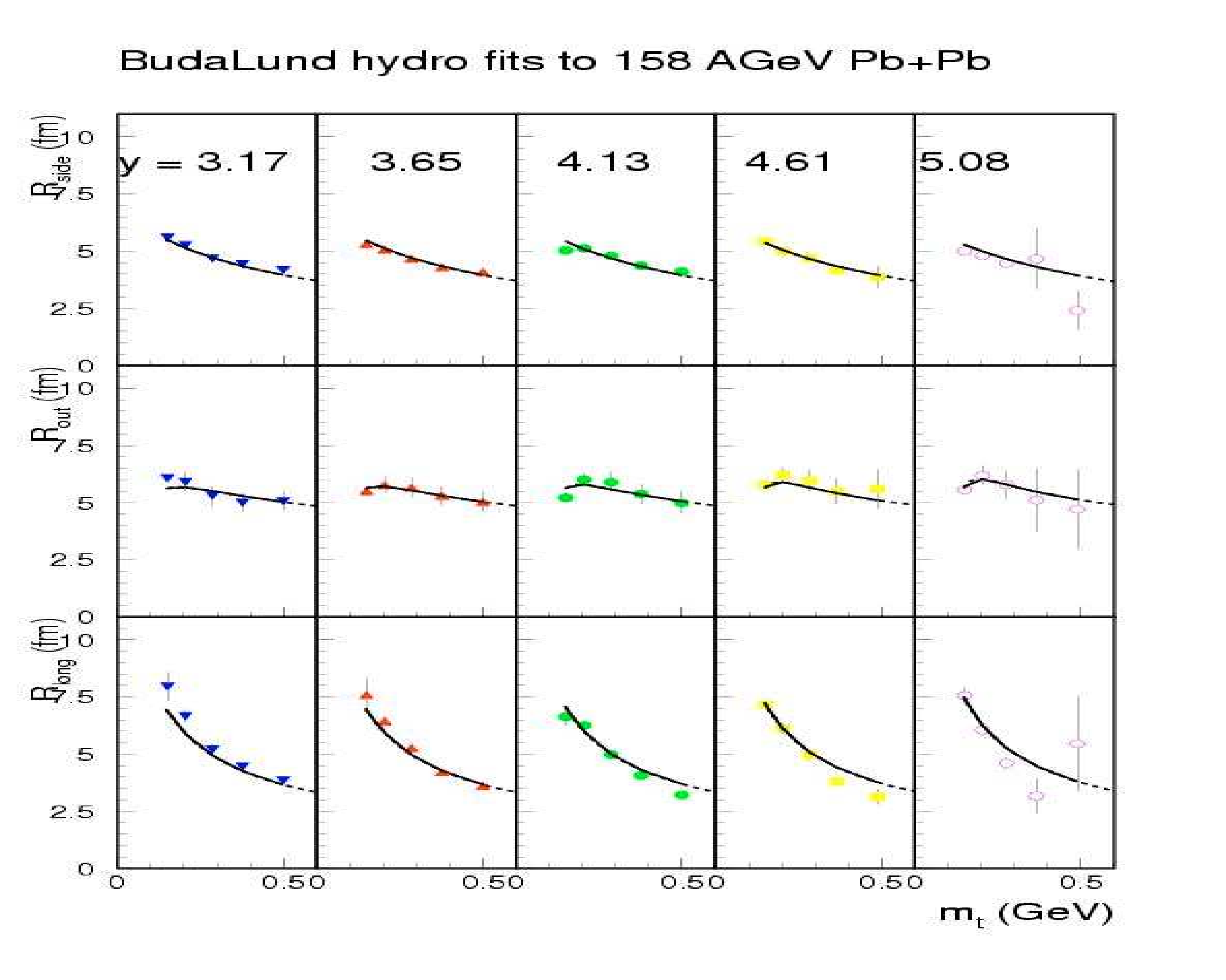
BL v1.5 parameters	RHIC 200 GeV (prel) p+p	RHIC 130 GeV Au+Au	Pb+Pb SPS	h+p SPS
T_0 [MeV]	289 ± 8	214 ±7	139 ± 6	140 ± 3
$\langle u_t \rangle$	0.04 ± 0.26	1.0 ± 0.1	0.55 ± 0.06	0.20 ± 0.07
R_G [fm]	1.2 ± 0.3	28.0 ± 5.5	7.1 ± 0.2	0.88 ± 0.13
R_s [fm]	1.13 ± 0.16	8.6 ± 0.4	28 ± 21	1.4 ± 0.3
T_{surf} [MeV]	0.5 T_0	0.5 T_0	0.5T_0	0.5T_0
τ_0 [fm/c]	1.1 ± 0.2	6.0 ± 0.2	5.9 ± 0.6	1.4 ± 0.1
$\Delta\tau$ [fm/c]	0.1 ± 0.5	0.3 ± 1.2	1.6 ± 1.5	1.3 ± 0.3
$\Delta\eta$	3.0 fixed	2.3 ± 0.4	2.1 ± 0.4	1.36 ± 0.02
T_{evap} [MeV]	90 ± 42	102 ± 11	87 ± 24	-
χ^2/NDF	89/71	158/180	342/277	642/683

BudaLund fits to NA49 data (preliminary HBT, work in progress)





BudaLund fits to NA49 data (preliminary HBT, work in progress)



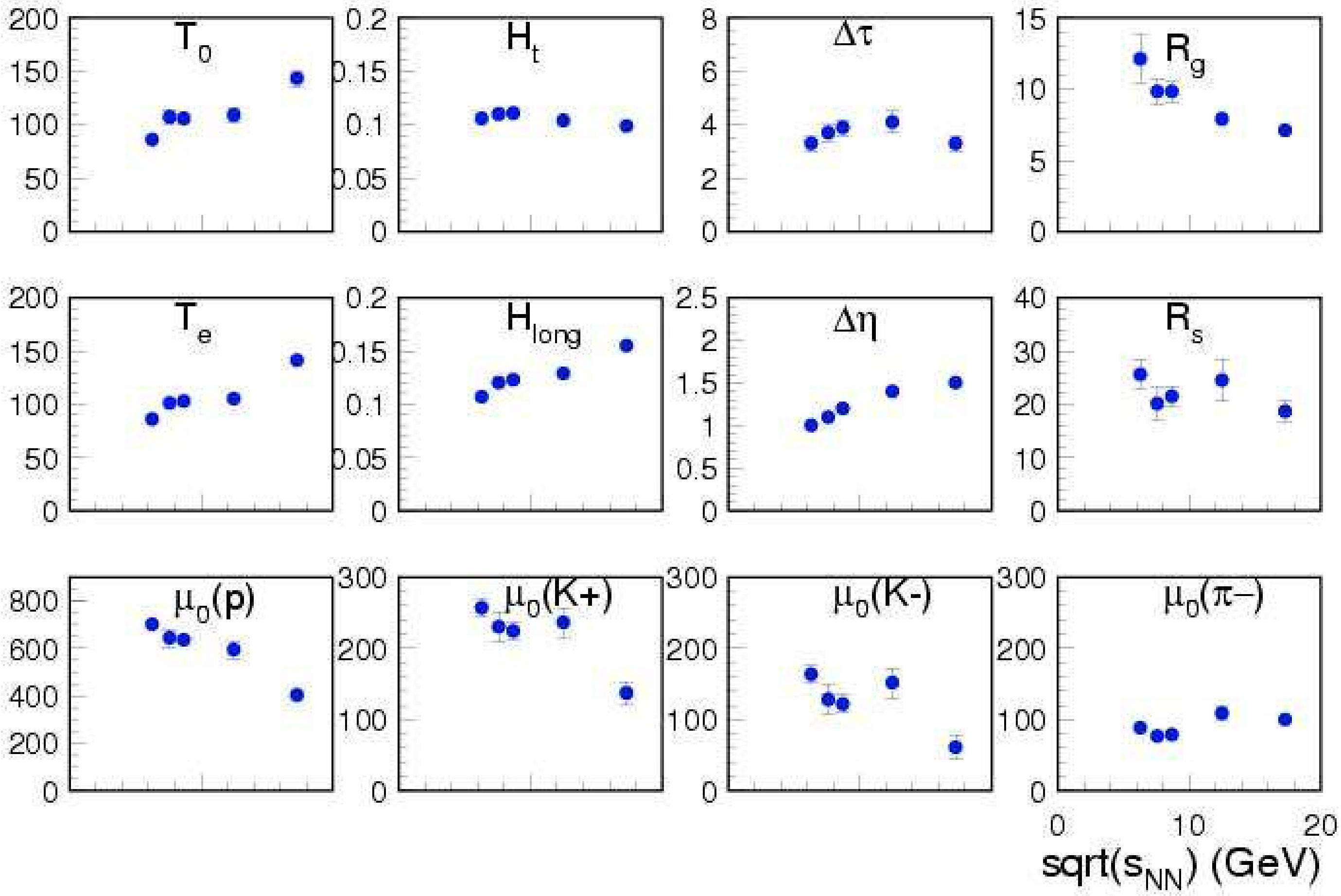
BudaLund fit results of NA49 data

BudaLund parameters	158 AGeV	80 AGeV	40 AGeV	30 AGeV	20 AGeV
T_0 [MeV]	143 ± 7	109 ± 7	106 ± 4	107 ± 7	86 ± 3
T_e [MeV]	141 ± 5	105 ± 3	103 ± 2	101 ± 3	86 ± 2
H_t [c/fm]	0.099 ± 0.003	0.104 ± 0.004	0.111 ± 0.003	0.110 ± 0.005	0.106 ± 0.004
H_l [c/fm]	0.155 ± 0.005	0.129 ± 0.005	0.123 ± 0.003	0.120 ± 0.004	0.107 ± 0.002
R_G [fm]	7.1 ± 0.2	7.9 ± 0.4	9.8 ± 0.7	9.8 ± 0.9	12.1 ± 1.7
R_s [fm]	18.7 ± 2.0	24.5 ± 3.8	21.5 ± 1.8	20.7 ± 3.0	25.5 ± 2.7
$\Delta\tau$ [fm/c]	3.3 ± 0.3	4.1 ± 0.4	3.9 ± 0.3	3.7 ± 0.3	3.2 ± 0.3
$\Delta\eta$	1.5 ± 0.2	1.4 ± 0.2	1.2 ± 0.1	1.1 ± 0.1	1.0 ± 0.1
$\mu_0^{\pi-}$ [MeV]	88 ± 7	109 ± 10	79 ± 6	77 ± 9	88 ± 6
μ_0^{K-} [MeV]	61 ± 16	151 ± 21	122 ± 12	128 ± 20	163 ± 12
μ_0^{K+} [MeV]	137 ± 16	236 ± 21	224 ± 12	230 ± 21	257 ± 12
μ_0^{p} [MeV]	403 ± 29	593 ± 38	635 ± 22	642 ± 38	700 ± 21
χ^2/NDF	198 /126 !	129 /128	200 /116 !	160 /116	128 /95

$$\langle u_t \rangle = \mathbf{H_t} \cdot \mathbf{R_G}$$

$$\tau_0 = 1/\mathbf{H_l}$$

source parameter excitation functions



Some other hydro models

Models with acceptable results:

nucl-th/0207016	Buda-Lund / core-halo model. T.Csörgő. A. Ster, Heavy Ion Phys. 17 (2003) 295-312.
nucl-th/0204054	Multiphase Trasport model (AMPT). Z. Lin, C. M. Ko, S. Pal.
nucl-ex/0307026	Blast wave model. F. Retiére for STAR.
nucl-th/0205053	Hadron cascade model. T . Humanic.

Other interesting models:

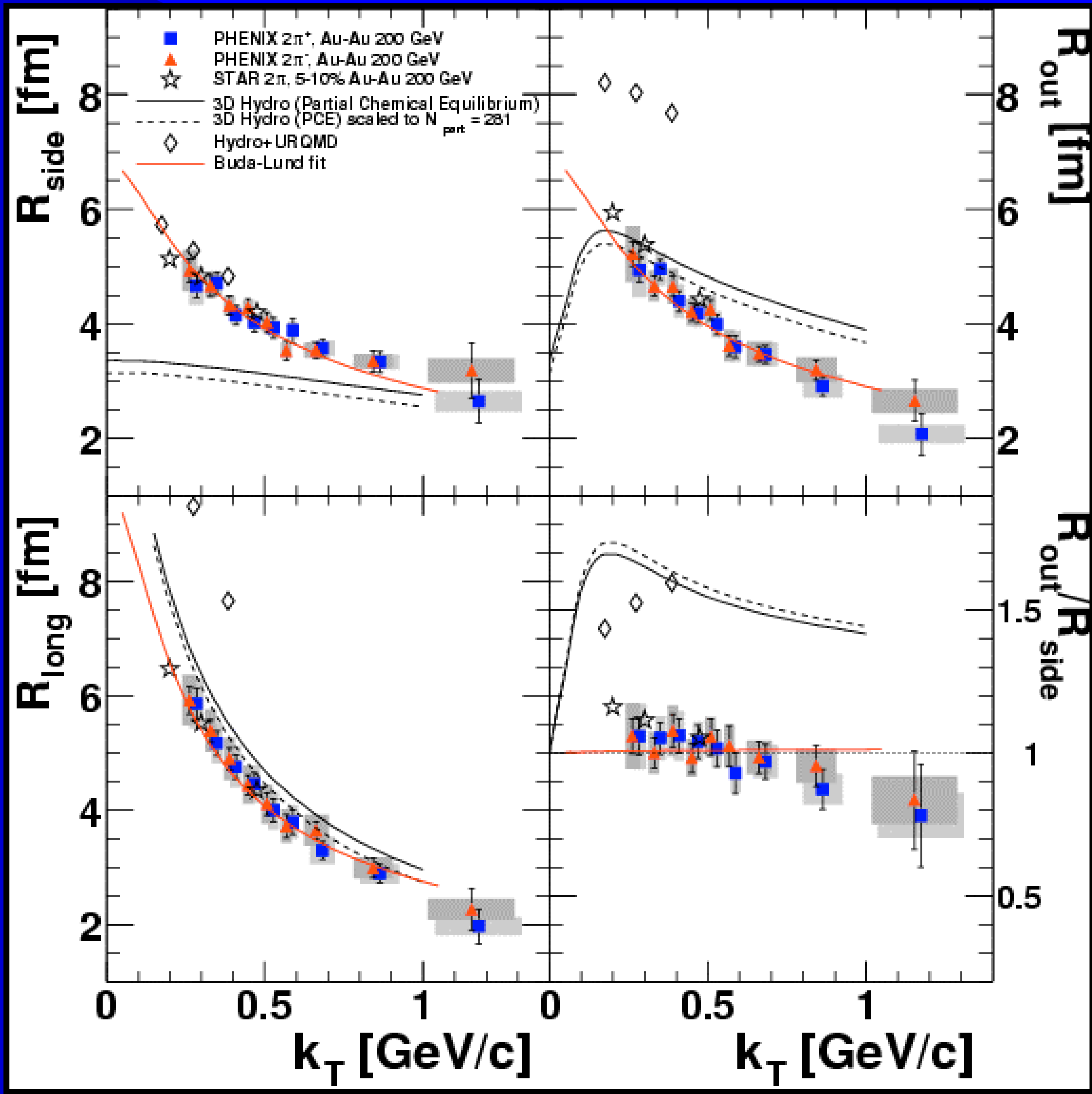
nucl-th/0208068	3D hydro model. T. Hirano, & T.Tsuda.
hep-ph/0209054	Hadron model. W.Broniowski, A. Baran W. Florkowski.

Femptoscopy of QGP

BudaLund fits indicate:

scaling of HBT radii

BudaLund prediction (1995):
each $R^2 \sim 1 / m_t$



Conclusions on central collisions

- **BudaLund model describes single particle distributions, rapidity distributions, HBT correlation function radii w/o puzzle in the following reactions:**

  h+p @SPS,   p+p @RHIC,   Pb+Pb @SPS,
  Au+Au @RHIC

- **Rings of fire in h+p @SPS and p+p @RHIC**
  Fireballs in Pb+Pb @SPS and Au+Au @RHIC

- **$T < T_c$ in h+p and Pb+Pb @SPS**
  $T > T_c$ in p+p and Au+Au @RHIC ; T_c (=172±3MeV)

asBuda-Lund fits to non-central RHIC data

Model extensions to ellipsoidal symmetry
for elliptic flow:

calculate 2nd harmonic coefficient of anisotropy:

$$\frac{N(\mathrm{p})}{\mathrm{d}\Phi} \propto 1 + 2v_2(\mathrm{p})\cos(2\Phi)$$

BudaLund fits to non-central RHIC data

Exact non-relativistic result for elliptic flow:

$$v_2 = \frac{I_1(w)}{I_0(w)} +$$

I_n: modified Bessel functions

$$w = \frac{p_t^2}{4\overline{m}_t}\left(\frac{1}{T_{*,y}} - \frac{1}{T_{*,x}}\right)$$

$$\overline{m}_t = m_t \cosh(\eta_s - y)$$

$$T_{*,i} = T_0 + m_t \dot{X}_i^2 \frac{T_0}{T_0 + m_t a^2}$$

Effective temperatures in reaction plane and in perp.

$$a^2 = \frac{T_0 - T_s}{T_s} = \left\langle \frac{\Delta T}{T} \right\rangle_r$$

For detailed calculations see: M. Csanád et al., hep-ph/0801.4434v2

BudaLund fits to non-central RHIC data

Model extensions to ellipsoidal symmetry for asHBT radii:

$$R_o^2 = R_{*,x}^2 \cos^2\varphi + R_{*,y}^2 \sin^2\varphi + \beta_o^2 \Delta\tau_*^2 \quad (36a)$$

$$= \frac{R_{*,x}^2 + R_{*,y}^2}{2} + \beta_o^2 \Delta\tau_*^2 - \frac{R_{*,y}^2 - R_{*,x}^2}{2}\cos(2\varphi)$$

$$R_s^2 = R_{*,x}^2 \sin^2\varphi + R_{*,y}^2 \cos^2\varphi \quad (36b)$$

$$= \frac{R_{*,x}^2 + R_{*,y}^2}{2} + \frac{R_{*,y}^2 - R_{*,x}^2}{2}\cos(2\varphi),$$

$$R_{os}^2 = \frac{R_{*,y}^2 - R_{*,x}^2}{2}\sin(2\varphi), \quad (36c)$$

$$R_l^2 = R_{*,z}^2, \quad (36d)$$

$$R_{*,x}^2 = X^2\left(1 + \frac{m_t\left(a^2 + \dot{X}^2\right)}{T_0}\right)^{-1},$$

$$R_{*,y}^2 = Y^2\left(1 + \frac{m_t\left(a^2 + \dot{Y}^2\right)}{T_0}\right)^{-1},$$

$$R_{*,z}^2 = Z^2\left(1 + \frac{m_t\left(a^2 + \dot{Z}^2\right)}{T_0}\right)^{-1}.$$

$X=R_X$, $Y=R_Y$

We found that „a" is different in each direction

asBuda-Lund fits to non-central RHIC data

Model extensions to ellipsoidal symmetry

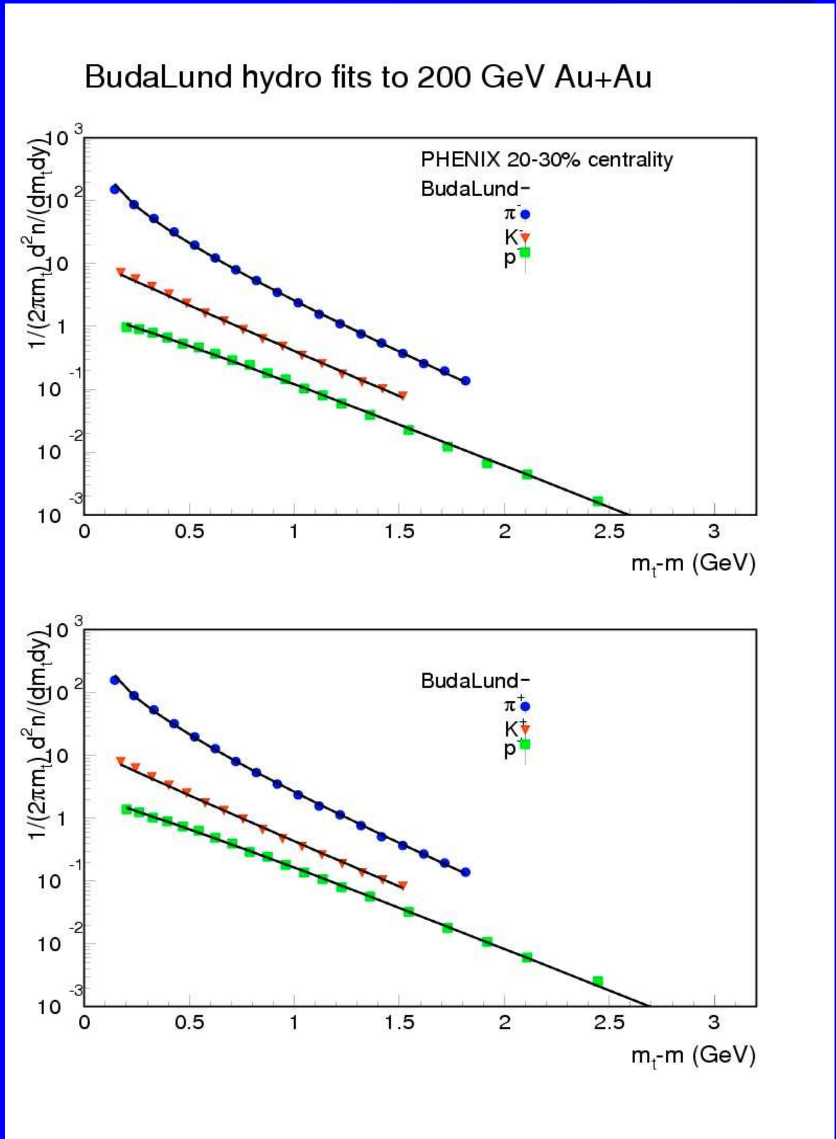
for azimuthally integrated spectra:

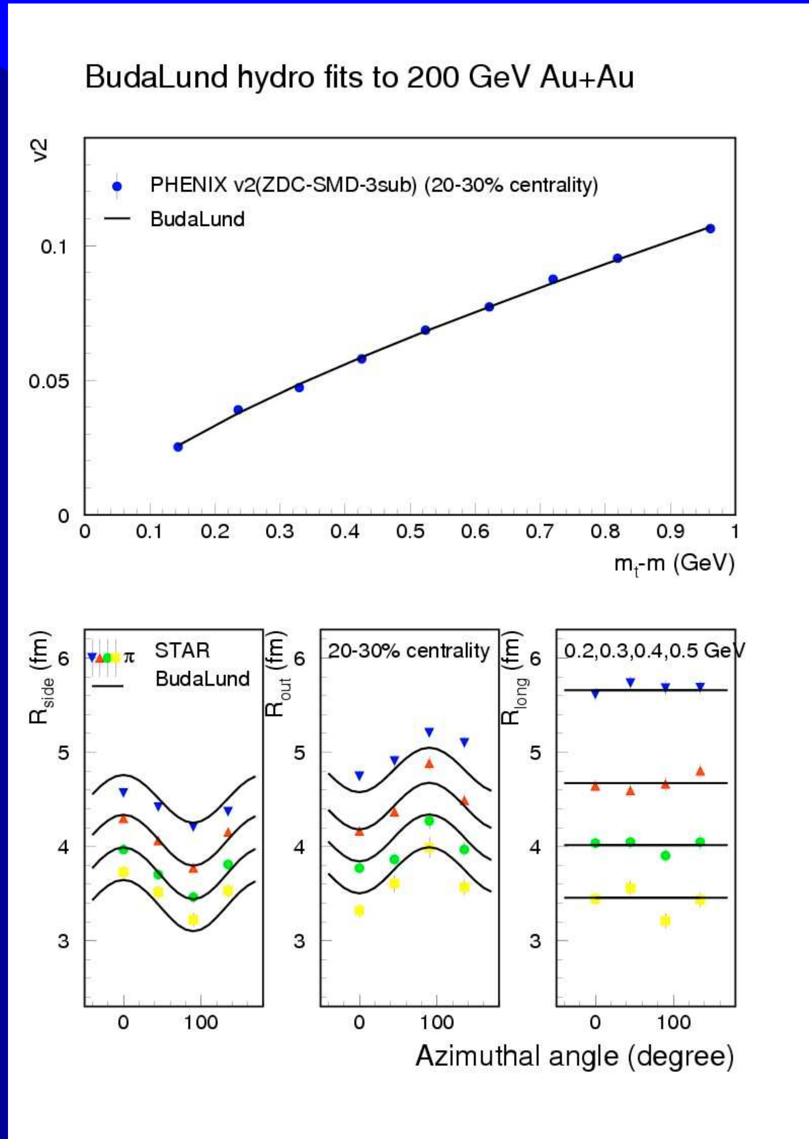
Calculate the volume term for ellipsoids :

$$\overline{V} = 2\pi^{(3/2)}\,\overline{R}_{par}\,\overline{R}_t^{\,2}\,\frac{\overline{\Delta\tau}}{\Delta\tau}$$

replaced $\overline{R}^2_t$ by $\overline{R}_x * \overline{R}_y$

asBudaLund fits to non-central RHIC data





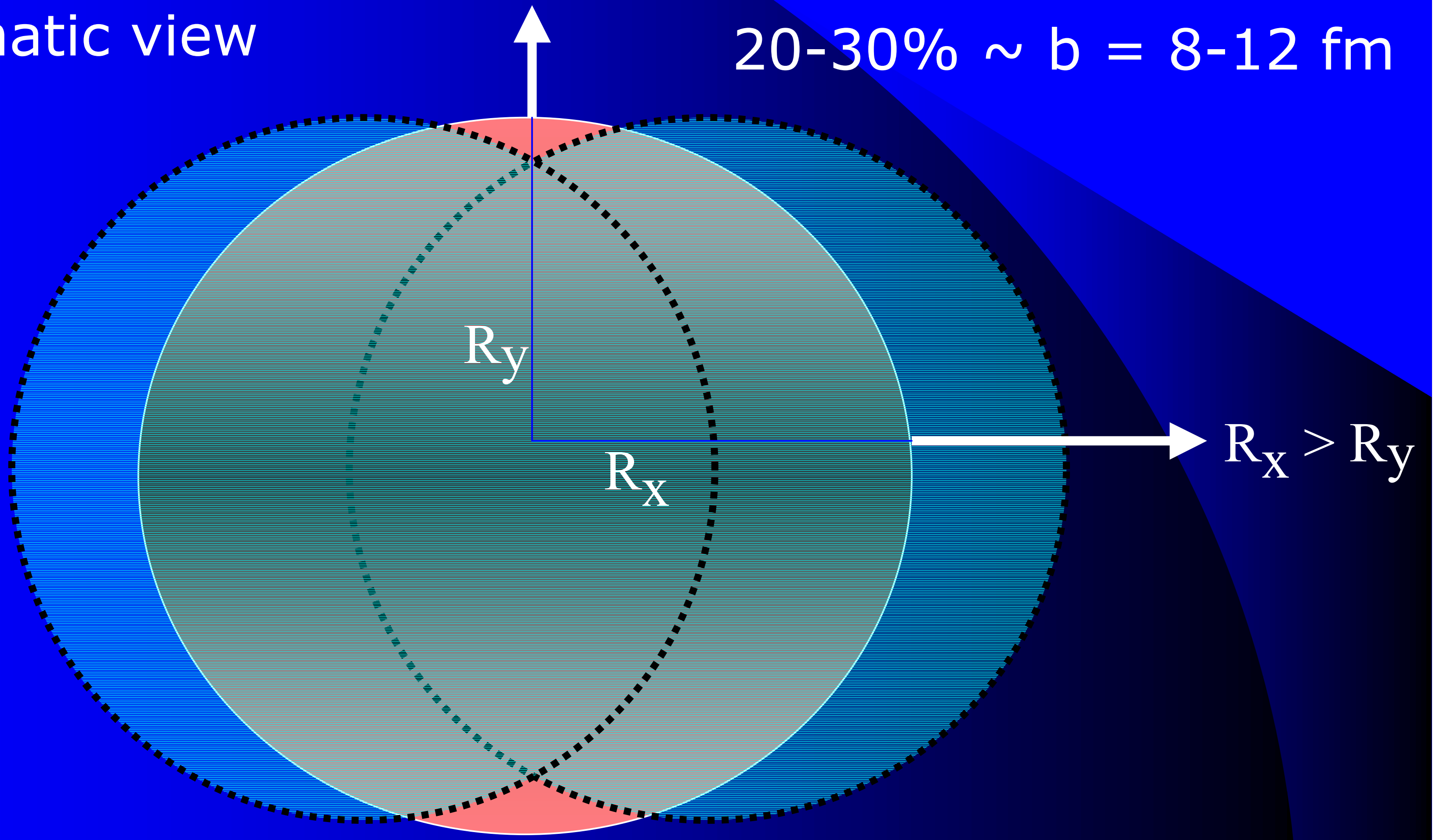
asBuda-Lund fit results of non-central RHIC data

BudaLund source parameters	RHIC 200 AGeV central (0-30%)	RHIC 200 AGeV non-central (20-30%)
T_0 [MeV]	196 ± 13	179 ± 7
T_e [MeV]	117 ± 12	119 ± 7
H_X [c/fm]	0.118 ± 0.013	0.150 ± 0.002
H_Y [c/fm]		0.111 ± 0.001
H_Z [c/fm]	0.172 ± 0.008	0.187 ± 0.005
R_X [fm]	13.5 ± 1.7	7.8 ± 0.3
R_Y [fm]		7.2 ± 0.2
R_{XS} [fm]	12.4 ± 1.6	12.2 ± 0.9
R_{YS} [fm]		12.5 ± 0.8
$\Delta\tau$ [fm/c]	0.9 ± 1.2	2.7 ± 0.2
$\Delta\eta$	3.1 ± 0.1	2.5 ± 0.3
μ_0^π [MeV]	-2 ± 14	40 ± 8
μ_0^K [MeV]	16 ± 19	55 ± 13
μ_0^p [MeV]	97 ± 28	178 ± 22
χ^2/NDF	114 /208	261 /152
		($CL_{\text{w/o radii}}$ ~10%)

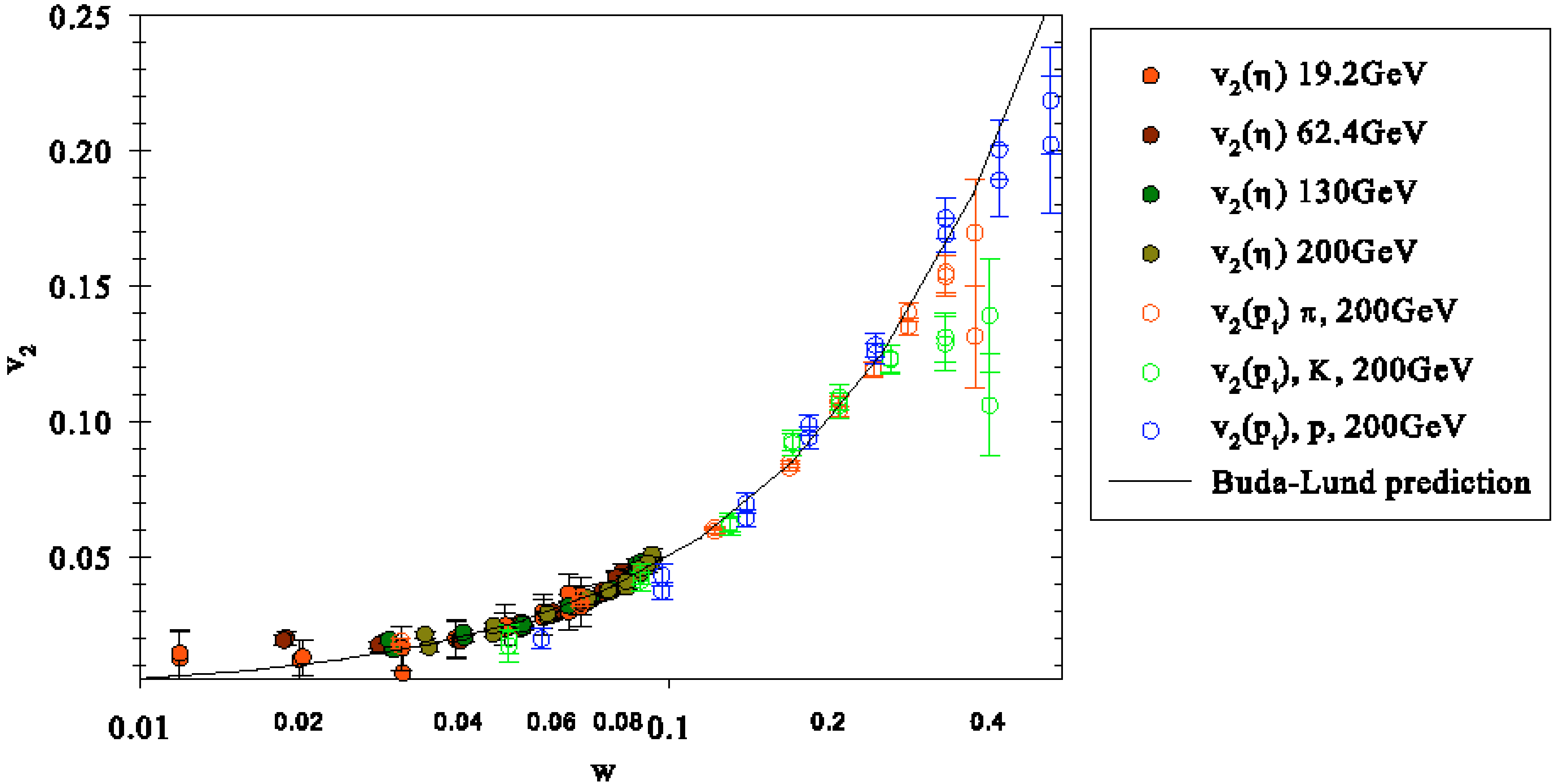
BudaLund geometrical picture of non-central RHIC reactions

Schematic view

20-30% ~ b = 8-12 fm



Universal v2 scaling predicted by BudaLund model (2003)



Conclusion

- **asBuda-Lund model describes single particle distributions, elliptic flow and asHBT correlation function radii of 20-30% centrality data of 200 GeV Au+Au reactions at RHIC.**
- **We find that T drops to 179 MeV from ~200 MeV of central collisions.**
- **The source at freeze-out was found to be slightly extended in the reaction plane ($R_x > R_y$), in agreement with M. Csanád's conclusion**