

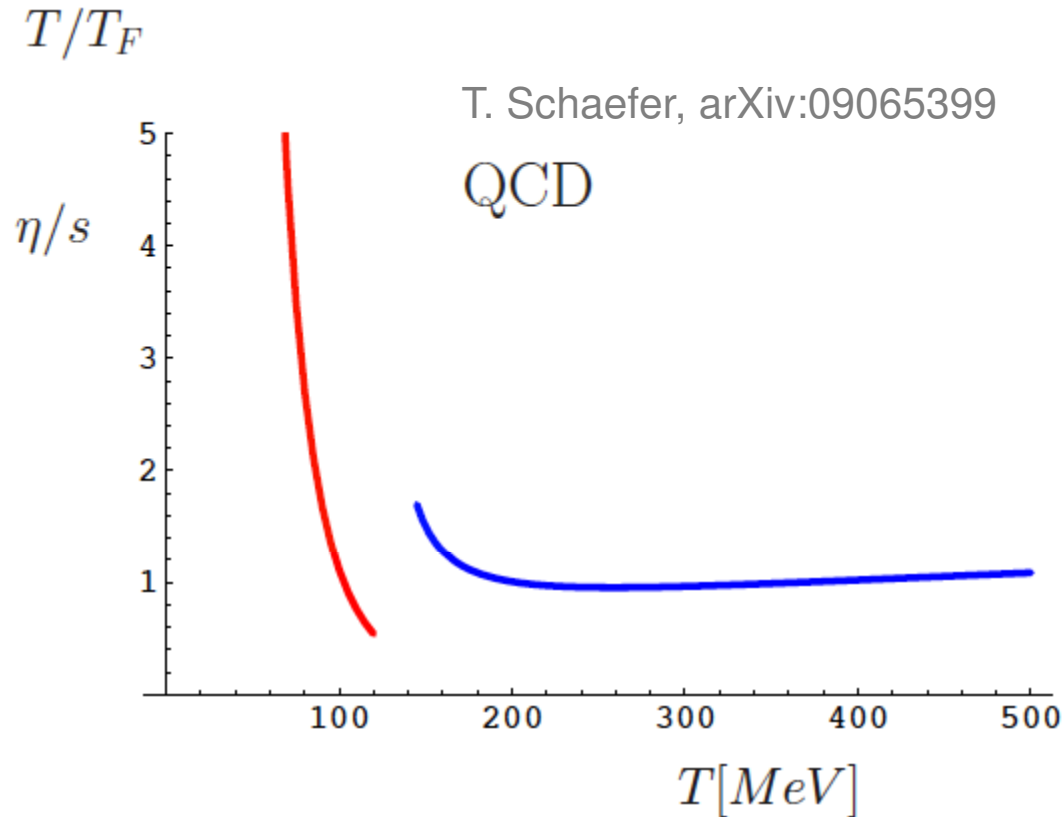
# Viscosity and HBT

*Dariusz Miśkowiec, GSI Darmstadt  
WPCF2009, CERN, 15-Oct-2009*

- ⊗ attempt of CERES to extract  $\eta/s$  from  $R_{\text{long}}(\mathbf{k}_t)$
- ⊗ problems with this approach
- ⊗ other approaches?

**prerequisites:  $R_{\text{out}}$ ,  $R_{\text{side}}$ ,  $R_{\text{long}}$ ,  $\eta/s$ ,  $v_2$ ,  $m_t$**

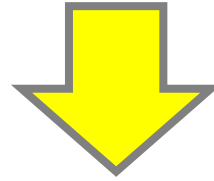
# Viscosity in nuclear collisions – what to expect



- 🌐  $\eta/s$  reaches minimum near the critical point
- 🌐 at the critical point it diverges
- 🌐 high viscosity at low energies

# Viscosity in nuclear collisions - observables

**finite viscosity reduces velocity gradients**



**less in-plane, more out-of-plane expansion**

⊗ **reduced  $v_2$**

**less longitudinal, more transverse expansion**

⊗ **narrower  $dN/dy$  distributions**

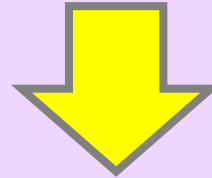
⊗ **harder  $p_t$  spectra**

⊗ **reduced  $R_{\text{long}}$**

⊗ **reduced  $R_{\text{out}}$**

# Viscosity in nuclear collisions - observables

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 reduced  $R_{\text{long}}$

 reduced  $R_{\text{out}}$

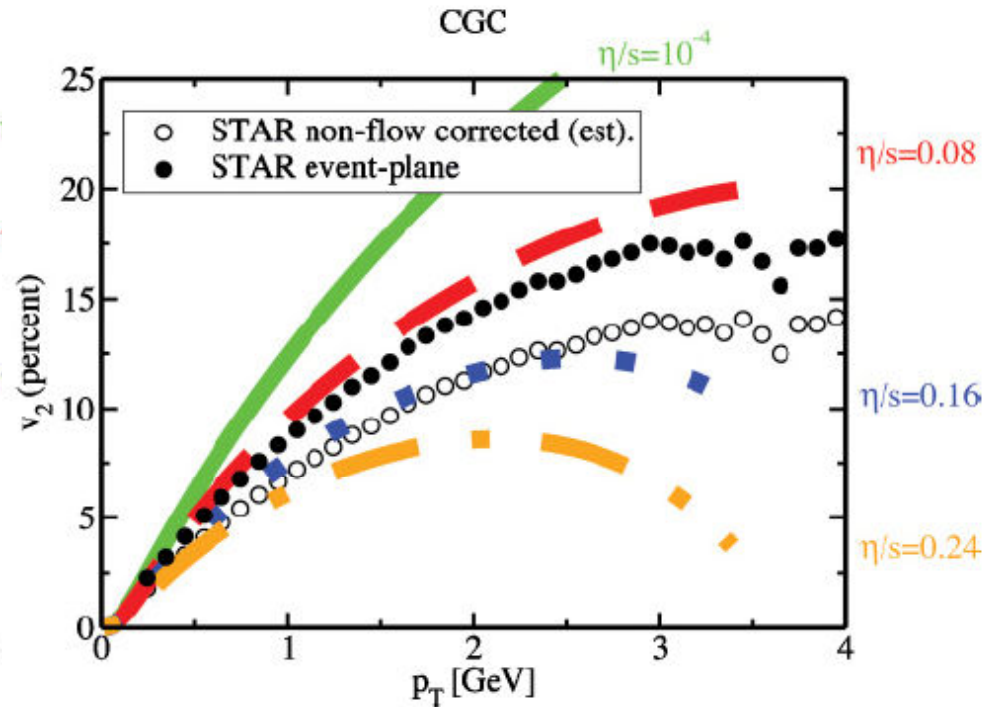
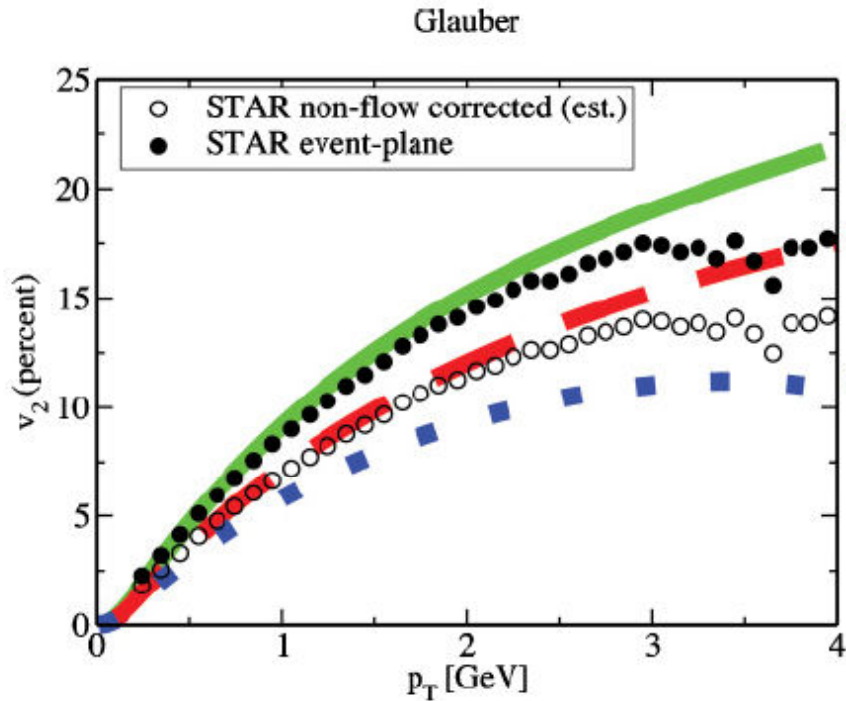
# Viscosity via $v_2$

	$\eta/s$
🌐 PRL 99 (2007) 172301	0.03, 0.08
🌐 PRC 78 (2008) 034915	$0.10 \pm 0.13$
🌐 PRL 98 (2007) 092301	0.1
🌐 PRC 76 (2007) 024905	0.19, 0.11
🌐 arXiv:0901.0460	$0.15 \pm 0.6$

**result close to the lower limit of  $\eta/s = 0.08$**

# Viscosity via $v_2$

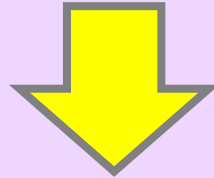
Luzum and Romatschke, PRC 79, 039903 (2009)



**result depends on the assumed initial conditions**

# Viscosity in nuclear collisions - observables

finite viscosity reduces velocity gradients



less in-plane, more out-of-plane expansion

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less longitudinal, more transverse expansion

🌐 narrower  $dN/dy$  distributions

🌐 harder  $p_t$  spectra

🌐 reduced  $R_{long}$

🌐 reduced  $R_{out}$

# $R_{long}$ basics

## Makhlin-Sinyukov

$$R_{long} = \tau_f \sqrt{\frac{T}{m_t}}$$

## Herrmann-Bertsch

$$R_{long} = \tau_f \sqrt{\frac{T}{m_t} \frac{K_2(m_t/T)}{K_1(m_t/T)}}$$

$\tau_f \sim$  inverse of the longitudinal Hubble constant

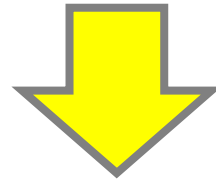


# How finite viscosity affects $R_{\text{long}}$

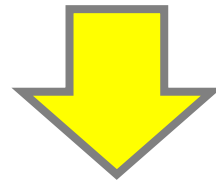
**enhanced transverse expansion**



**reduced lifetime**



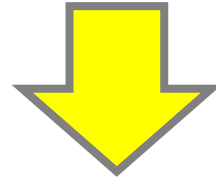
**reduced longitudinal length at freeze-out**



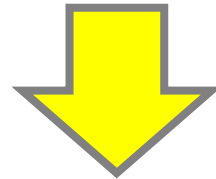
**smaller  $R_{\text{long}}$**

# How finite viscosity affects $R_{\text{long}}$

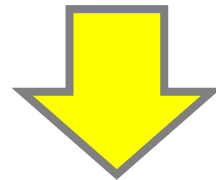
enhanced transverse expansion



reduced lifetime



~~reduced longitudinal length at freeze-out~~  
larger Hubble constant



smaller  $R_{\text{long}}$

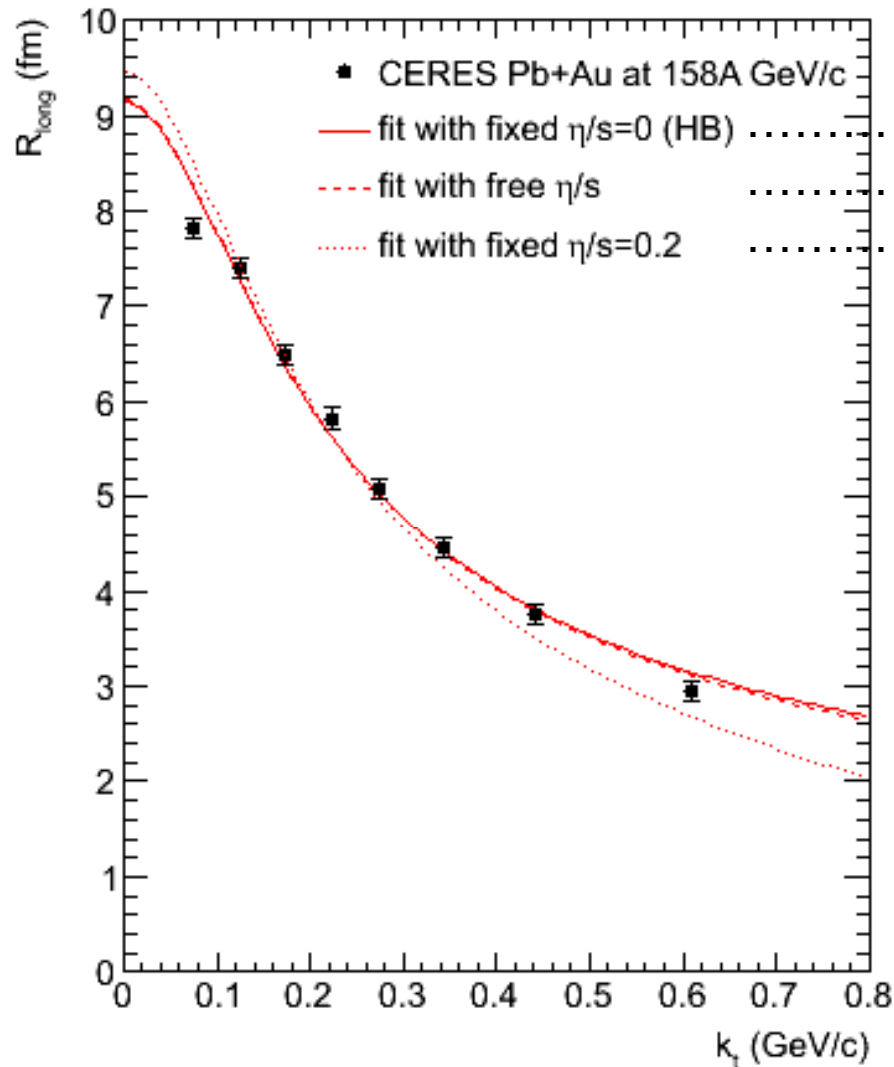
# Effect of viscosity on $R_{\text{long}}$ quantitatively

viscous correction, D. Teaney, PRC 68, 034913 (2003)

$$\frac{\delta R_L^2}{(R_L^2)^{(0)}} = -\frac{\Gamma_s}{\tau} \left[ \frac{6}{4} \frac{x K_3(x)}{K_2(x)} - x^2 \frac{1}{8} \left( \frac{K_3(x)}{K_2(x)} - 1 \right) \right], \quad \Gamma_s \equiv \frac{4}{3} \frac{\eta}{sT}$$

$$x \equiv \sqrt{m^2 + K_T^2} / T$$

# Viscosity via $R_{\text{long}}$ – fit to CERES data



$T = 120$  MeV (fixed)

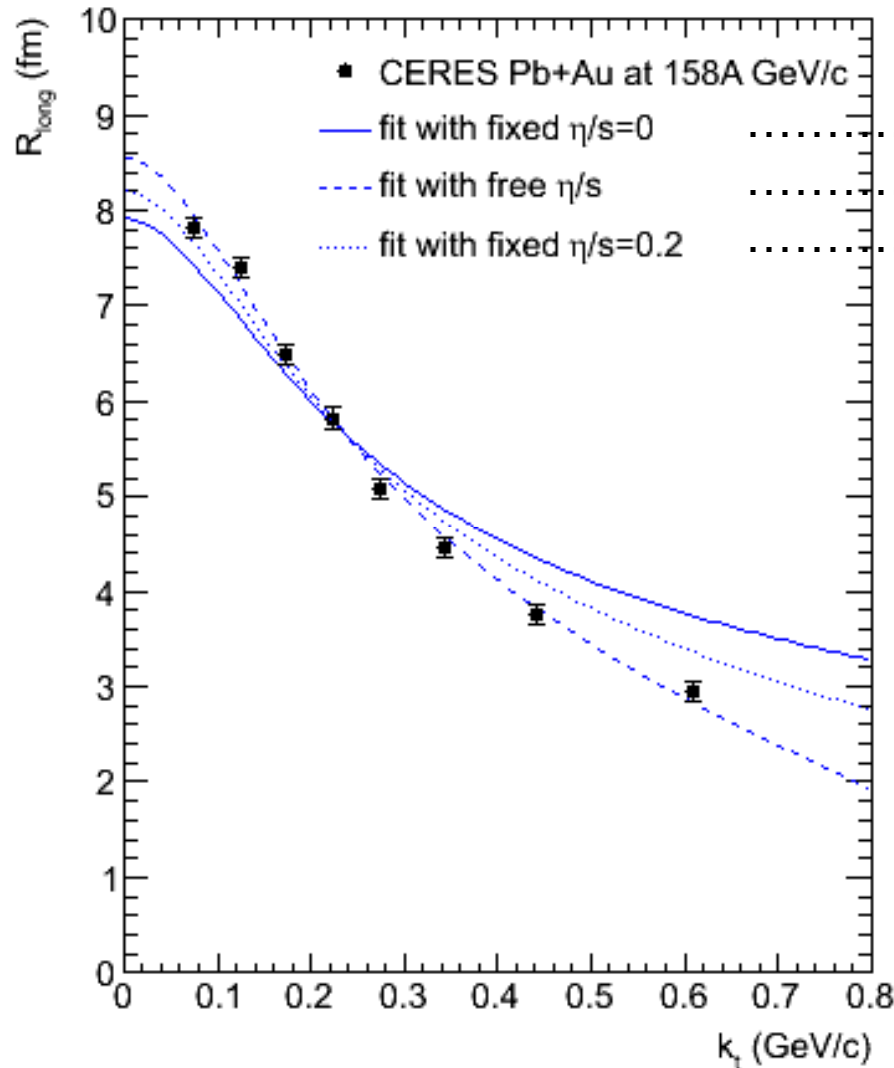
fit function:

Herrmann-Bertsch with viscous correction

$$\frac{\delta R_L^2}{(R_L^2)^{(0)}} = -\frac{\Gamma_s}{\tau} \left[ \frac{6}{4} \frac{x K_3(x)}{K_2(x)} - x^2 \frac{1}{8} \left( \frac{K_3(x)}{K_2(x)} - 1 \right) \right]$$

**result:  $\eta/s$  below 0.1  
(recall lower bound 0.08)**

# Viscosity via $R_{\text{long}}$ – fit to CERES data



.....  $\tau_f = 8.5$  fm  
 .....  $\tau_f = 9.9$  fm     $\eta/s = 0.42$   
 .....  $\tau_f = 9.2$  fm

$T = 120$  MeV (fixed)

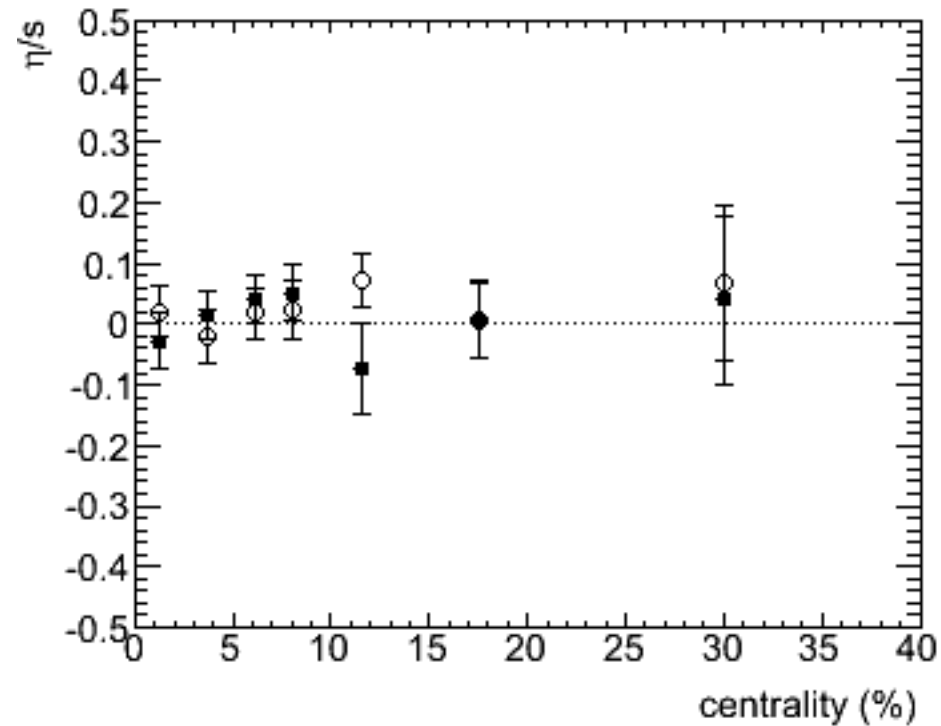
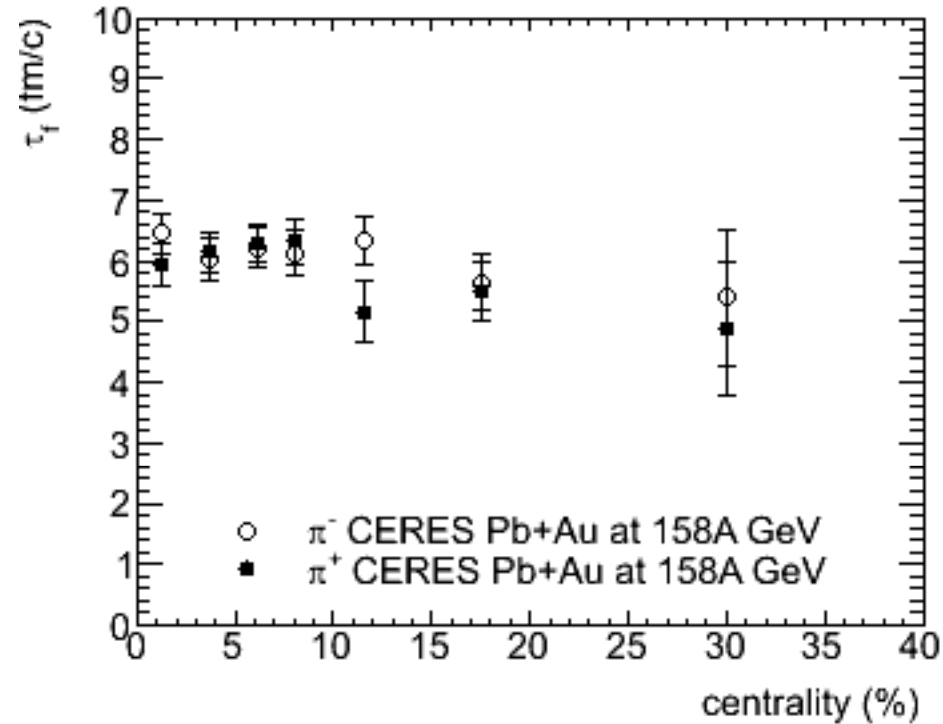
fit function:

Makhlin-Sinyukov with appr. visc. cor.

$$(R_L^2)^{(0)} + \delta R_L^2 = \tau_o^2 \left( \frac{T}{m_T} - \frac{19}{16} \frac{\Gamma_s}{\tau_o} \right)$$

**very different result,  
beware!**

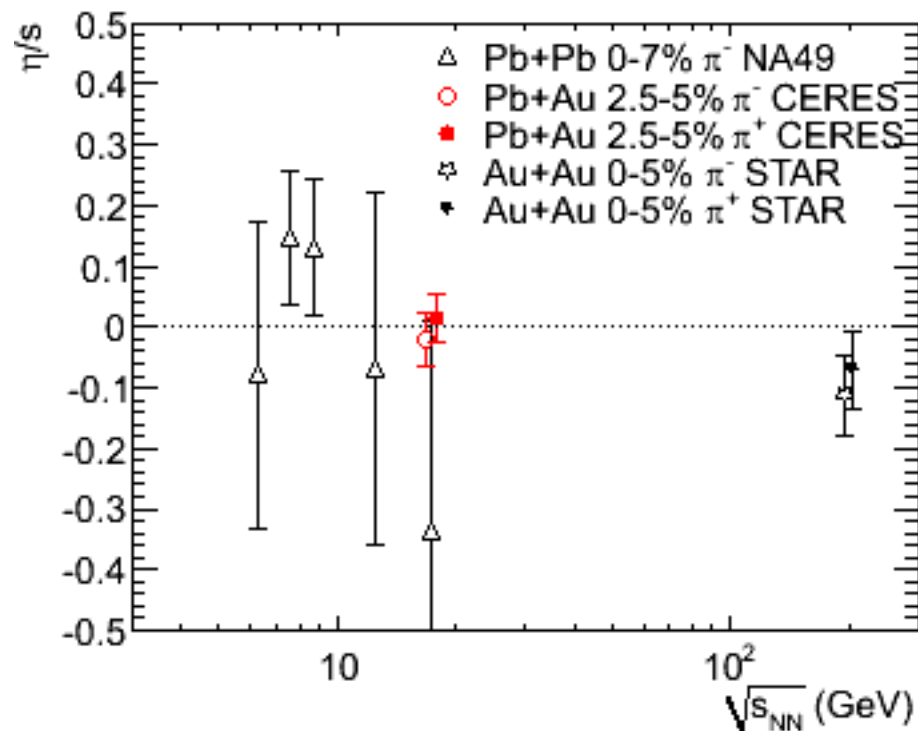
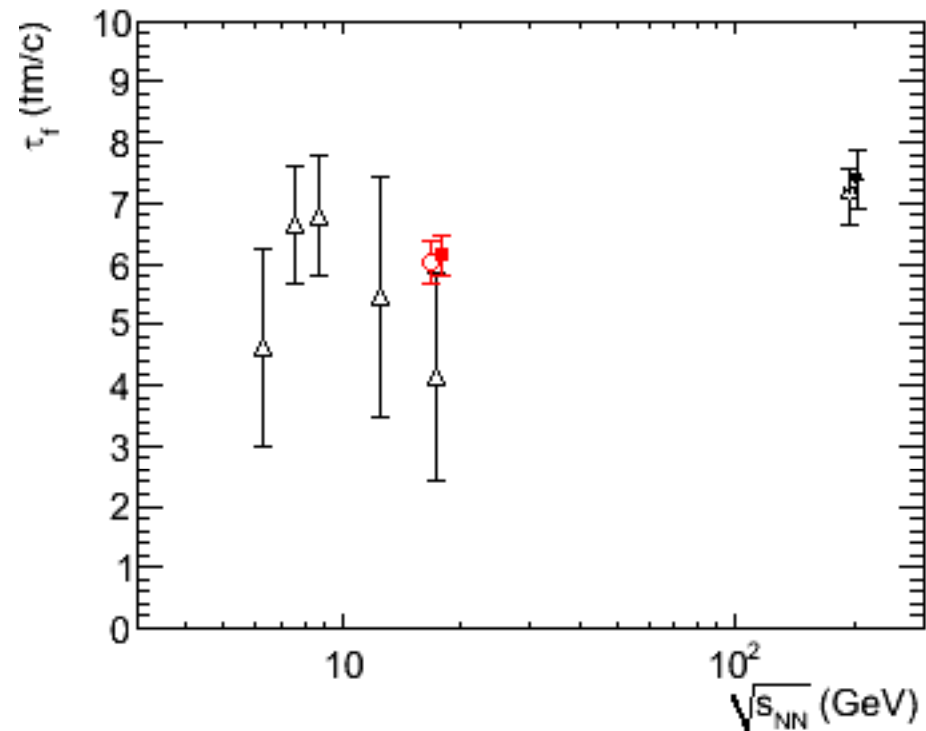
# Viscosity via $R_{long}$ – fit to CERES data



**$\eta/s$  is small for both charges and all centralities**

# Viscosity via $R_{\text{long}}$ – fit to NA49, CERES, STAR data

CERES Collaboration, arXiv:0907.2799



**$\eta/s$  is small for all energies**

# Viscosity low at all energies? Why not!

**N. Auerbach and S. Shlomo**  
**“The  $\eta/s$  ratio in finite nuclei”**  
**arXiv:0908.4441v1 [nucl-th], 31-Aug-2009**

- 🌐 giant resonance width  $\rightarrow \eta \approx 0.5-2.5 \times 10^{-23} \text{ MeV fm}^{-3} \text{ s}$
- 🌐 fission  $\rightarrow \eta \approx 0.9-1.9 \times 10^{-23} \text{ MeV fm}^{-3} \text{ s}$
- 🌐 Fermi gas of nucleons in Woods-Saxon well  $\rightarrow s$

$\rightarrow \eta/s \sim 0.3-1.5$  for large nuclei  
 $\eta/s \sim 0.2-1.0$  for small nuclei

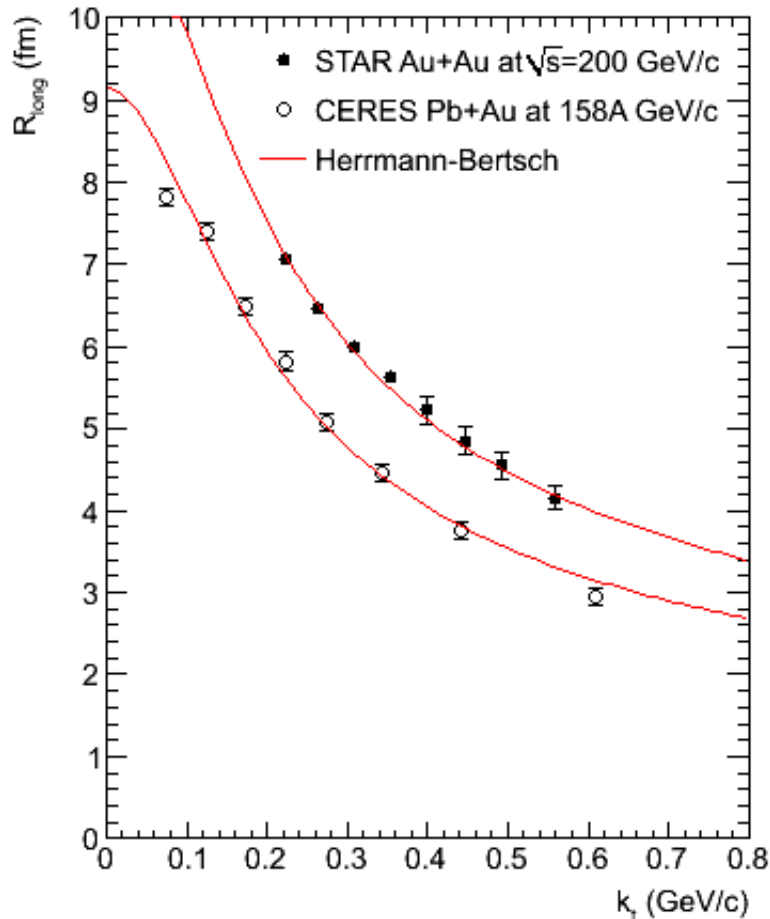


# However, serious problems in our analysis:

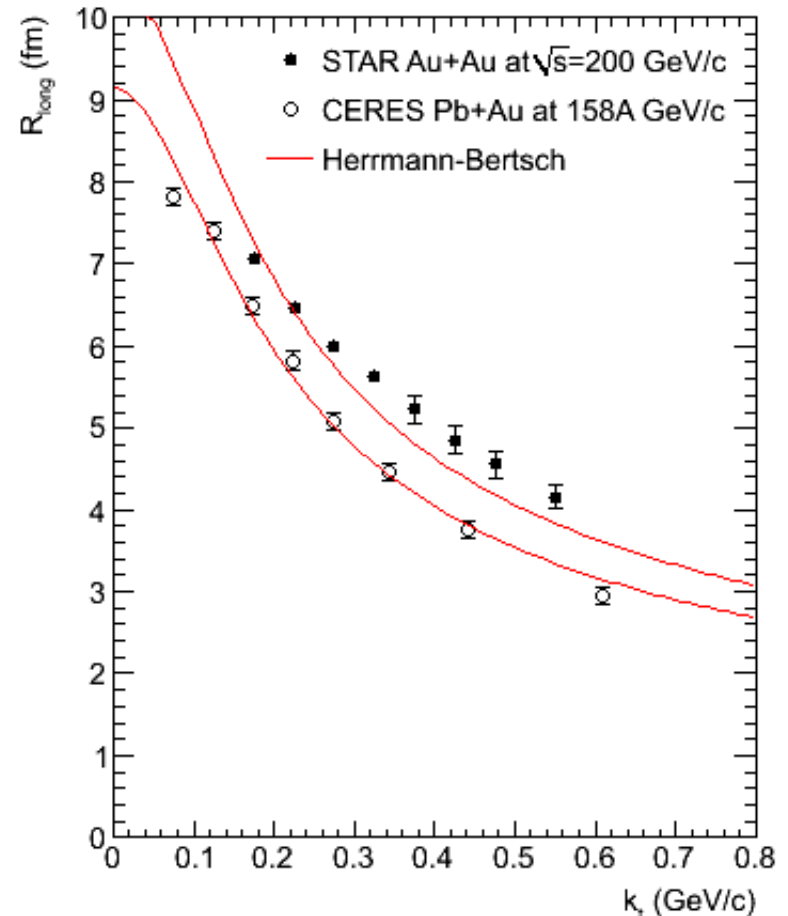
- ⊛ neglected transverse expansion
- ⊛ Teaney's formula accounts for the modified distribution at freeze-out but not for flow! (M. Lisa, U. Heinz)
- ⊛ even the freeze-out part is not clear:
  - Bożek/Wyskiel see no effect on HBT radii when using the same method with  $\eta/s=0.16$
  - Song/Heinz get opposite modification of  $p_t$  spectra
- ⊛ last but not least:
  - my mistake when interpreting STAR data ( $m_t$  vs  $k_t$ )**

# mistake when interpreting STAR data ( $m_t$ vs $k_t$ )

## STAR points misplaced



## STAR points placed correctly

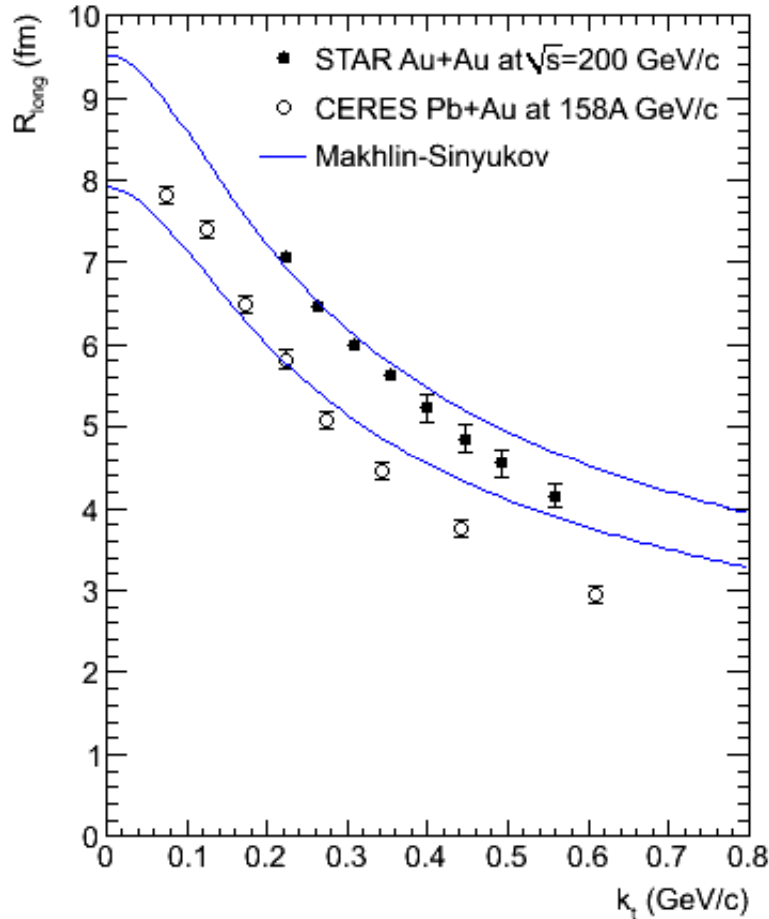


$$R_{long} = \tau_f \sqrt{\frac{T}{m_t} \frac{K_2(m_t/T)}{K_1(m_t/T)}}$$

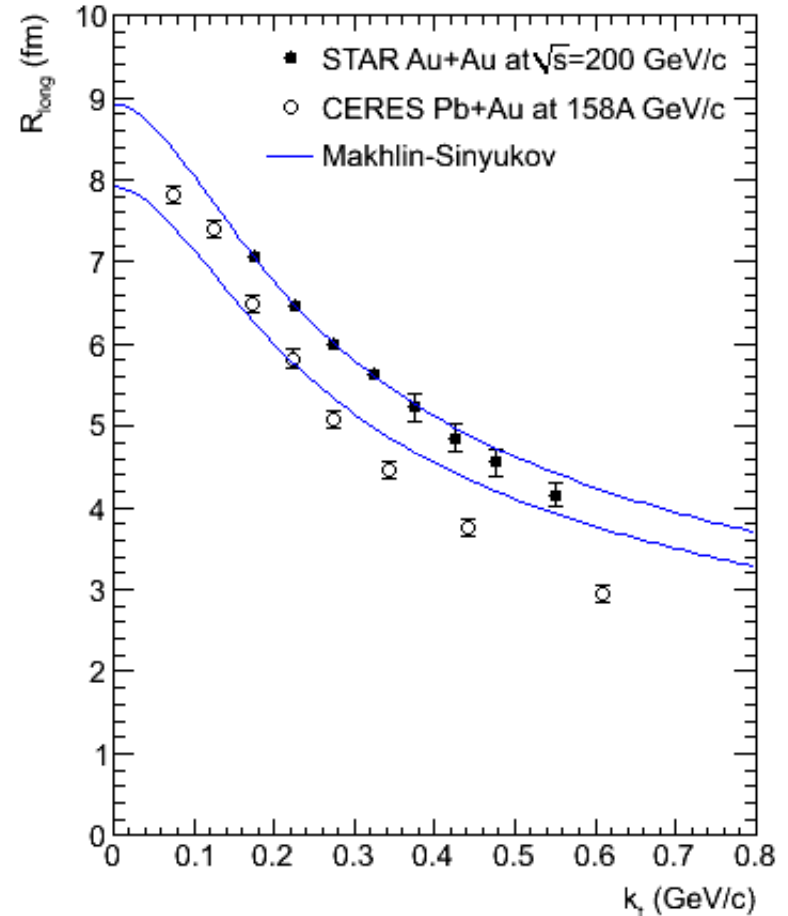
**no room for viscosity, even the pure Herrmann-Bertsch curve is steeper than the data**

# mistake when interpreting STAR data ( $m_t$ vs $k_t$ )

## STAR points misplaced



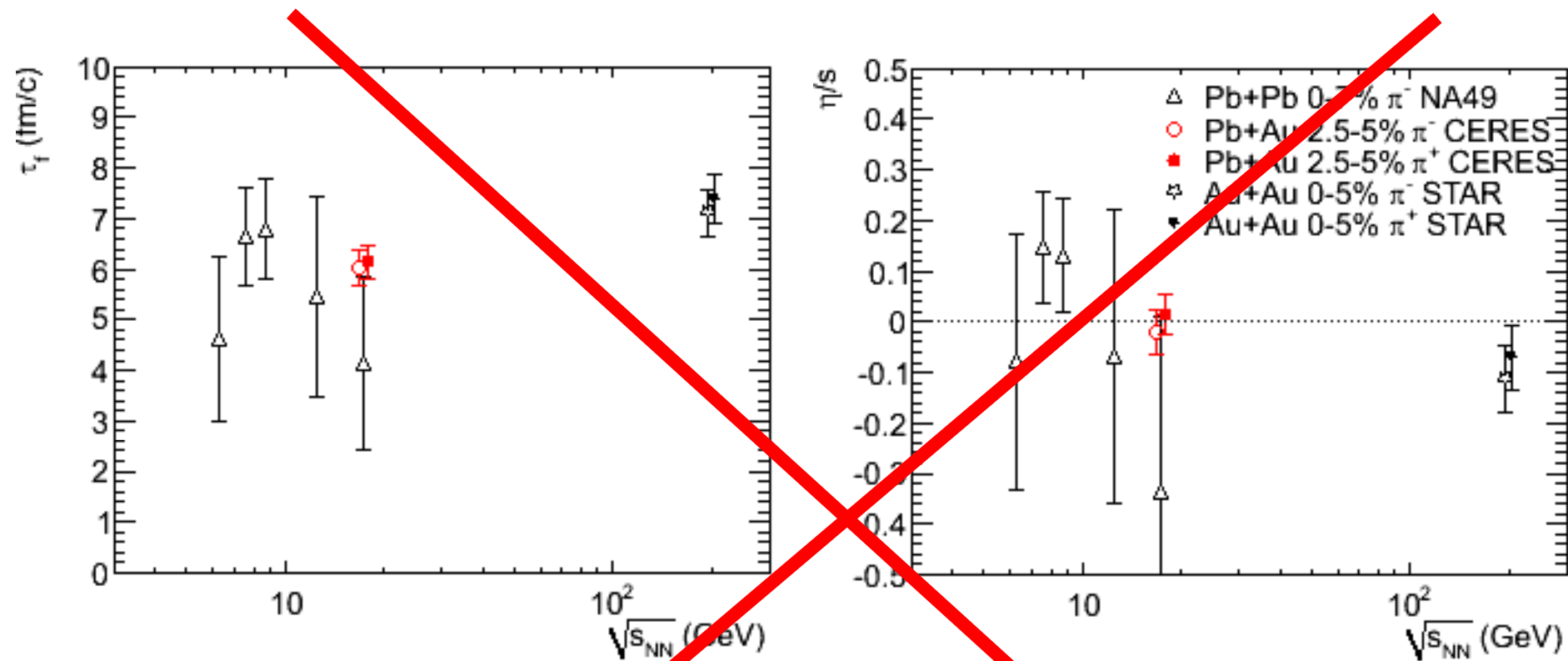
## STAR points placed correctly



$$R_{long} = \tau_f \sqrt{\frac{T}{m_t}}$$

**btw., Makhlin-Sinyukov  
fits much better...**

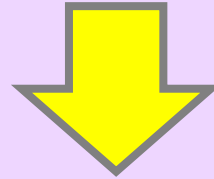
# Viscosity via $R_{\text{long}}$ – fit to NA49, CERES, STAR data



**$\eta/s$  is small for all energies**

# Viscosity in nuclear collisions - observables

finite viscosity reduces velocity gradients



less in-plane, more out-of-plane expansion

🌐 reduced  $v_2$

less longitudinal, more transverse expansion

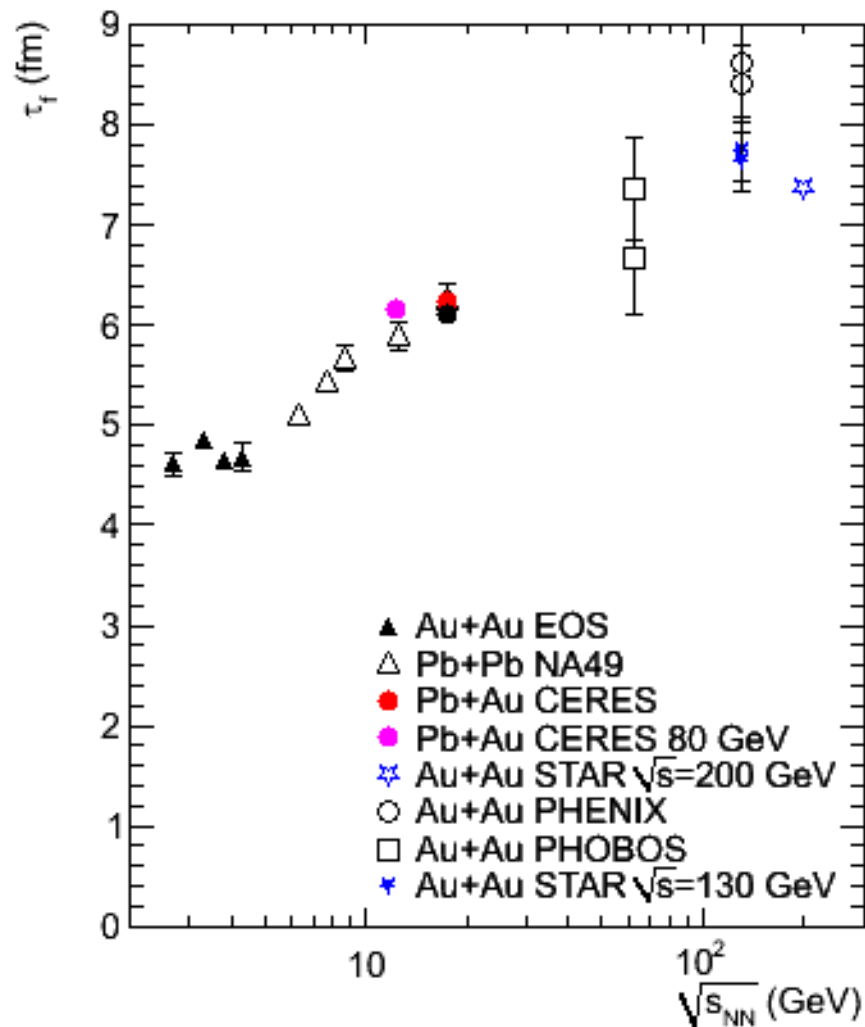
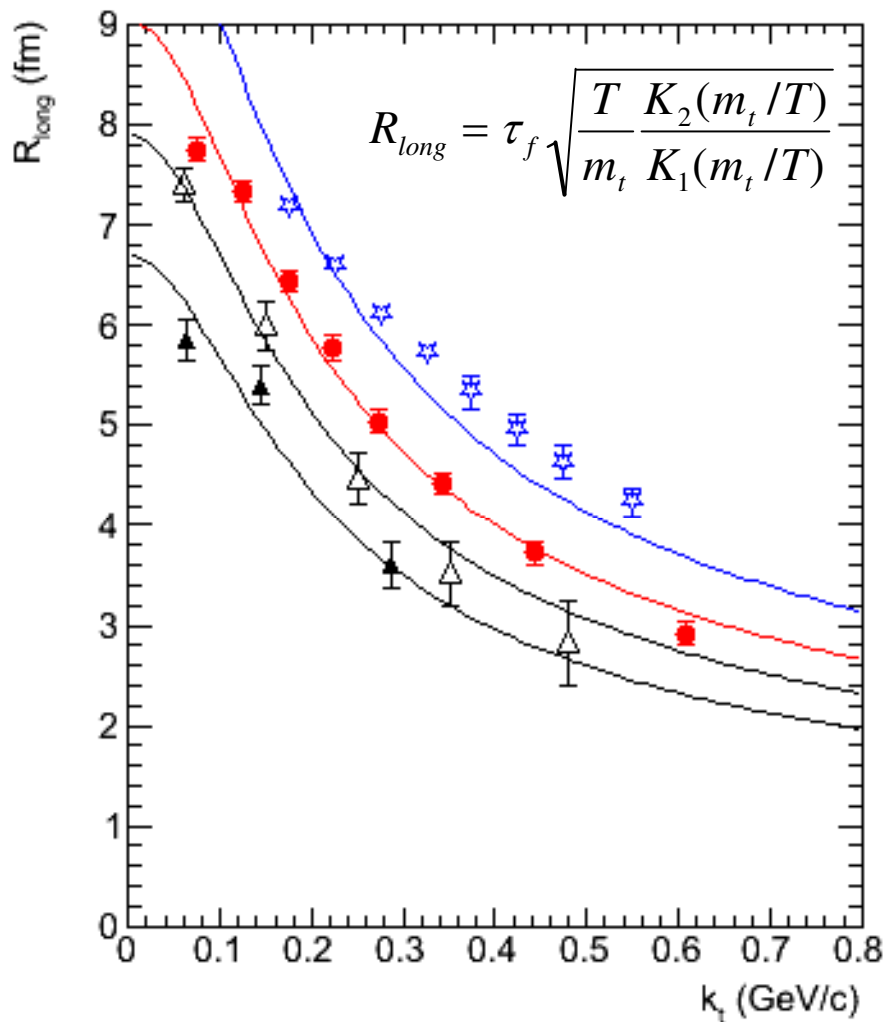
🌐 narrower  $dN/dy$  distributions

🌐 harder  $p_t$  spectra

🌐 reduced  $R_{\text{long}}$

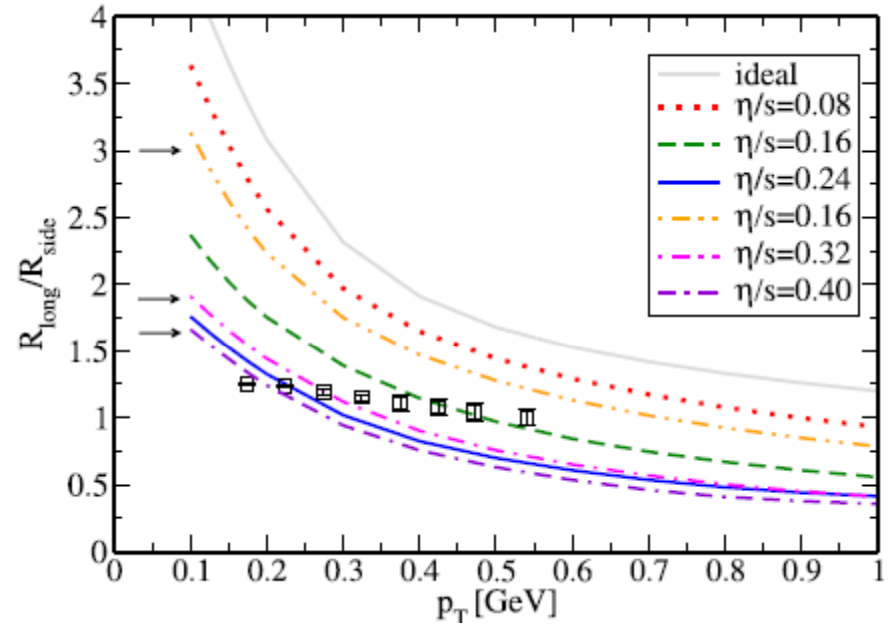
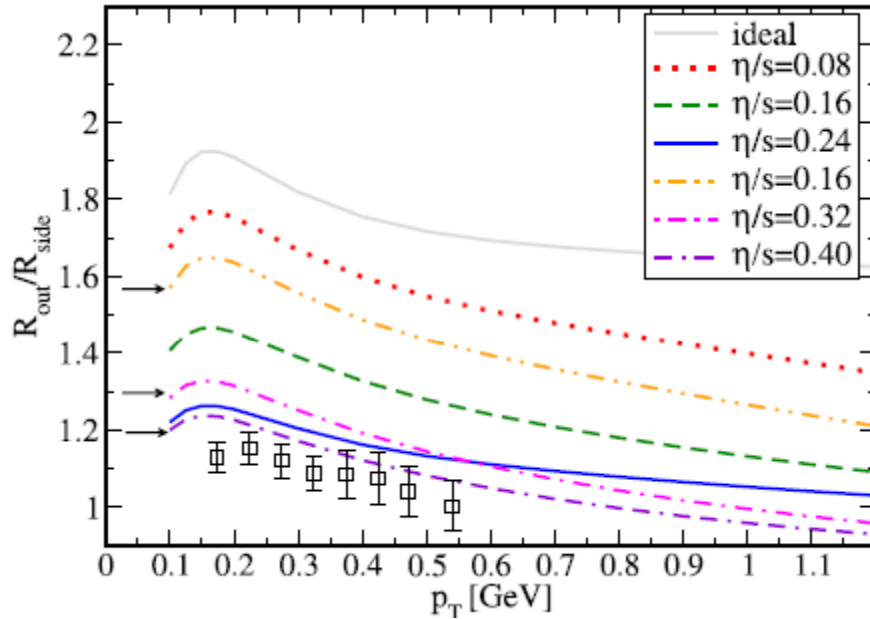
🌐 reduced  $R_{\text{out}}$

# $R_{long}$ systematics



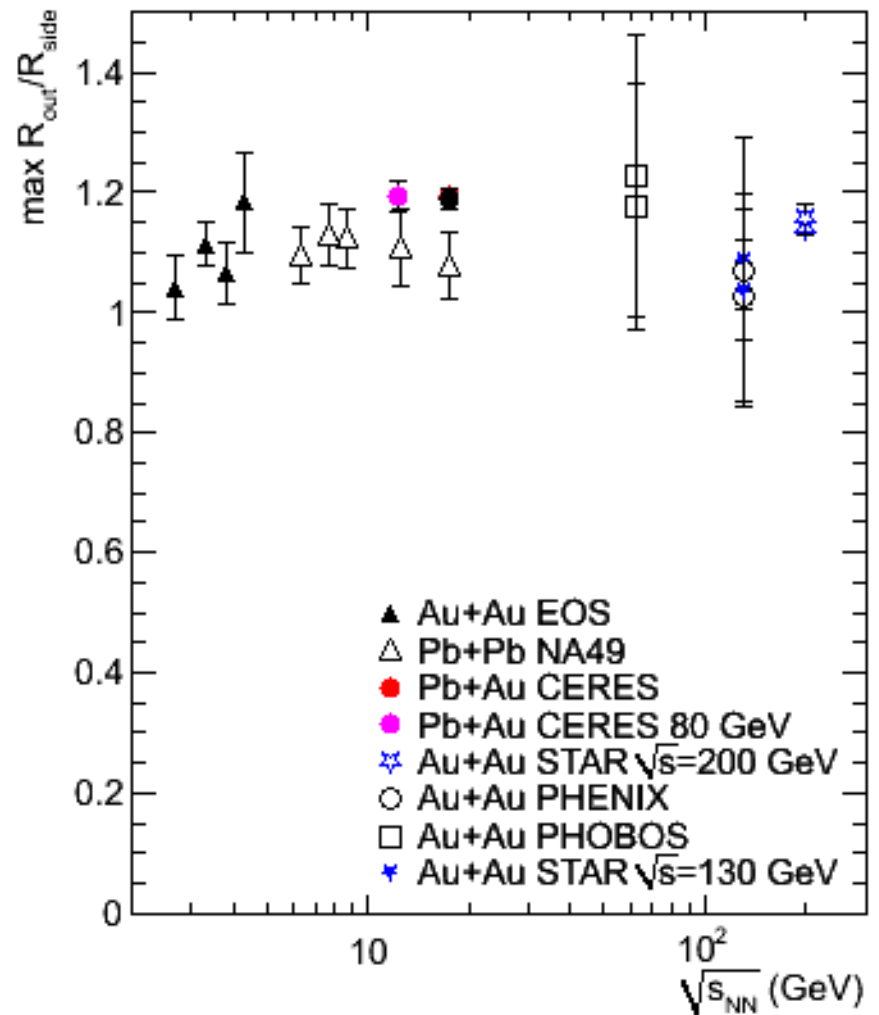
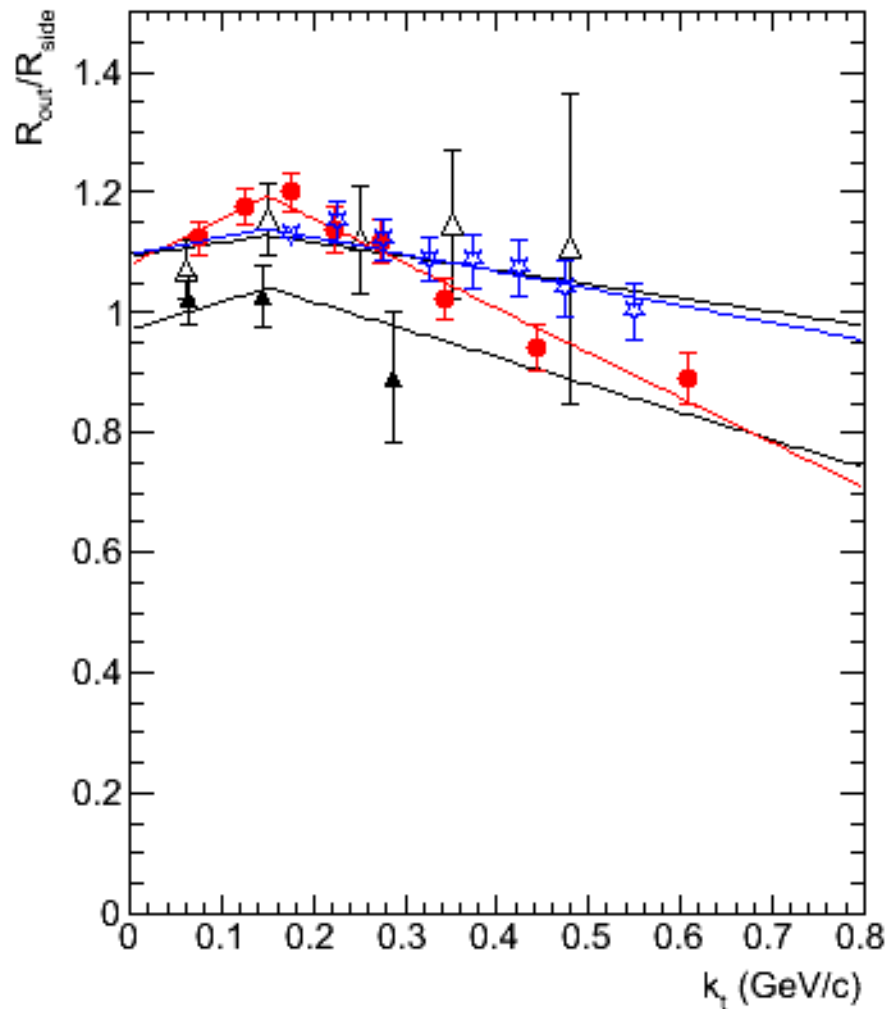
# Viscosity via $R_{out}/R_{side}$ and $R_{long}/R_{side}$ ratios

*P. Romatschke, Eur. Phys. J C 52 (2007) 203*



**$R_{out}/R_{side}$  and  $R_{long}/R_{side}$  ratios are sensitive to viscosity**

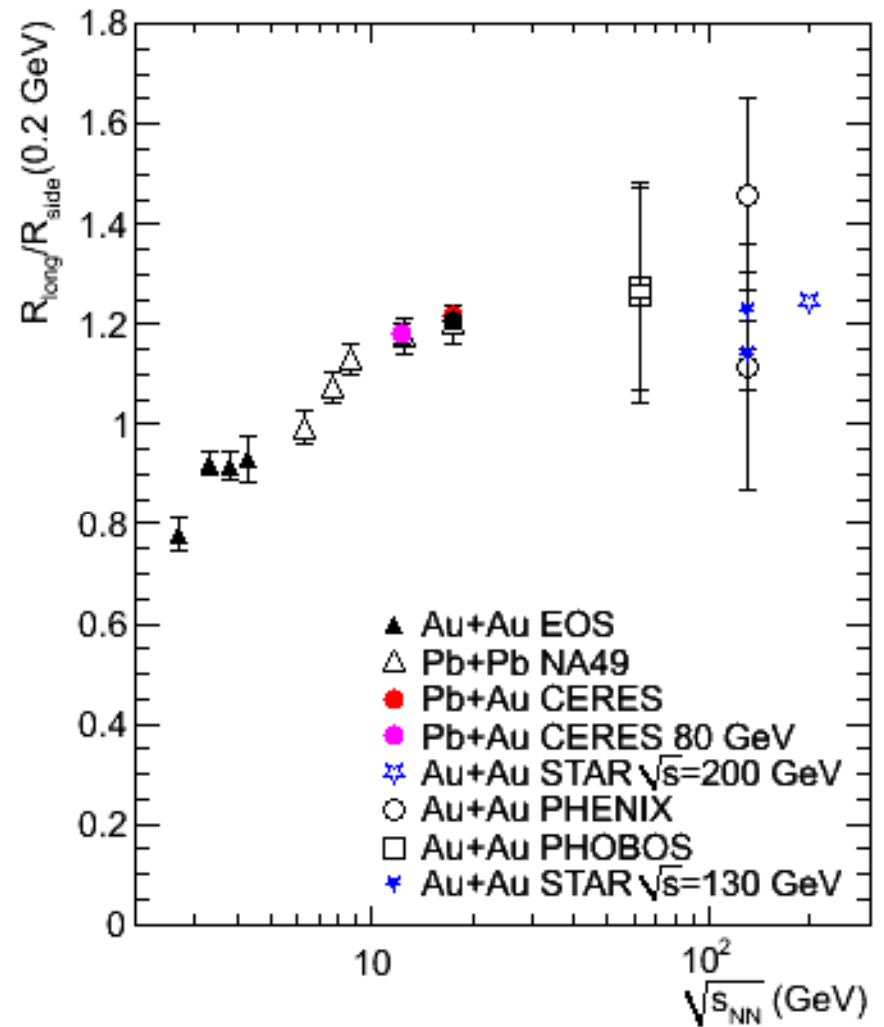
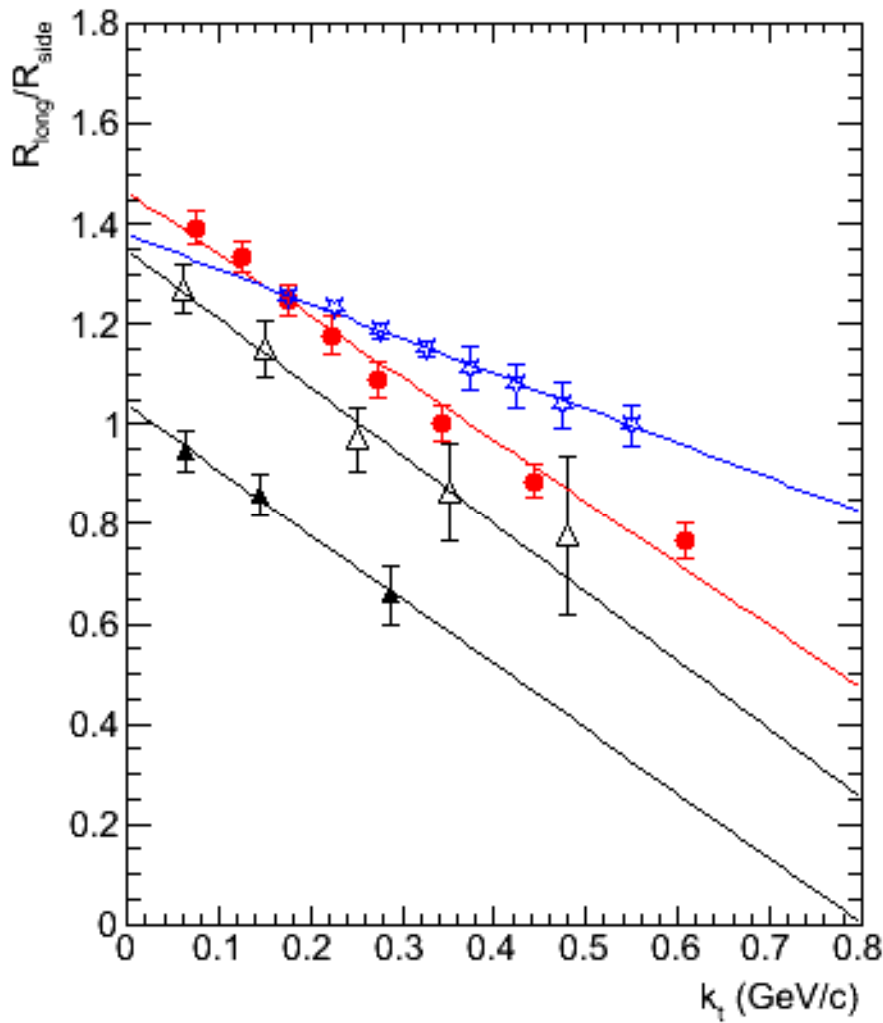
# $R_{out}/R_{side}$ systematics



**$R_{out}/R_{side}$  ratio is constant from AGS to RHIC**



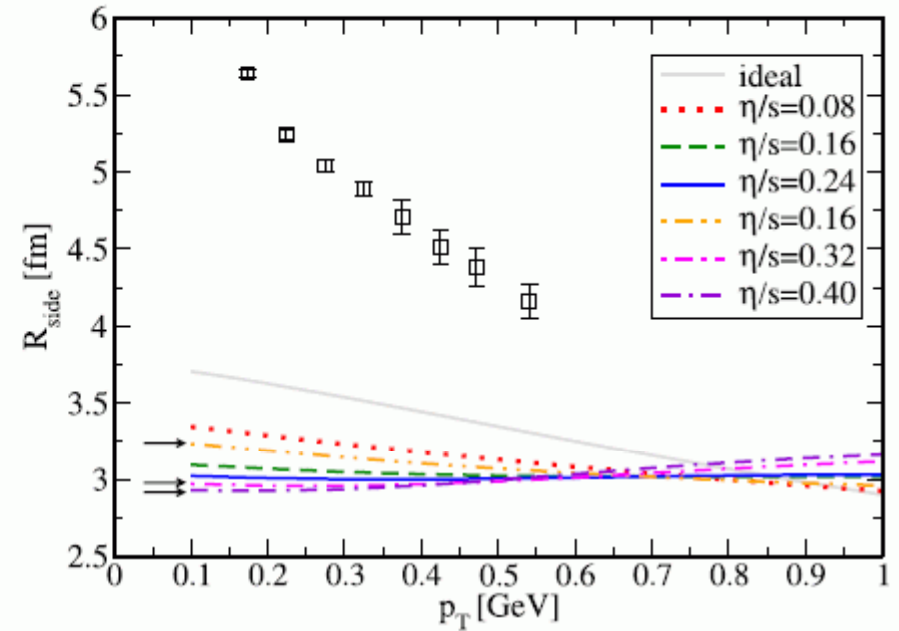
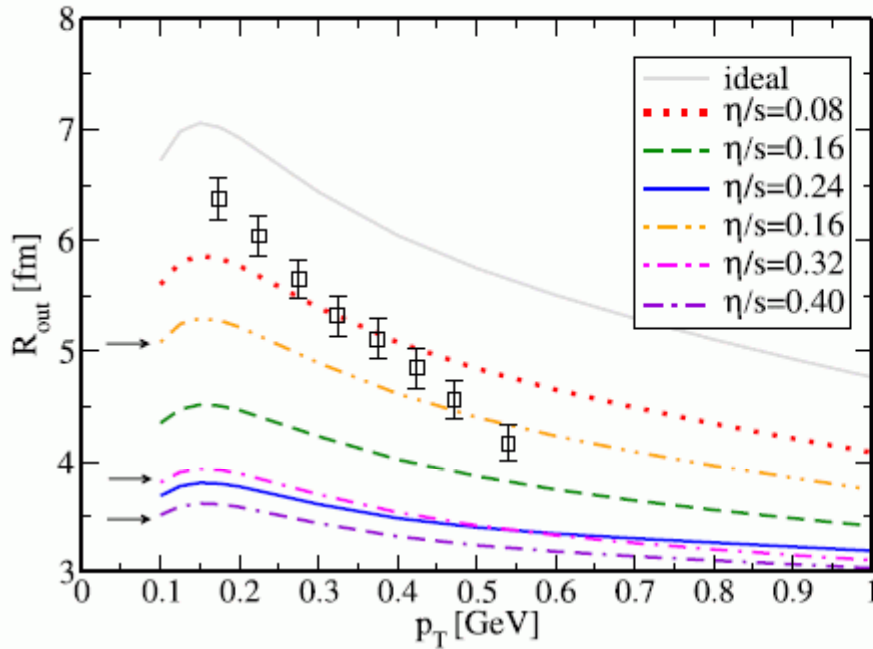
# $R_{\text{long}}/R_{\text{side}}$



**$R_{\text{long}}/R_{\text{side}}$  ratio is constant from top SPS to RHIC**

# Viscosity via $R_{out}/R_{side}$ and $R_{long}/R_{side}$ ratios

P. Romatschke, *Eur. Phys. J C* 52 (2007) 203



## Quantitatively:

**no statement can be made given the calculation does not reproduce the HBT radii but only their ratios**

## Qualitatively:

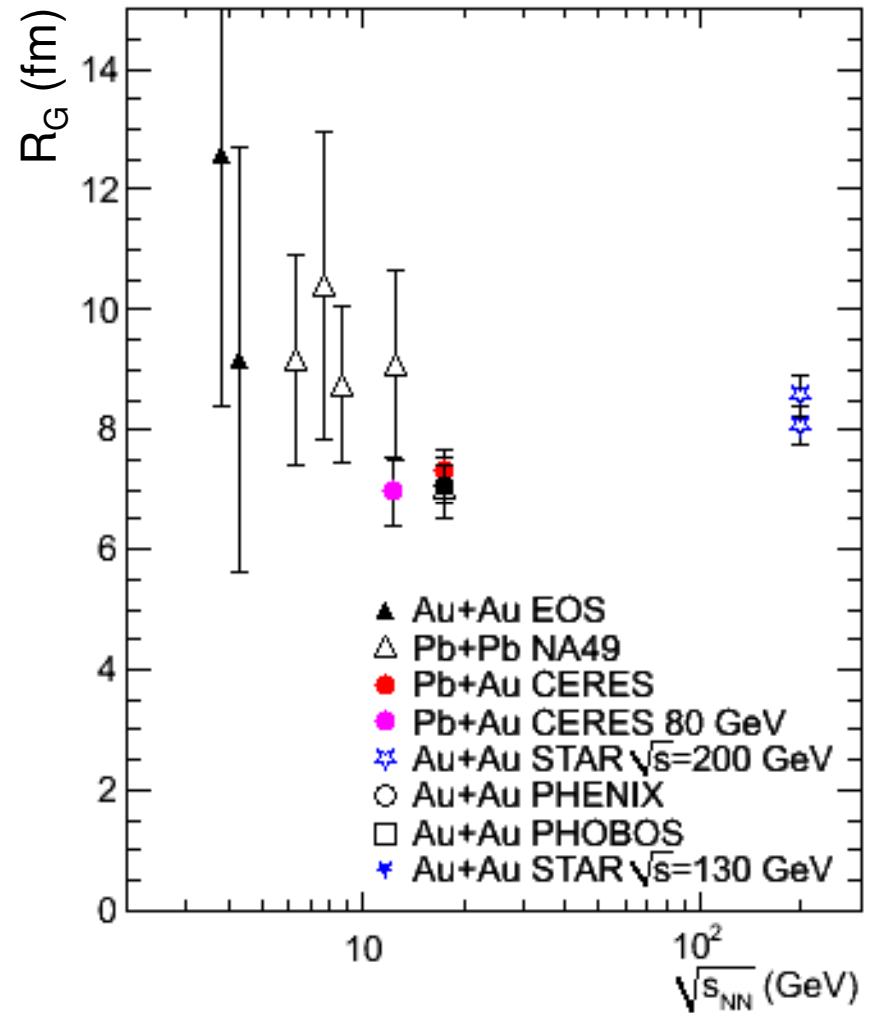
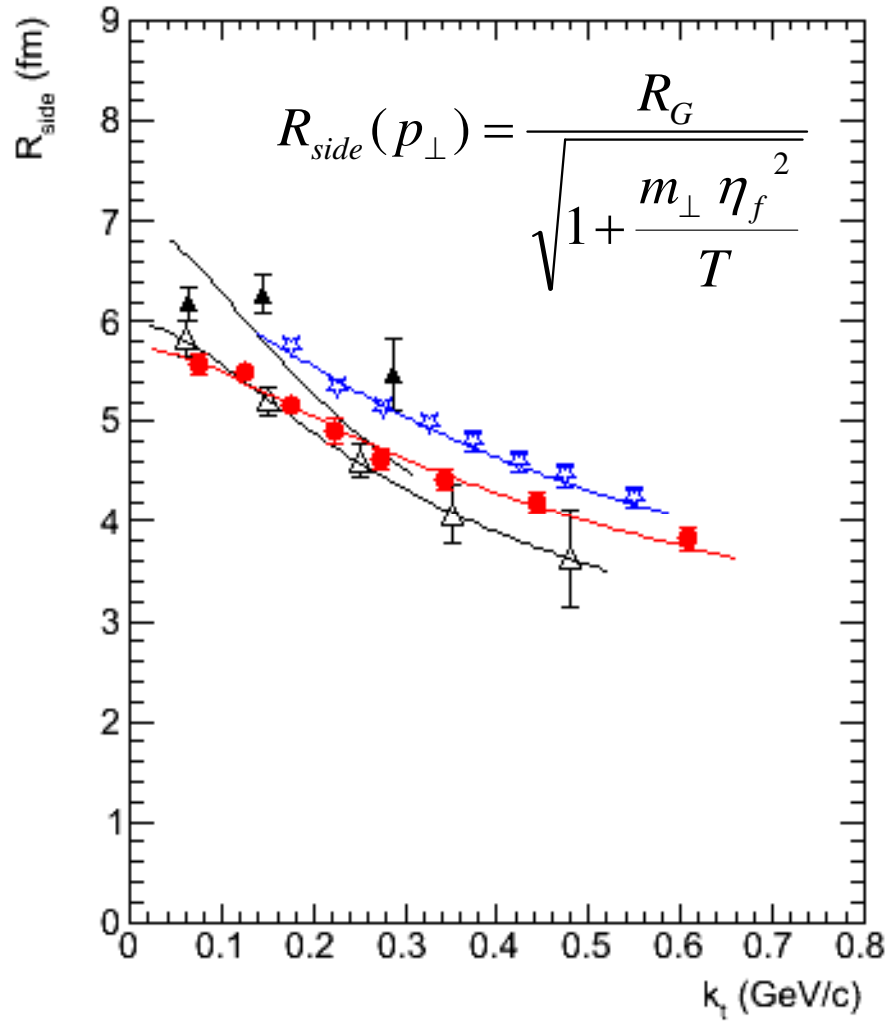
**no indication of RHIC viscosity being lower than at SPS**

# summary

- 🌐 **our quantitative method was wrong for several reasons; the preprint will be withdrawn**
- 🌐 **however, the conclusion seems to hold:  
no indication of increasing viscosity  
when going from RHIC down to SPS**

**BACKUP**

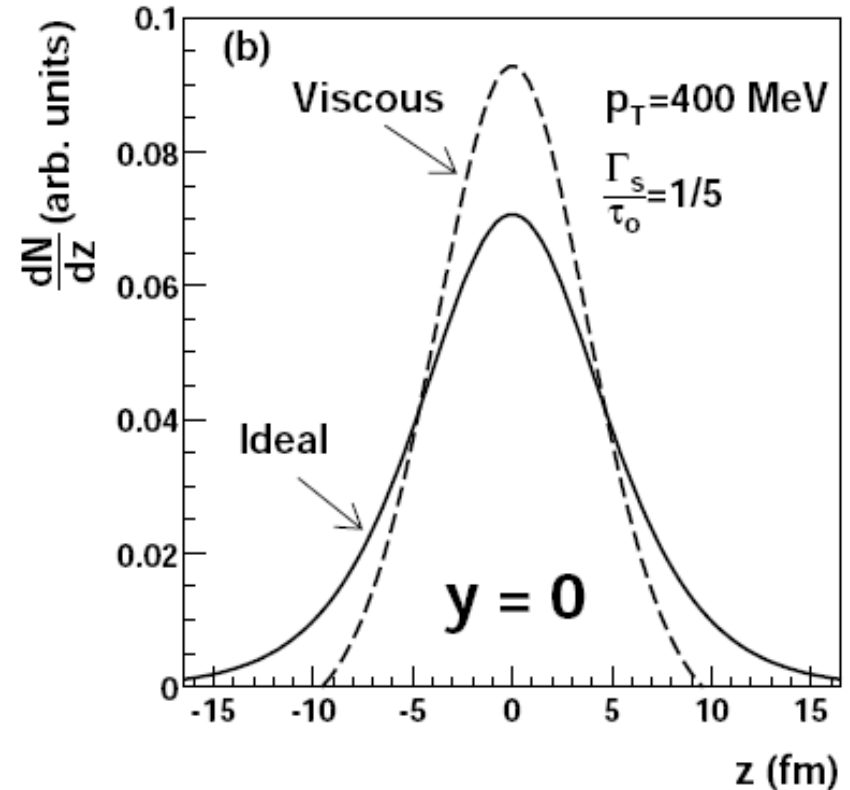
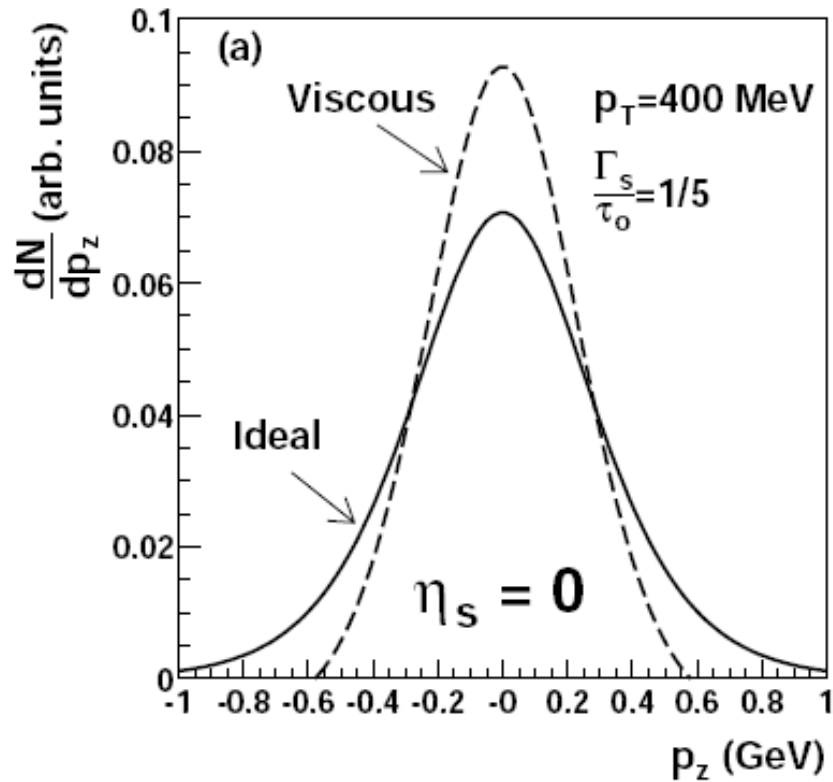
# $R_{side}$ systematics



# Viscosity via $R_{\text{long}}$

D. Teaney, *Phys. Rev. C* 68,034913 (2003)

“Viscous corrections to a Bjorken expansion”



# Viscous gas

$$\lambda = \frac{1}{n\sigma} \quad \begin{array}{l} \text{mean free path} \\ \text{hydrodynamics applies when } \lambda \text{ small} \end{array}$$

$$\eta = \frac{np\lambda}{3} = \frac{p}{3\sigma} \quad \begin{array}{l} \text{dynamical viscosity} \\ \text{(independent on } n) \\ \text{(infinite for ideal gas)} \\ \text{(not quite 0 for inf. } \sigma) \end{array}$$

$$\nu = \frac{\eta}{\rho} = \frac{\langle v \rangle \lambda}{3} \quad \text{kinematical viscosity}$$

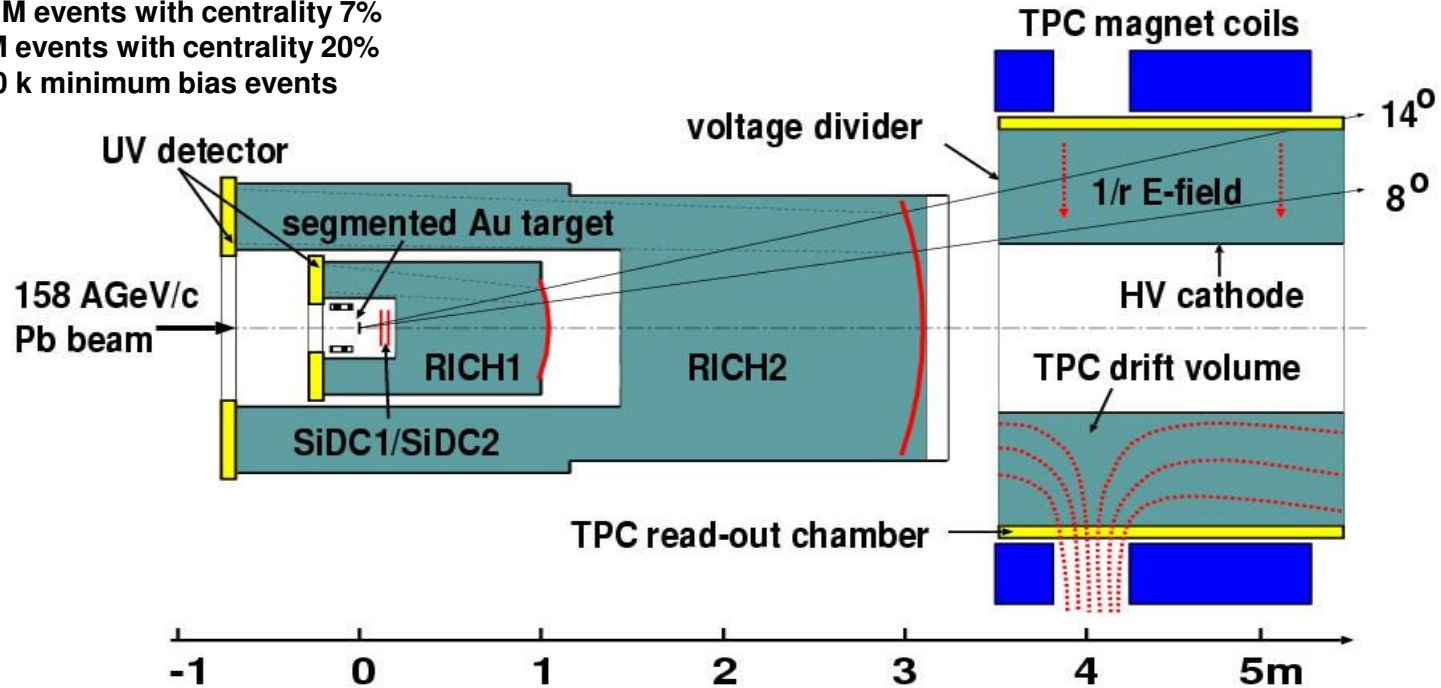
$$\begin{array}{l} \eta \sim np\lambda \\ s \sim n \end{array} \quad \begin{array}{l} \text{uncertainty principle provides} \\ \text{lower bound} \end{array}$$

$$\frac{\eta}{s} \sim p\lambda \sim \frac{\text{mean free path}}{\text{de Broglie wavelength}} \geq \text{const}$$

$$\frac{\eta}{s} = \frac{3}{4} \lambda T \quad \text{Teaney}$$

# CERES setup in 2000

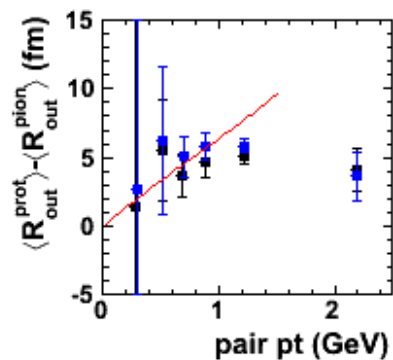
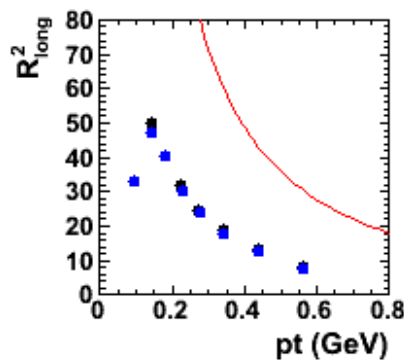
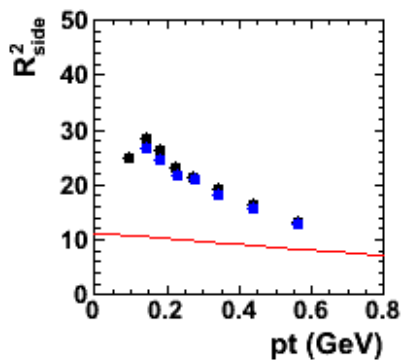
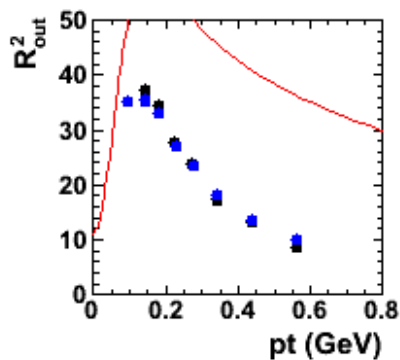
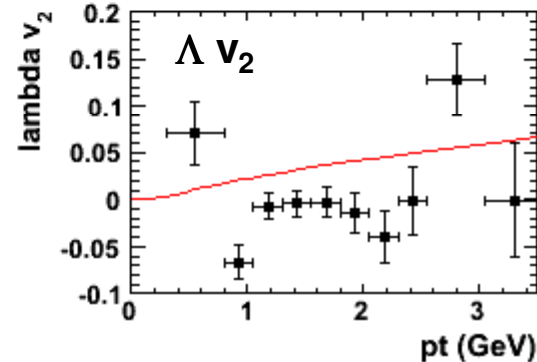
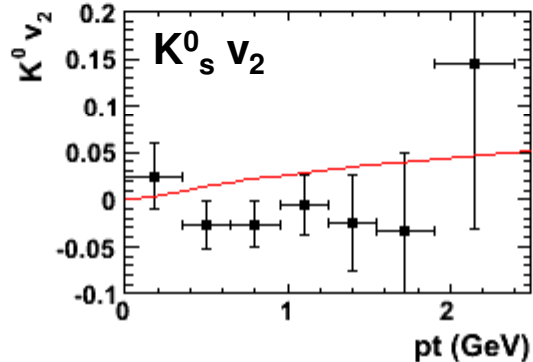
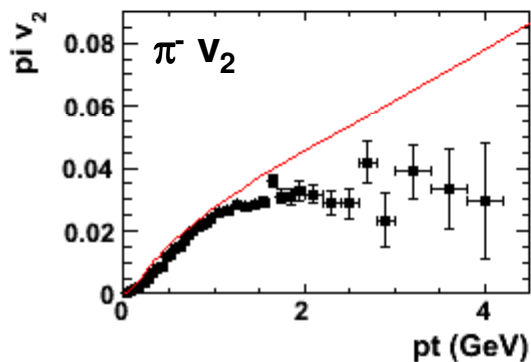
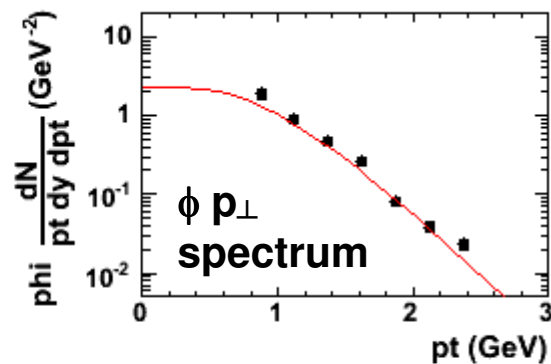
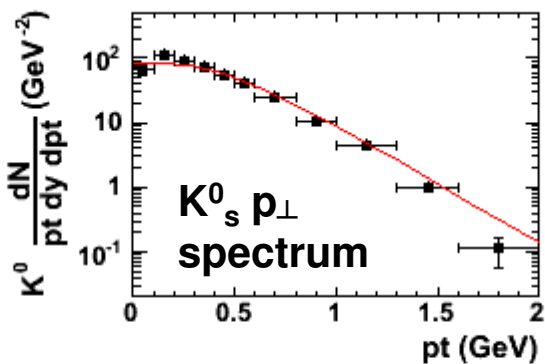
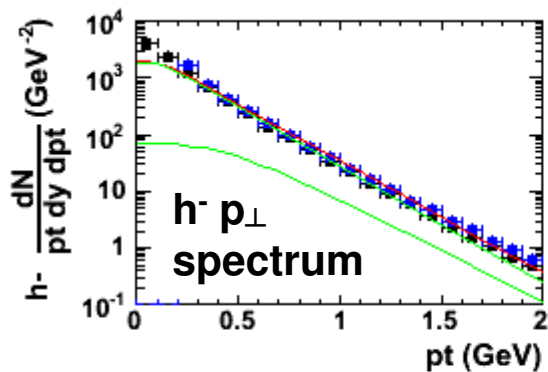
run 2000: 30 M events with centrality 7%  
2 M events with centrality 20%  
500 k minimum bias events



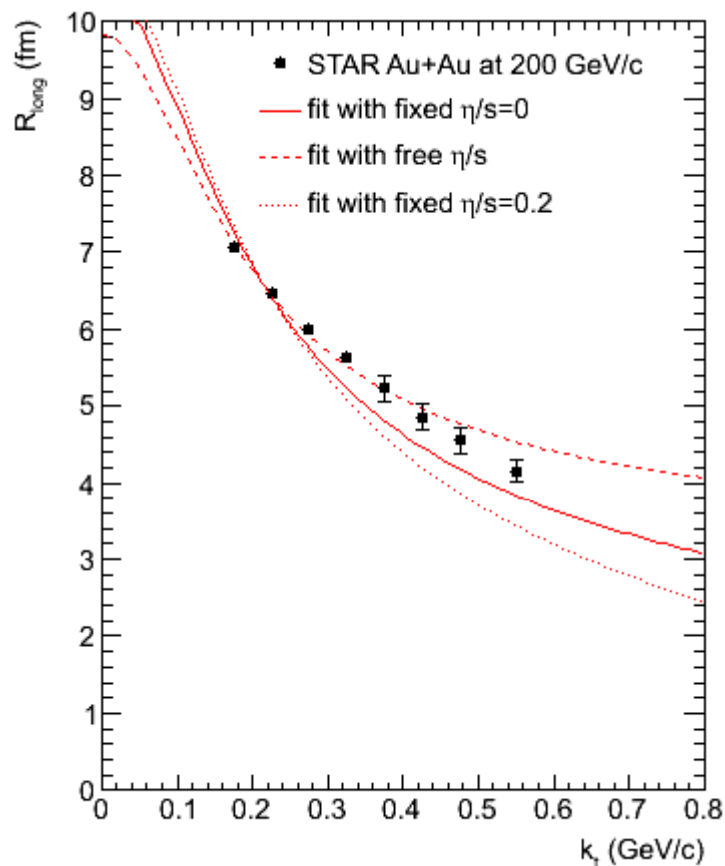
**CERES built and upgraded for leptons; but also good for...  
pt spectra, elliptic flow, two-particle correlations of hadrons**



# CERES (points) and hydro T=120 MeV (lines)

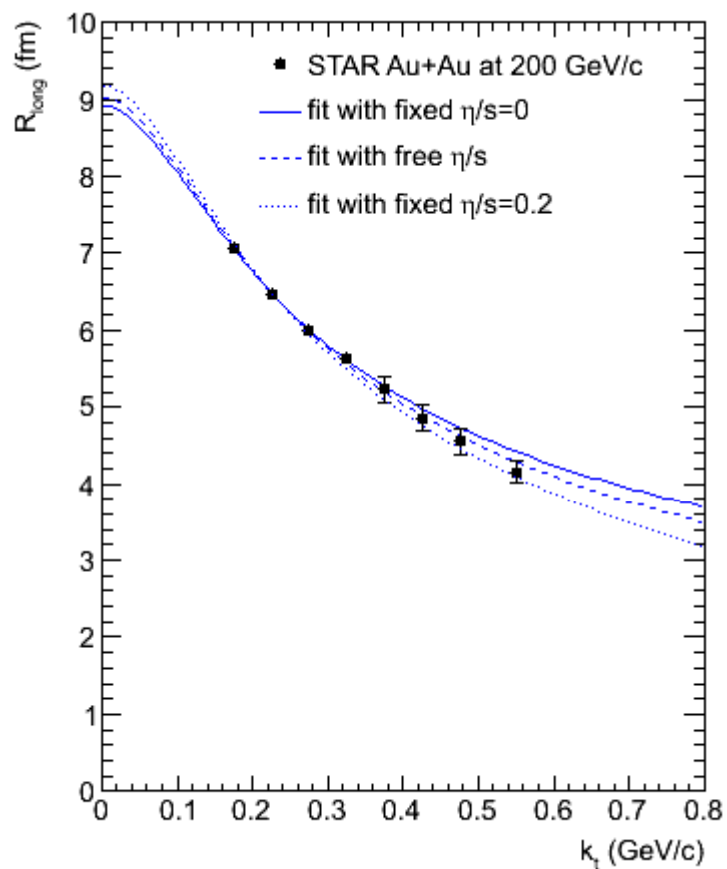


# Fit to STAR Rlong, Herrmann-Bertsch + visco



fitfun	tau0 (fm)	T (GeV)	eta/s
fixed eta/s=0	7.224+-0.022	0.120+-0.000	0.000+-0.000
free eta/s	1.850+-1.721	0.120+-0.000	1.956+-1.596
fixed eta/s=0.2	8.926+-0.022	0.120+-0.000	0.200+-0.000

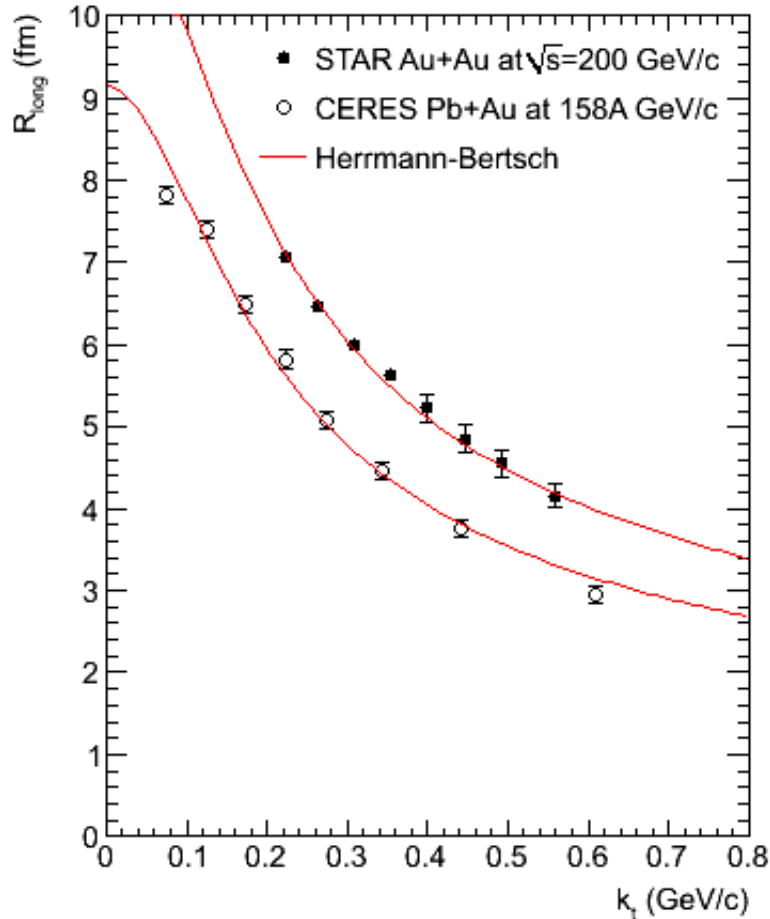
# Fit to STAR Rlong, Makhlin-Sinyukov + approx. visco



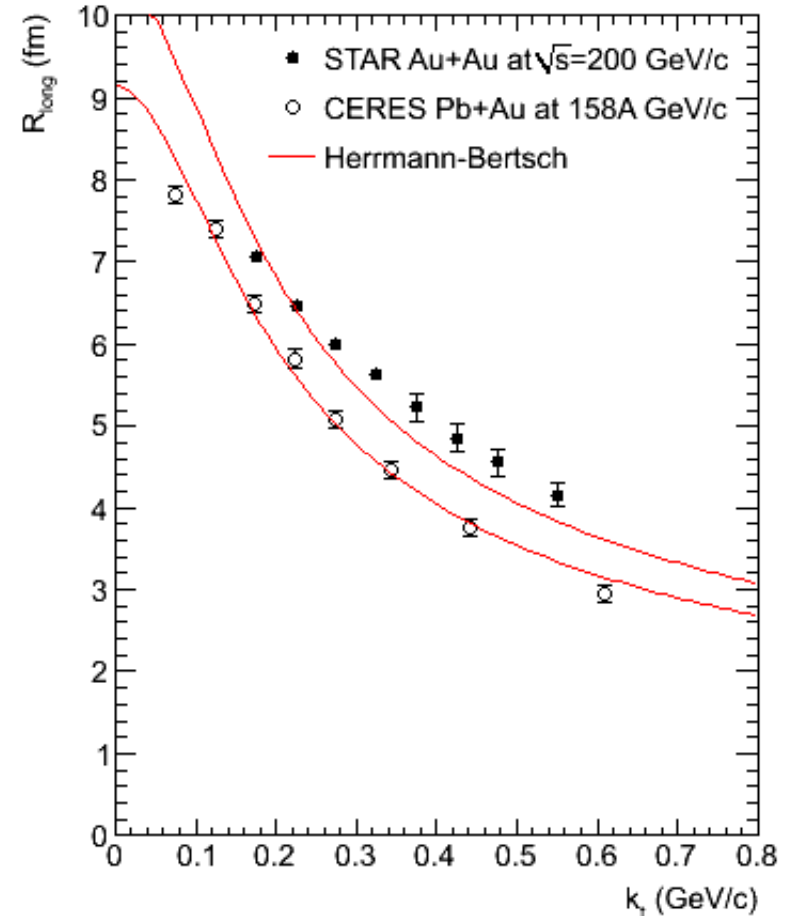
fitfun	tau0 (fm)	T (GeV)	eta/s
fixed eta/s=0	9.618+-0.030	0.120+-0.000	0.000+-0.000
free eta/s	9.861+-0.131	0.120+-0.000	0.084+-0.044
fixed eta/s=0.2	10.205+-0.030	0.120+-0.000	0.200+-0.000

# mistake when interpreting STAR data ( $m_t$ vs $k_t$ )

## STAR points misplaced



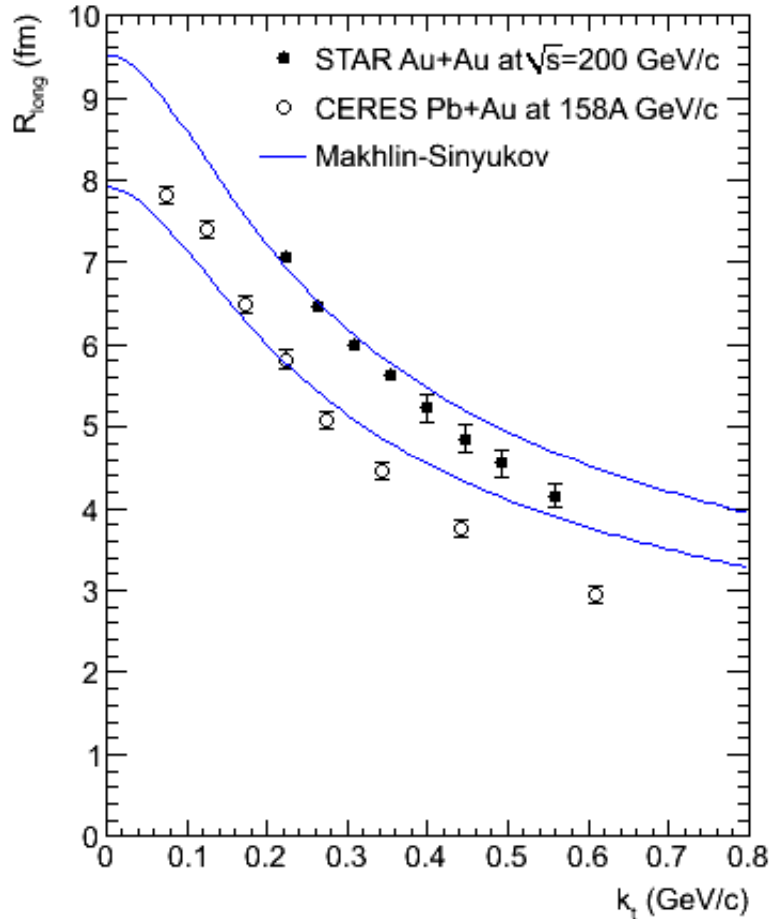
## STAR points placed correctly



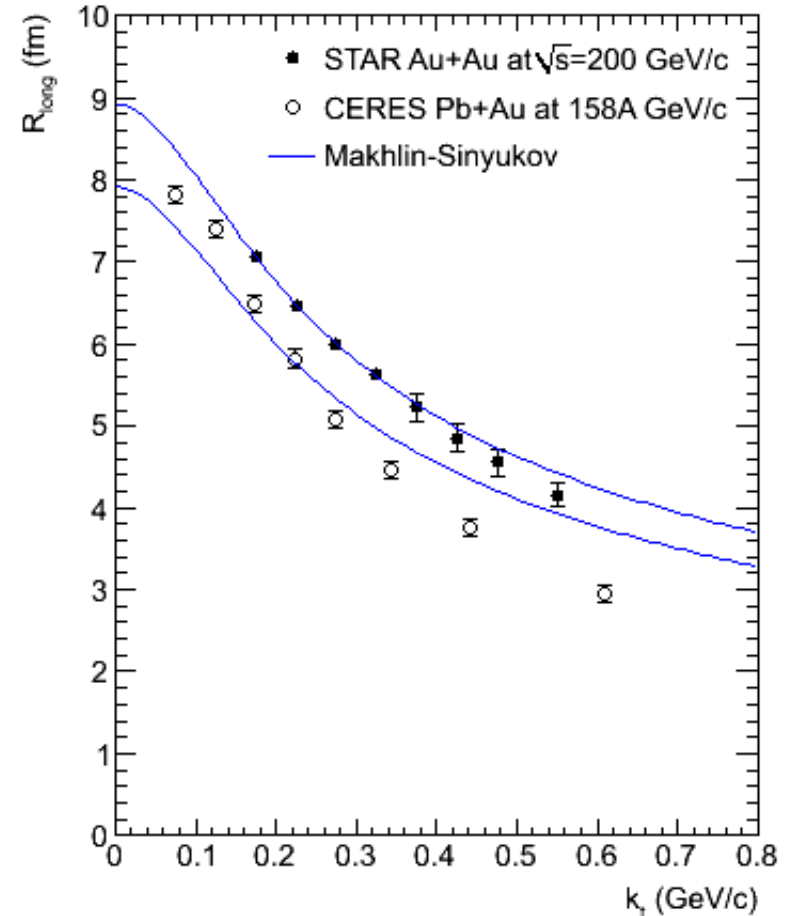
**no room for viscosity, even the pure Herrmann-Bertsch curve is steeper than the data**

# mistake when interpreting STAR data ( $m_t$ vs $k_t$ )

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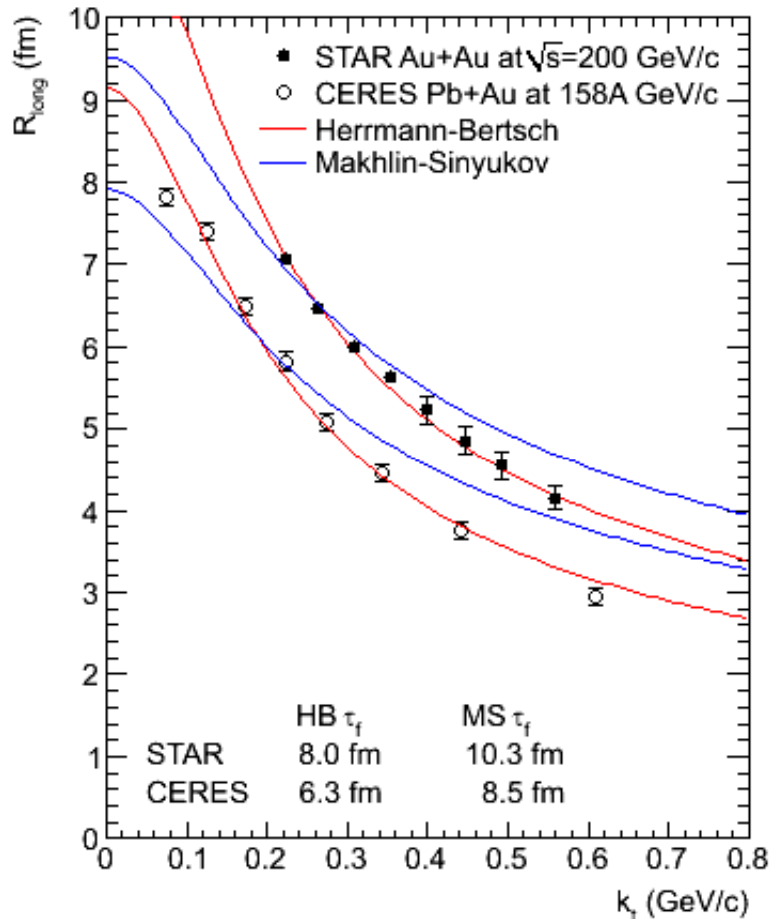
## STAR points placed correctly



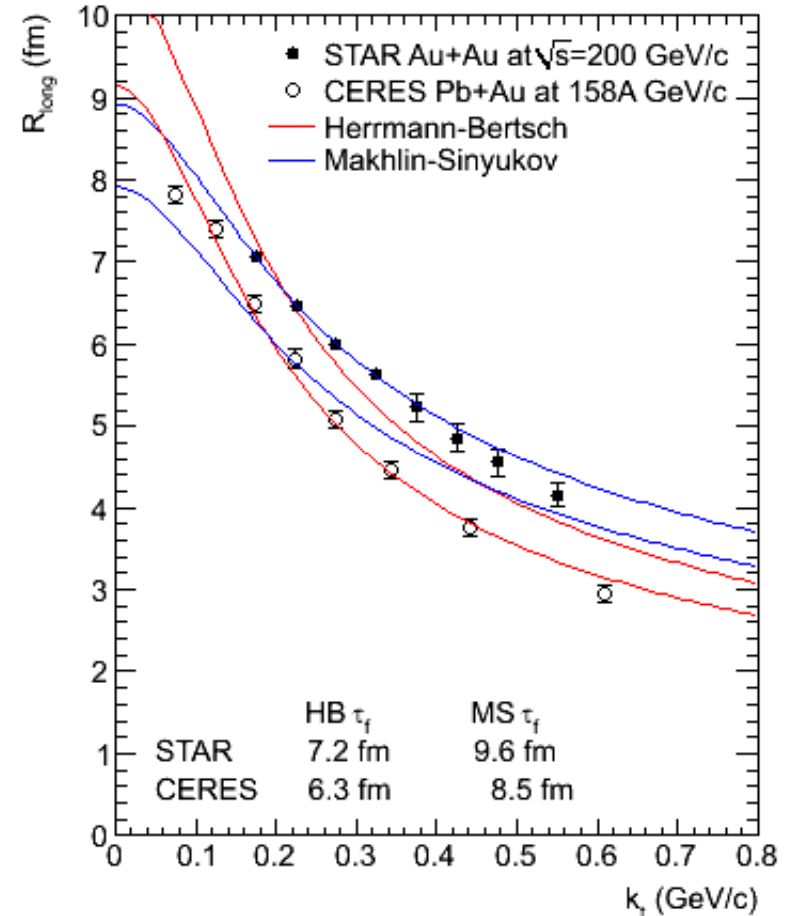
**Makhlin-Sinyukov fits much better...**

# mistake when interpreting STAR data ( $m_t$ vs $k_t$ )

## STAR points misplaced

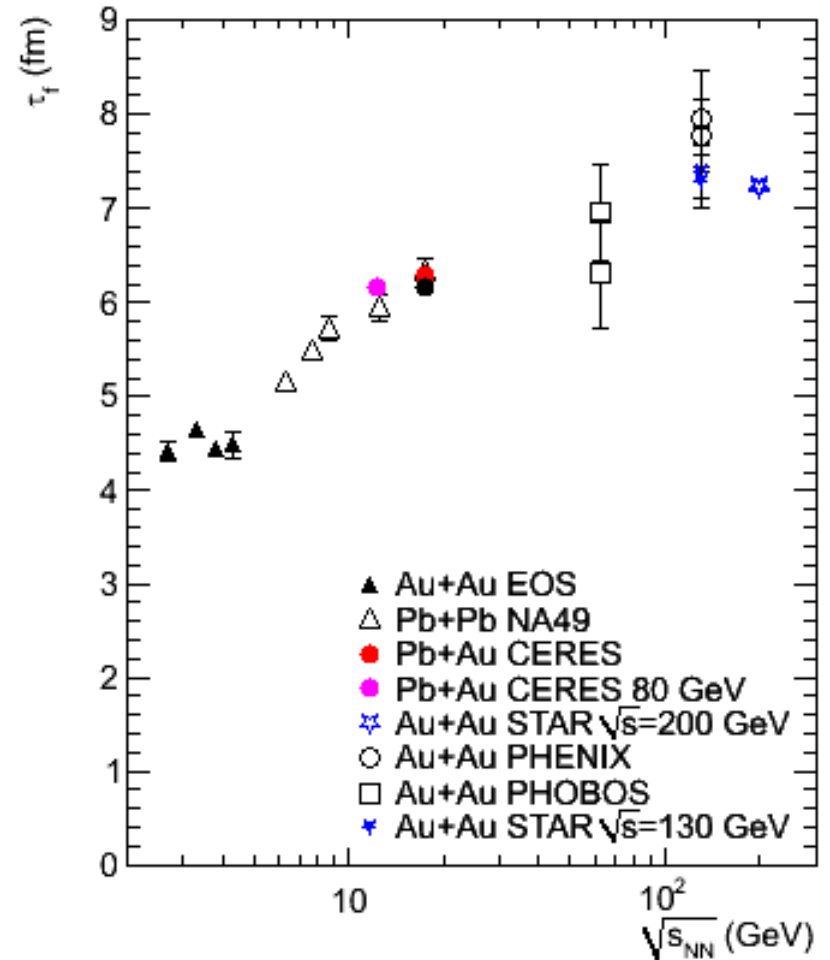
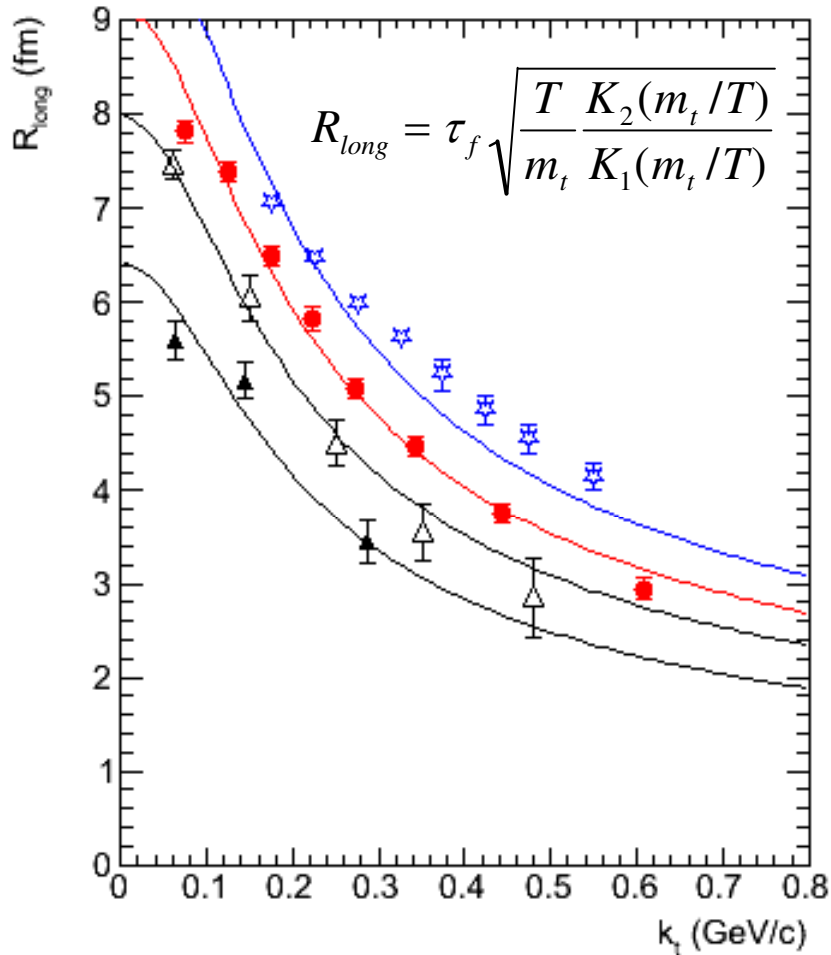


## STAR points placed correctly

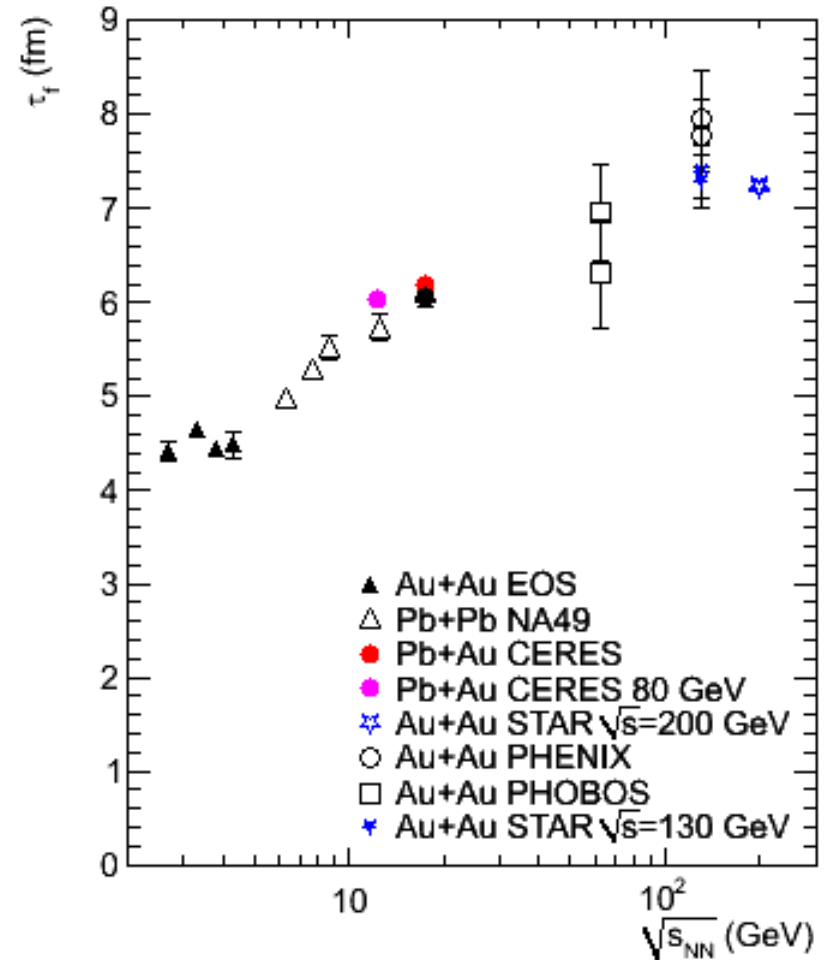
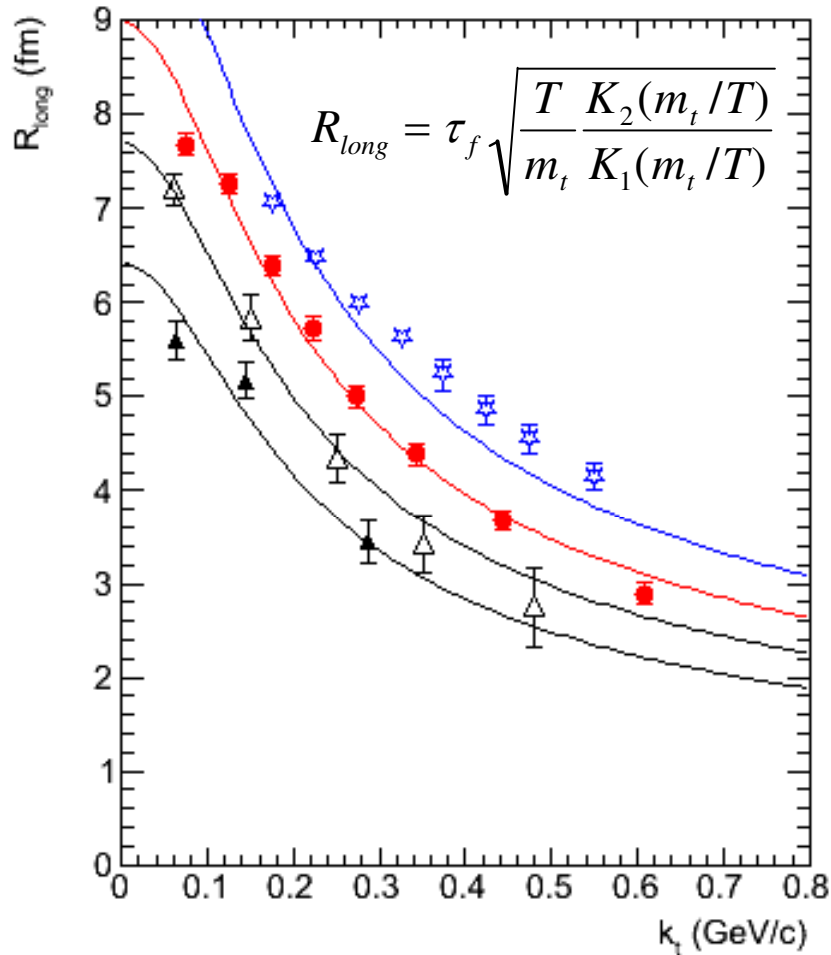


**Herrmann-Bertsch vs Makhlin-Sinyukov**

# $R_{long}$

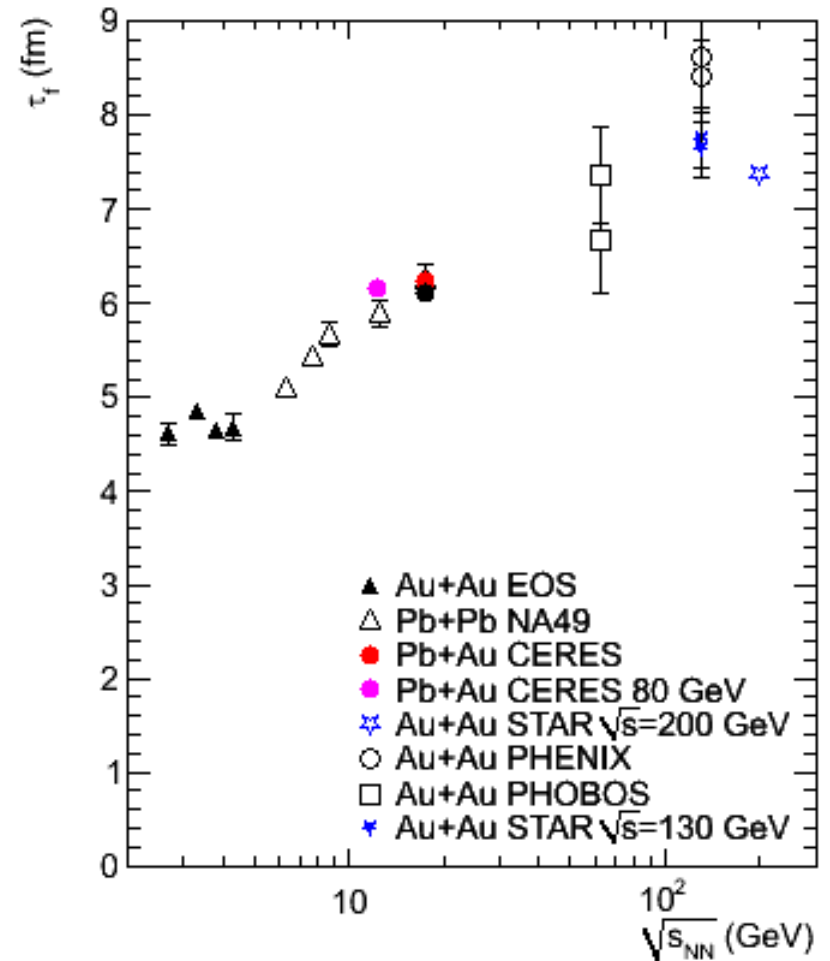
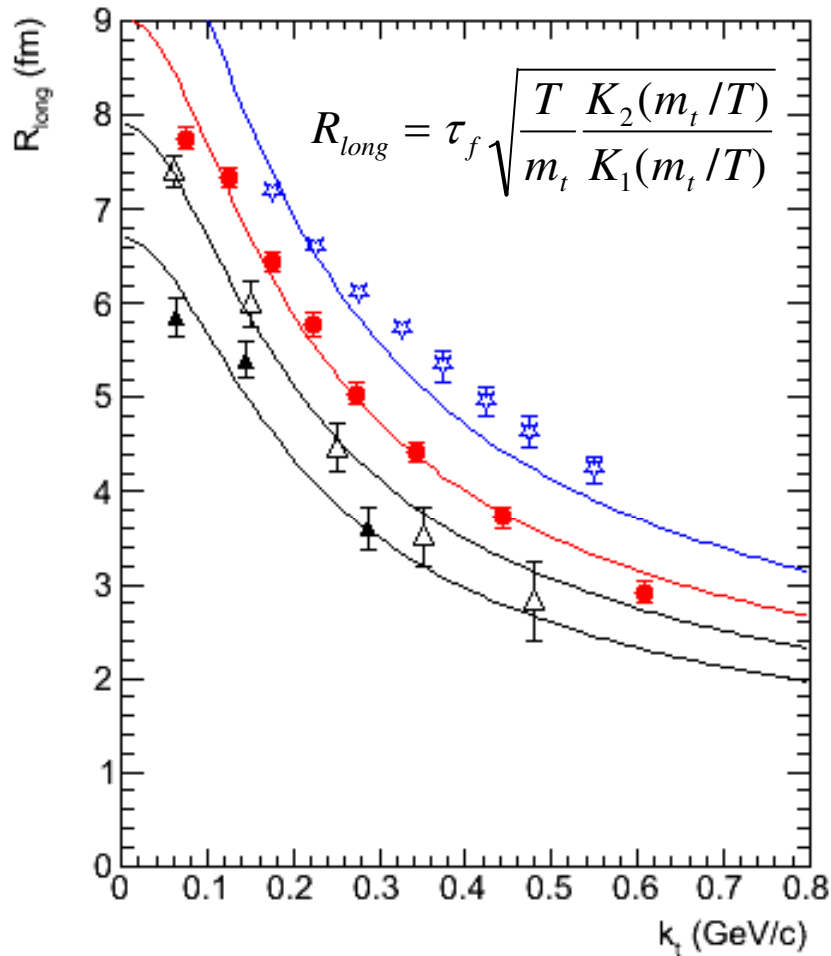


# $R_{long}$ corrected by $(A/197)^{1/3}$

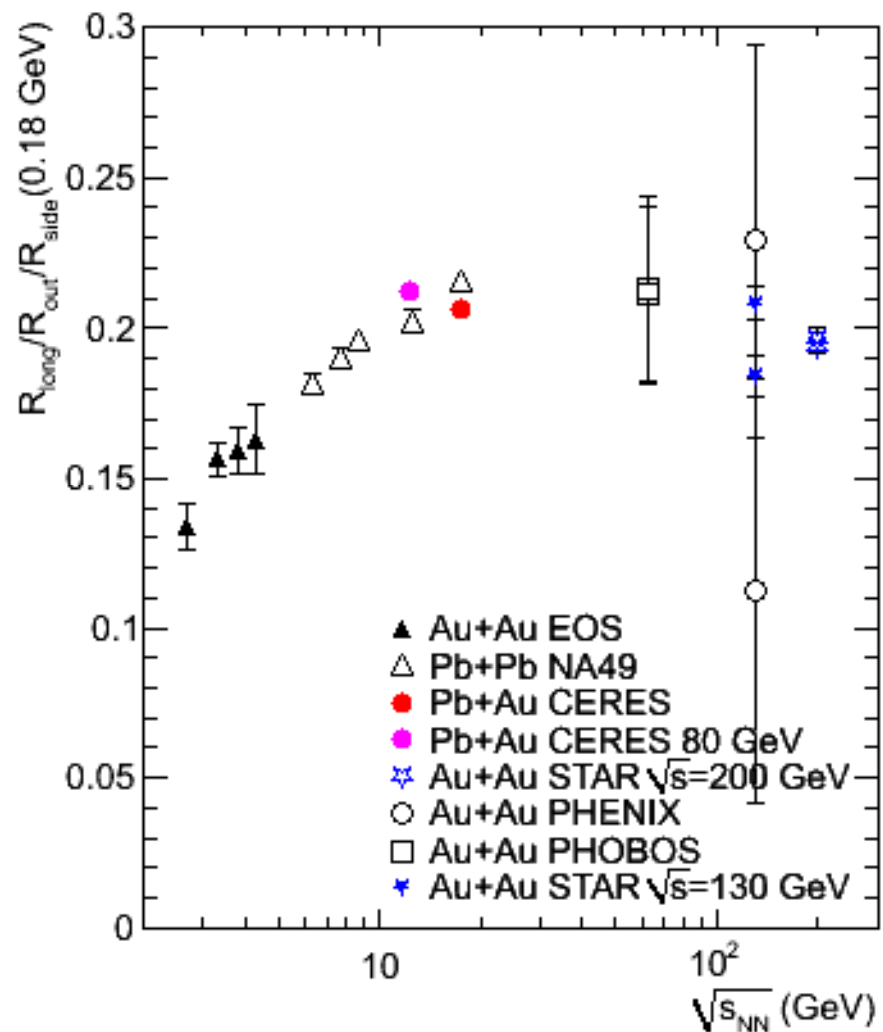
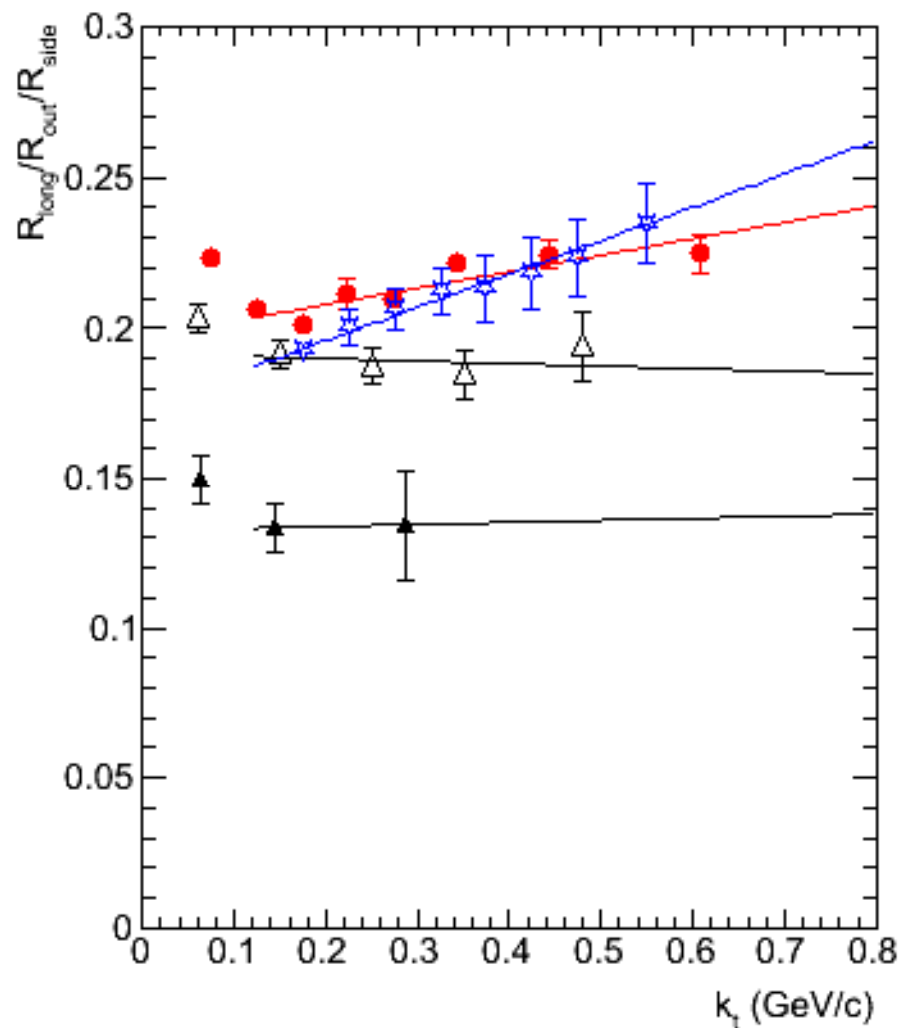




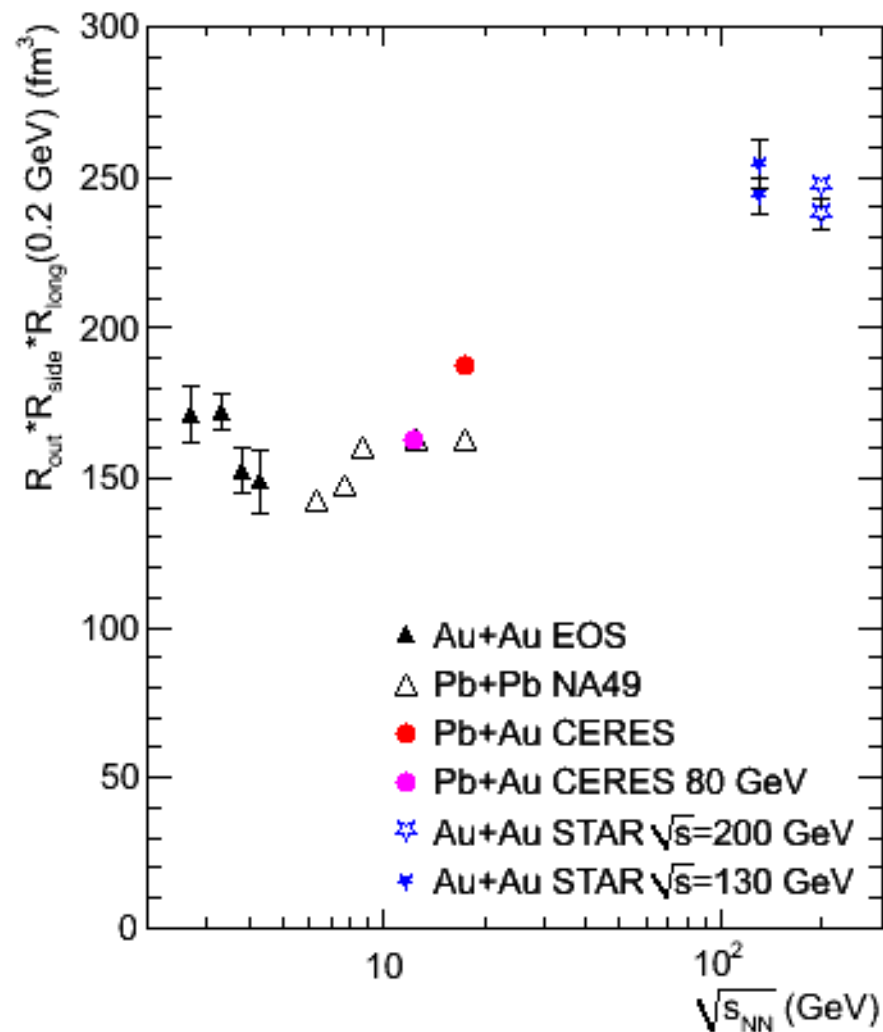
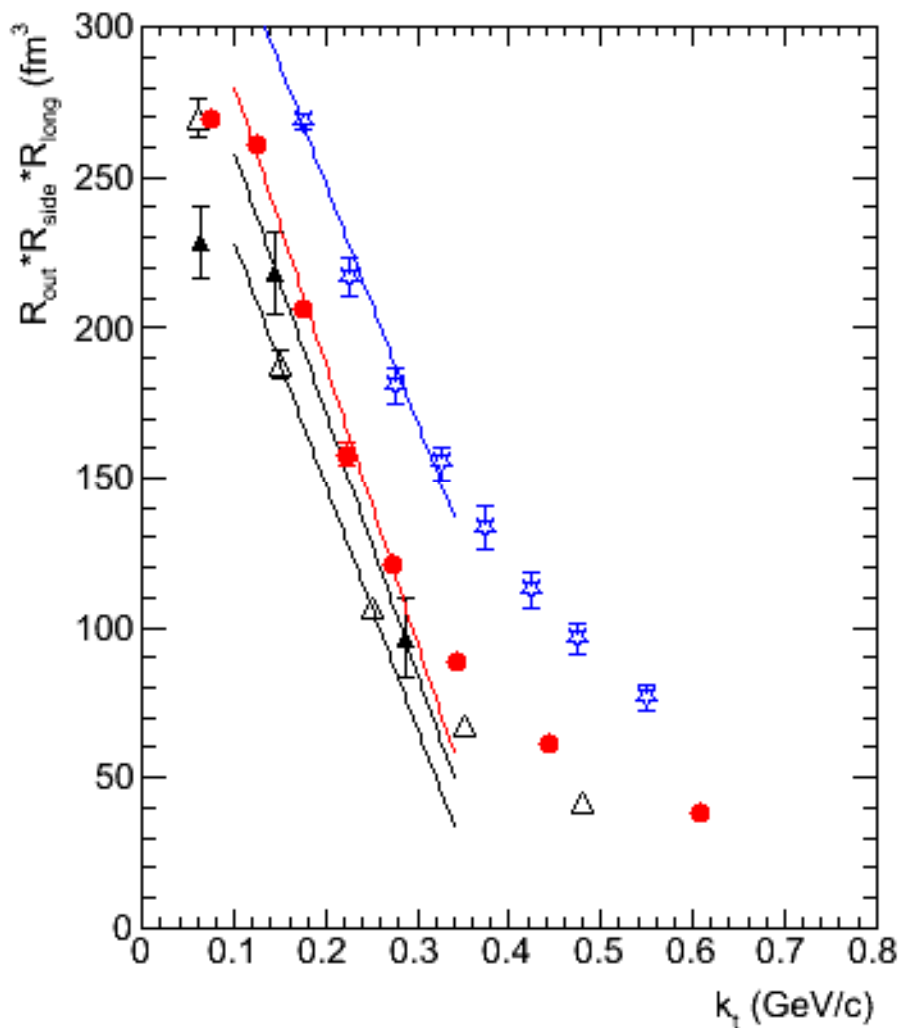
# $R_{long}$ corrected by $(A/197)^{1/3}$ and for centrality



# $R_{\text{long}}/R_{\text{out}}/R_{\text{side}}$



$$R_{\text{out}} * R_{\text{side}} * R_{\text{long}}$$



$$R_{\text{side}} * R_{\text{side}} * R_{\text{long}}$$

