Size and $\langle p_T \rangle$ fluctuations [1]

Mikołaj Chojnacki

Institute of Nuclear Physics
Polish Academy of Sciences
Kraków, Poland

V Workshop on Particle Correlations and Femtoscopy
14-17 October 2009

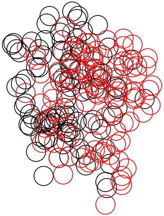
[1] based on: Wojciech Broniowski, MCh, Łukasz Obara; arXiv:0907.3216 [nucl-th]

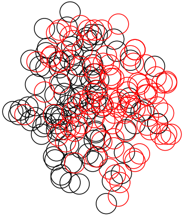
Motivation

Size fluctuations of the initial conditions

- An event with the same number of wounded nucleons N_w may have different shape and size.

$\langle r \rangle$ = 2.95 fm





$\langle r \rangle$ = 2.83 fm

Two examples[2] of non-central ^{197}Au + ^{197}Au collision with $N_w = 198$.

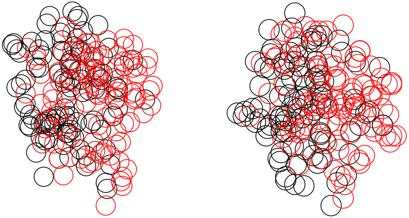
[2] GLISSANDO WB, M. Rybczyński, P. Bożek, Comput. Phys. Commun. **180** (2009) 69

Motivation

Size fluctuations of the initial conditions

- An event with the same number of wounded nucleons N_w may have different shape and size.

$\langle r \rangle = 2.95$ fm



$\langle r \rangle = 2.83$ fm

Two examples[2] of non-central ^{197}Au + ^{197}Au collision with $N_w = 198$.

smaller size $\rightarrow$ larger gradients $\rightarrow$ larger hydrodynamic flow $\rightarrow$ $\rightarrow$ larger p_T (and vice versa)

[2] GLISSANDO WB, M. Rybczyński, P. Bożek, Comput. Phys. Commun. **180** (2009) 69

Event-by-event fluctuations

average size fluctuations

- average of the transverse size in a given event

$$\langle r \rangle = \sum_{i=1}^{N_w} \sqrt{x_i^2 + y_i^2}$$

- e-by-e average of transverse size

$$\langle\langle r \rangle\rangle = \frac{1}{N_{events}} \sum_{k=1}^{N_{events}} \langle r \rangle_k$$

Event-by-event fluctuations

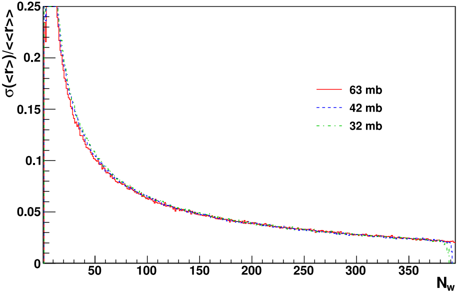
average size fluctuations

- average of the transverse size in a given event

$$\langle r \rangle = \sum_{i=1}^{N_w} \sqrt{x_i^2 + y_i^2}$$

- e-by-e average of transverse size

$$\langle\langle r \rangle\rangle = \frac{1}{N_{events}} \sum_{k=1}^{N_{events}} \langle r \rangle_k$$



- convenient measure — scaled standard deviation for set N_w

$$\sigma_{scaled} = \frac{\sigma\left(\langle r \rangle\right)}{\langle\langle r \rangle\rangle}$$

In the wounded nucleon model the σ_{scaled} is insensitive to σ_{NN}.

Event-by-event fluctuations

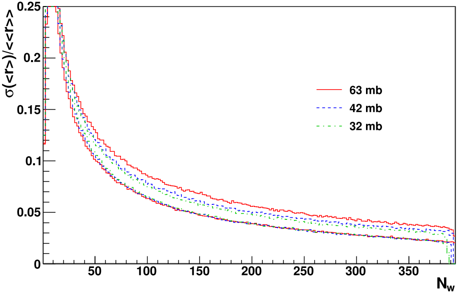
average size fluctuations

- average of the transverse size in a given event

$$\langle r \rangle = \sum_{i=1}^{N_w} \sqrt{x_i^2 + y_i^2}$$

- e-by-e average of transverse size

$$\langle\langle r \rangle\rangle = \frac{1}{N_{events}} \sum_{k=1}^{N_{events}} \langle r \rangle_k$$



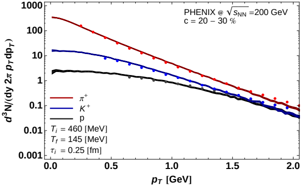
- convenient measure — scaled standard deviation for set N_w

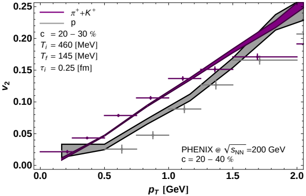
$$\sigma_{scaled} = \frac{\sigma(\langle r \rangle)}{\langle\langle r \rangle\rangle}$$

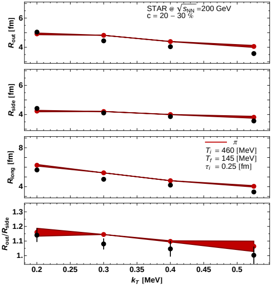
In the mixed model $(\frac{\alpha}{2} N_w + (1-\alpha) N_{bin})$ a moderate change with σ_{NN} is caused by the different admixture of the binary collisions profile which is much more sensitive to fluctuations.

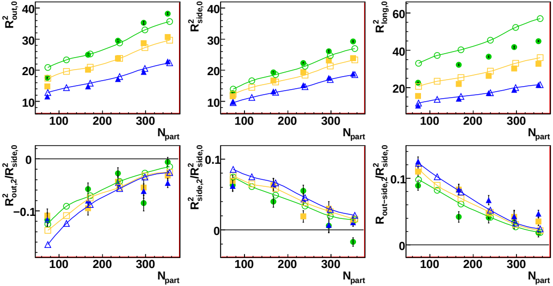
Hydrodynamics with statistical hadronization [3]

(2+1) perfect fluid hydro + THERMINATOR









[3] WB, MCh, W. Florkowski and A. Kisiel, Phys. Rev. Lett. **101**, 022301 (2008)
AK, WB, MCh and WF, Phys. Rev. **C 79**, 014902 (2009)

e-by-e hydrodynamics

fluctuating initial conditions

- Instead of 100 000 events, two are enough!

e-by-e hydrodynamics

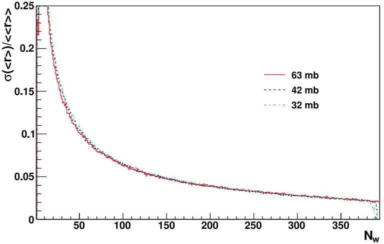
fluctuating initial conditions

- Instead of 100 000 events, two are enough!
- Size of the initial condition for hydrodynamics (energy density profile) is scaled up and down according to the scaled variance.
- No e-by-e energy fluctuations (could be included).

e-by-e hydrodynamics

fluctuating initial conditions

- Instead of 100 000 events, two are enough!
- Size of the initial condition for hydrodynamics (energy density profile) is scaled up and down according to the scaled variance.
- No e-by-e energy fluctuations (could be included).



Example:

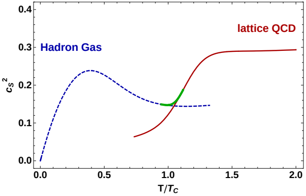
centrality 20-30% corresponds to:
$N_w = 165$
$\sigma(\langle r \rangle)/\langle\langle r \rangle\rangle = 4.4\%$

initial central temperature is changed from 455 MeV to 466 MeV (stretched) or 445 MeV (squeezed) profile → total energy the same.

Hydrodynamics with statistical hadronization

LHYQUID – 2+1 perfect fluid hydrodynamics

- realistic Equation of State [a]

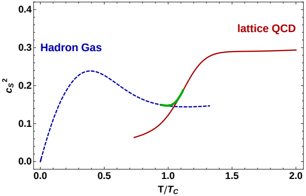


- initial time τ_i = 0.25 fm/c
- freeze-out temperature T_f = 145 MeV

Hydrodynamics with statistical hadronization

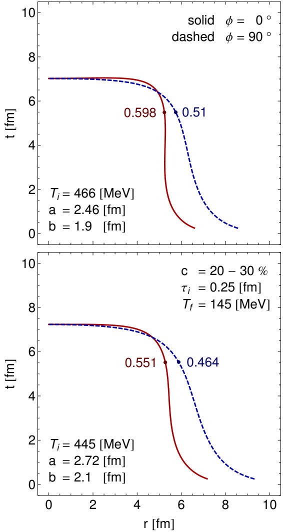
LHYQUID – 2+1 perfect fluid hydrodynamics

- realistic Equation of State [a]



- initial time τ_i = 0.25 fm/c
- freeze-out temperature T_f = 145 MeV

squeezed profile

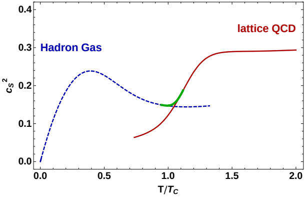


stretched profile

Hydrodynamics with statistical hadronization

LHYQUID – 2+1 perfect fluid hydrodynamics

- realistic Equation of State [a]

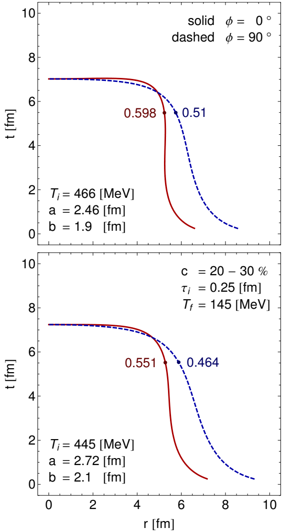


- initial time τ_i = 0.25 fm/c
- freeze-out temperature T_f = 145 MeV

THERMINATOR – statistical hadronization

- Cooper-Frye formalism used to produce particles
- resonance decays included

squeezed profile



stretched profile

[a] MCh, WF, WB and AK, Phys. Rev. **C 78**, 014905 (2008)
MCh, WF, Acta Phys. Polon. **B 38** (2007) 3249

Results

distributions of $\langle r \rangle$ and $\langle p_T \rangle$

- The distribution of the $\langle r \rangle$ is approximately Gaussian

$$f(\langle r \rangle) \sim \exp\left(-\frac{(\langle r \rangle - \langle\langle r \rangle\rangle)^2}{2\sigma^2(\langle r \rangle)} \right)$$

  Imagine we ran simulations with fixed $\langle r \rangle$ (no size fluctuations). Then particles would have some average momentum $\bar{p}_T$

- Since hydrodynamic evolution is deterministic, $\bar{p}_T$ is a (very complicated) function of $\langle r \rangle$.
- Now let us include fluctuations of $\langle r \rangle$. We can use Taylor expansion

$$\bar{p}_T - \langle\langle p_T \rangle\rangle = \left.\frac{d\bar{p}_T}{d\langle r \rangle}\right|_{\langle r \rangle = \langle\langle r \rangle\rangle} (\langle r \rangle - \langle\langle r \rangle\rangle) + \ldots$$

- The statistical distribution of $\langle \bar{p}_T \rangle$ is

$$f(\bar{p}_T) \sim \exp\left(-\frac{(\bar{p}_T - \langle\langle p_T \rangle\rangle)^2}{2\sigma^2(\langle r \rangle)\left(\frac{d\bar{p}_T}{d\langle r \rangle}\right)^2} \right)$$

Results

scaled variance of $\langle p_T \rangle$

- The full statistical distribution $f(\langle p_T \rangle)$ in a given centrality class is a folding of the statistical distribution of $\langle p_T \rangle$ at a fixed initial size, centered around a certain $\bar{p}_T$, with the distribution of $\bar{p}_T$ centered around $\langle\langle p_T \rangle\rangle$.

$$f(\langle p_T \rangle) \sim \int d^2\bar{p}_T \exp\left(-\frac{(\langle p_T \rangle - \bar{p}_T)^2}{2\sigma^2_{stat}}\right) \exp\left(-\frac{(\bar{p}_T - \langle\langle p_T \rangle\rangle)^2}{2\sigma^2_{dyn}}\right)$$

$$\sim \exp\left(-\frac{(\langle p_T \rangle - \langle\langle p_T \rangle\rangle)^2}{2\left(\sigma^2_{stat} + \sigma^2_{dyn}\right)}\right)$$

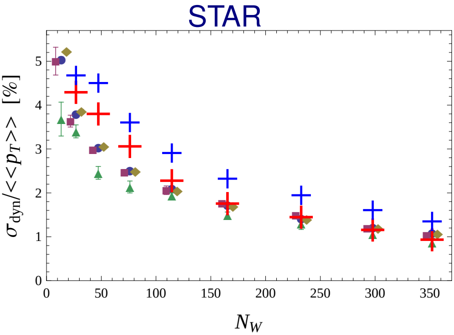
where $\sigma_{dyn}(\langle p_T \rangle) = \sigma(\langle r \rangle) \left.\frac{d\bar{p}_T}{d\langle r \rangle}\right|_{\langle r \rangle = \langle\langle r \rangle\rangle}$ is extracted by the experimentalists.

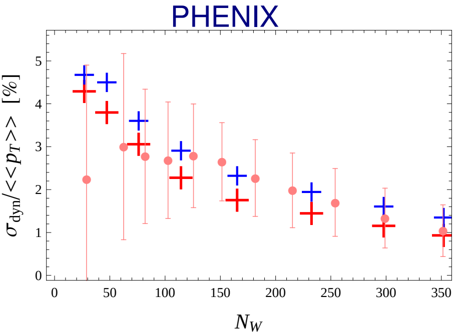
- Scaled dynamical variance

$$\frac{\sigma_{dyn}}{\langle\langle p_T \rangle\rangle} = \frac{\sigma(\langle r \rangle)}{\langle\langle r \rangle\rangle} \frac{\langle\langle r \rangle\rangle}{\langle\langle p_T \rangle\rangle} \left.\frac{d\bar{p}_T}{d\langle r \rangle}\right|_{\langle r \rangle = \langle\langle r \rangle\rangle}$$

Results

comparison with experiments





- scaled variation for Wounded Nucleon model (red crosses) and mixed model (blue crosses)
- overal amazing agreement!
- mixed model overshoots data by 20% which can possibly be reduced by modified evolution (i.e. viscosity in hydrodynamics)
- proper scaling is reproduced, $\sigma_{dyn}/\langle\langle p_T \rangle\rangle \sim 1/\sqrt{N_W}$

Results

connection to the EoS[4]

- scaled variance of $\langle p_T \rangle$ is connected to thermodynamics

$$\frac{\sigma_{dyn}}{\langle\langle p_T \rangle\rangle} = \frac{P}{\varepsilon}\frac{\sigma(\langle s \rangle)}{\langle\langle s \rangle\rangle} = 2\frac{P}{\varepsilon}\frac{\sigma(\langle r \rangle)}{\langle\langle r \rangle\rangle}$$

where s is the entropy density, ε energy density, and P the pressure

[4]Jean-Yves Ollitrault, Phys. Lett. **B 273** (1991)

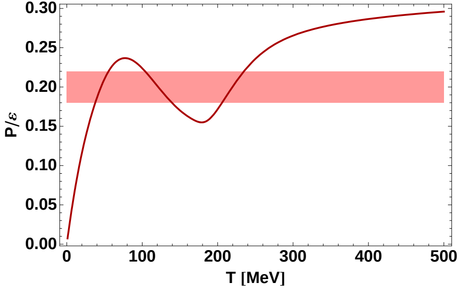
Results

connection to the EoS[4]

- scaled variance of $\langle p_T \rangle$ is connected to thermodynamics

$$\frac{\sigma_{dyn}}{\langle\langle p_T \rangle\rangle} = \frac{P}{\varepsilon} \frac{\sigma(\langle s \rangle)}{\langle\langle s \rangle\rangle} = 2 \frac{P}{\varepsilon} \frac{\sigma(\langle r \rangle)}{\langle\langle r \rangle\rangle}$$

where s is the entropy density, ε energy density, and P the pressure



- We can study this way the average properties of the equation-of-state i.e. its stiffness

[4] Jean-Yves Ollitrault, Phys. Lett. **B 273** (1991)

Conclusions

- a few percent fluctuations at the initial size of the collision explains the bulk of the experimental $\langle p_T \rangle$ fluctuations (even too much!)
- proper scaling with the number of wounded nucleons $\sigma_{dyn}/\langle\langle p_T \rangle\rangle \sim 1/\sqrt{N_W}$ – proper dependence on centrality
- a weak dependence on energy
- our $\langle p_T \rangle$ fluctuations should be considered as the main geometric background for studying further effects like: (mini) jets, clusters, temperature fluctuations, etc.
- average information on P/ε according to Ollitrault's formula

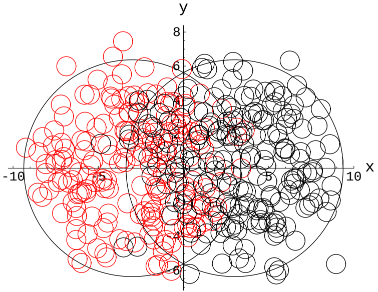
backup slides

GLISSANDO

GLauber Initial-State Simulation AND mOre

The algorithm:

- nucleon positions generated according to the Woods-Saxon distribution,
- a short-range repulsion is simulated by keeping the distance ($d \geq 0.4$ fm) between the nucleons,



Overlapping nucleons in the transverse plane

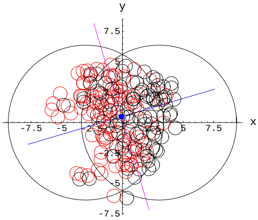
GLISSANDO

Nuclear density profiles

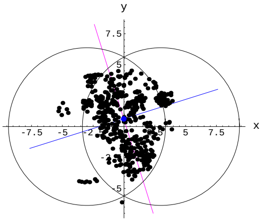
Nucleons interact if the distance $d = \sqrt{\sigma_{\mathrm{NN}}/\pi}$.
Three models for constructing the nuclear density profile are consider:

- Wounded Nucleons [Bialas, Bleszynski, Czyz, 1976],
- Binary Collisions,
- mixture of the two above, where α is the fraction of the binary collisions taken.

The inelastic cross-section σ_{NN} varies from 32 mb (SPS), 42 mb (RHIC) to 63 mb at the LHC.



Wounded Nucleons



Binary Collisions

Hydrodynamics with statistical hadronization

Initial condition

initial transverse energy density profile — Gaussian fit to GLISSANDO

$$\varepsilon(x,y) = \varepsilon_0(T_i)\exp\left(-\frac{x^2}{2a^2} - \frac{y^2}{2b^2}\right)$$

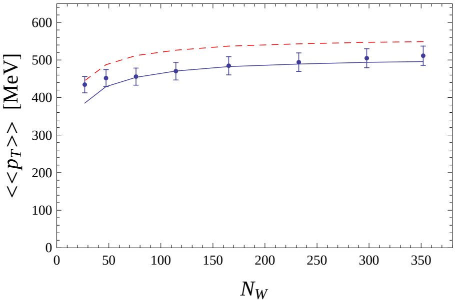
- parameters a, b and T_i depend on centrality,
- eccentricity fluctuations are included,
- a and b are fitted to reproduce the GLISSANDO's $\langle x \rangle$ and $\langle y \rangle$,
- T_i is fitted to reproduce the correct particle multiplicity

c	[%]	0-5	5-10	10-20	20-30	30-40	40-50	50-60	60-70
a	[fm]	2.70	2.54	2.38	2.00	1.77	1.58	1.40	1.22
b	[fm]	2.93	2.85	2.74	2.59	2.45	2.31	2.16	2.02
T_i	[MeV]	500	491	476	455	429	398	354	279

Results

average transverse momentum

- event-averaged transversed momentum $\langle\langle p_T \rangle\rangle$



- solid line: averaged over whole p_T range,
  dashed line: STAR cuts 0.2 GeV < p_T < 2 GeV
- experimental points from STAR Collaboration Phys. Rev. **C** 79, 034909 (2009) extrapolated to full p_T range