# Elliptic flow fluctuations and non-flow correlations

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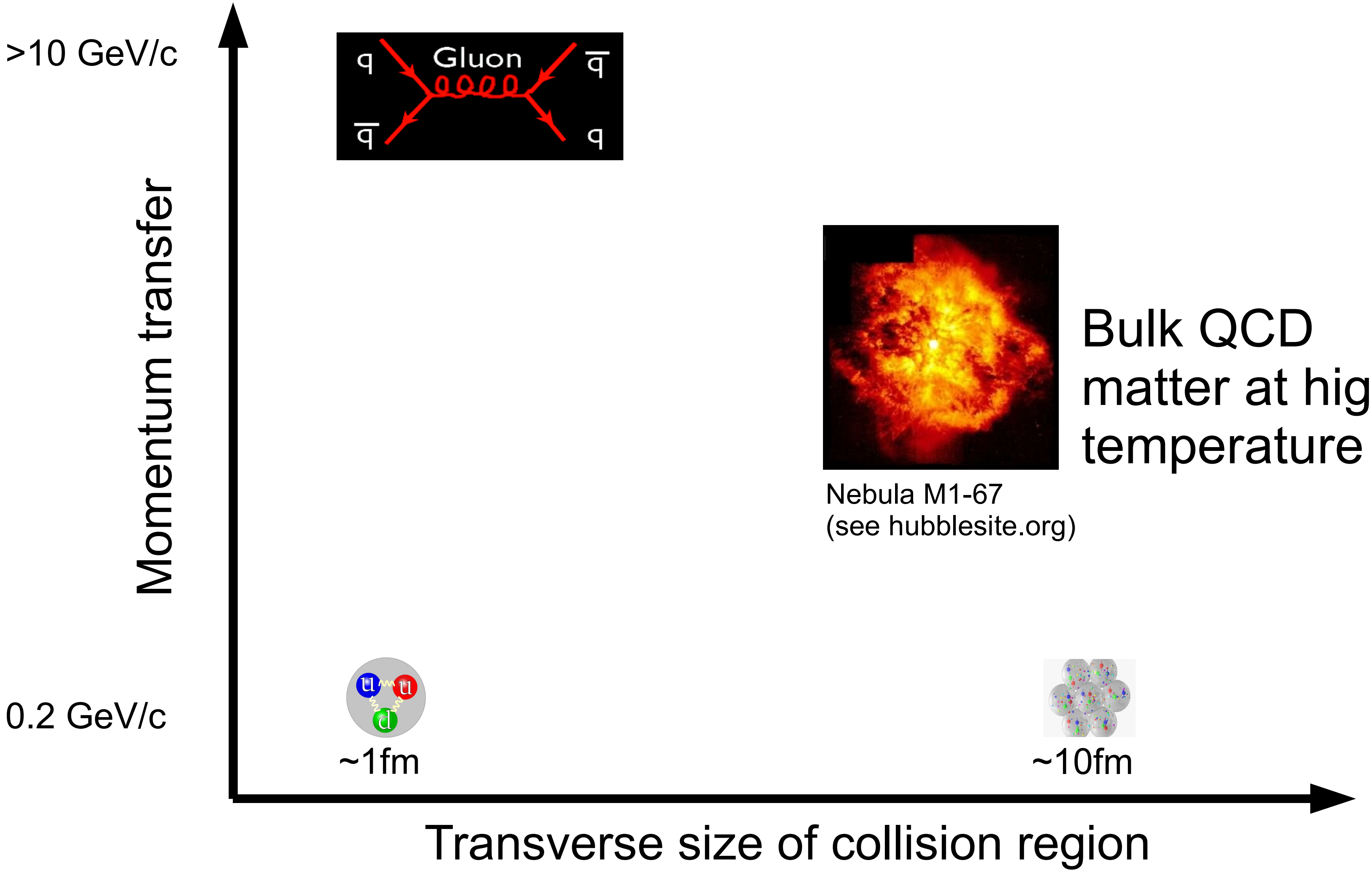
(MIT)

V Workshop on Particle Correlations and Femtoscopy

CERN, 15 October 2009

# Study of bulk QCD matter



>10 GeV/c

0.2 GeV/c

Momentum transfer

Bulk QCD matter at high temperature

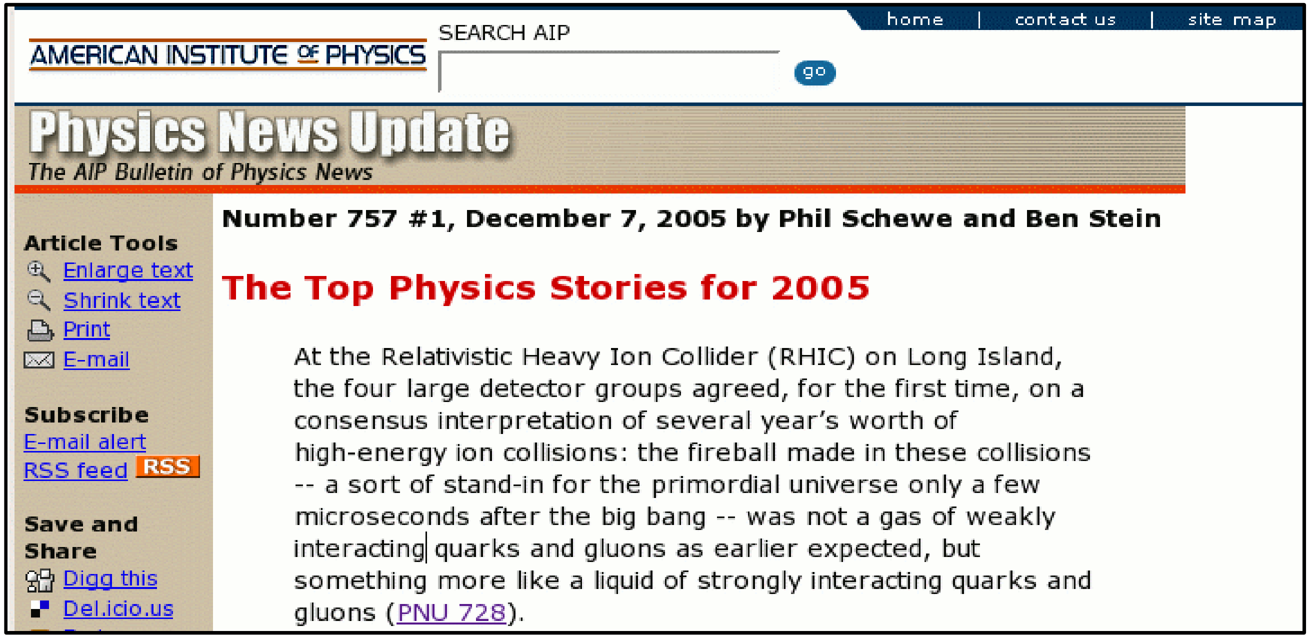
Nebula M1-67
(see hubblesite.org)

~1fm

~10fm

Transverse size of collision region

# AIP Top Physics Story, Dec 2005



**Physics News Update**

*The AIP Bulletin of Physics News*

**Number 757 #1, December 7, 2005 by Phil Schewe and Ben Stein**

**The Top Physics Stories for 2005**

At the Relativistic Heavy Ion Collider (RHIC) on Long Island, the four large detector groups agreed, for the first time, on a consensus interpretation of several year's worth of high-energy ion collisions: the fireball made in these collisions -- a sort of stand-in for the primordial universe only a few microseconds after the big bang -- was not a gas of weakly interacting quarks and gluons as earlier expected, but something more like a liquid of strongly interacting quarks and gluons (PNU 728).

"... the fireball made in these [heavy-ion] collisions ... was not a gas of weakly interacting quarks and gluons as earlier expected, but something more like a liquid of strongly interacting quarks and gluons"

http://www.aip.org/pnu/2005/split/757-1.html
RHIC whitepapers: NPA 757 (2005) 1-283

# Initial anisotropy and elliptic flow

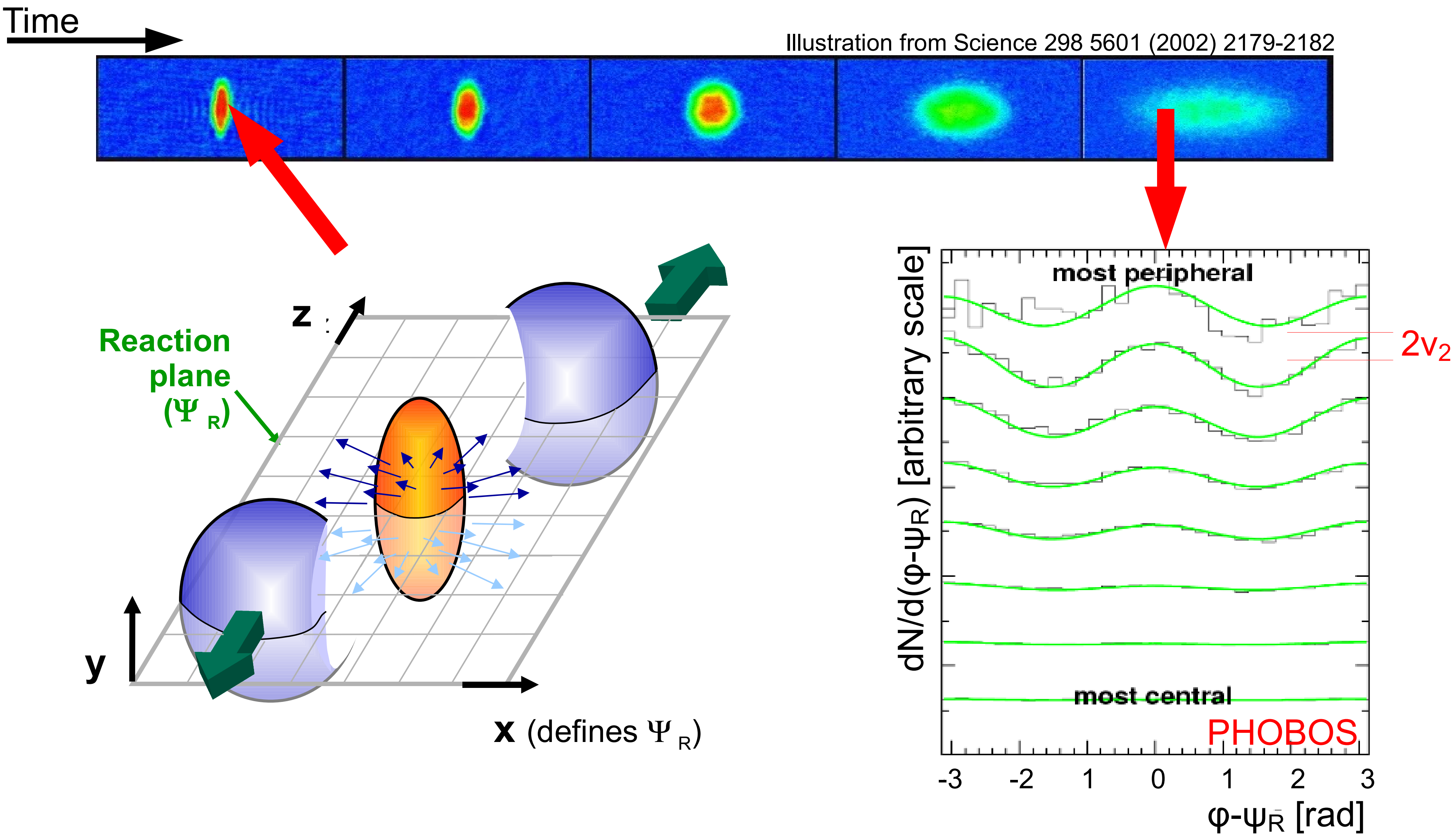


Illustration from Science 298 5601 (2002) 2179-2182

Initial spatial anisotropy eccentricity ε

Interactions present early

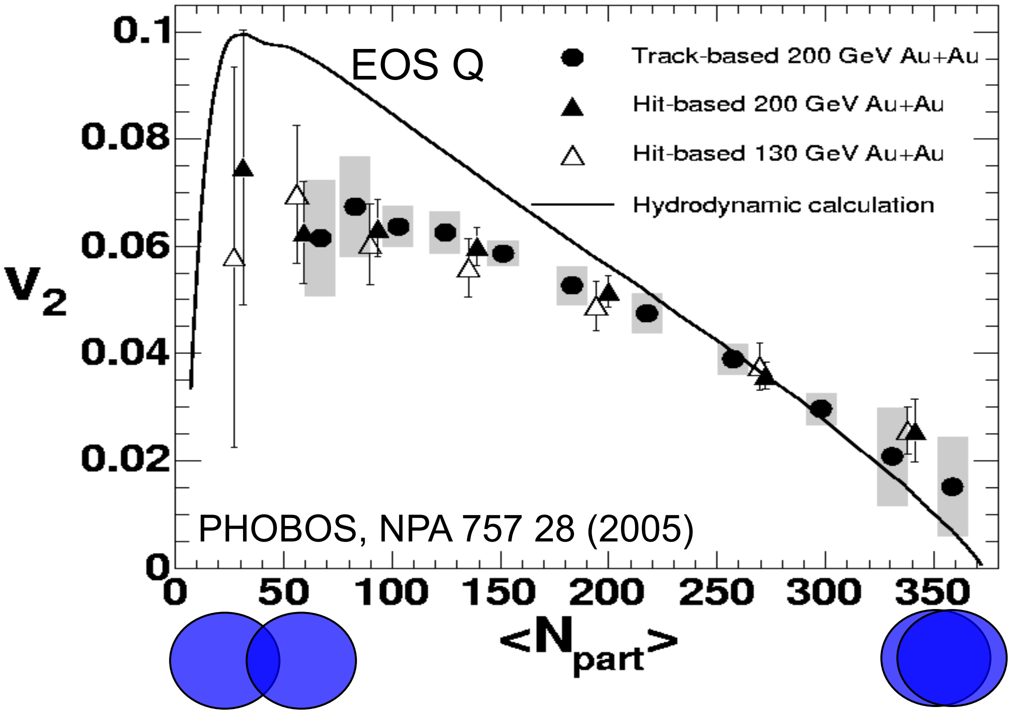
Momentum space anisotropy $v_2 = \langle \cos(2\phi - 2\Psi_R) \rangle$

# Elliptic flow and ideal hydro

**Ideal relativistic hydrodynamics**

$$T^{\mu\nu}=(e+p)u^{\mu}u^{\nu}-p\,g^{\mu\nu}$$
$$\delta_{\mu}T^{\mu\nu}=0$$
$$\delta_{\mu}N_{i}^{\mu}=0,\ i=B,S,\ldots$$
$$p=p(e,n)$$

Closure with EoS

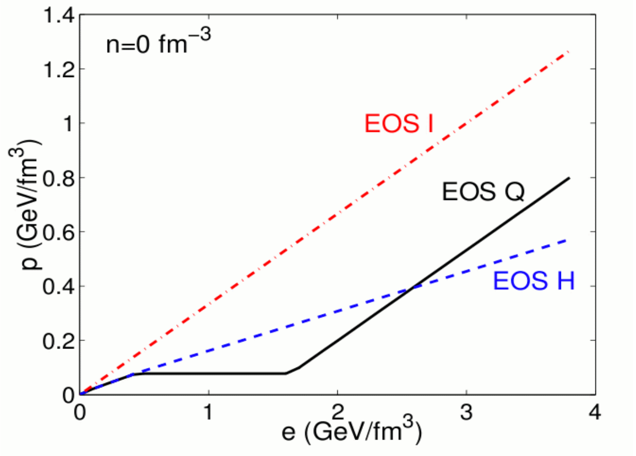


PHOBOS, NPA 757 28 (2005)

Assumption:

After a short thermalization time (≤1fm/c) a system in local equilibrium with zero mean free path and zero viscosity is created

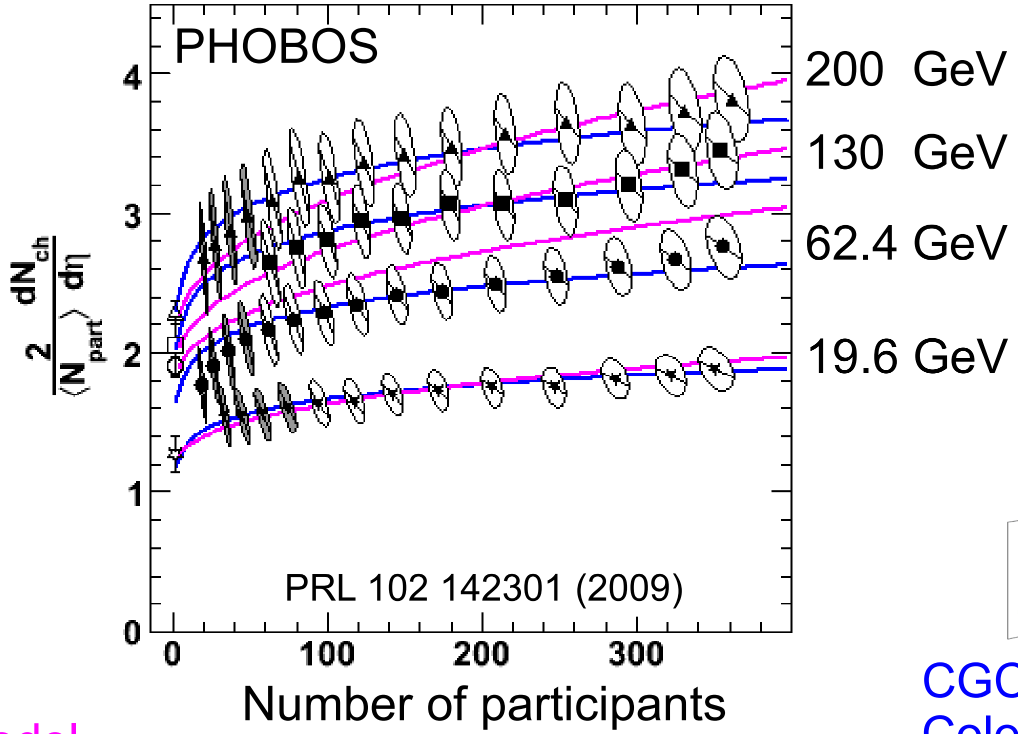
Initial conditions (IC) →
Equation of state (EOS) → Hydro → Observables
Freeze-out cond. (FO) →



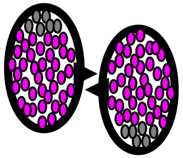
# Ambiguity of initial conditions

Collision energy

Mid-rapidity density



PRL 102 142301 (2009)

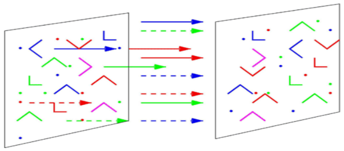


Glauber IC

Two-component model

$$\frac{dN}{d\eta}=\frac{dN}{d\eta^{pp}}((1-x)N_{coll}+x\,N_{part}/2)$$

PRC 70 021902 (2004)



CGC IC

Color glass condensate

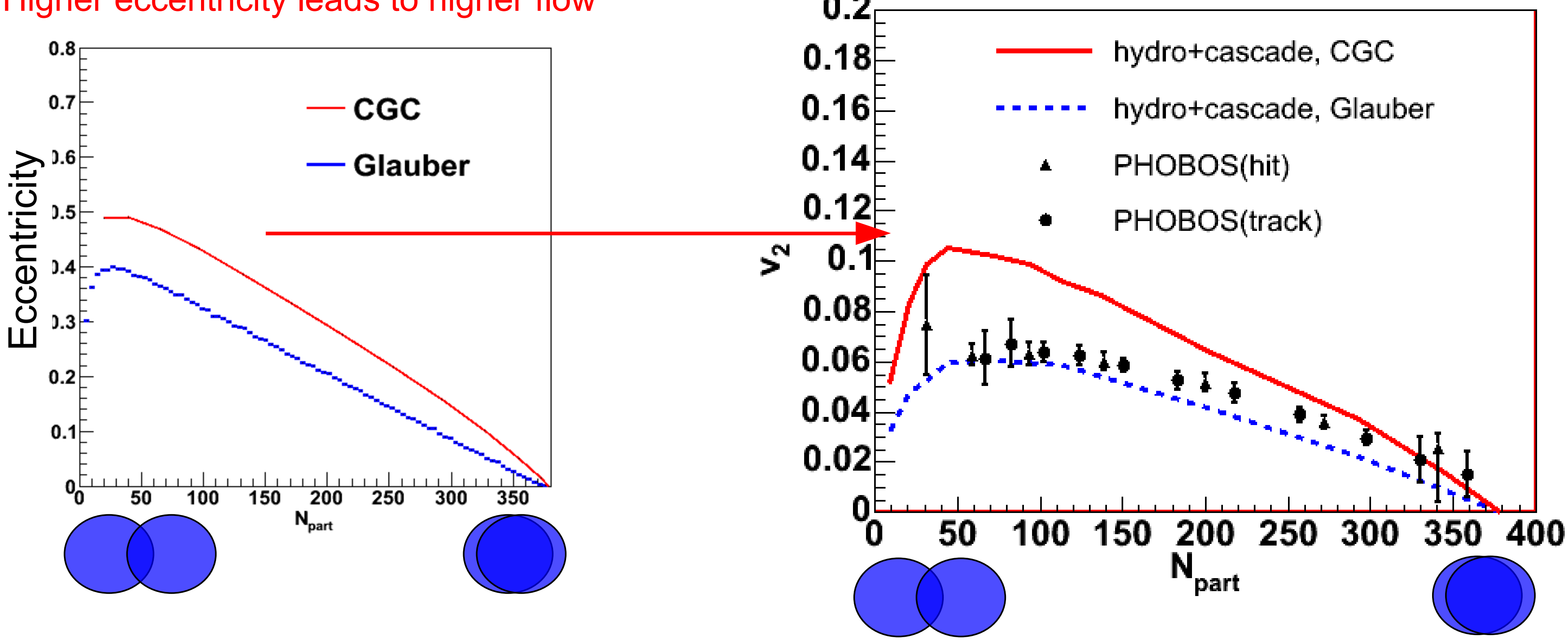
$$\frac{dN}{d\eta}\propto N_{part}^{\alpha}\sqrt{s}^{\lambda}$$

PRL 94 022002 (2005)

Collision centrality

# Ambiguity of conclusions

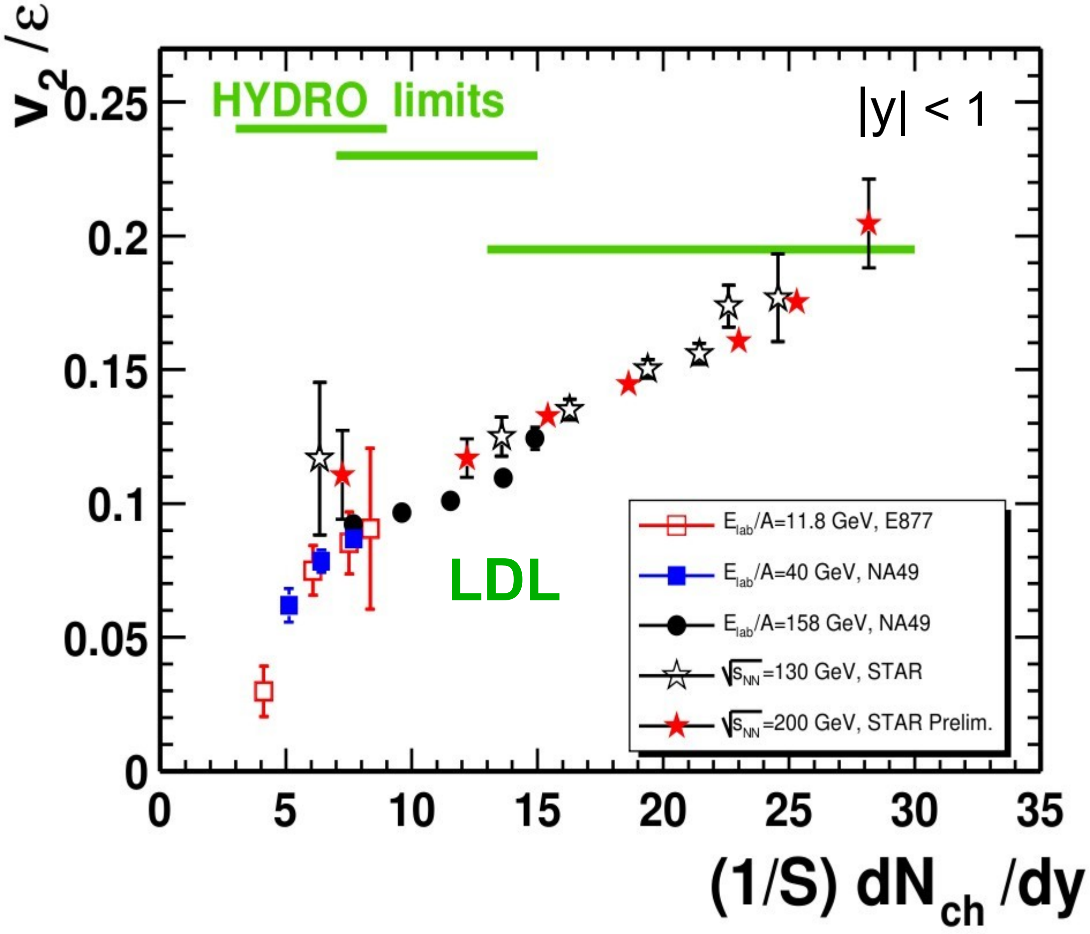
Higher eccentricity leads to higher flow



Ambiguity in description of initial state leads to ambiguity of conclusions: viscous corrections and/or soft equation of state?

Hirano et al., PLB 636 299 (2006)

# “... something more like a liquid ...”

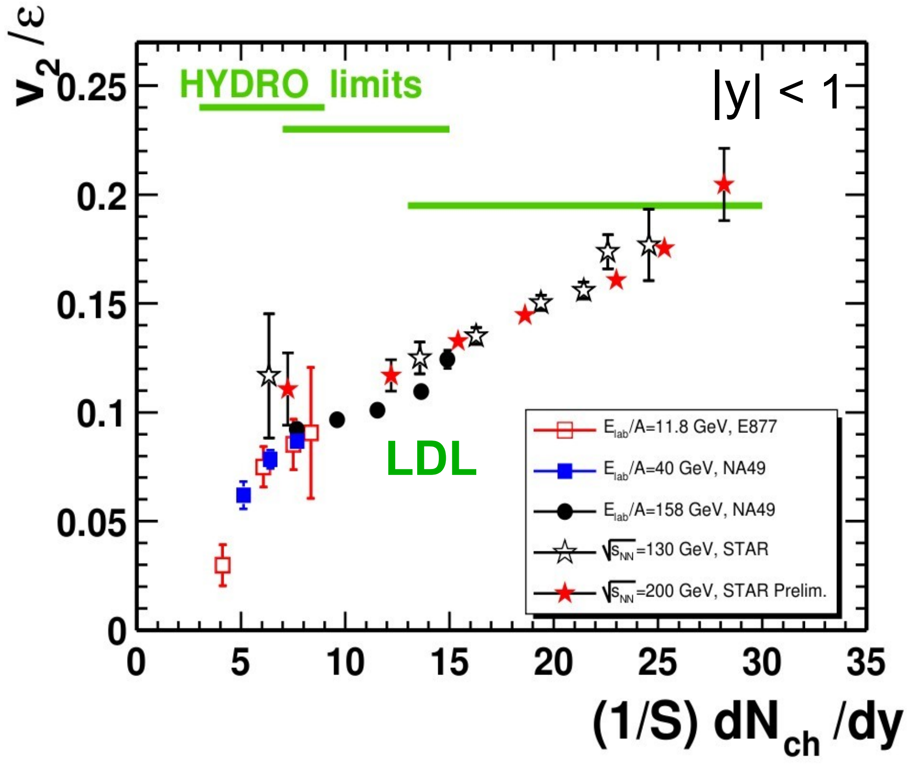


Geometry should cancel out in the $v_2/\varepsilon$ ratio.
In the low density limit region, the ratio rises with the number of collisions per particle, until it is expected to saturate in the hydro limit.

Heiselberg, Levy, PRC 59 (1999) 2716
Voloshin, Poskanzer, PLB 474 (2000) 27
STAR, PRC 66 034904 (2002)
NA49, PRC 68 (2003) 034903

# “... something more like a liquid ...”

There are many components in this picture: Each of them has and still is been questioned!



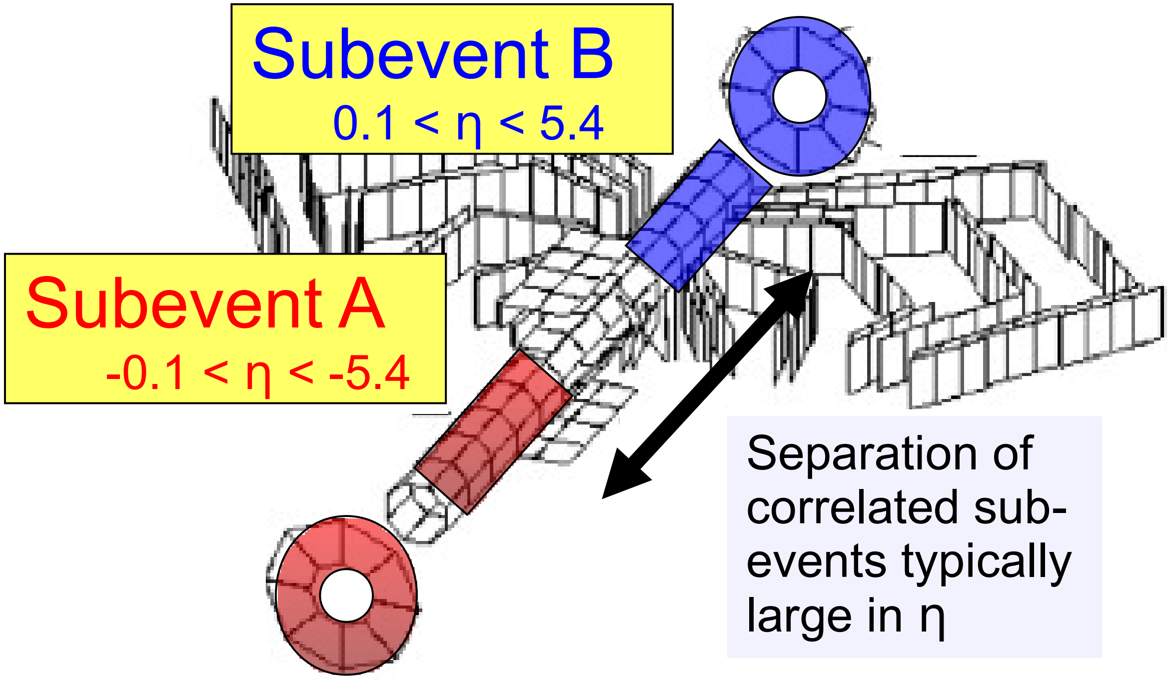
Note:
“Data points” in this plot depend on initial state model for ε and S.

- How well do we measure and understand flow
  - System/species comparison
  - Flow and non-flow fluctuations
  - Sensitivity of mean flow methods to underlying fluctuations/correlations
- How well do we model the initial conditions
  - Definition of eccentricity and eccentricity fluctuations
  - Glauber vs CGC
- What is the role of (shear) viscosity
- Does $v_2$ saturate at the hydrodynamical limit

(Not really covered in this talk)

# Elliptic flow measurement

- Reaction-plane / Sub-event technique
  - Correlate reaction plane determined from azimuthal pattern of hits in one part of the detector with information from other parts a of the detector



**Subevent B**

$0.1 < \eta < 5.4$

**Subevent A**

$-0.1 < \eta < -5.4$

Separation of correlated sub-events typically large in η

$$\tan(2\psi_A) = \frac{\langle \sin(2\phi) \rangle_A}{\langle \cos(2\phi) \rangle_A}$$

$$v_2^{obs} = \langle \cos(2\phi - 2\psi_A) \rangle_B$$

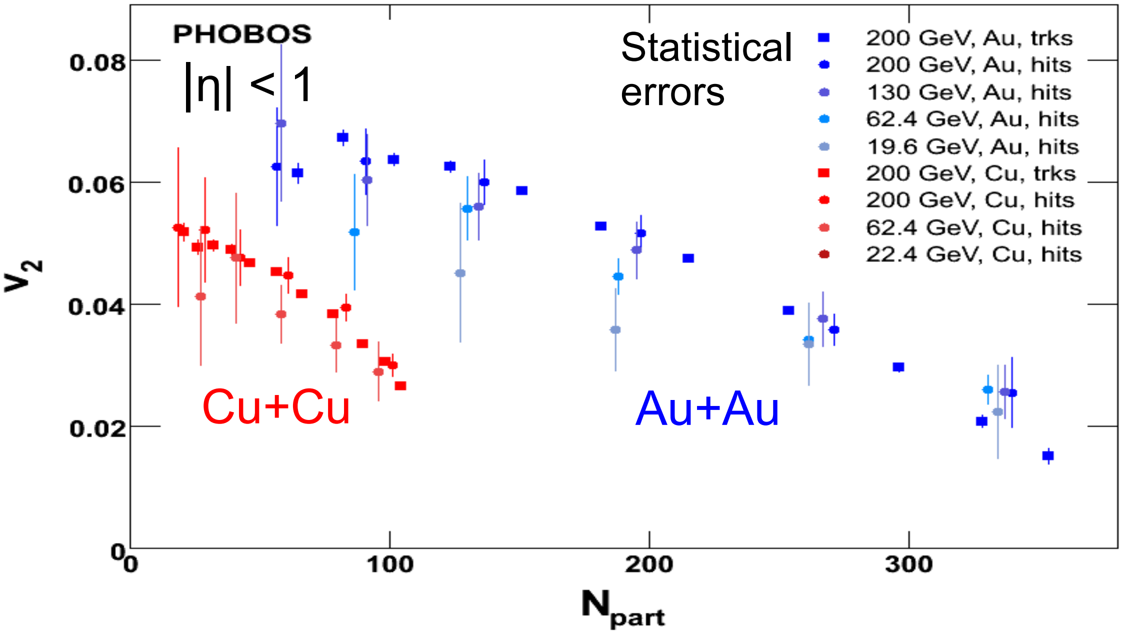
$$v_2 = \frac{\langle v_2^{obs} \rangle_{events}}{\sqrt{\langle \cos(2\psi_A - 2\psi_B) \rangle_{events}}}$$

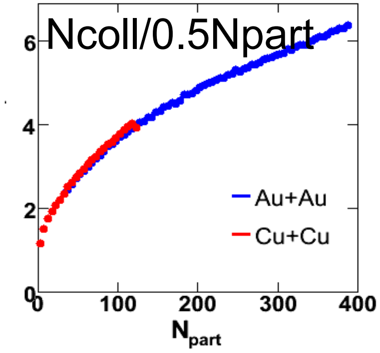
Poskanzer, Voloshin, nucl-ex/9805001

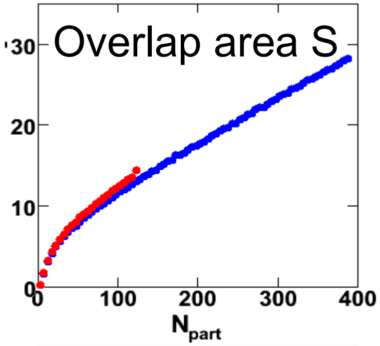
Resolution correction

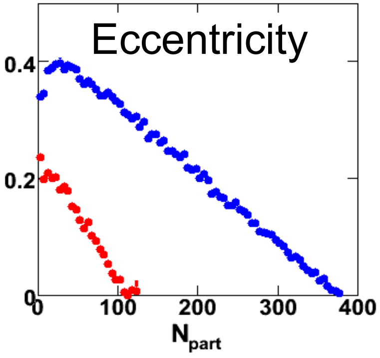
# Elliptic flow and collision geometry

QM05







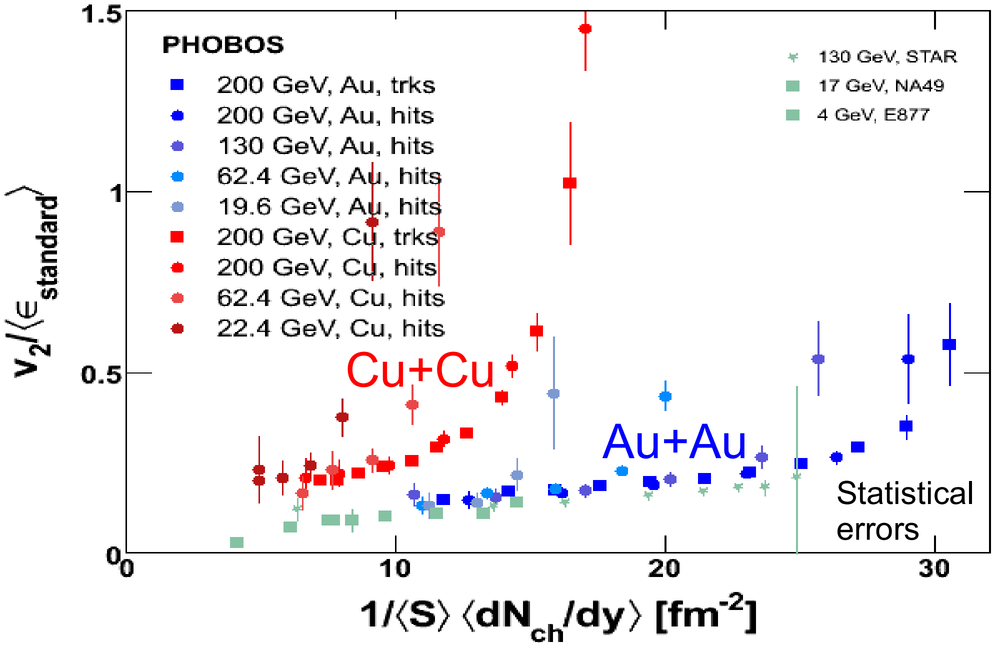


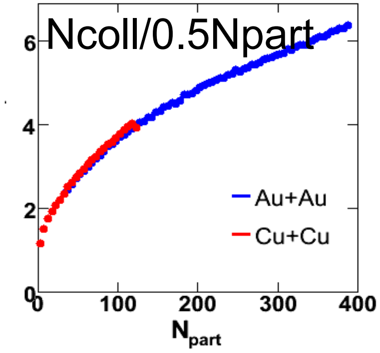
Geometry should cancel out in the $v_2/\varepsilon$ ratio

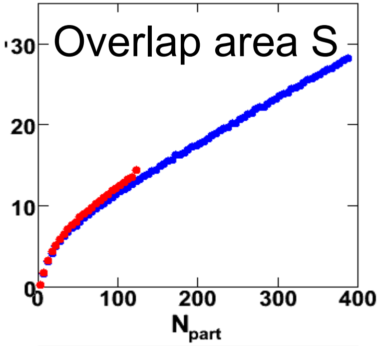
PHOBOS, Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
PHOBOS, Cu+Cu, 200+62.4 GeV: PRL 98 242302 (2007)
PHOBOS, Cu+Cu, 22.4 GeV: prel. QM06

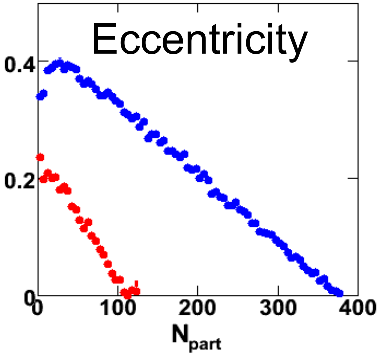
# Elliptic flow and collision geometry

QM05







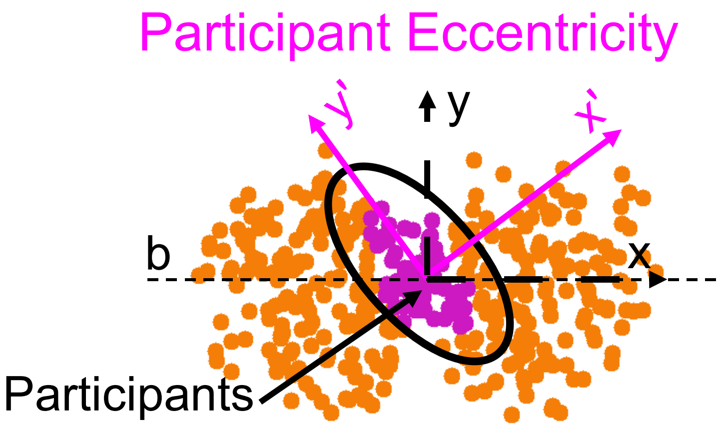


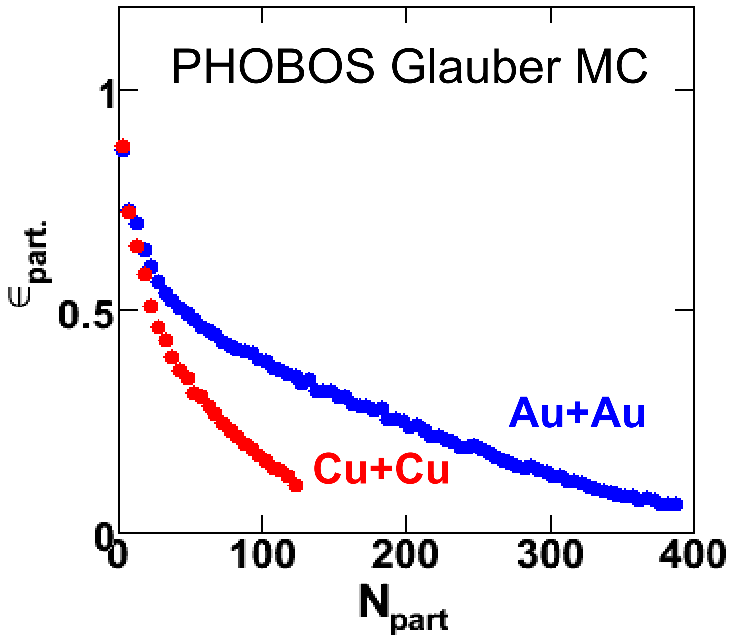
No scaling between Cu+Cu and Au+Au
using the standard eccentricity definition

PHOBOS, Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
PHOBOS ,Cu+Cu, 200+62.4 GeV: PRL 98 242302 (2007)
PHOBOS, Cu+Cu, 22.4 GeV: prel. QM06
STAR+NA49+E877, PRC 66 034904 (2002)
(data taken with no adjustments)

# Participant eccentricity

QM05





The spatial distribution of the interaction points of participating nucleons for the same b varies from event-to-event. Thus, define

$$\epsilon_{\mathrm{part}} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\boxed{\sigma_{xy}^2}}}{\sigma_y^2 + \sigma_x^2} \longleftarrow$$

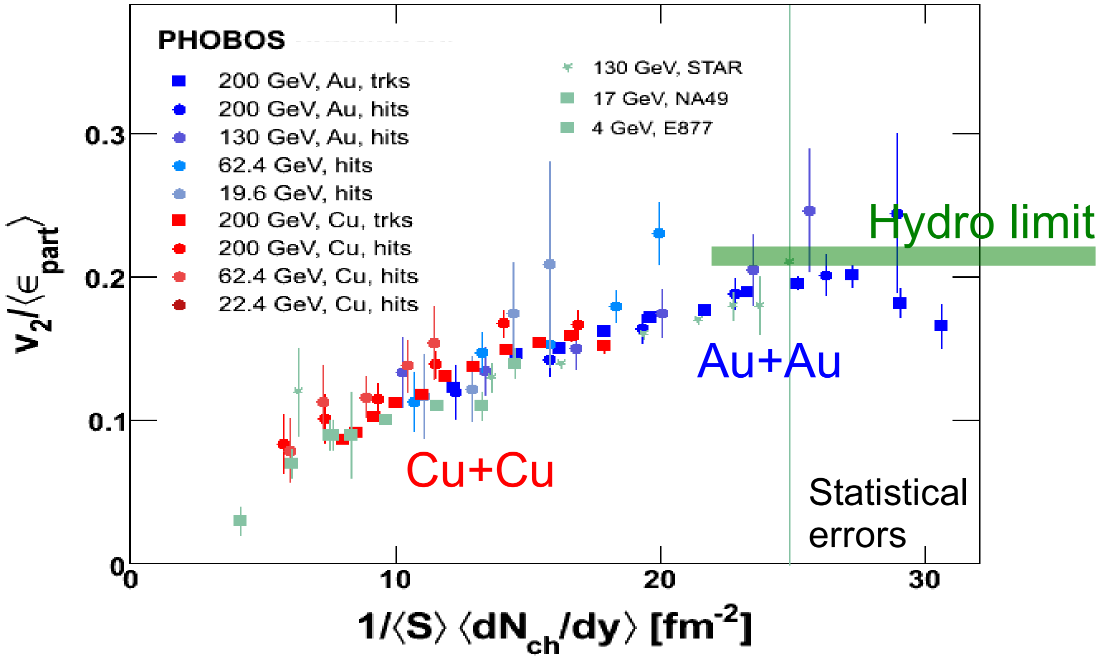
Correlation term missing in standard definition

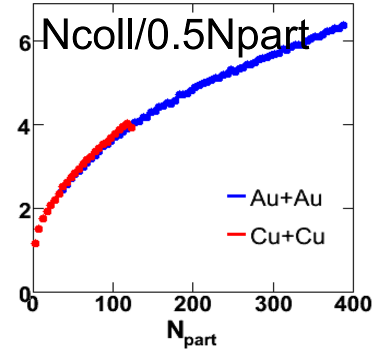
$$(0 < \epsilon_{\mathrm{part}} \leq 1)$$

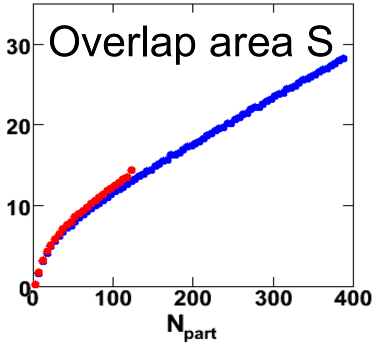
Introduced at QM05,
PHOBOS, PRL 98 242302 (2007)

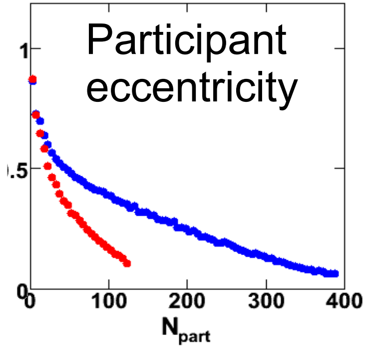
# Elliptic flow and collision geometry

QM05







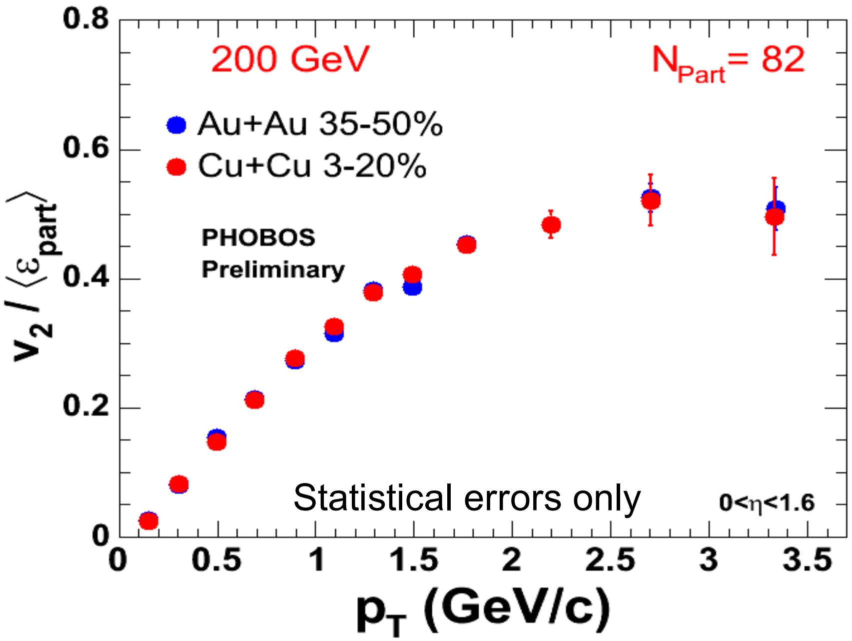


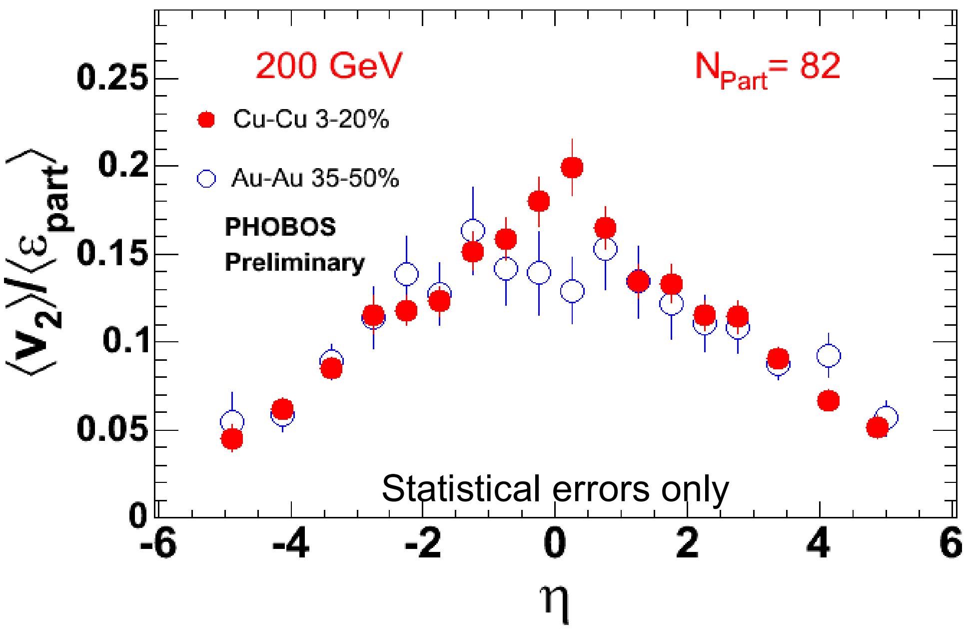
Scaling between Cu+Cu and Au+Au
using participant eccentricity definition

PHOBOS, Au+Au, 200,130,62.4+19.6 GeV: PRL 94 122303 (2005)
PHOBOS ,Cu+Cu, 200+62.4 GeV: PRL 98 242302 (2007)
PHOBOS, Cu+Cu, 22.4 GeV: prel. QM06
STAR+NA49+E877, PRC 66 034904 (2002)
(data taken with no adjustments)

# Eccentricity scaling is global

QM06



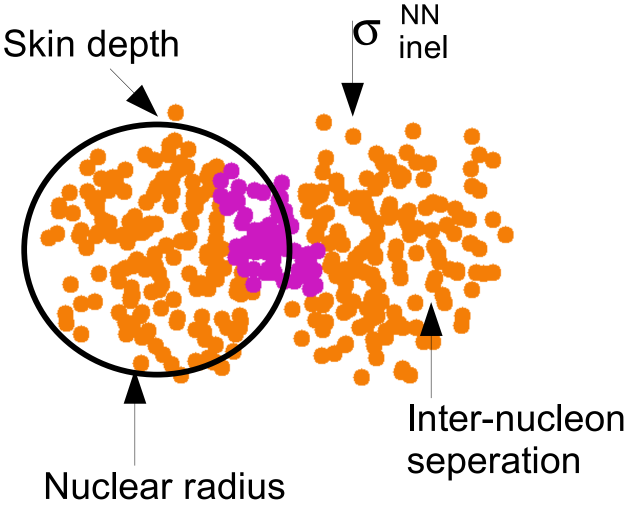


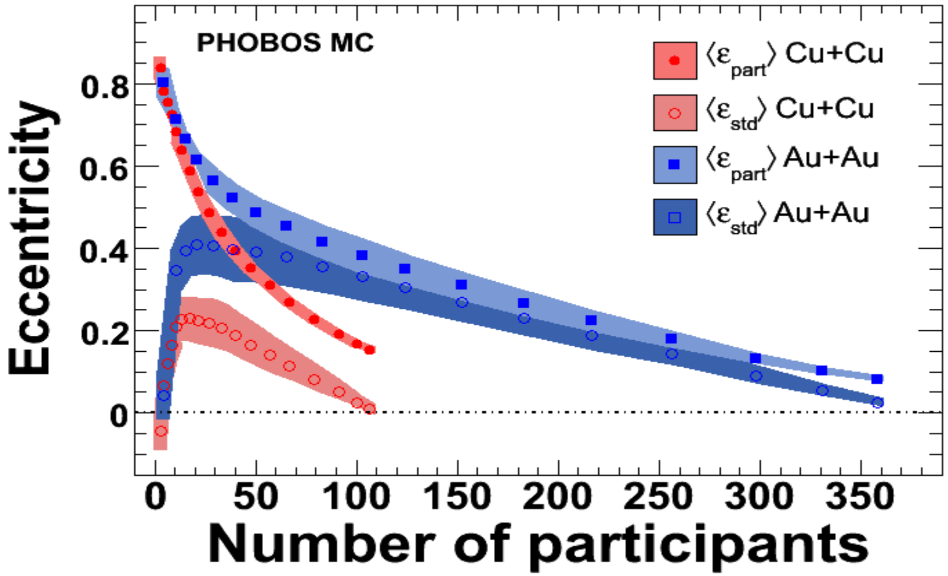
Unity of geometry, system, energy, transverse momentum and pseudorapidity for the same $N_{part}$ (~area density)

PHOBOS, JPG 34 887 (2007)

# Robustness of eccentricity definition

QM06





Baseline parameters:

- Nucleon-nucleon cross section: $\sigma_{NN}$=42mb
- Skin depth: a=0.535fm
- Wood-saxon radius: $R_A$=6.38fm
- Inter-nucleon separation distance: d=0.4fm

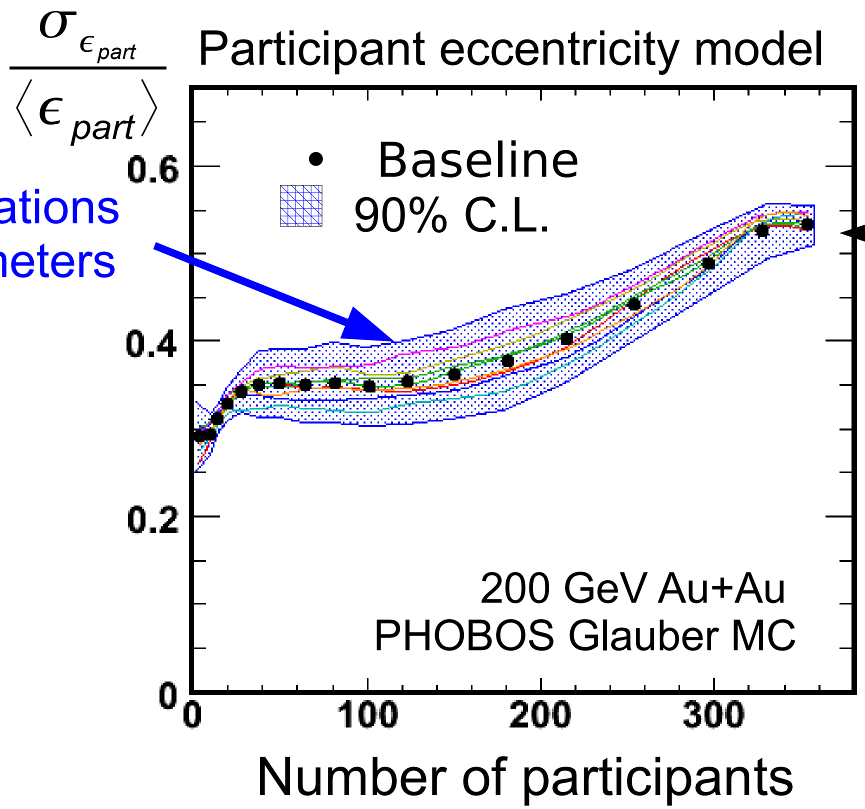
Robust definition wrt variation of Glauber parameters and to varying assumptions about matter production (not shown)

Alver et al., PRC 77 014906 (2008)

# Expected relative flow fluctuations

QM06

Uncertainty from variations of Glauber MC parameters



Analytic (b=0fm)

$$\sqrt{\frac{4}{\pi}-1} \approx 0.52$$

Broniowski et al., PRC 76 054905 (2007)

Baseline parameters:
- Nucleon-nucleon cross section: $\sigma_{NN}$=42mb
- Skin depth: a=0.535fm
- Wood-saxon radius: $R_A$=6.38fm
- Inter-nucleon separation distance: d=0.4fm

If initial state fluctuations are present, expect large relative flow fluctuations:

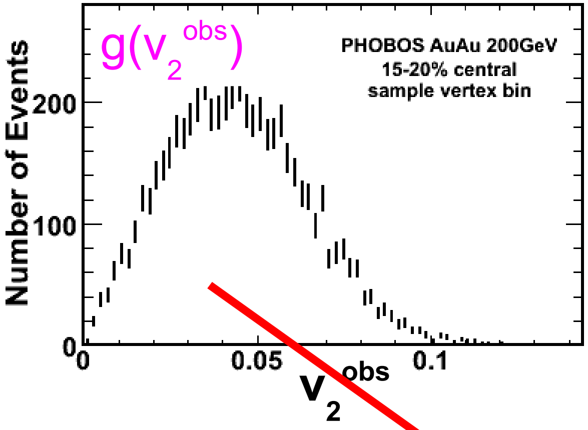
$$\frac{\sigma_{v_2}}{\langle v_2 \rangle} \sim \frac{\sigma_{\epsilon_{part}}}{\langle \epsilon_{part} \rangle}$$

PHOBOS, nucl-ex/0608025

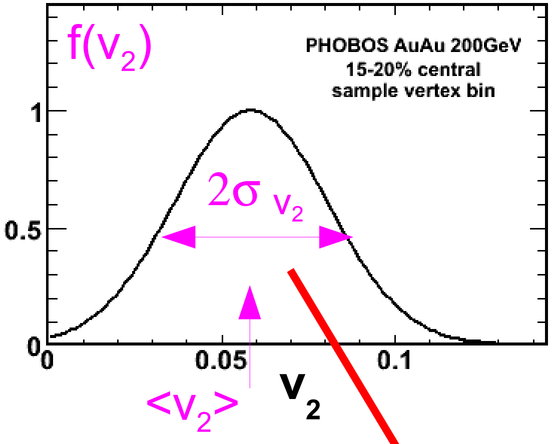
# Measuring elliptic flow fluctuations

QM06

Observed $v_2$ distribution



Parametrized $v_2$ distribution



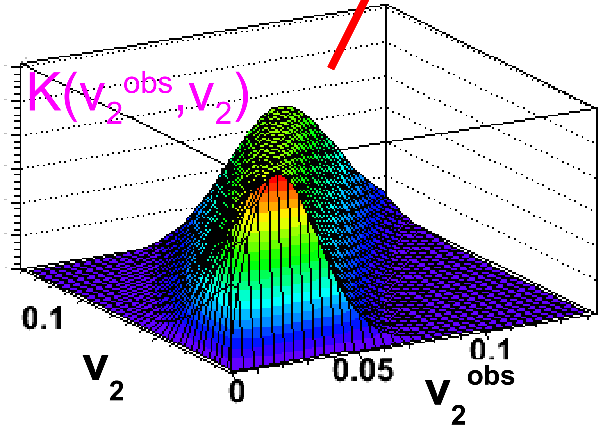
Max-Likelihood fit to determine: $<v_2>$ and $\sigma_{v_2}$

Kernel

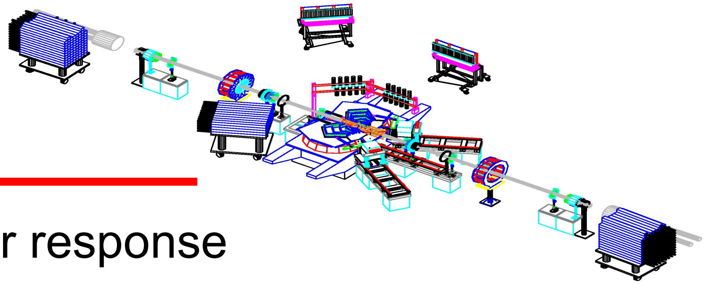
- Detector and acceptance effects
- Finite-number fluctuations
- Multiplicity fluctuations

$$g(v_2^{obs}) = \int_0^1 K(v_2^{obs}, v_2)\, f(v_2)\, dv_2$$

Kernel



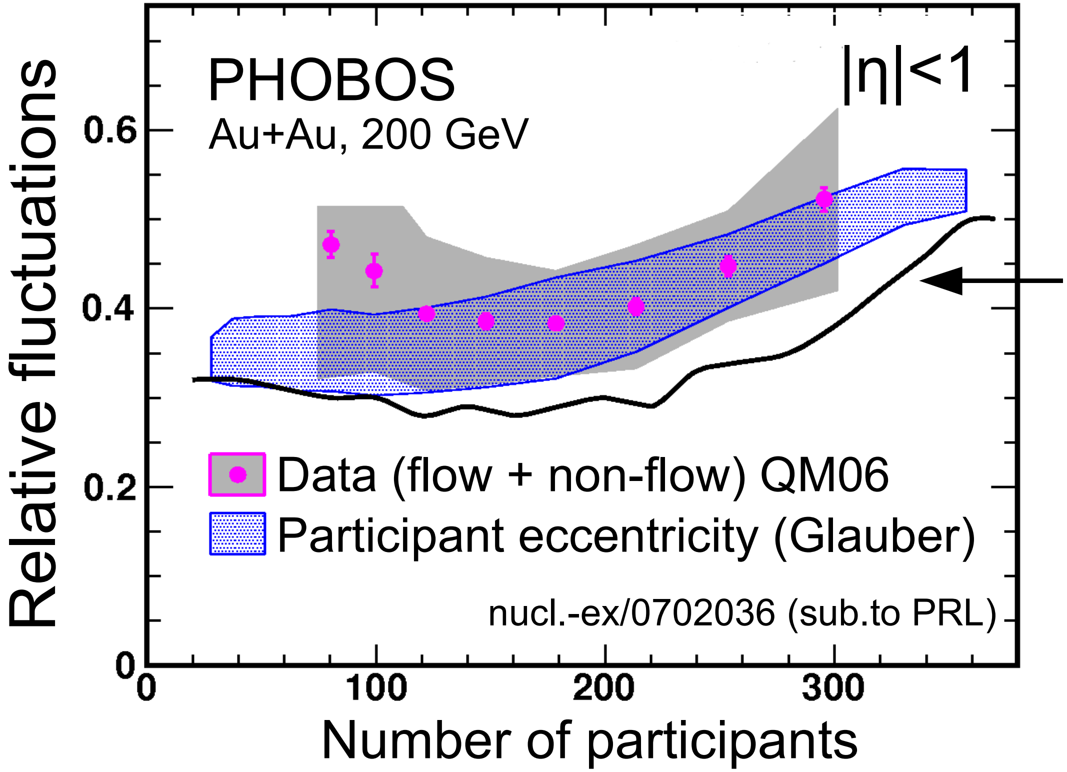
Detector response



PHOBOS, nucl.-ex/0702036 (sub.to PRL)

# Measured relative total fluctuations

QM06



CGC-MC (fKLN)
Drescher, Nara,
PRC 76 (2007) 41903

Shown at QM06 as flow fluctuations, however non-flow contribution (included in sys.error) was underestimated.

# Contribution from non-flow correlations

QM08, WORK IN PROGRESS

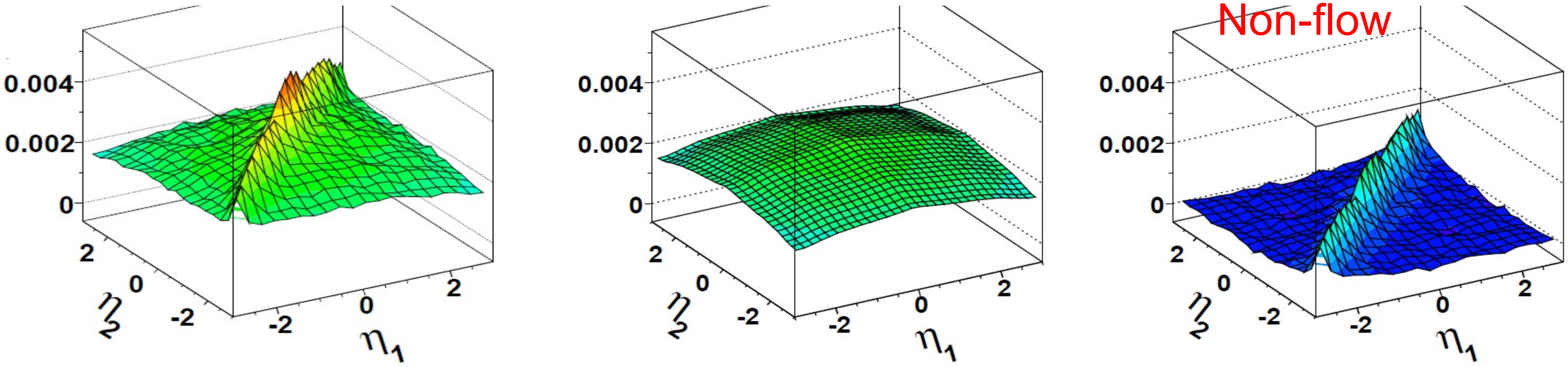
- PHOBOS has data driven analysis to measure the contribution of non-flow
  - Flow is a function of η and correlates particles at all Δη
  - Non-flow (δ) is dominated by short range correlations (small Δη)
  - Study correlations at different Δη

$$v_2^2(\eta_1,\eta_2)\equiv\langle\cos(2\Delta\phi)\rangle(\eta_1,\eta_2)$$
$$=v_2(\eta_1)*v_2(\eta_2)+\delta(\eta_1,\eta_2)$$

- Assume non-flow to be zero for Δη>2
- Fit $v_2^2(\eta_1,\eta_2)=v_2^{fit}(\eta_1)*v_2^{fit}(\eta_2)$, $|\eta_2-\eta_1|>2$
- Subtract fit results at all $(\eta_1,\eta_2)$
- Integrate over particle pairs to obtain $\delta/v_2^2$
- Numerically relate $\delta/v_2^2$, $\sigma_{tot}/\langle v_2\rangle$ and $\sigma_{flow}/\langle v_2\rangle$

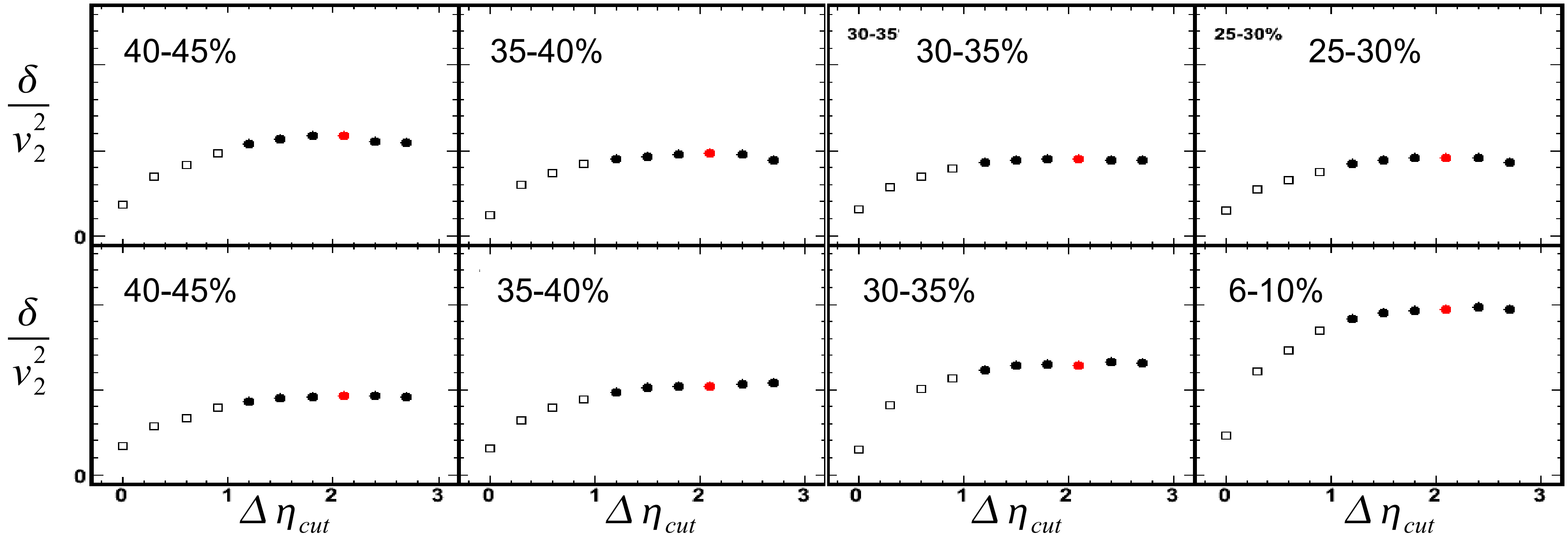
$$v_2^2(\eta_1,\eta_2) - v_2^{fit}(\eta_1)\times v_2^{fit}(\eta_2) = \delta(\eta_1,\eta_2)$$

Non-flow



# Contribution from non-flow correlations

QM08, WORK IN PROGRESS



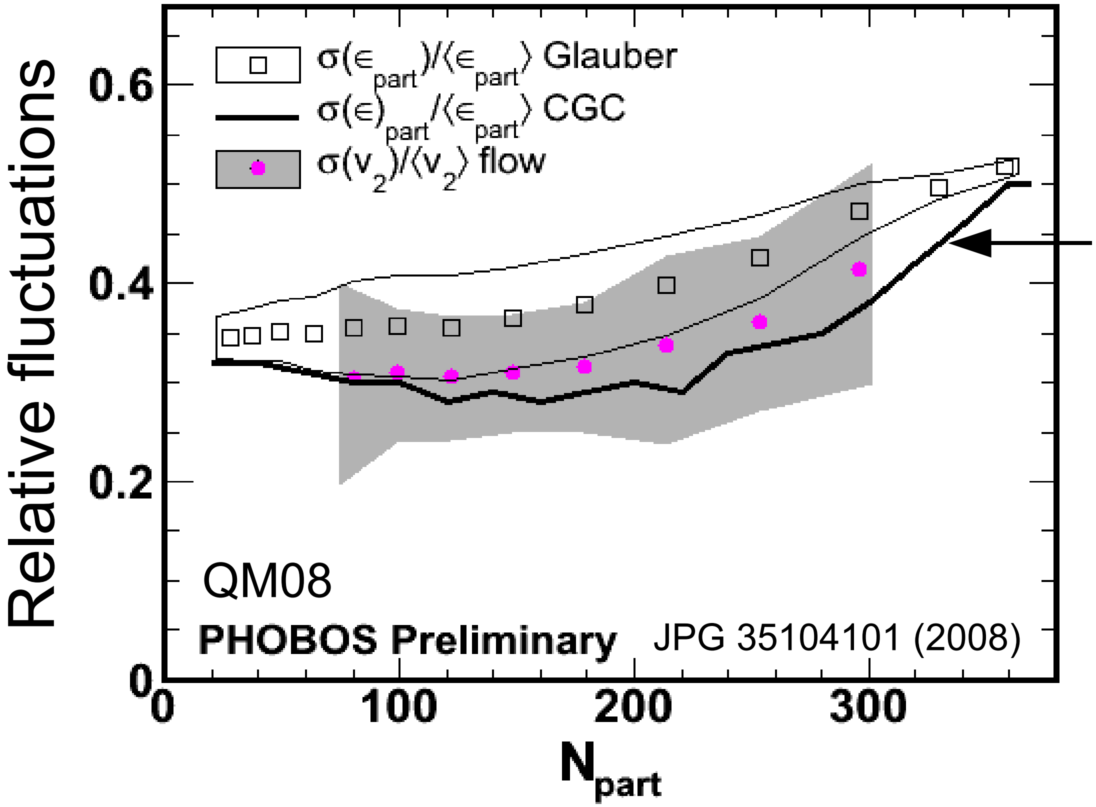
Non-flow ratio as a function of Δη cut used to obtain the fit.

Saturation is very encouraging, although does not rule out contributions with very little Δη dependence.

Red-point is baseline for analysis, while black points are used for systematic error

# Measured relative flow fluctuations

QM08, WORK IN PROGRESS



CGC-MC (fKLN)

Drescher, Nara,
PRC 76 (2007) 41903

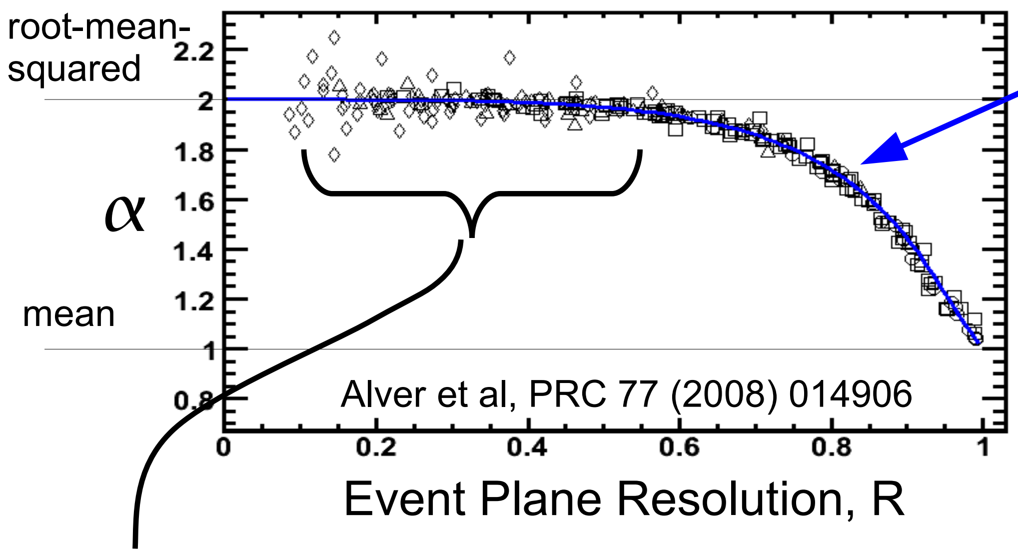
Initial state fluctuations if indeed present seem not to be significantly enhanced in later stages of the collision

Short-range non-flow contribution is taken out with method as shown at QM08

# Which moment of $v_2$ is measured?

Define

$$v_2 \equiv \langle v_2^{\alpha} \rangle^{1/\alpha}$$



Alver et al, PRC 77 (2008) 014906

PHOBOS R:
0.13 – 0.55

By now α is known:

$$\alpha = 2 - 4 i_1^2 / (i_0 + i_1)^2$$

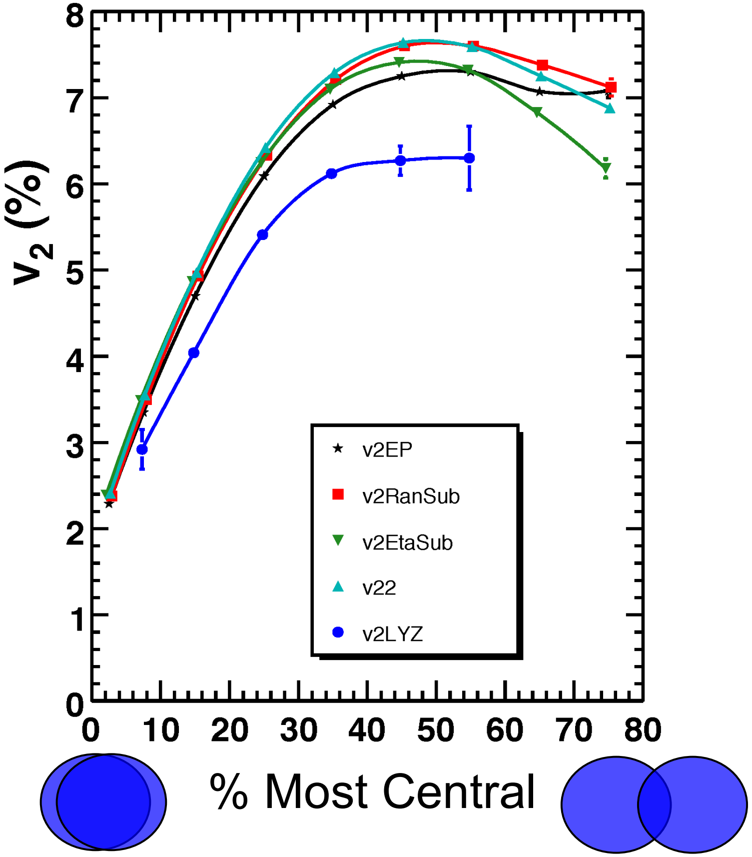
Ollitrault et. al.,
PRC 80 80 014904 (2009)

For PHOBOS standard event-plane method $v_2\{EP\} = \sqrt{\langle v_2^2 \rangle}$

(For the observed fluctuations this implies about 10% difference)

# Correction for non-flow and fluctuations

Published STAR results



Derive analytic correction for non-flow and fluctuations in leading order of $\delta$ and $\sigma_{v_2}^2$

Eg, for 2-particle correlations:

$$\langle\cos(2\Delta\phi)\rangle = \langle v_2\rangle^2 + \sigma_{v_2}^2 + \delta$$

Non-flow term

Differences between methods proportional to

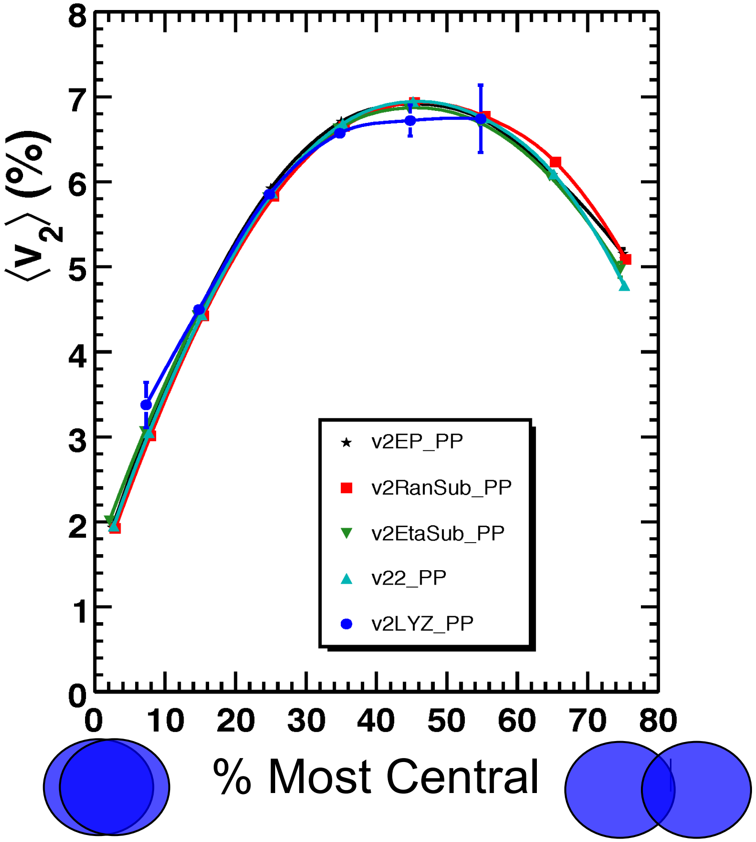
$$\sigma_{tot} = \delta + 2\sigma_{v_2}^2$$

Need additional assumption or information to separate between non-flow and fluctuations

Ollitrault, Poskanzer, Voloshin
PRC 80 80 014904 (2009)

# Correction for non-flow and fluctuations

Corrected mean results



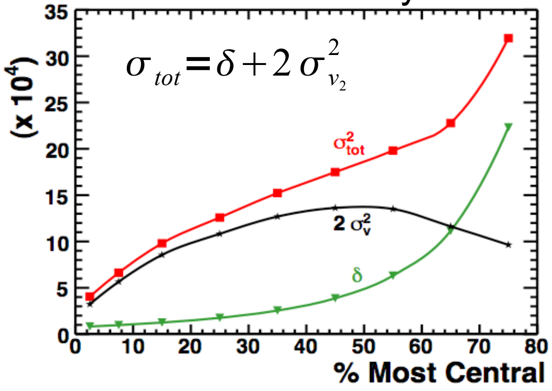
Model assuming:

$$\sigma_{v_2} = \frac{\sigma_{\epsilon_{\text{part}}}}{\langle \epsilon_{\text{part}} \rangle} \langle v_2 \rangle$$

$$\delta = \frac{2}{N_{\text{part}}} \delta_{pp}$$

with $\delta_{pp} = 0.0145$

Glauber eccentricity

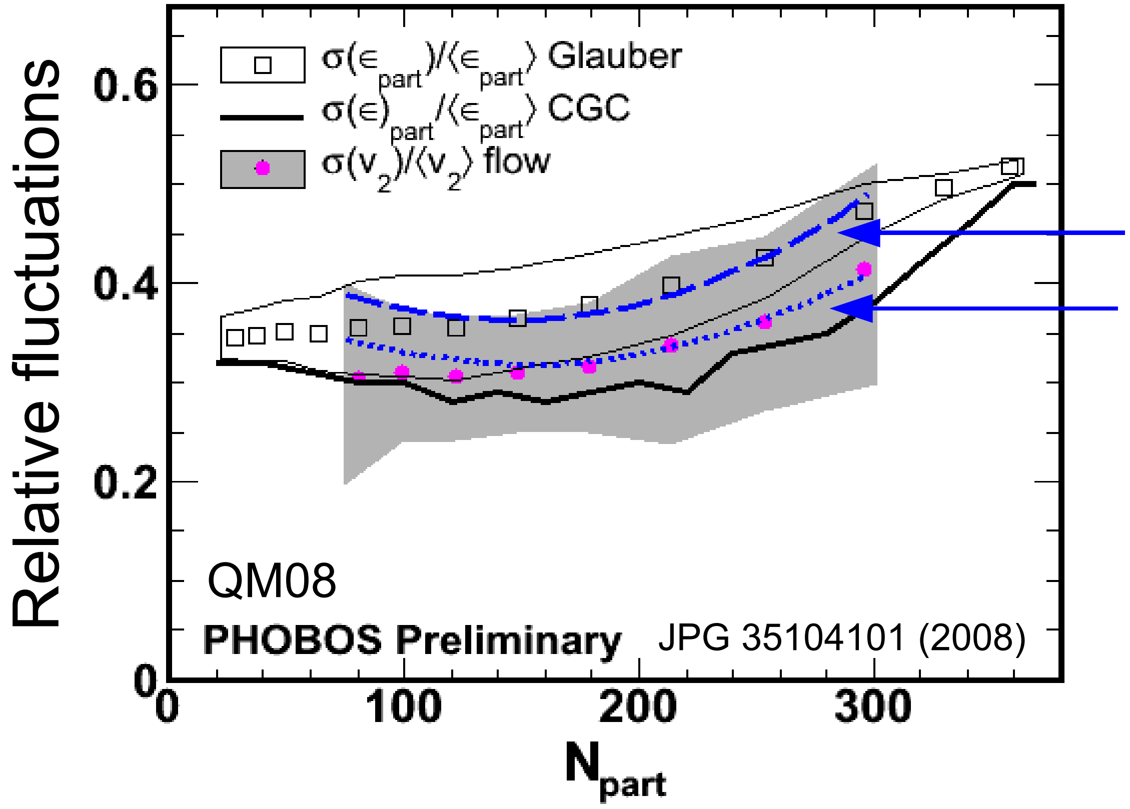


Corrected mean values agree in participant frame.
Reduces errors on $v_2$ measurements by about 20%.

Eccentricity values are calculated for standard Glauber and a mix of 30:70 CGC (not shown)

Ollitrault, Poskanzer, Voloshin
PRC 80 80 014904 (2009)

# Measured relative flow fluctuations



Analytic correction:

Glauber

CGC (30:70)

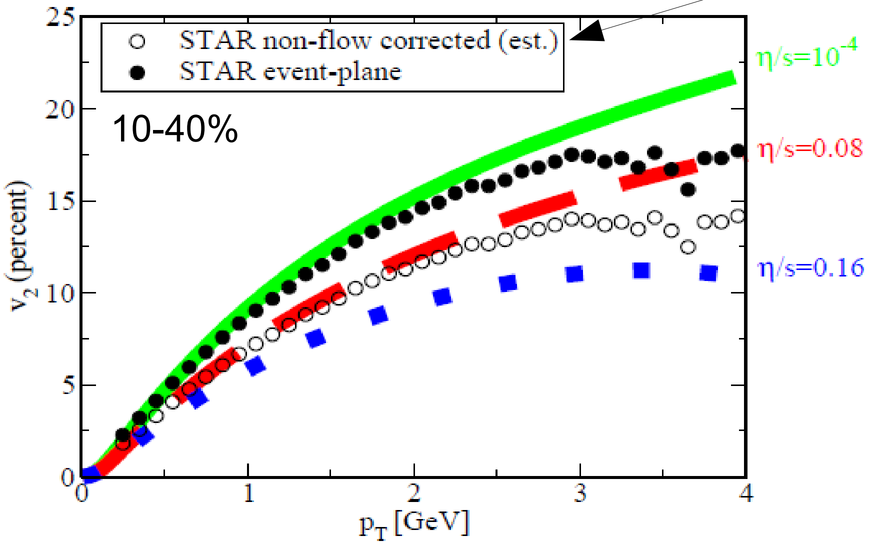
Ollitrault et. al.,
PRC 80 80 014904 (2009)

Results based on analytic model corrections are consistent with data.

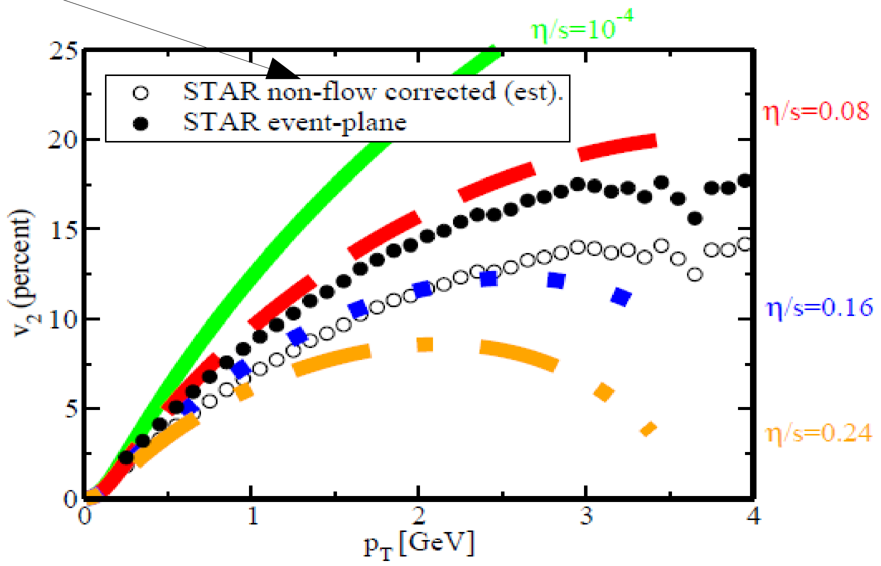
# How viscous is the liquid?

20% reduced

Glauber IC



CGC IC



State-of-art results from second-order conformal hydrodynamics (2+1D) yield a low shear viscosity to entropy ratio.

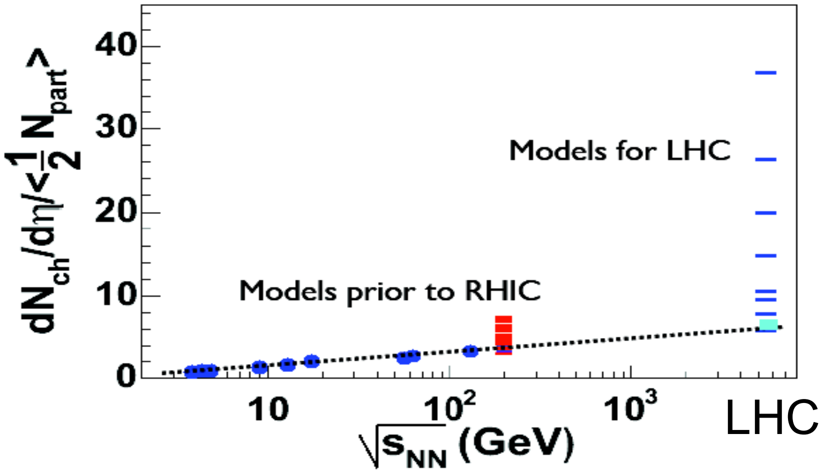
“General consensus” (from QM09) that: $\frac{\eta}{s} < 6 \times \frac{1}{4\pi}$

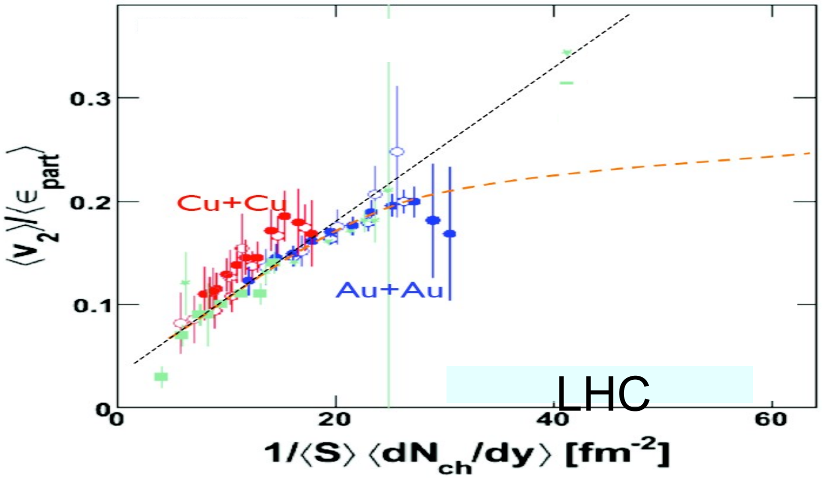
Reduced errors on $v_2$ data allows to study 20% effects.

Luzum, Romatschke,
PRC 78 034915 (2008);
PRC 79 039903 (2009)

# Summary

- Significant progress in understanding and quantification of ”something more like a liquid”
  - Understanding of flow, non-flow correlations, flow- and eccentricity-fluctuations seems to converge
    - Remaining caveat is role of long-range non-flow contributions
  - Contribution of (short-ranged) non-flow correlations to total relative fluctuations is about 10% (absolute)
  - Initial state fluctuations if present are not significantly enhanced throughout the collision evolution
- Exiting times ahead with p+p and Pb+Pb @ LHC in 2009/10





# The PHOBOS collaboration

I'd like to give credit to my PHOBOS colleagues:

Burak Alver, Birger Back, Mark Baker, Maarten Ballintijn, Donald Barton, Russell Betts, Richard Bindel, Wit Busza (Spokesperson), Vasundhara Chetluru, Edmundo García, Tomasz Gburek, Joshua Hamblen, Conor Henderson, David Hofman, Richard Hollis, Roman Hołyński, Burt Holzman, Aneta Iordanova, Chia Ming Kuo, Wei Li, Willis Lin, Constantin Loizides, Steven Manly, Alice Mignerey, Gerrit van Nieuwenhuizen, Rachid Nouicer, Andrzej Olszewski, Robert Pak, Corey Reed, Christof Roland, Gunther Roland, Joe Sagerer, Peter Steinberg, George Stephans, Andrei Sukhanov, Marguerite Belt Tonjes, Adam Trzupek, Sergei Vaurynovich, Robin Verdier, Gábor Veres, Peter Walters, Edward Wenger, Frank Wolfs, Barbara Wosiek, Krzysztof Woźniak, Bolek Wysłouch

ARGONNE NATIONAL LABORATORY
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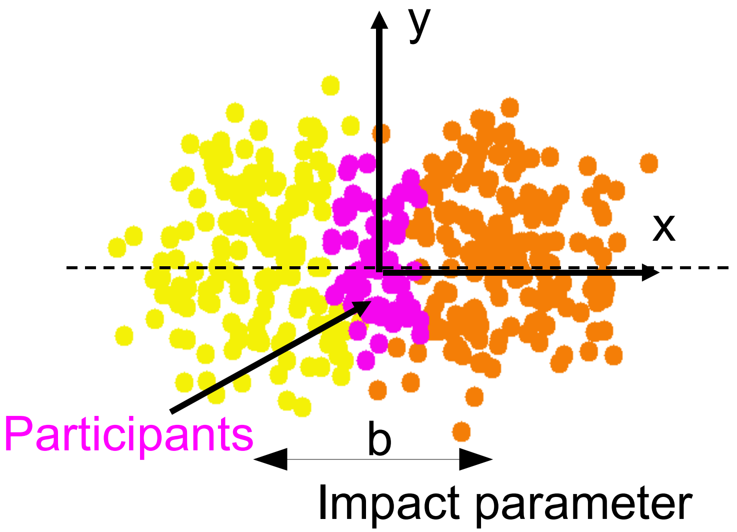
NATIONAL CENTRAL UNIVERSITY, TAIWAN
UNIVERSITY OF MARYLAND

BROOKHAVEN NATIONAL LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
UNIVERSITY OF ILLINOIS AT CHICAGO
UNIVERSITY OF ROCHESTER

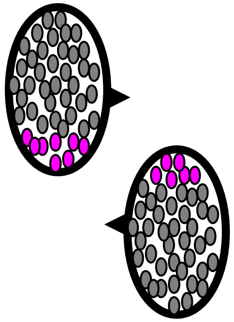
# Extras

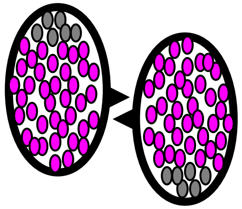
# External control parameters

- Centrality classes
  - Cross section percentile
  - Impact parameter (<b>)
  - #Participants ($<N_{part}>\sim A$)
    - Nucleons struck at least once
  - #NN-collisions ($<N_{coll}>\sim A^{4/3}$)
    - Total number of collisions
- Relate to data via Glauber MC based detector simulations



Collision energy

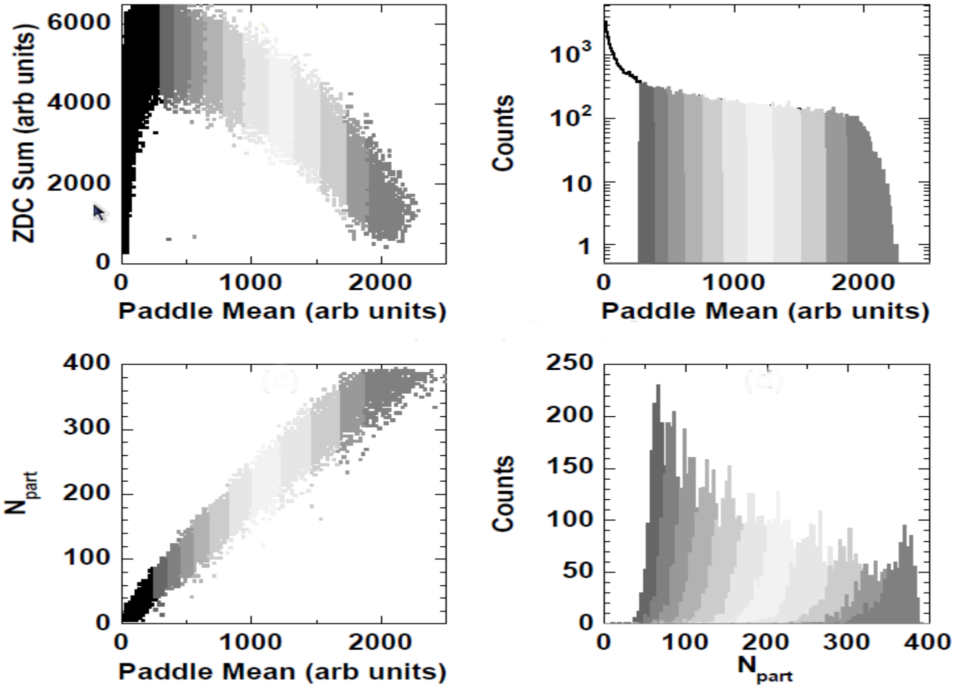


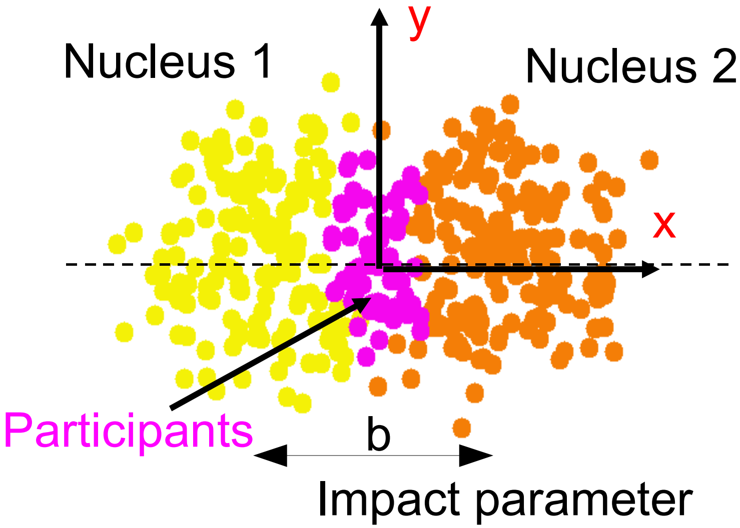


Collision centrality

# Centrality determination

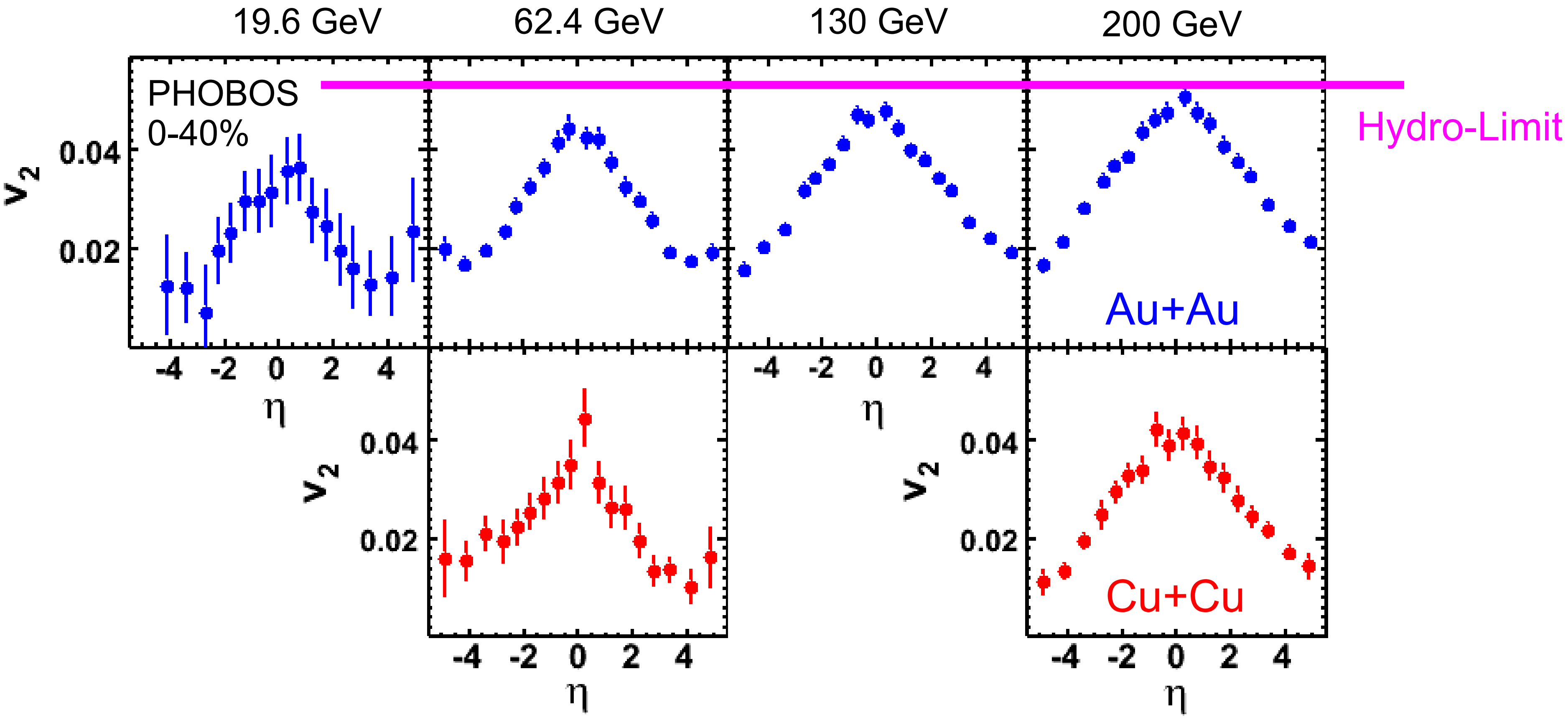
- Makeup of nuclei
  - Made up of nucleons drawn from Wood-Saxon distribution
  - Separate by b (with dN/db~b)
- Collision of nuclei
  - Assume: Nucleons travel along z on straight-line paths and interact when their centers are within $\sqrt{\sigma_{inel}^{NN}/\pi}$
  - #Participants is number of nucleons that interact at least once ($N_{part}$~A)
  - #NN-collisions is total number of collisions ($N_{coll}$~$A^{4/3}$)
- Relate to data via Glauber MC based detector simulations





NPA 757 28 (2005)

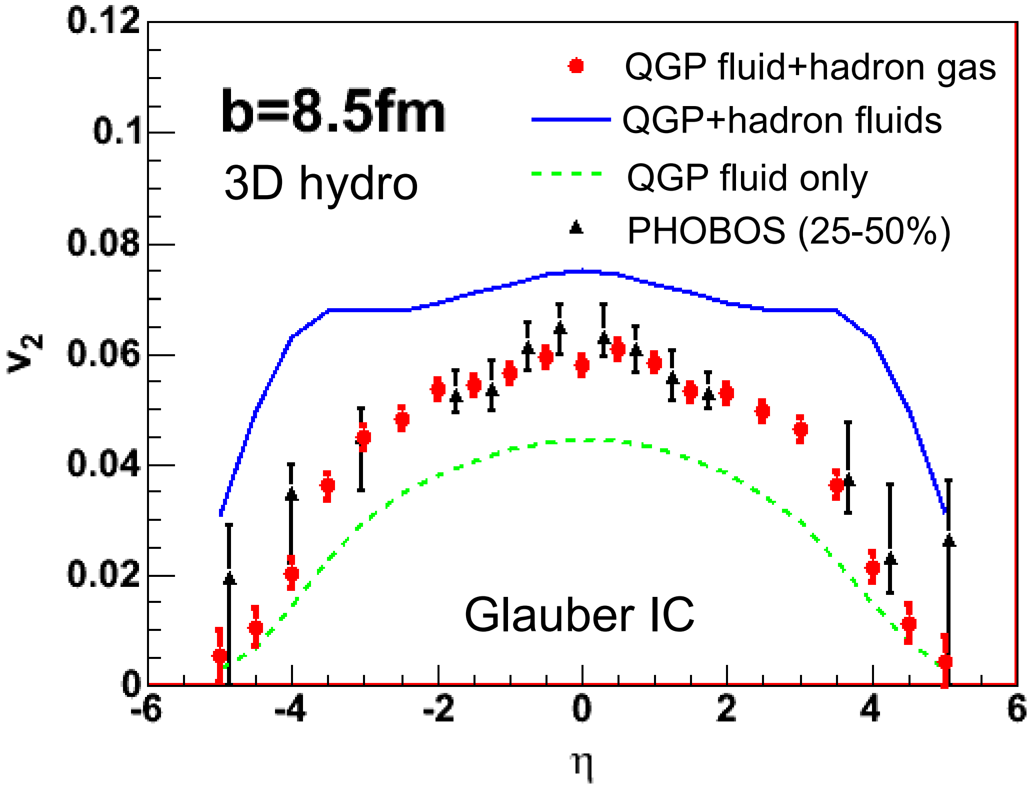
# Equilibrium only at mid-rapidity?



Hydro-limit reached at mid-rapidity for highest energies?

Au+Au: PRL 94 122303 (2005)
Cu+Cu: PRL 98 242302 (2007)

# Hydrodynamic model



Hadronic corona is important

Remark: Hydrodynamic model ≠ ideal hydrodynamics (Boltzmann transport for hadrons includes effective viscosity through finite mean free path)

Hirano et al., PLB 636 299 (2006)

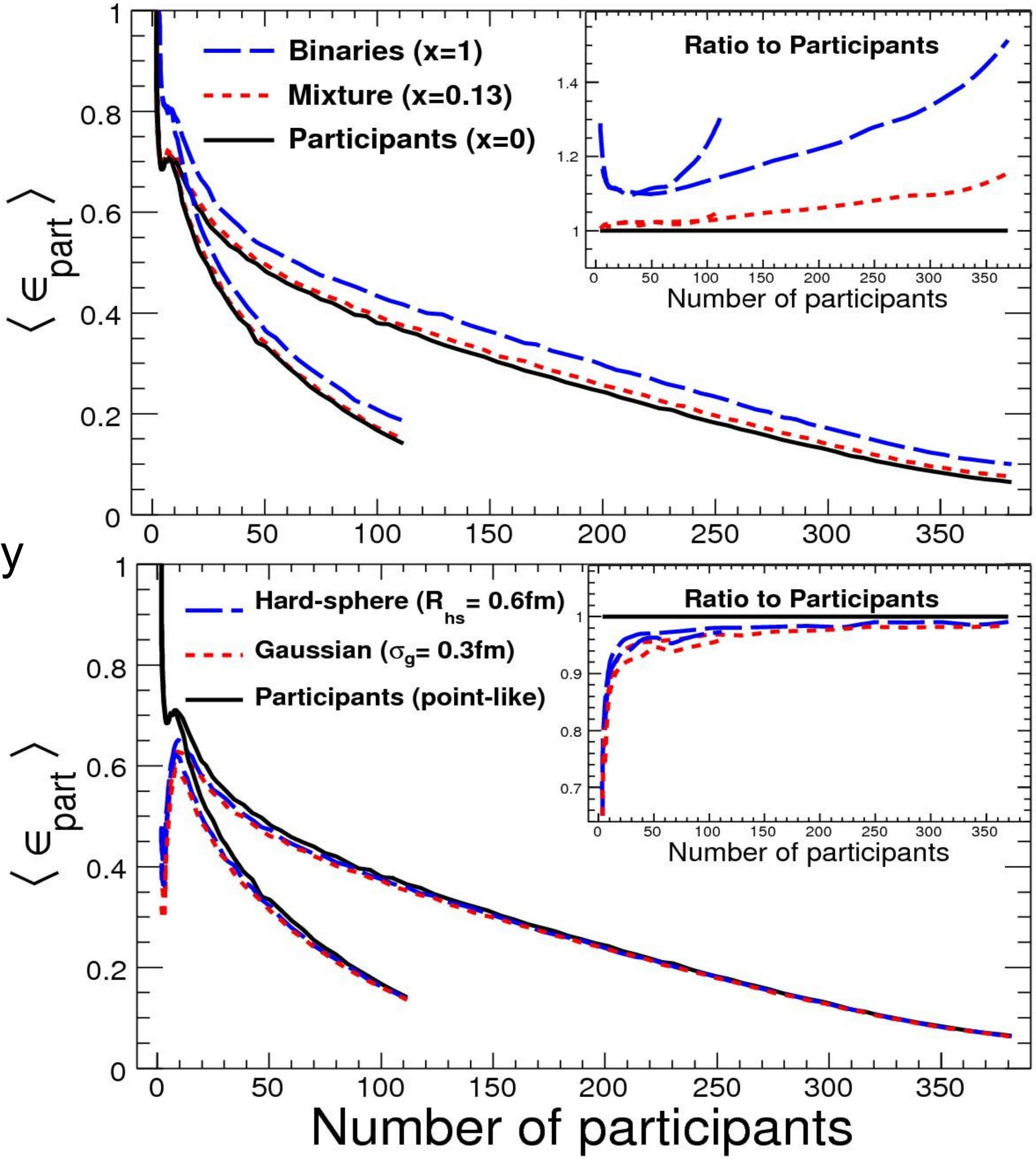
# Assumptions of particle production

- Model two component scenario
  - Matter production via participants and binary collisions

$$\frac{dN^{AA}}{d\eta}=\frac{dN^{pp}}{d\eta}\left(\frac{1-x}{2}N_{part}+x\,N_{coll}\right)$$

  - Mixture with x=0.13 describes mid-rapidity dN/dη quite well
    - 10% increase in eccentricity for central Au+Au
- Include thermalization time by smearing the matter around the original production point
  - Hard-sphere and Gaussian
    - For chosen set of parameters only a very small effect

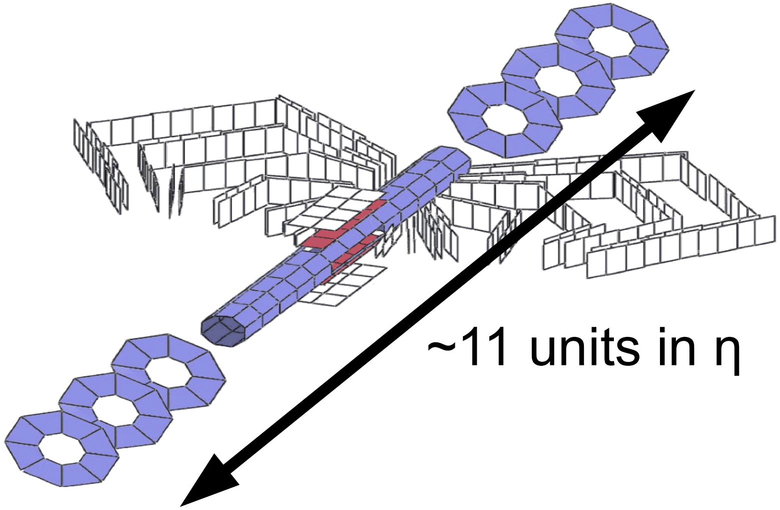
NB: More generalized studies also done, see Broniowski et al., PRC 76 (2007) 054905

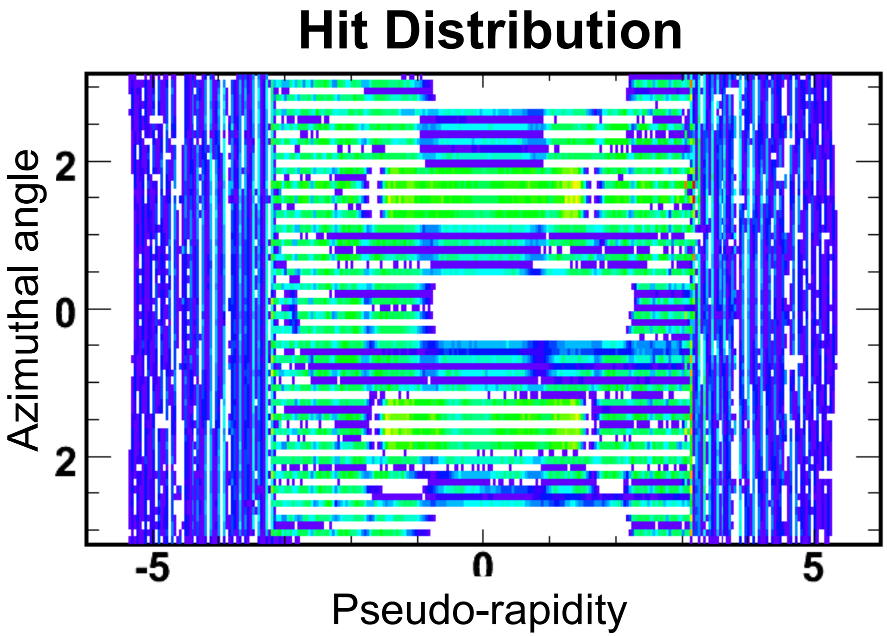


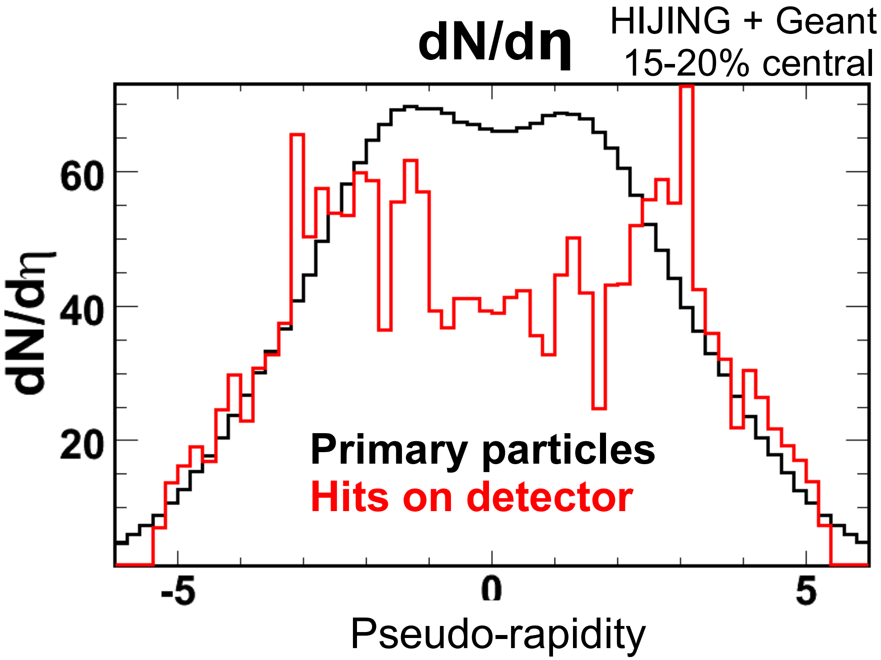
Alver et al., PRC 77 (2008) 014906

# Challenges of event-by-event $v_2^{obs}$

- PHOBOS Multiplicity Array
  - $-5.4<\eta<5.4$ coverage
  - Holes and granularity differences
- Usage of all available information in event to determine event-by-event a single value for $v_2^{obs}$







# Event-by-event measurement of $v_2^{obs}$

- Event-by-event measurement of $v_2^{obs}$
  - Deal with acceptance effects
  - Use all available hit information

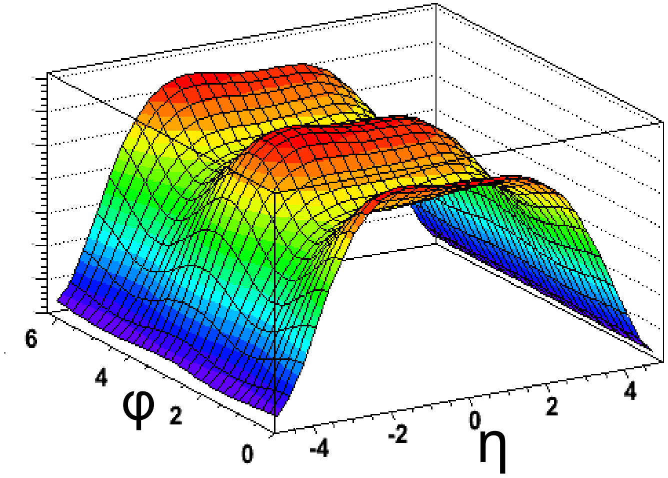
- Probability distribution function for hit positions:

$$P(\eta,\phi;\ v_2^{obs},\phi_0)=\underbrace{p(\eta)}_{\text{Normalization incl. acceptance}}\underbrace{[1+2v_2(\eta)\cos(2\phi-2\phi_0)]}_{\textbf{Probability of hit in }(\varphi,\eta)}$$

- Maximize the likelihood function to obtain $v_2^{obs}$ and $\varphi^0$ (event plane angle)

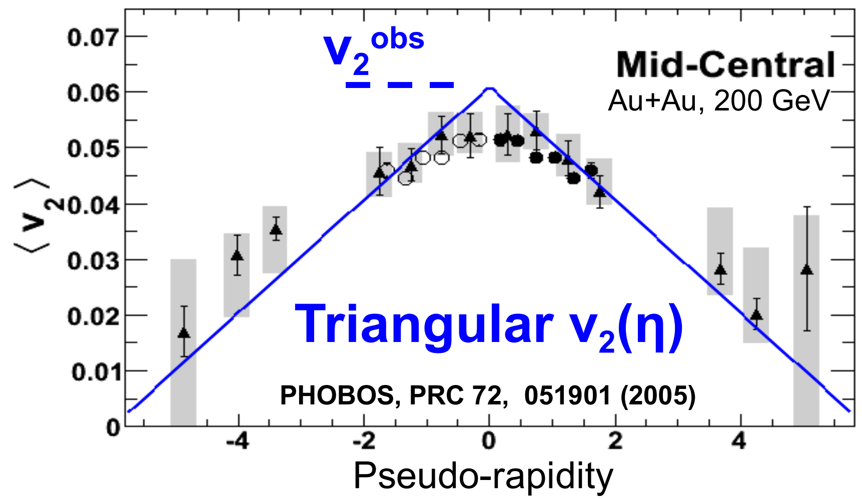
$$L(v_2^{obs},\ \phi_0)=\prod_{i=1}^{n}P(\eta_i,\phi_i;\ v_2^{obs},\phi_0)$$

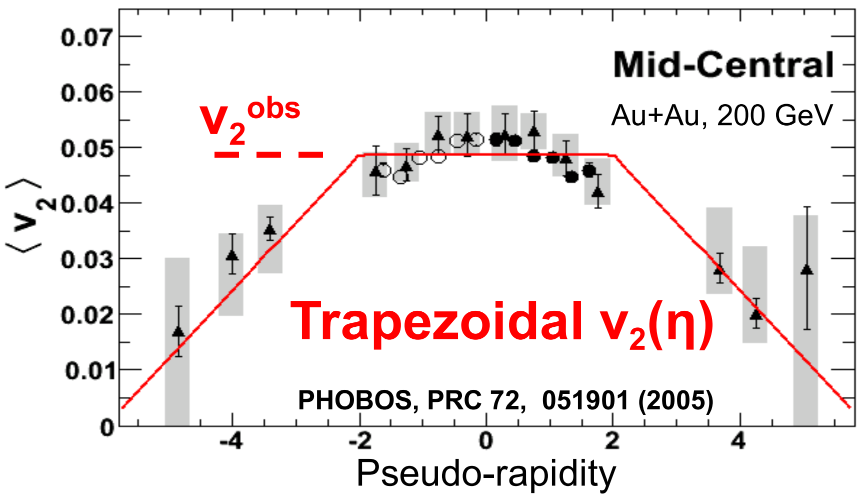
**Probability distribution function**



nucl.-ex/0702036 (sub.to PRL),
nucl-ex/0608025 (Proceedings of Science)

# Event-by-event measurement of $v_2^{obs}$





$$P(\eta,\phi;\ v_2^{obs},\phi_0)=p(\eta)[1+2\underline{v_2(\eta)}\cos(2\phi-2\phi_0)]$$

↑

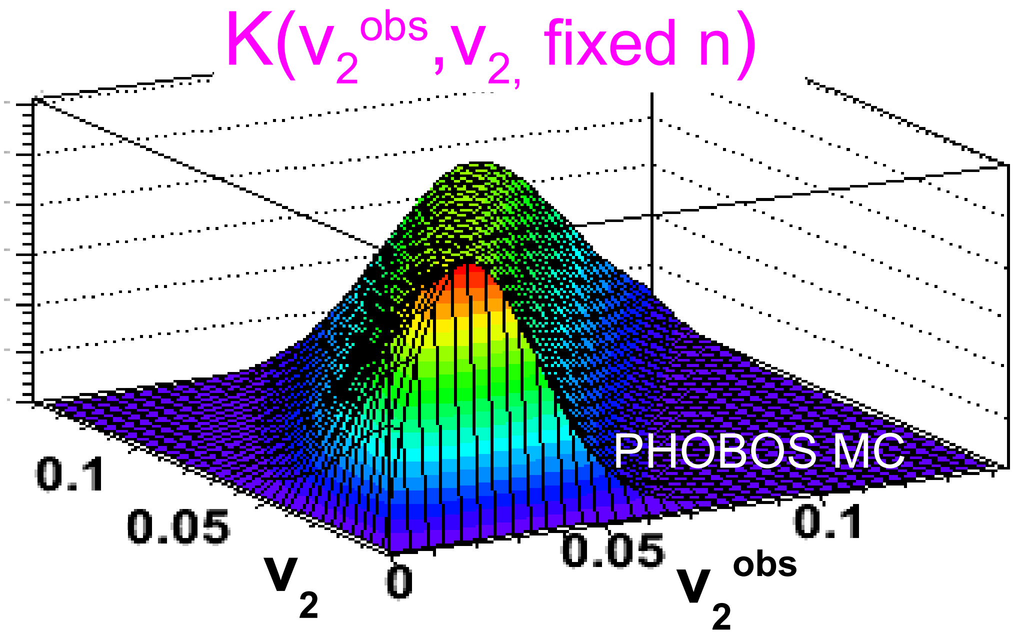
Use known, measured shape

Analysis is run on triangular and trapezoidal shape.
Results are averaged at the end.

nucl.-ex/0702036 (sub.to PRL),
nucl-ex/0608025 (Proceedings of Science)

# Determining the kernel

- "Measure" and record the $v_2^{obs}$ distribution in bins of $v_2$ and multiplicity (n) from large MC samples
    - $1.5 \cdot 10^6$ HIJING events
    - Modified φ to include triangular or trapezoidal flow



- Fit response function (ideal case)

$$K(v_2^{obs}, v_2, n) = \frac{v_2^{obs}}{\sigma^2} e^{-\left(\frac{v_2^{obs}+v_2^2}{2\sigma^2}\right)} I_0\left(\frac{v_2^{obs} v_2}{\sigma^2}\right)$$

(J.-Y.Ollitrault, PRD (1992) 46, 226)

- Changed to account for detector effects

$$v_2 \rightarrow (An+B)\, v_2$$

(suppression)

$$\sigma = \frac{C}{\sqrt{n}} + D$$

(finite resolution)

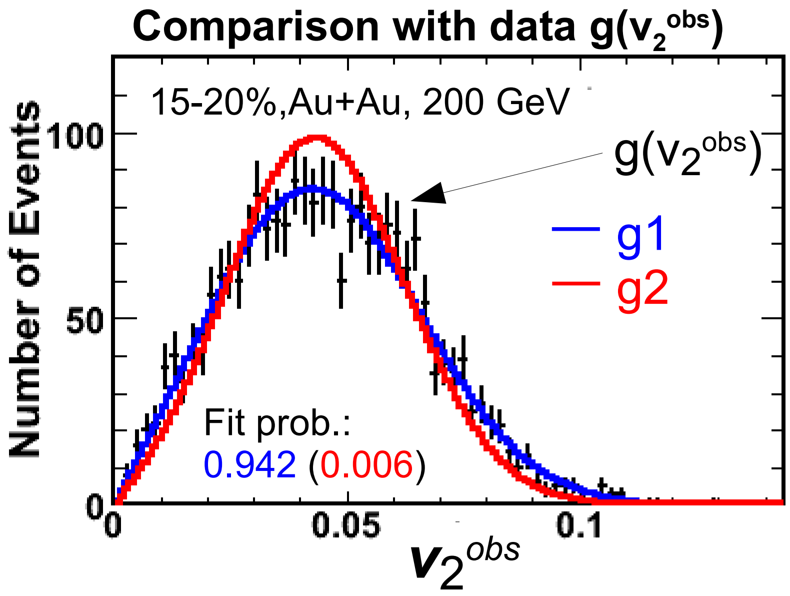
nucl.-ex/0702036 (sub.to PRL),
nucl-ex/0608025 (Proceedings of Science)

# Extracting dynamical fluctuations

$$\underbrace{g(v_2^{obs})}_{\text{Measured}} = \int_0^1 \underbrace{K(v_2^{obs}, v_2)}_{\text{Constructed from MC}} \underline{f(v_2)}\, dv_2$$

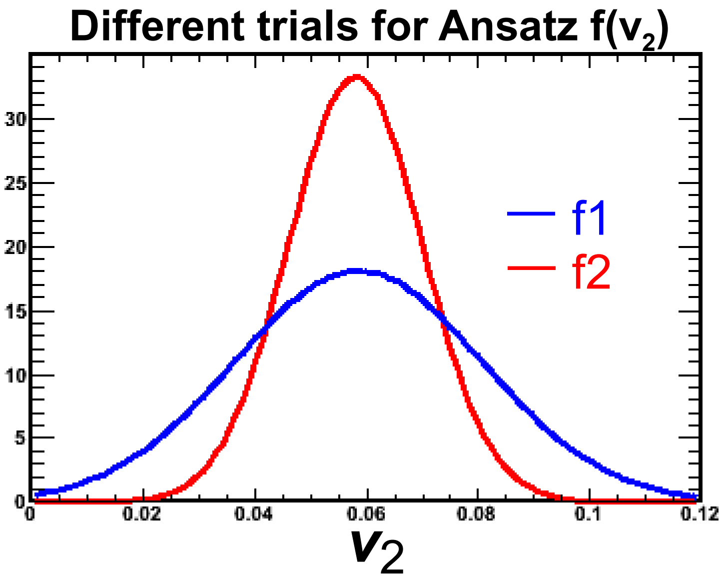
Gaussian Ansatz:

$$f(v_2) = \exp\left[\frac{-(v_2 - \langle v_2 \rangle)^2}{2\sigma_{v_2}^2}\right]$$



**Comparison with data $g(v_2^{obs})$**

Use kernel + integrate



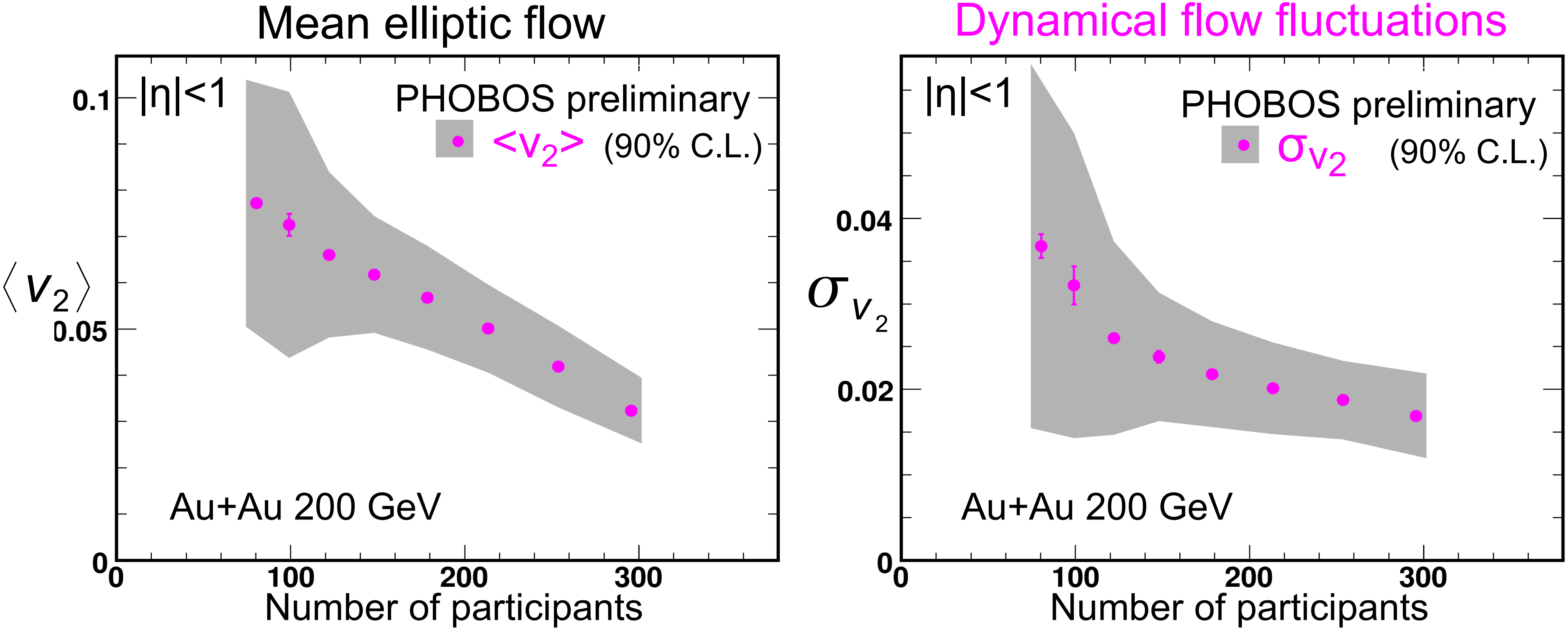
**Different trials for Ansatz $f(v_2)$**

Compare expected $g(v_2^{obs})$ for trials with data:

Maximum-Likelihood fit → $\langle v_2 \rangle$ and $\sigma_{v_2}$

nucl.-ex/0702036 (sub.to PRL),
nucl-ex/0608025 (Proceedings of Science)

# Elliptic flow fluctuations: $\langle v_2 \rangle$ and $\sigma_{v_2}$



Systematic errors:

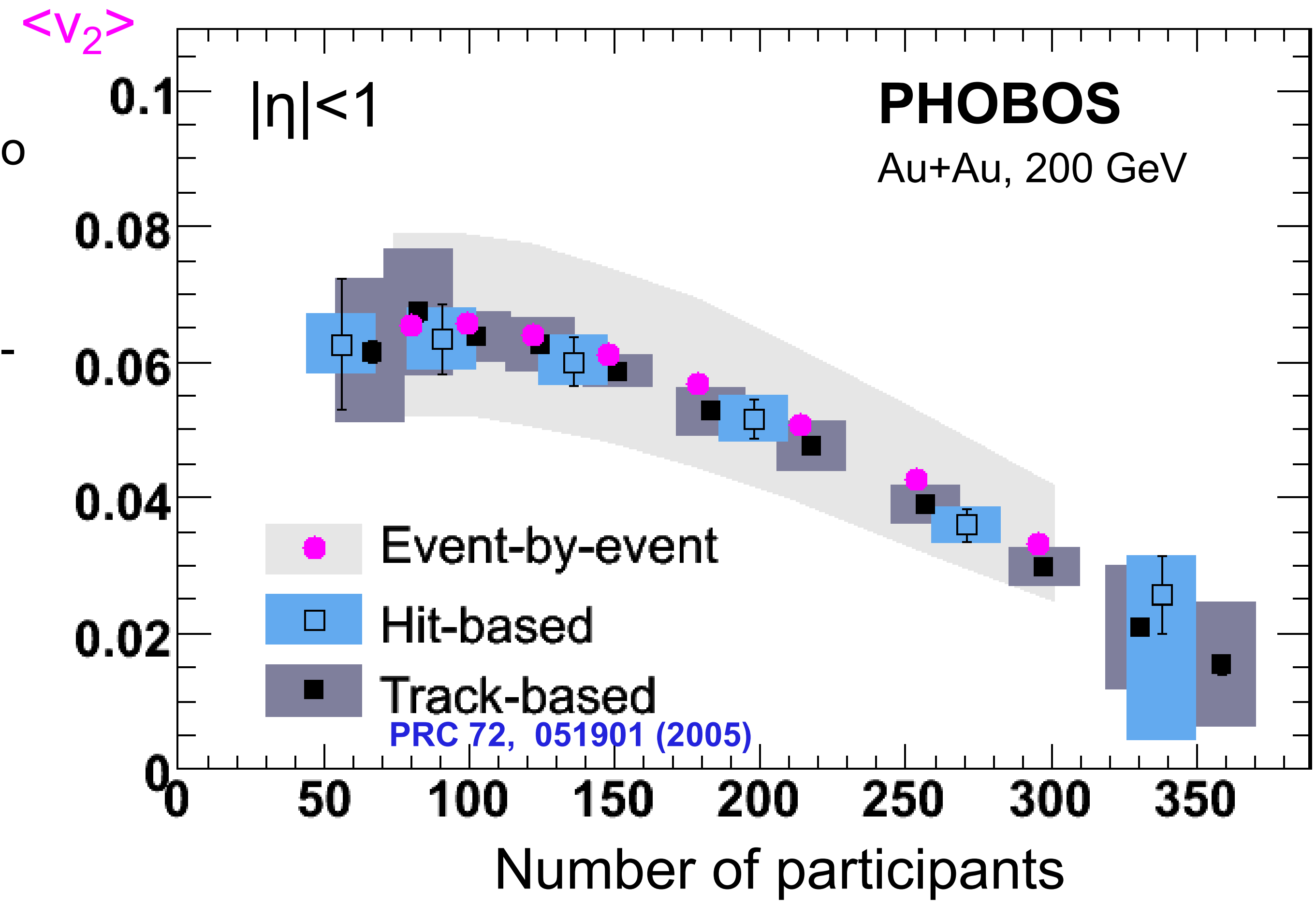
- Variation in η-shape
- Variation of f($v_2$)
- MC response
- Vertex binning
- $\Phi_0$ binning

"Scaling" errors cancel in the ratio: relative fluctuations, $\sigma_{v_2}/\langle v_2 \rangle$

nucl.-ex/0702036 (sub.to PRL)

# Event-by-event $v_2$ vs published results

- Standard methods
    - Averaged over events to measure the mean
    - Hit- and track-based
    - Use reaction plane sub-event technique



Very good agreement of the event-by-event measured mean $v_2$ with the hit- and tracked-based, event averaged, published results

nucl.-ex/0702036 (sub.to PRL)

# Numerical subtraction

QM08, WORK IN PROGRESS

$$K(v_2^{obs}, v_2, n) = BG(v_2^{obs}, v_2, \sigma_n), \quad \sigma_n = 1/\sqrt{2n}$$

$$K_\delta(v_2^{obs}, v_2, n) = BG(v_2^{obs}, v_2, \sqrt{\sigma_n^2 + \sigma_\delta^2}), \quad \sigma_n = 1/\sqrt{2n}, \sigma_\delta = \sqrt{\delta/2}$$

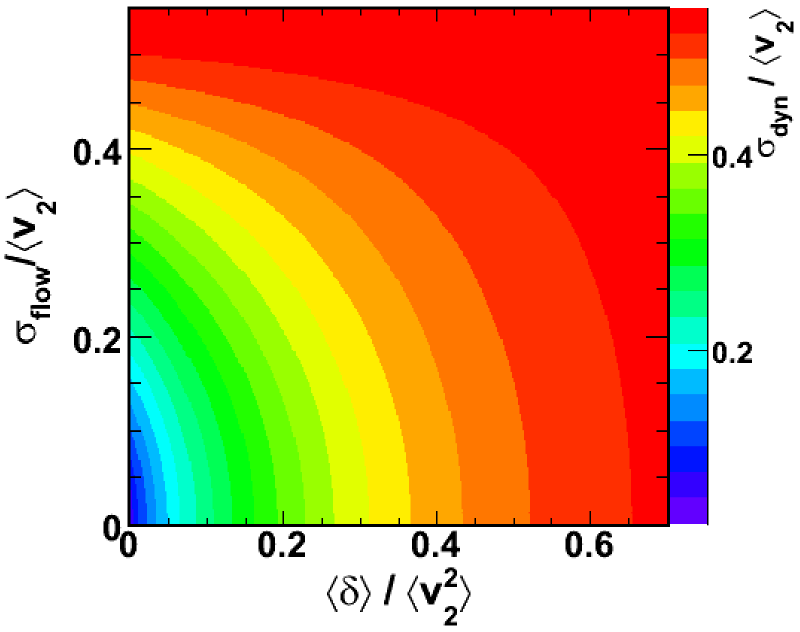
$$g(v_2^{obs}) = \int K_\delta(v_2^{obs}, v_2, n)\, f_{flow}(v_2)\, \mathrm{d}v_2$$

$$g(v_2^{obs}) = \int K(v_2^{obs}, v_2, n)\, f(v_2)\, \mathrm{d}v_2$$

Generate g($v_2^{obs}$) using this

Do a fit using this

Lookup table



- Keep results as lookup table
- Results slightly depend on $\sigma_n$
  - Use $\sigma_n$ = 0.4, 0.6 and 0.8