Multiplicity fluctuations as signature of the temperature fluctuations

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(*) *G.Wilk, Z.Włodarczyk,*

[#] Multiplicity fluctuations due to the temperature fluctuations in high-energy nuclear collisions *PRC79(2009)054903*

[#] Power laws in elementary and heavy-ion collisions; - A story of fluctuations and nonextensivity? - *EPJA40(2009)299*

[#] Fluctuations, correlations and the nonextensivity *Physica A376(2007)279*

(*) *M.Rybczyński, Z.Włodarczyk, G.Wilk,*

[#] Possible signal for critical point in hadronization processes *APPB35(2004)819*

(*) *O.V.Utyuzh, G.Wilk, Z.Włodarczyk,*

[#] Limitations of the Φ measure of fluctuations in event-by-event analysis *PRC64(2001)027901*

Fluctuations

(*) The large (and increasing) number of particles produced make it possible to study fluctuations in different physical observables on event-by-event basis

(*) Fluctuations are potentially very important source of information on the thermodynamic properties of strongly interacting systems formed in heavy ion collisions like, for example, specific heat, its chemical potential, matter compressibility,….

(*) Fluctuations of multiplicity observed in heavy ion collisions exhibit spectacular and unexpected features as function of the number of participants:

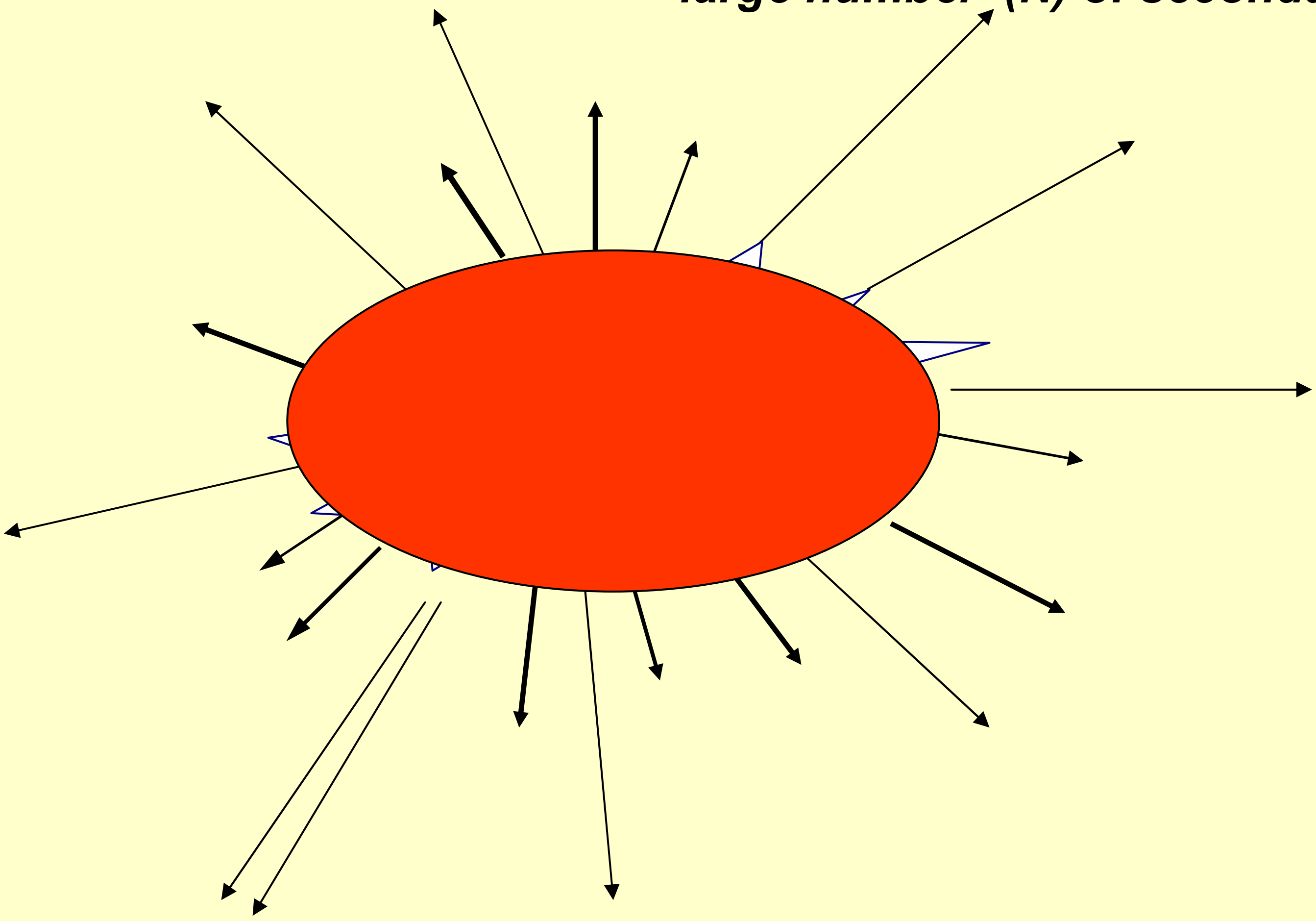
scaled variance of the multiplicity distributions, $Var(N)/\langle N\rangle$, increases when proceeding from the central toward peripheral collisions, i.e., when the number of participants N_P decreases.

(*) So far models aimed at describing all essential features of multiparticle production processes fail to describe the above observation of behaviour of $Var(N)/\langle N\rangle$ versus N_P.

(*) We shall analyse fluctuations using nonextensive statistical mechanics approach.

Why statistical approach …

Hadronic production processes:
large number (N) of secondaries is produced



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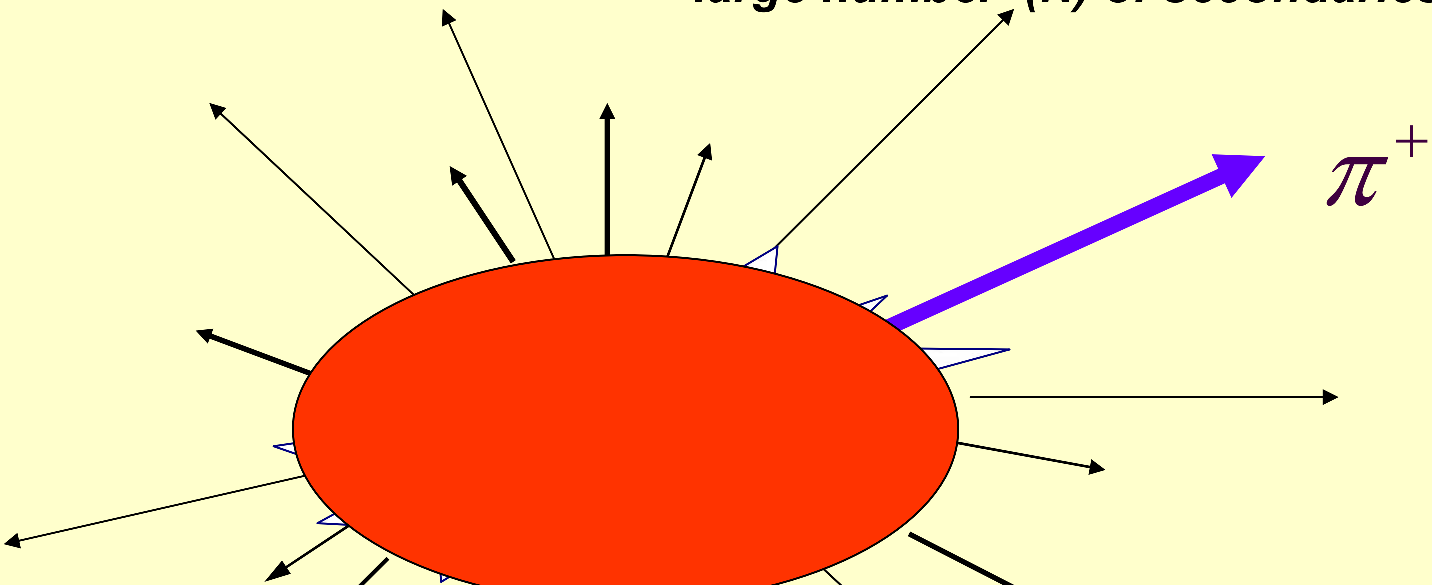

... but usually only one of them is observed and single particle distributions are formed

$$\left.\frac{d\sigma}{d^3p}\right|_{\pi^+} = f\left(\vec{p}_T, p_L\right)$$

L.Van Hove, Z.Phys. C21 (1985) 93, Z.Phys. C27 (1985) 135.

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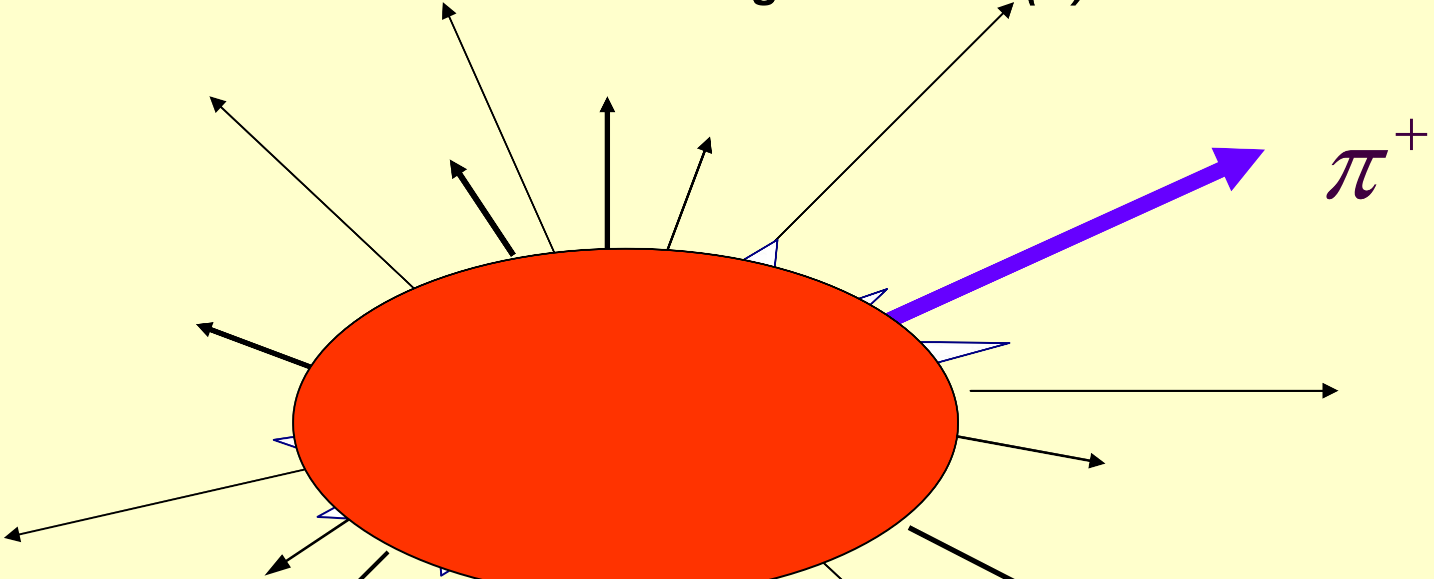

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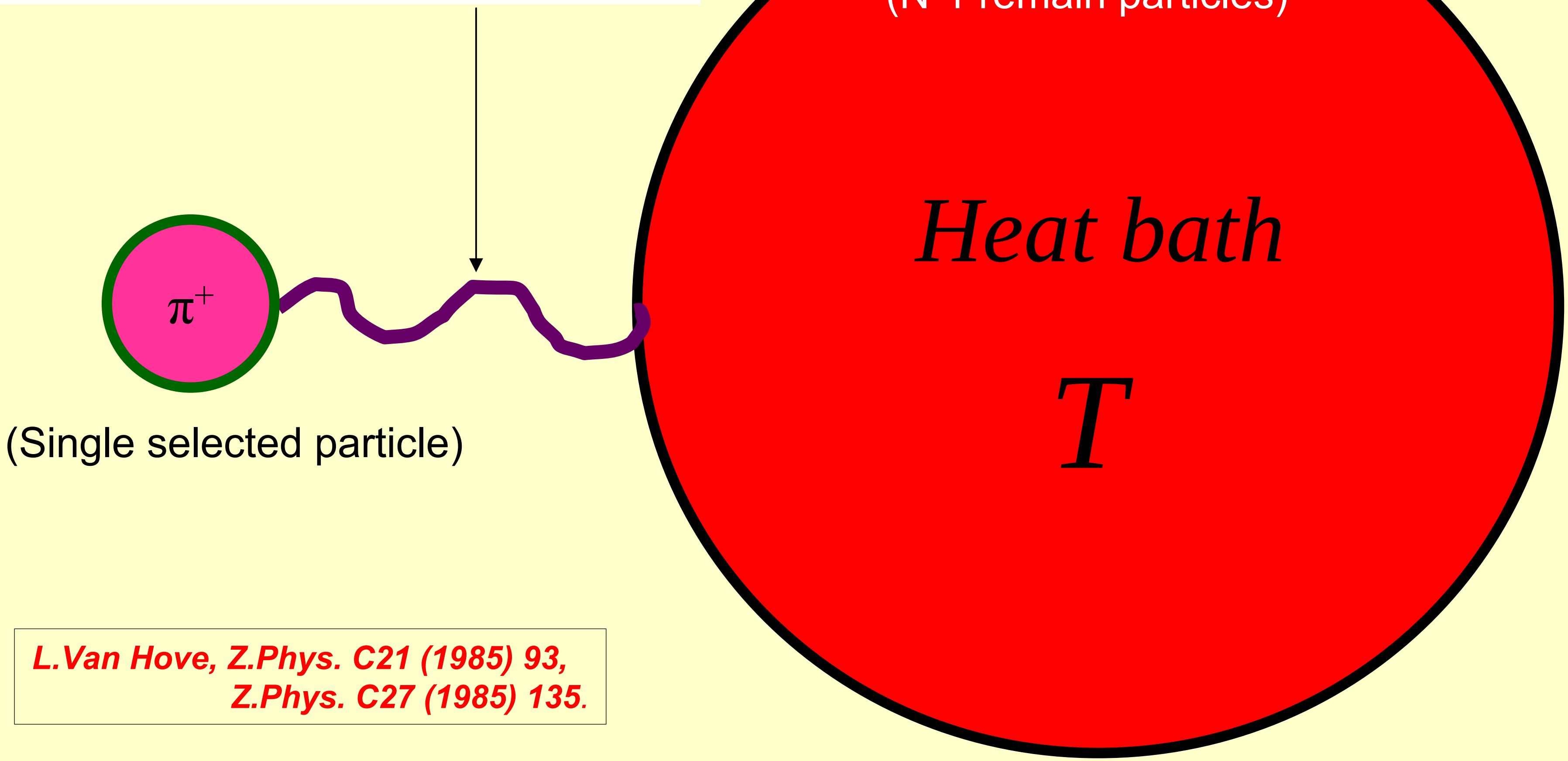

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$$\left.\frac{d\sigma}{d^3p}\right|_{\pi^+} = f\left(\vec{p}_T, p_L\right) = C\cdot\exp\left(-\frac{E}{T}\right) = C\cdot\exp\left(-\frac{\sqrt{\vec{p}_T^{\,2}+p_L^2+m^2}}{T}\right)$$

But: In such ”thermodynamical” approach one has to remember tacit assumptions of infinity and homogenity made when formulating this approach - only then behaviour of the observed particle will be characterised by a single parameter - the ”temperature” T

In reality: This is true only approximately and in most cases we deal with systems which are neither infinite nor homogeneous

In both cases: Fluctuations occur and new parameter(s) in addition to T is(are) necessary

Can one introduce it keeping simple structure of statistical model approach?

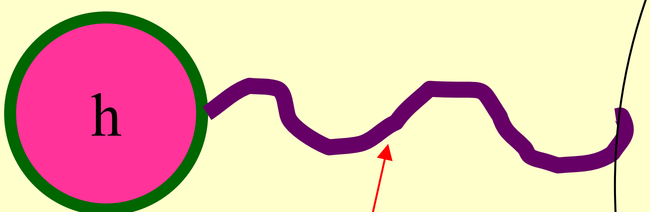
Yes, one can, by applying nonextensive statistical model.

[*G.Wilk, Z.Włodarczyk, EPJA40(2009)299*]

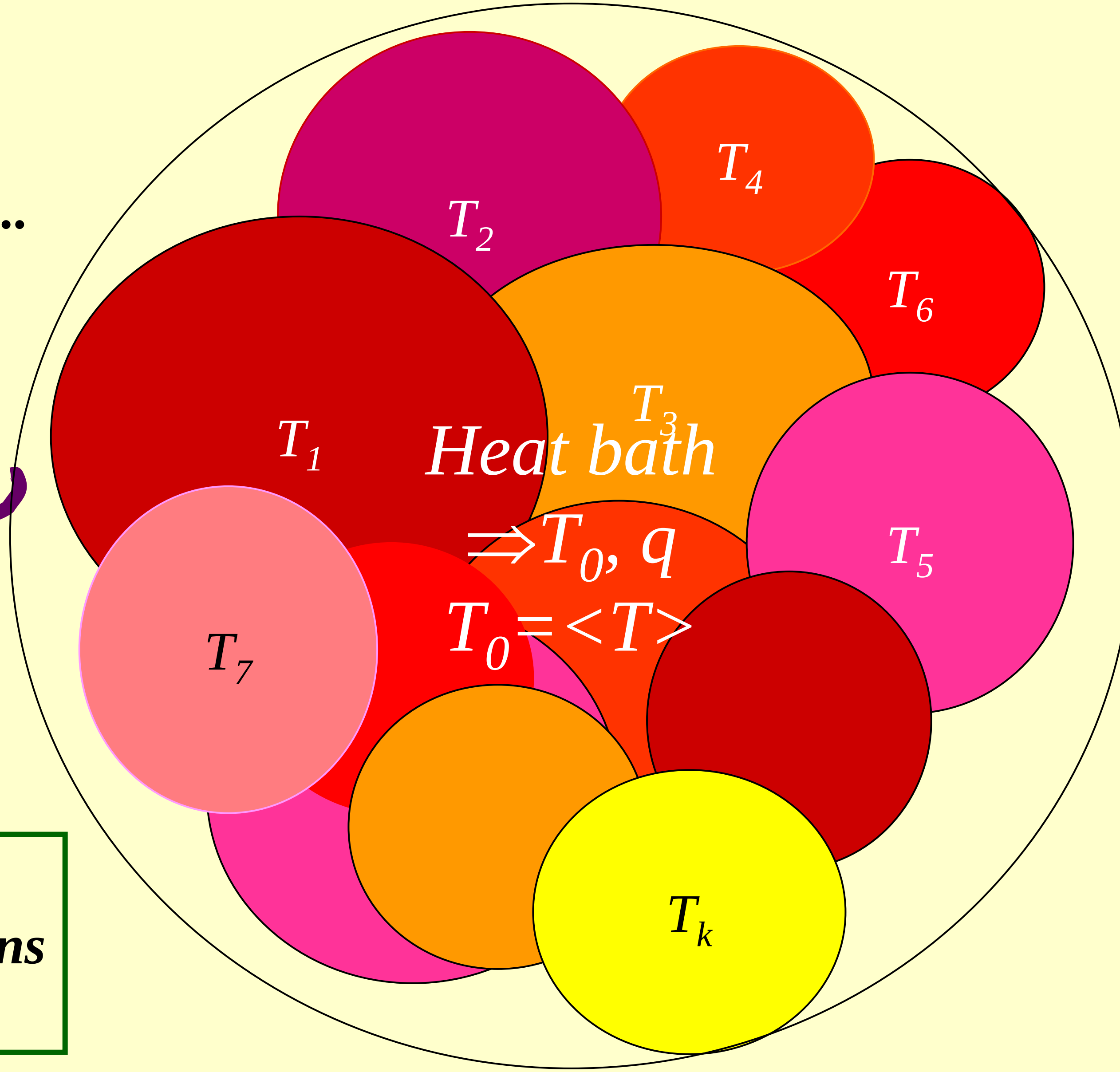
Heath bath is not homogeneous ......

***T* varies** $\Leftrightarrow$

intrinsic fluctuations...



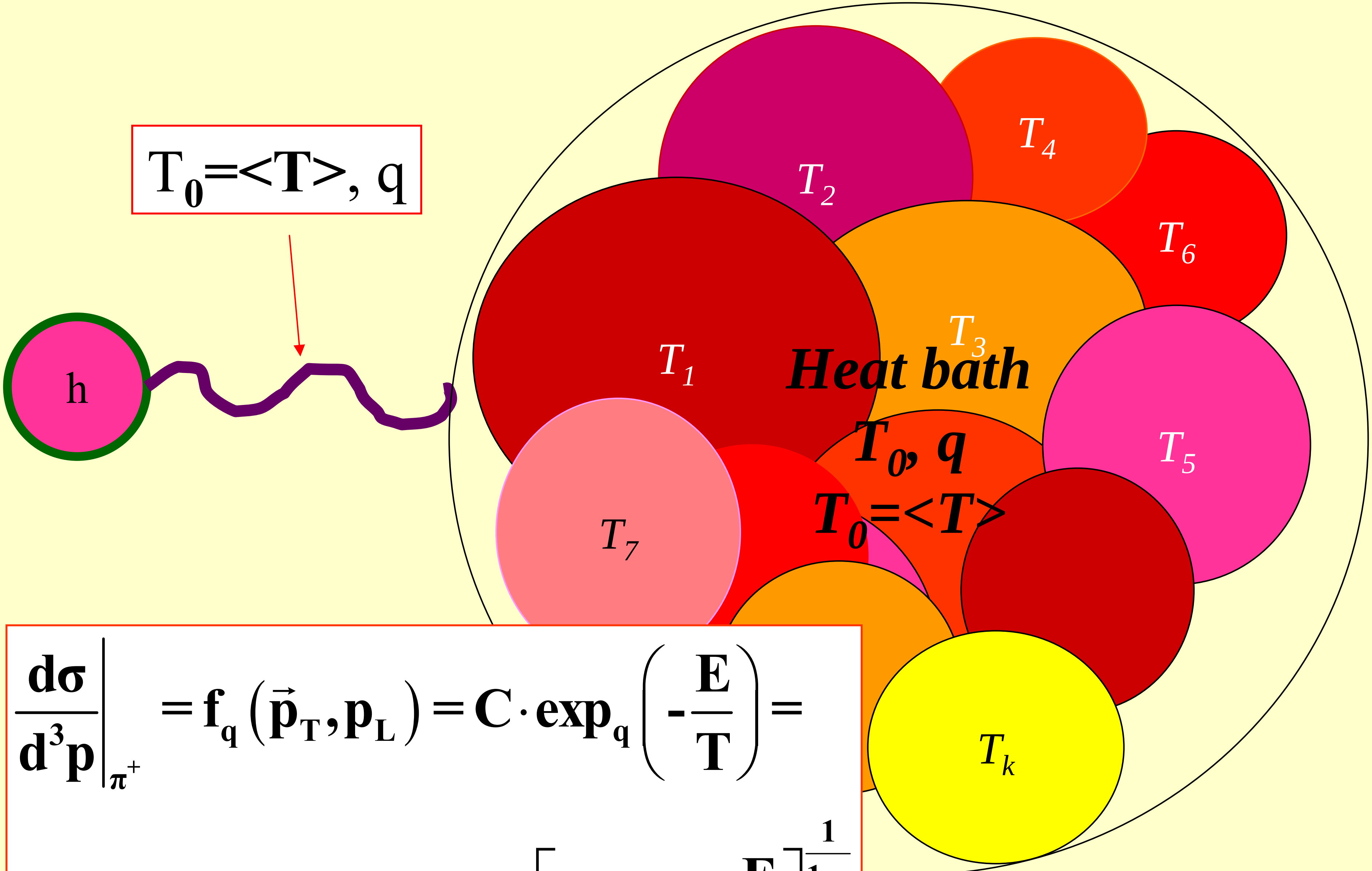
$T_0 = \langle T \rangle$, q



q = 1+Var(1/T)/<1/T>²

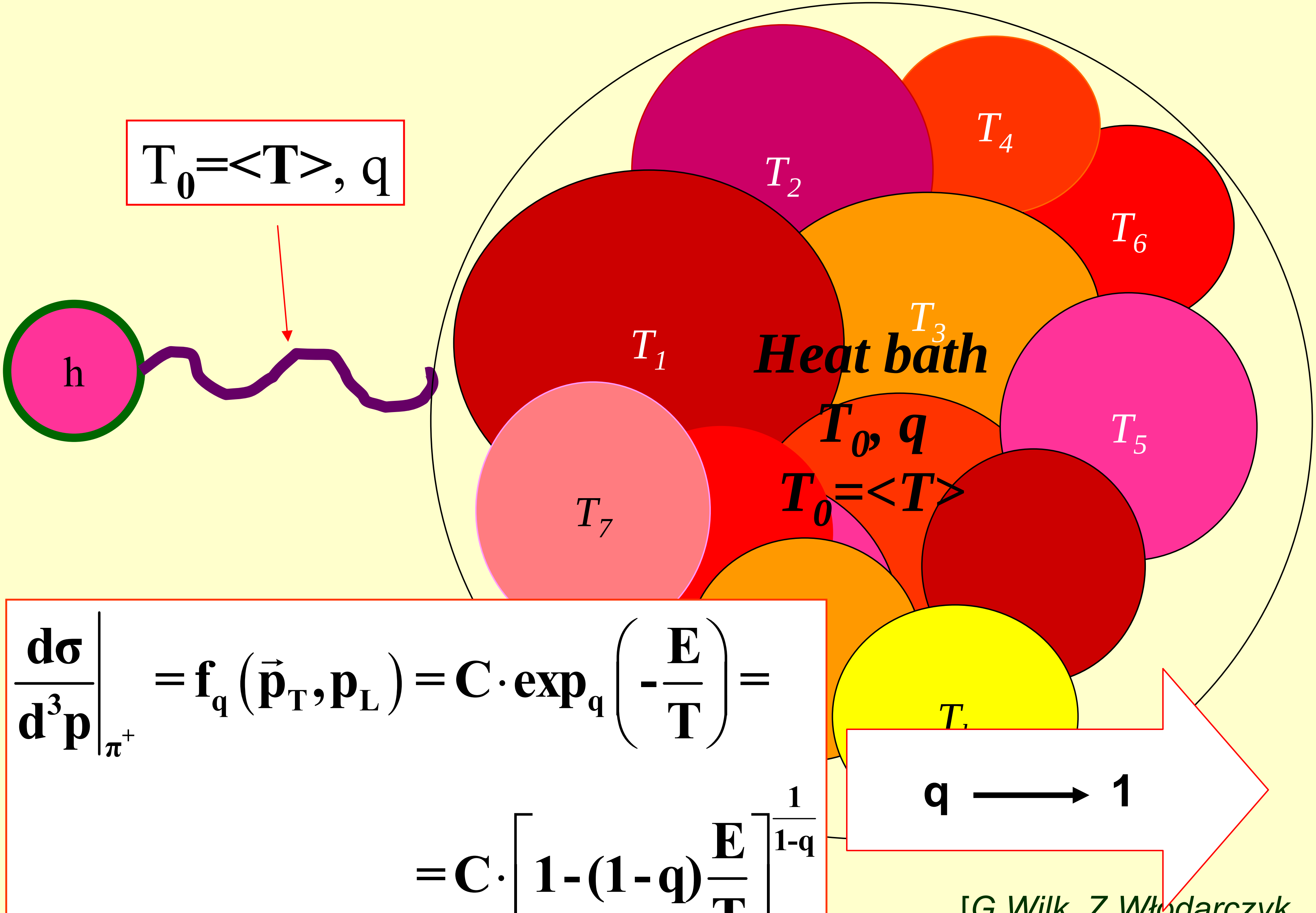
- measure of fluctuations

q-statistics (Tsallis)



$$\left.\frac{d\sigma}{d^3p}\right|_{\pi^+} = f_q\left(\vec{p}_T, p_L\right) = C\cdot \exp_q\left(-\frac{E}{T}\right) = C\cdot\left[1-(1-q)\frac{E}{T}\right]^{\frac{1}{1-q}}$$

[*G.Wilk, Z.Włodarczyk, EPJA40(2009)299*]



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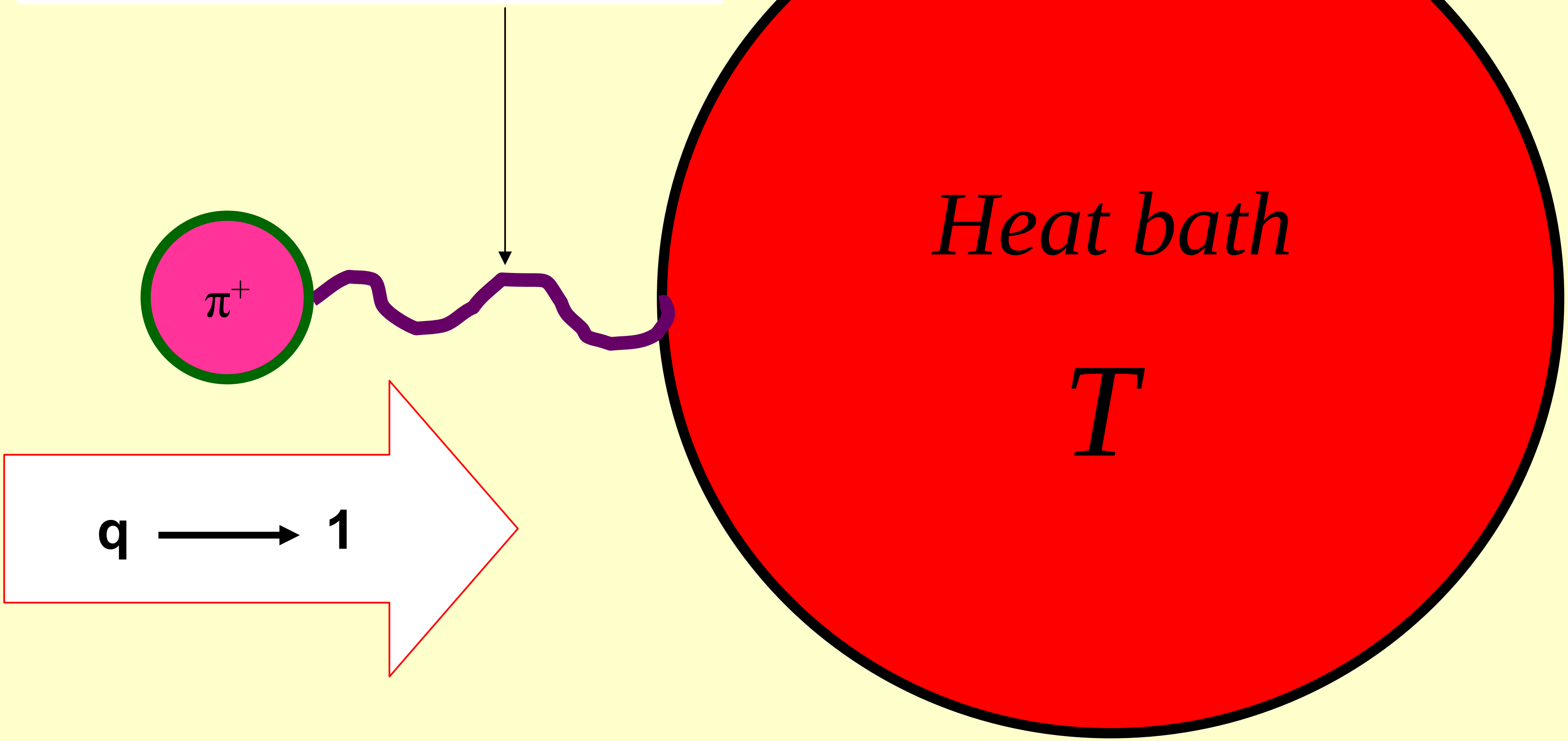
[G.Wilk, Z.Włodarczyk, EPJA40(2009)299]

Statistical approach emerges as simplest model independent tool with a single parameter T



$$\left.\frac{d\sigma}{d^3p}\right|_{\pi^+} = f\left(\vec{p}_T, p_L\right) = C \cdot \exp\left(-\frac{E}{T}\right) = C \cdot \exp\left(-\frac{\sqrt{\vec{p}_T^{\,2} + p_L^2 + m^2}}{T}\right)$$

In many cases there is still one more possibility, namely the

energy exchange between the local heat bath and a still bigger environment (surroundings).

In this case some energy can be transfered either

(*) to the system from this environment (as in cosmic rays case) or

(*) from the system to this environment (as in nuclear collisions case)

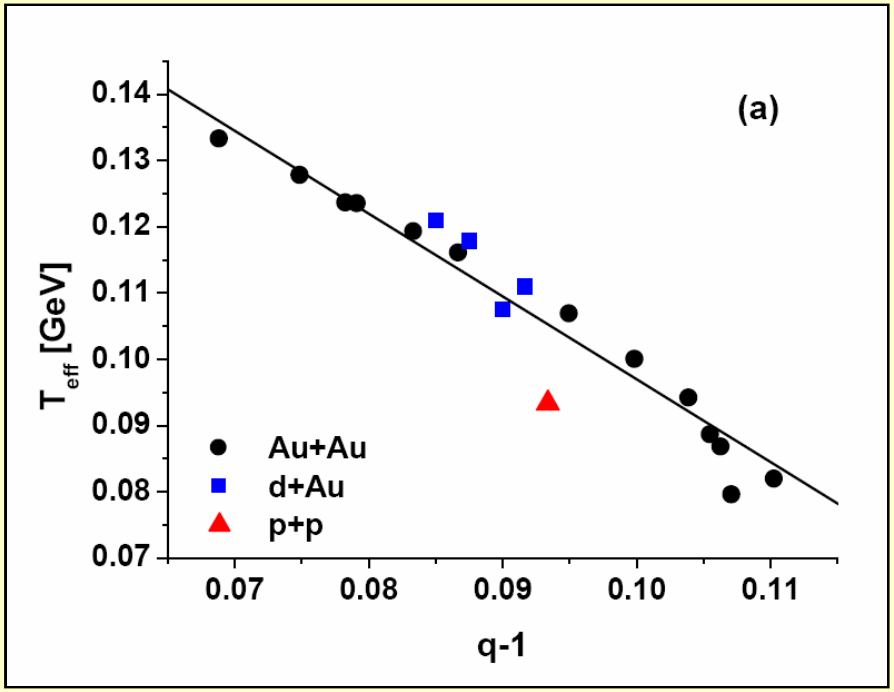
[WW, PRC79(2009)054903;EPJA40(2009)299; hep-ph/0904.0528].

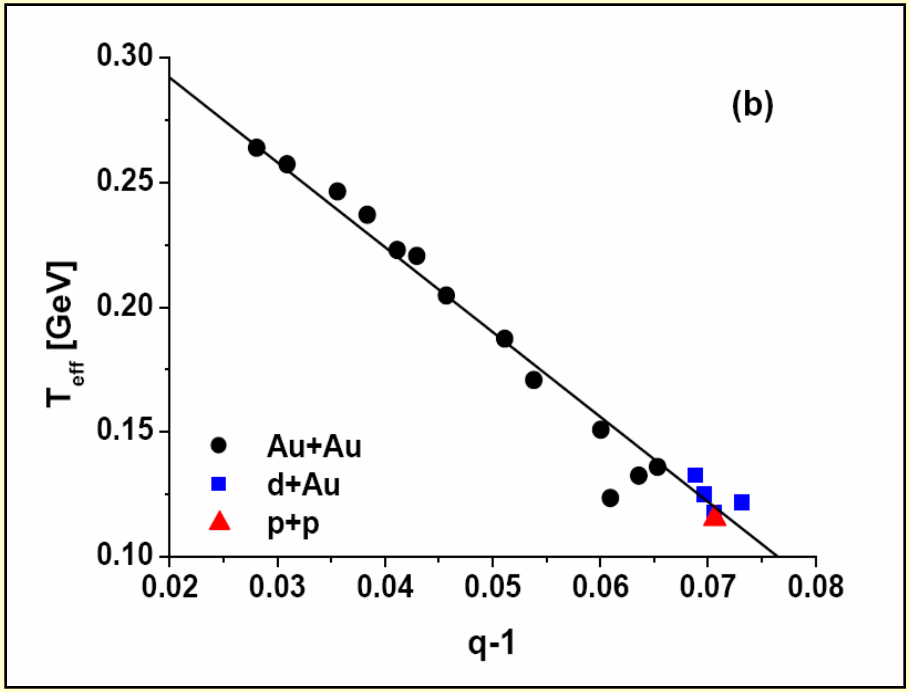
In this case $T \rightarrow T_{eff} = T_0 + (q-1)T_{visc}$ **with other formulae remaing intact. Notice the new parameter,** T_{visc}**, here.**

★

Not covered here: Other types of fluctuations concerning, for example, the whole energy available (inelasticity); notice that they are fluctuations from event to event rather than in a given event.

Illustration of $T_{eff} = T_0 + (q-1)T_{visc}$ phenomenon

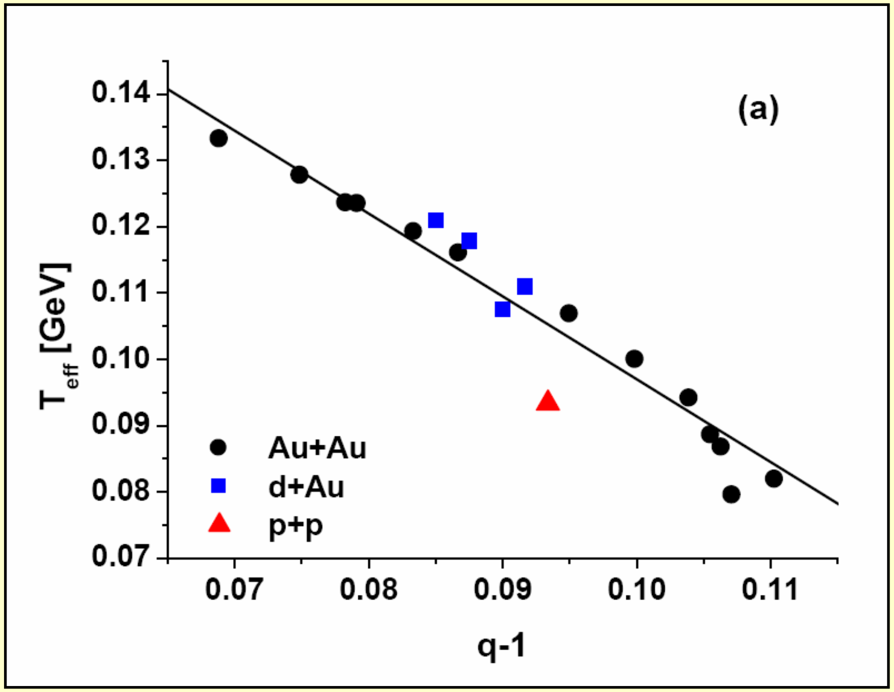


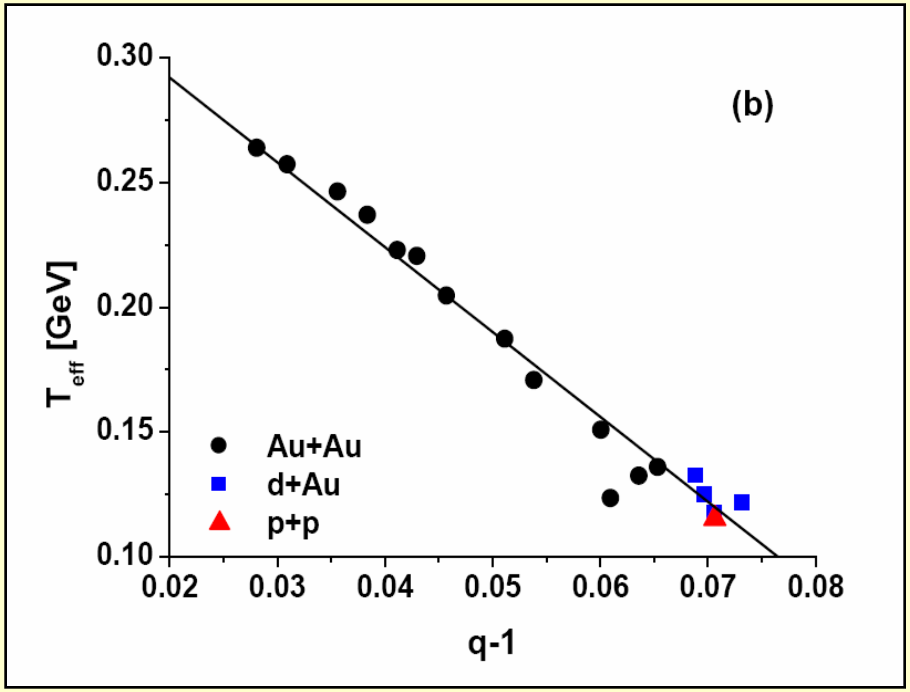


From RHIC data on 200 GeV for p_T distributions of pions (a) and antiprotons (b)

$T_{eff}=0.22-1.25(q-1)$ for (a) and $T_{eff}=0.36-3.4(q-1)$ for (b)

Illustration of $T_{eff} = T_0 + (q-1) T_{visc}$ phenomenon





Notice that T_{visc} <0, i.e., the energy is transferred from the interaction region to the surroundings – here represented by the spectators (noninteracting nucleons)

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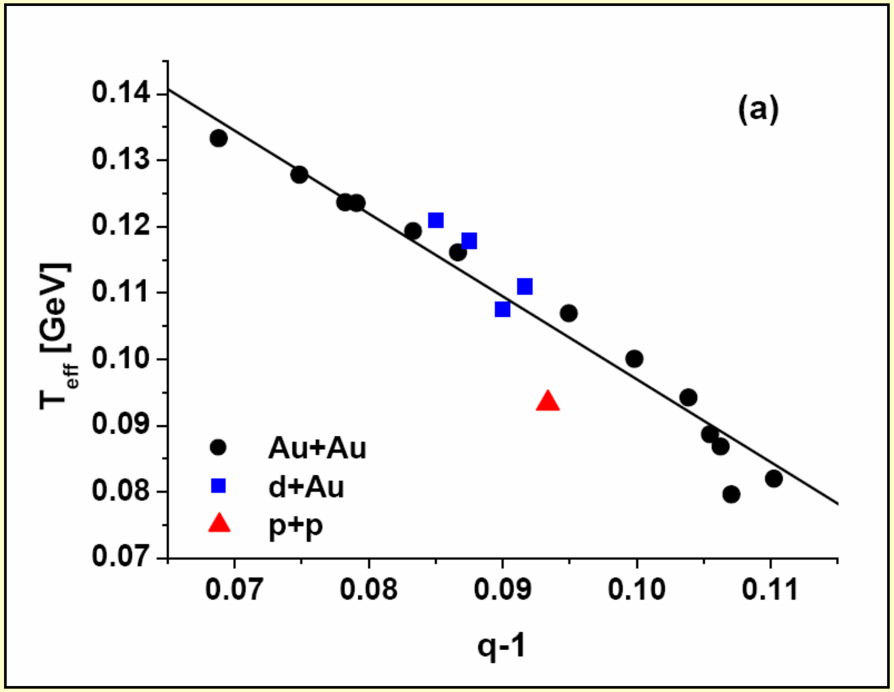
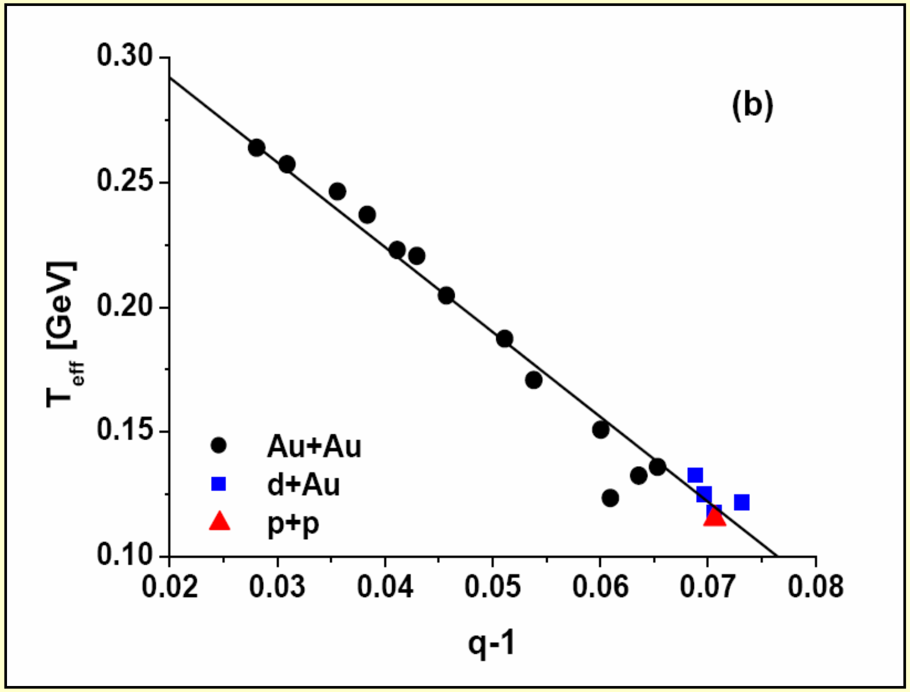




Notice that $T_{visc} < 0$, i.e., the energy is transferred from the interaction region to the surroundings – here represented by the spectators (noninteracting nucleons)

Also this will not be discussed here in detail

(cf. *G.Wilk, Z.Włodarczyk, PRC79(2009)054903; EPJA40(2009)299* **)**

♦ Multiplicity fluctuations

Multiplicity fluctuations are essentially the main (if not the only one …) source of information on the fluctuations taking place in the hadronizing system.

Notice: P(N) reflect global properties of the system whereas p_T distribution are sensitive rather to the local thermal-like properties. The corresponding values of q-1 are different: $(q-1) \gg (q_T-1)$. In what follows we shall use only q.

Parameter q measures also dynamical fluctuations in P(N)

(*) **Experiment:** P(N) is adequately described by NBD depending on <N> and k (k≥1) affecting its width:

$$\frac{1}{k} = \frac{\sigma^2(N)}{<N>^2} - \frac{1}{<N>}$$

(*) If 1/k is understood as a measure of fluctuations of <N>, then

$$P(N) = \int_0^\infty d\bar{n} \frac{\bar{n}^n \exp(-\bar{n})}{n!} \cdot \frac{\gamma^k \bar{n}^{k-1} \exp(-\gamma \bar{n})}{\Gamma(k)}$$

$$= \frac{\Gamma(k+n)}{\Gamma(1+n)\Gamma)\Gamma(} \cdot \frac{\gamma^k}{(\gamma+1)^{k+n}} \qquad \text{with} \qquad \gamma = \frac{k}{<\bar{n}>}$$

(P.Carruthers,C.C.Shih, Int.J.Phys. A4 (1989)5587)

$$\frac{1}{k} = D(\bar{n}) = \frac{\sigma^2(\bar{n})}{<\bar{n}>^2} = q - 1$$

(*) → one expects: q=1+1/k what indeed is observed

[G.Wilk, Z.Włodarczyk,EPJA40(2009)299; F.Navarra,O.Utyuzh,WW, PRD67(2003)114002]

Parameter q measures also dynamical fluctuations in P(N)

Digression:

C.Vignat and A.Plastino in

Estimation in a fluctuating medium and power-law distributions ,

PlA360(2007)415,

demanded to give credit to

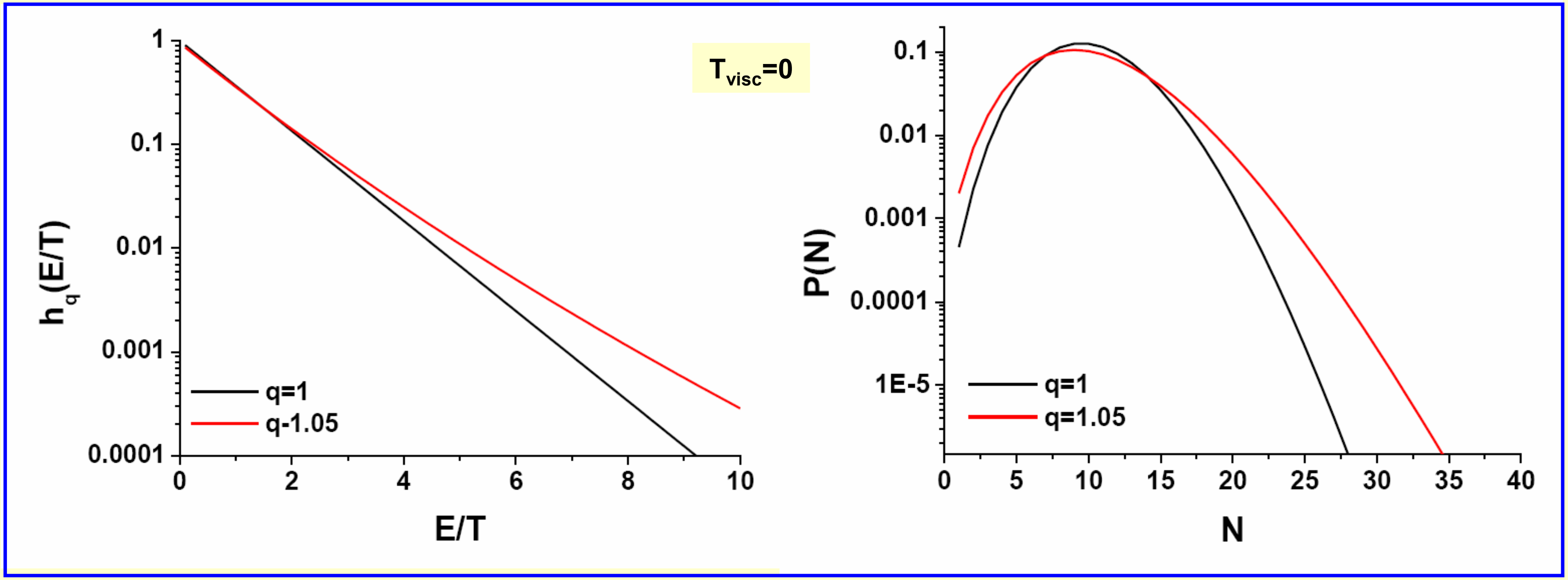
R.A.Fisher, Ann.Eugenics 11(1941)182, available at

http://digital.library.adelaide.edu.au/coll/special/fisher/stat_math.html

as the first person to propose and discus such connection between

the Poisson and NB distributions.

Temperature fluctuations vs. Multiplicity fluctuations



$$g(E_i) = C\exp\left(-\frac{E_i}{T}\right) \Rightarrow$$

$$h_q(E_i) = C_q\left[1-(1-q)\frac{E_i}{T_{eff}}\right]^{\frac{1}{1-q}}$$

where $q = 1 + \frac{Var(T)}{\langle T\rangle^2}$

and $T_{eff} = T_0 + (q-1)T_{visc}$

$$P(N) = \frac{\langle N\rangle^N}{N!}\exp(-\langle N\rangle); \quad \langle N\rangle = \frac{E}{T} \Rightarrow$$

$$P(N) = \frac{\Gamma(N+k)}{\Gamma(N+1)\Gamma(k)}\frac{\left(\frac{\langle N\rangle}{k}\right)^N}{\left(1+\frac{\langle N\rangle}{k}\right)^{N+k}}$$

where $k = \frac{1}{q-1}$ *and* $Var(N) = \langle N\rangle + (q-1)\langle N\rangle^2$

[G.Wilk, Z.Włodarczyk, PRC79(2009)054903; EPJA40(2009)299; Physica A376(2007)279]

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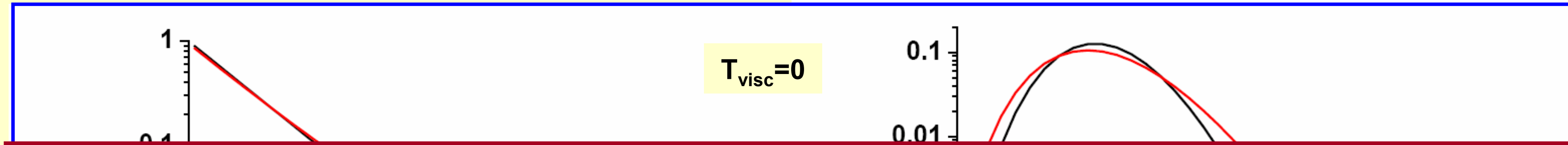

Whenever N variables $E_{1,\ldots,N}$ follow exponential distribution and satisfy condition

$$\sum_{i=0}^{N} E_i \le E \le \sum_{i=0}^{N+1} E_i$$

then the corresponding multiplicity N has a Poissonian distribution.

If, instead, they follow the q-exponential distribution,
then N has negative binomial distribution

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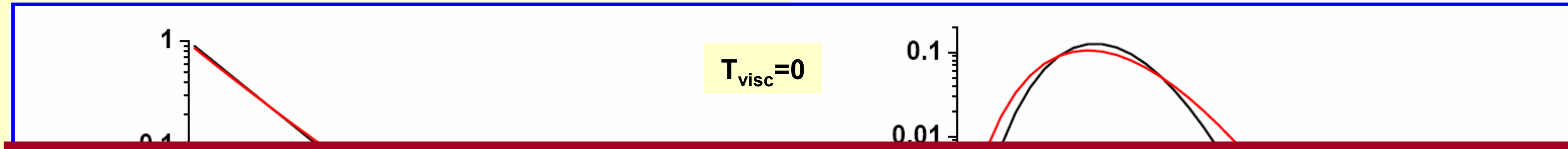

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or $$\frac{Var\left(\frac{1}{T}\right)}{\left\langle \frac{1}{T}\right\rangle^2} = q-1 = \frac{\frac{Var(N)}{\langle N\rangle} - 1}{\langle N\rangle}$$

For a system with finite size remaining in contact with a heat bath one has a kind of uncertainty relation (in the sense that it expresses the truth that in the case of conjugate variables one standard deviation in some measurement can only become small at the expense of the increase of some other standard deviation):

$$\mathrm{Var(U)} + C_V^2\,\mathrm{Var(T)} = \langle T \rangle^2 C_V \qquad [\bullet]$$

This is valid all the way from the canonical ensemble where

$\mathrm{Var(T)} = 0$ and $\mathrm{Var(U)} = \langle T \rangle^2 C_V$

to the microcanonical ensemble where

$\mathrm{Var(U)} = 0$ and $\mathrm{Var(T)} = \langle T \rangle^2 / C_V$

[•] expresses both the complementarity between the temperature and energy, and between the canonical and the microcanonical description of the system.

Canonically distributed system: $Var(U) = \langle T\rangle^2 C_V$

Isolated system: $Var(U) = 0$

Let us assume that realistic (intermediate) case interpolates linearly between these two and that the energy fluctuations In the system is equal: $Var(U)=\langle T\rangle^2 C_V\,\xi$

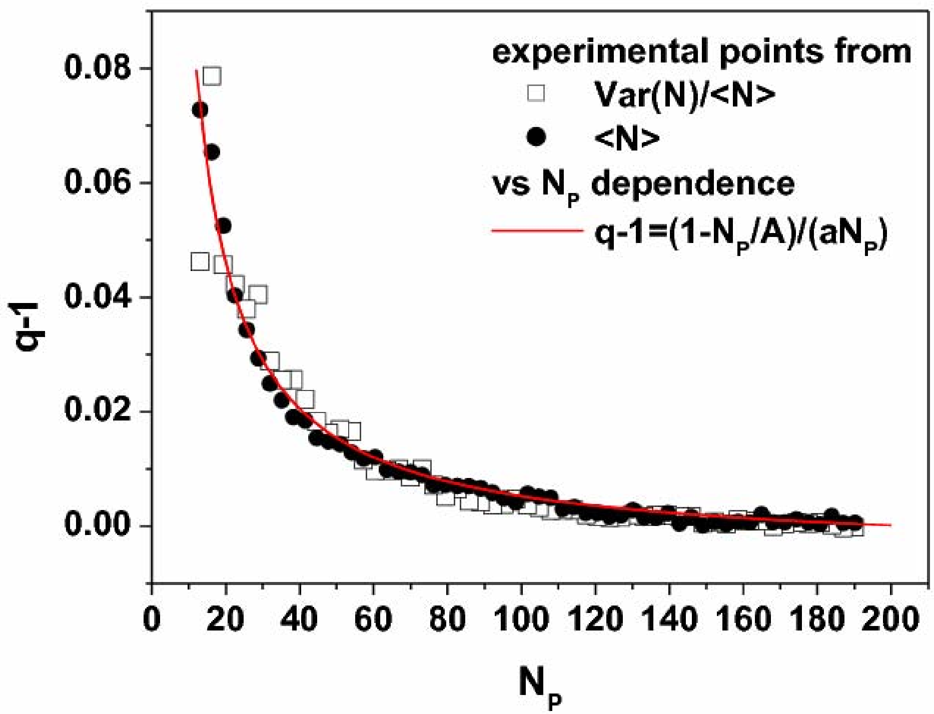
$0 \le \xi \le 1$

Parameter ξ depends on the size of the hadronizing source and is assumed to be given by the number N_P of nucleons participating in the collision: $\xi \approx N_P/A$. Because $C_V \approx aN_P$ we obtain

$$q-1=\frac{Var(T)}{\langle T\rangle^2}=\frac{1-\xi}{C_V}=$$

$$=\frac{1}{aN_P}\left(1-\frac{N_P}{A}\right)$$

Centrality dependence of fluctuations (measured by q-1)



$Var(U) = \langle T\rangle^2 C_V$

$Var(U) = 0$

$Var(U)=\langle T\rangle^2 C_V \xi$

$0 \leq \xi \leq 1$

$$q-1=\frac{1}{aN_P}\left(1-\frac{N_P}{A}\right) \quad \textit{where} \quad \xi \approx \frac{N_P}{A} \ \textit{and} \ C_V \approx aN_P$$

Universal participant dependence of multiplicity fluctuations

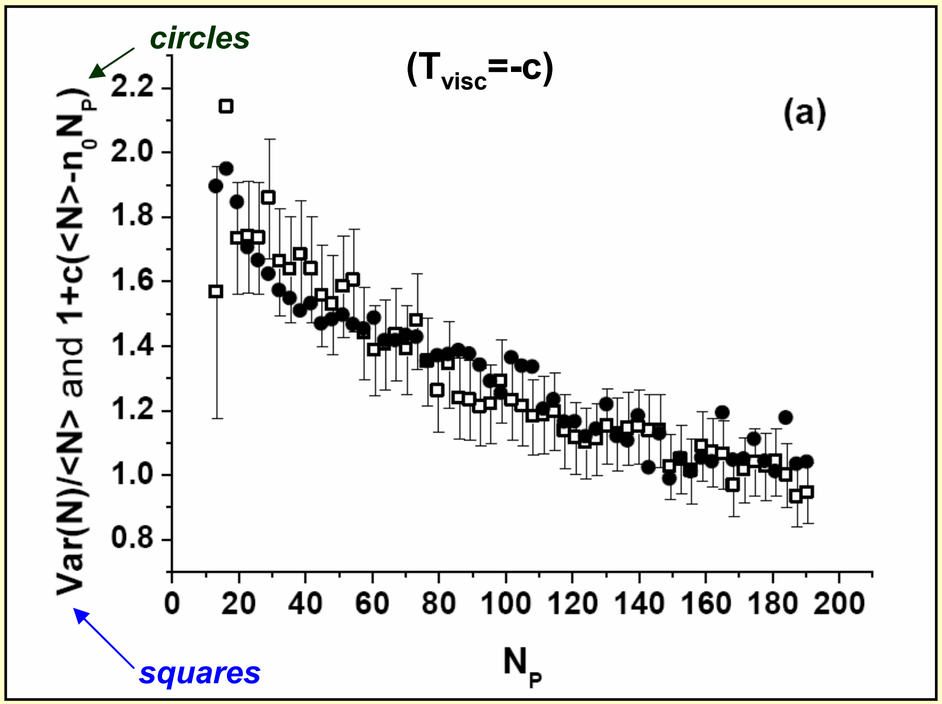
We expect that:

$$Var(N)/\langle N\rangle - 1 = \langle N\rangle(q-1)$$

If U is the accessible energy and gT_{eff} is energy per particle (with g being a parameter and $T_{eff} = T_0 + (q-1)T_{visc}$) then $\langle N\rangle = \langle U\rangle/[gT_{eff}]$ and

n_0 is the multiplicity in the single NN collision measured in the region of acceptance and defined by constraint $\langle U\rangle = n_0 g T_0 N_P$

$$\frac{Var(N)}{\langle N\rangle} = 1 - T_{visc}\left(\langle N\rangle - n_0 N_P\right)$$



Notice:

(*) Var(N)/<N> and <N> are mutually connected

(*) Var(N)/<N> = 1 if $T_{visc} = 0$ (or $T_{eff} = T_0$) or if $\langle N\rangle = n_0 N_P$

Therefore observed decreasing of Var(N)/<N> with N_P indicates nonlinear dependence of <N> on the number of participants N_P

♦ Multiplicity fluctuations

To recapitulate:

(*) we know that if U=const and T=const then P(N) is Poissonian

(*) we know how fluctuations of T lead to NB multiplicity distributions

(*) we want to see how big are fluctuations of T in our hadronizing systems formed in a collision process:

Assuming that the variance expressing energy fluctuations in the system for the intermediate case is equal to $Var(U) = \langle T \rangle^2 C_V \xi$, $0 < \xi < 1$, ***where parameter*** ξ ***depends on the size of the hadronizing source, one gets finally that*** $q - 1 = (1 - \xi)/C_V$

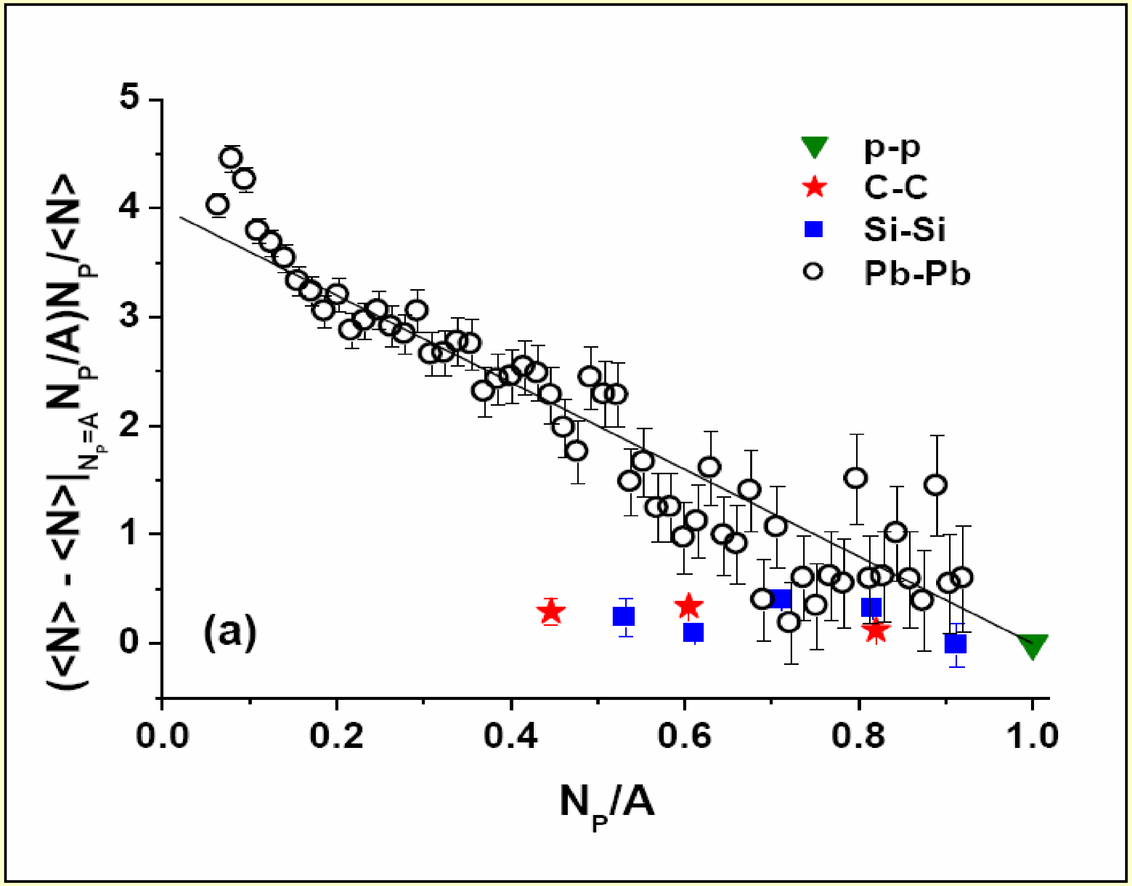
(*) knowing how big the fluctuations of T are, we attempt to deduce the fluctuations of the multiplicity N.

♦ Multiplicity fluctuations: *- system size dependence [1]*

The system size enters through C_V and $\xi \approx N_P/A$:

- for $C_V = a N_P$

$$\left(\langle N \rangle - \frac{N_P}{A} \langle N \rangle \Big|_{N_P = A} \right) \frac{N_P}{\langle N \rangle} = \frac{c}{a} \left(1 - \frac{N_P}{A} \right) = c N_P (q - 1)$$



$\langle N \rangle_{NP=A} = n_0 A$ is multiplicity extrapolated to $N_P = A$.
Notice different dependencies for Pb+Pb collision and lighter nuclei and deviation from the linear fit for most peripheral Pb+Pb collisions.

- for $C_V = a' \langle N \rangle$

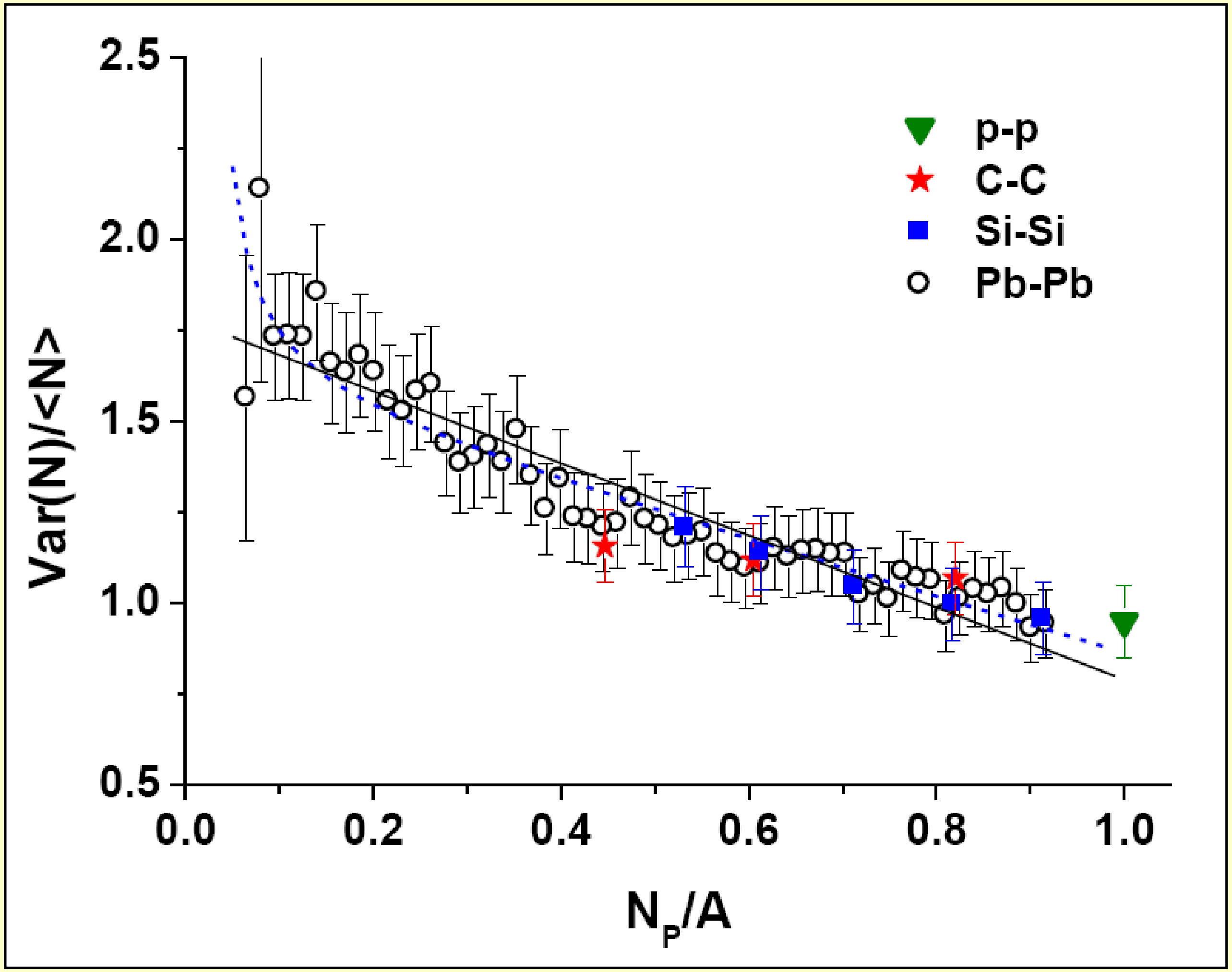
one gets similar picture

$$\left(\langle N \rangle - \frac{N_P}{A} \langle N \rangle \Big|_{N_P = A} \right) = \frac{c}{a'} \left(1 - \frac{N_P}{A} \right)$$

♦ Multiplicity fluctuations: *- system size dependence [2]*

One gets in our approach simple scaling relation (dashed line):

$$\frac{Var(N)}{\langle N\rangle} = 1 + \langle N\rangle(q-1) = 1 + \frac{n_0\left(1-\frac{N_P}{A}\right)}{a-\frac{c}{A}\left(\frac{A}{N_P}-1\right)}$$

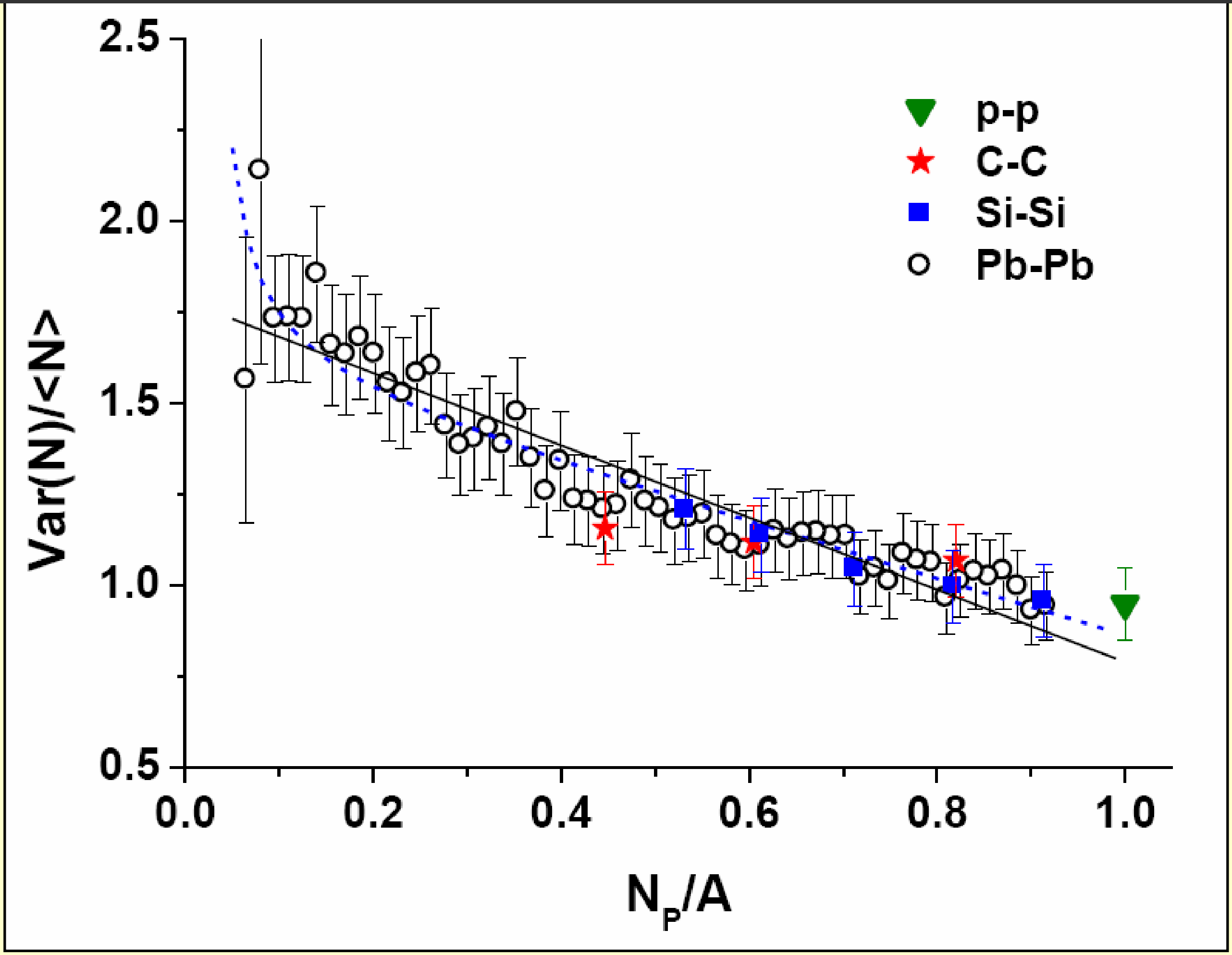


♦ Multiplicity fluctuations: *- system size dependence [2]*

The solid line represents the phenomenological fit to the simple formula

$$\frac{\mathrm{Var(N)}}{\langle \mathrm{N} \rangle} = \omega_0 + \frac{1}{a'}\left(1 - \frac{N_P}{A}\right) \quad where \quad \omega_0 = \left[\frac{\mathrm{Var(N)}}{\langle \mathrm{N} \rangle}\right]_{N_P = A}$$

(full line)
ω_0=0.79



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(full line) $\omega_0=0.79$

Parameter ω_0 (which within a statistical model with fixed volume varies in the range 0.5-1.0) accounts here for the possible effects expected from the following factors:

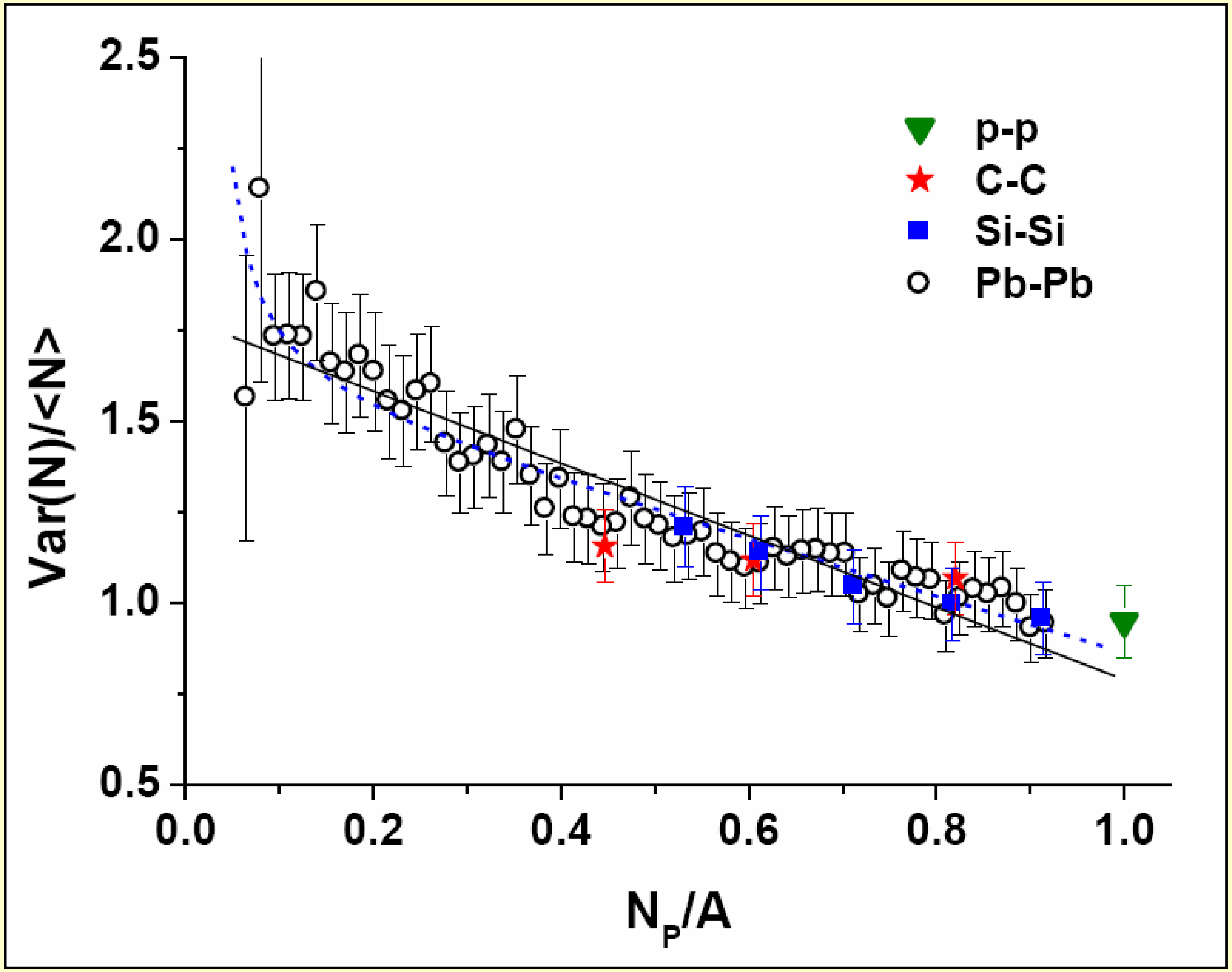
(1) The global conservation laws – when imposed on each microscopic state of the statistical system lead to suppression of the particle number fluctuation (by 0.8 for canonical ensemble down to 0.3 for microcanonical one).

(2) Effect of quantum statistics – it is observed in the primordial values of the scaled variables (i.e., calculated before decays of resonances), they enhance of particle number fluctuations by 1.05-1.06.

(3) The inclusion of known resonance decays (in either ensemble) results in the enhancement of multiplicity fluctuations by about 1.1 (depending on the acceptance).

♦ Multiplicity fluctuations: *- system size dependence [2]*

$$\frac{Var(N)}{\langle N \rangle} = 1 + \langle N \rangle (q-1) = 1 + \frac{n_0 \left(1 - \frac{N_P}{A}\right)}{a - \frac{c}{A}\left(\frac{A}{N_P} - 1\right)}$$



♦ Multiplicity fluctuations: *- system size dependence [2]*

$$\frac{Var(N)}{\langle N\rangle} = 1 + \langle N\rangle(q-1) = 1 + \frac{n_0\left(1 - \frac{N_P}{A}\right)}{a - \frac{c}{A}\left(\frac{A}{N_P} - 1\right)}$$

Notice:

(*) We observe a weak dependence on the mass number A of colliding nuclei.

(*) For a small number of participants, N_P << A, we observe an additional increase of relative variance, Var(N)/<N>.

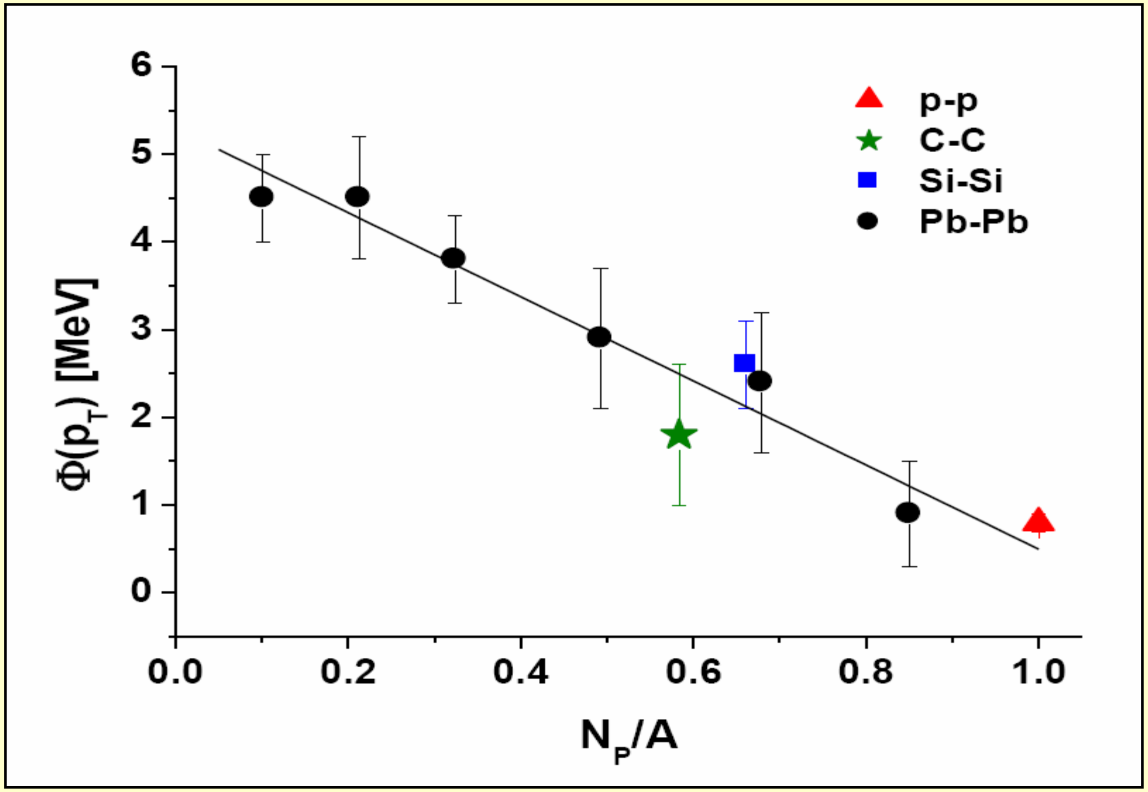
(*) However, should we use T_{visc} > 0 instead, one would observe the opposite trend – resulting in a nonmonotonic behavior of Var(N)/<N> with increasing number of participants N_P .

(The question about the sign of the heat source term or T_{visc} is still open because of the lack of data in the region of small number of participants).

♦ Some other results (1)

Transverse momentum fluctuations measured in nuclear collisions at 158 A GeV and quantified by the measure $\Phi(p_T)$ shows similar centrality dependence,

$$\Phi(p_T) = \Phi_{N_P=A} + b\left(1 - \frac{N_P}{A}\right) = \Phi_{N_P=A} + aN_P(q-1)$$



It is determined by the pattern of multiplicity fluctuations

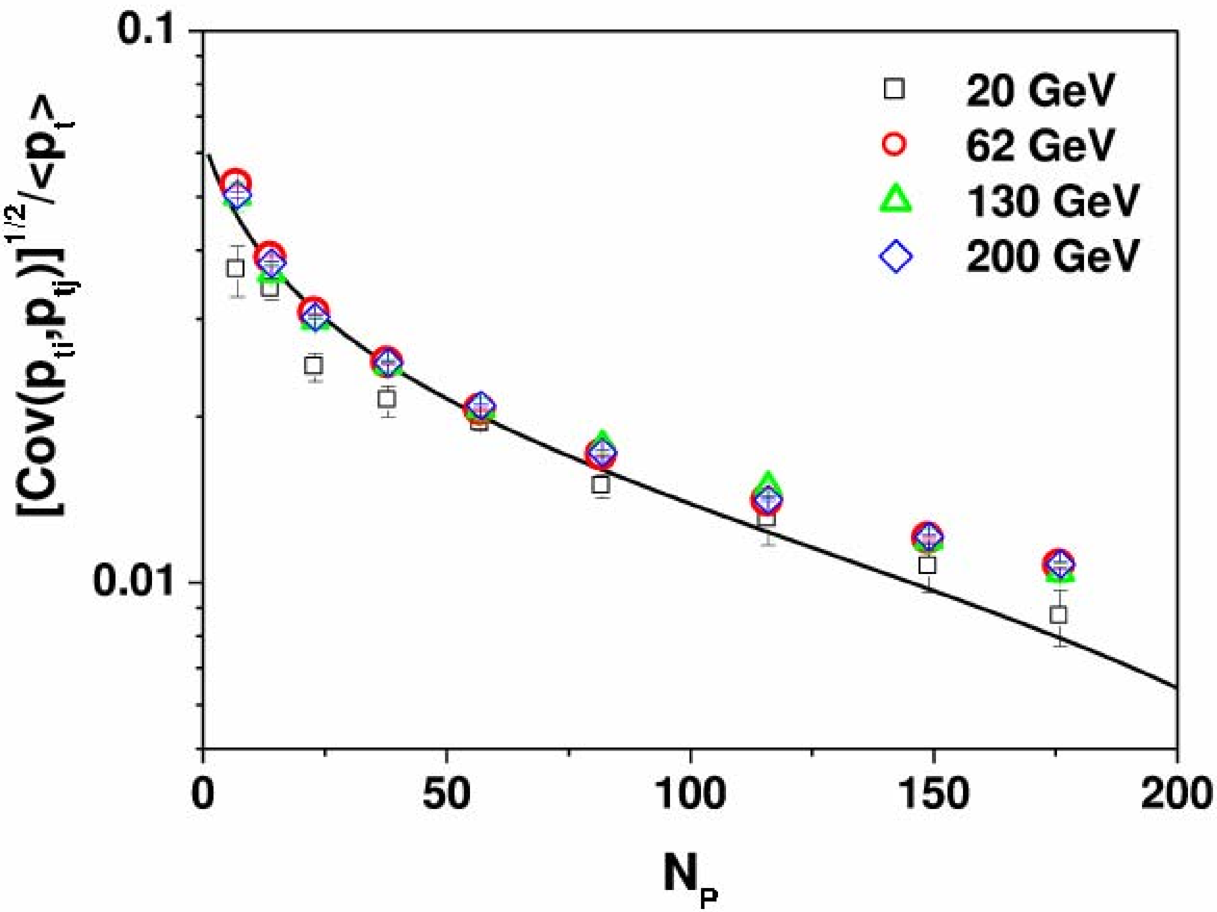
♦ Some other results (2)

Our predictions (full line) for the scaled covariance for:

$Var(p_T)/\langle p_T\rangle=0.43$
$\langle p_T\rangle=400$ MeV,
$b=4.8$ MeV
$n_0=0.642$,
$c=4.1$,
$a'=1$,
$\Phi_{Np=A}=1.44$ MeV
(*PRC79(2009)054903)*

Data points are from STAR ***(PRC72(2005)044902)***

$$\frac{Cov\left(p_{Ti},p_{Tj}\right)}{2\sqrt{Var\left(p_T\right)}} \cong \frac{\Phi_{N_P=A}+b\left(1-\frac{N_P}{A}\right)}{n_0 N_P+\frac{c+1}{a'}\left(1-\frac{N_P}{A}\right)} = \frac{\Phi_{N_P=A}+aN_P b(q-1)}{n_0 N_P+\frac{c+1}{a'}aN_P(q-1)}$$



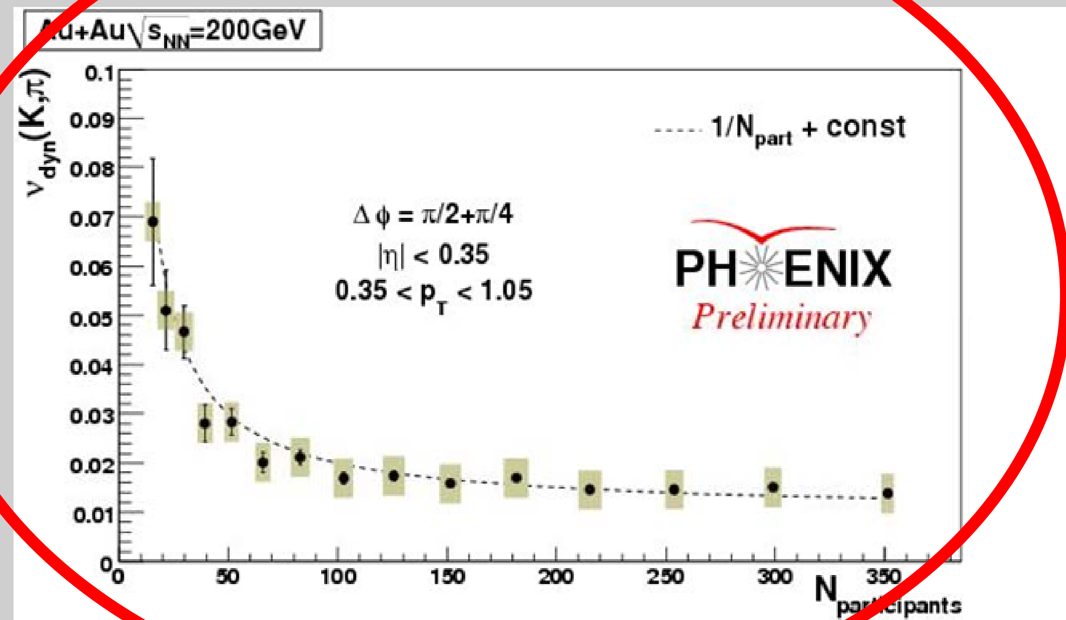
Ratio Fluctuations

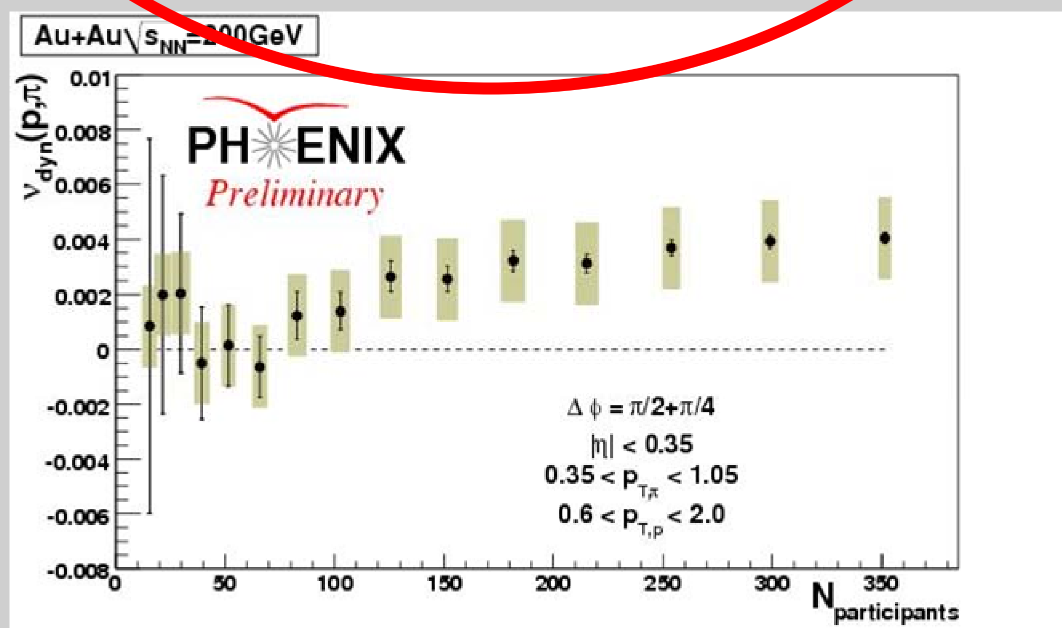
- **Some other results (3) – particle ratio fluctuations**

For example – as presented at QM08

$\langle K\pi\rangle$ / $K\rangle\langle\pi\rangle$

ies:





C. Pruneau, S.Gavin, S.Voloshin, Phys. Rev. C 66 044904 (2002)

Particle ratio fluctuation show no indication of critical behaviour

K.Homma, Session XVII, 02/09 15:20h
Poster by E.Haslum

♦ Some other results (3) – particle ratio fluctuations

$$\sigma^2_{N_\pi/N_K} = \frac{\left\langle (\Delta N_\pi)^2 \right\rangle}{\left\langle N_\pi \right\rangle^2} + \frac{\left\langle (\Delta N_K)^2 \right\rangle}{\left\langle N_K \right\rangle^2} - 2\frac{\left\langle \Delta N_\pi \Delta N_K \right\rangle}{\left\langle N_\pi \right\rangle \left\langle N_K \right\rangle} =$$

$$= (q_\pi - 1) + (q_K - 1) - 2\frac{Cov(K\pi)}{\left\langle N_\pi \right\rangle \left\langle N_K \right\rangle} =$$

$$= \left(\frac{1}{a_K} + \frac{1}{a_\pi} \right) \frac{1}{N_P} \left(1 - \frac{N_P}{A} \right) + \kappa = \alpha \frac{1}{N_P} \left(1 - \frac{N_P}{A} \right) + \kappa$$

$$\alpha = \frac{1}{a_K} + \frac{1}{a_\pi}; \qquad \kappa = -2\frac{Cov(K\pi)}{\left\langle N_\pi \right\rangle \left\langle N_K \right\rangle}; \qquad a_{\pi,K} = \frac{C_{\pi,K}}{N_P}$$

Assumption: fluctuations of both types of particles behave in the same way as function of centrality

♦ Some other results (3) – particle ratio fluctuations

$$\sigma^2_{N_\pi/N_K} = \frac{\left\langle (\Delta N_\pi)^2 \right\rangle}{\left\langle N_\pi \right\rangle^2} + \frac{\left\langle (\Delta N_K)^2 \right\rangle}{\left\langle N_K \right\rangle^2} - 2\frac{\left\langle \Delta N_\pi \Delta N_K \right\rangle}{\left\langle N_\pi \right\rangle \left\langle N_K \right\rangle} =$$

$$= (q_\pi - 1) + (q_K - 1) - 2\frac{\mathrm{Cov}(K\pi)}{\left\langle N_\pi \right\rangle \left\langle N_K \right\rangle} =$$

$$= \left(\frac{1}{a_K} + \frac{1}{a_\pi}\right)\frac{1}{N_P}\left(1 - \frac{N_P}{A}\right) + \kappa = \alpha \frac{1}{N_P}\left(1 - \frac{N_P}{A}\right) + \kappa$$

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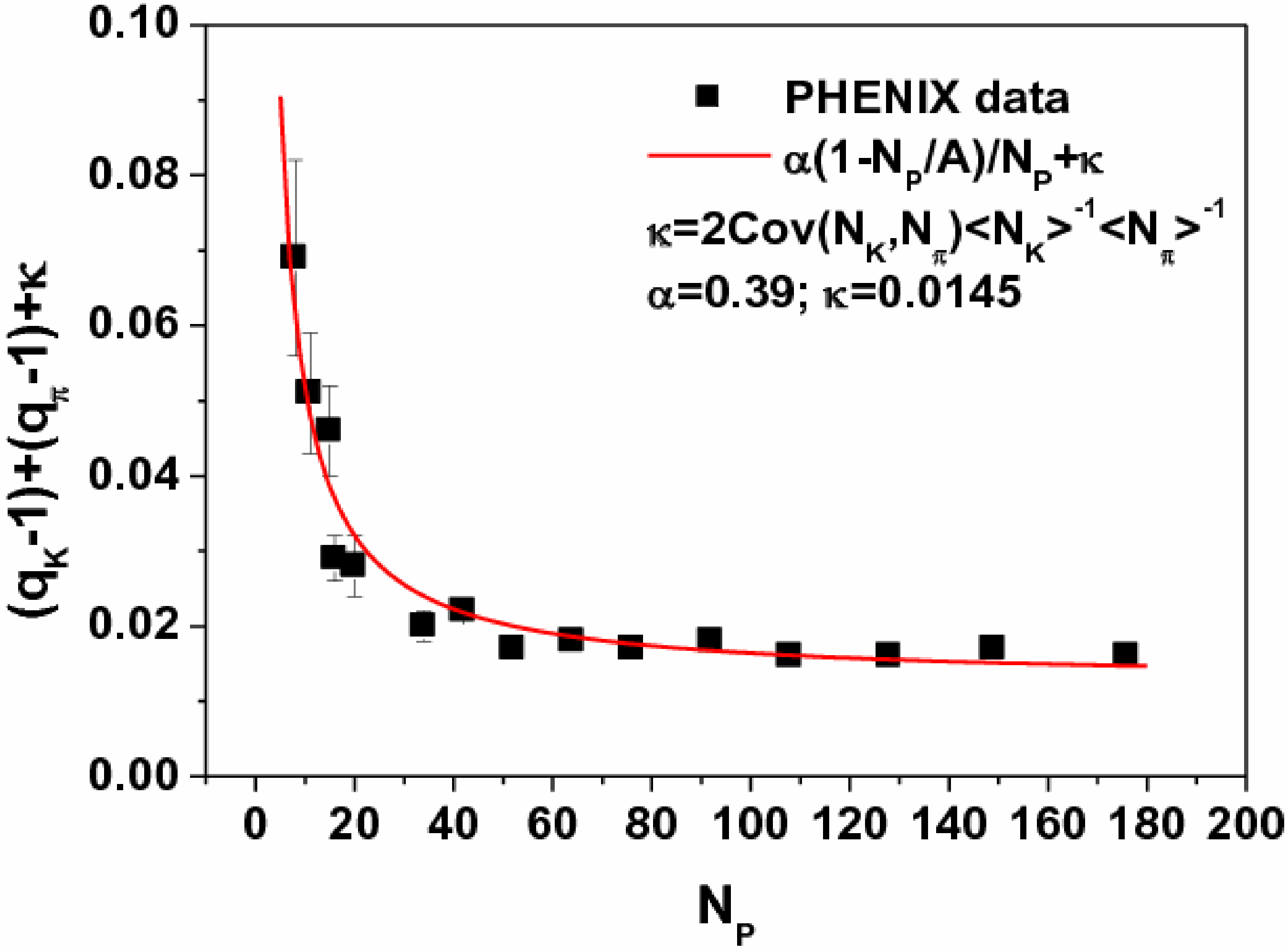
Assumption: fluctuations of both types of particles behave in the same way as function of centrality

♦ Some other results (3) – particle ratio fluctuations

our fit



$$(q_\pi - 1) + (q_K - 1) + \kappa = \frac{\alpha}{N_P}\left(1 - \frac{N_P}{A}\right) + \kappa = \left(\frac{1}{C_\pi} + \frac{1}{C_K}\right)\left(1 - \frac{N_P}{A}\right) + \kappa$$

$C_{\pi,K}$ – heat capacities

Summary [1]:

(*) Some features of the observed multiplicity fluctuations are due to the intrinsic, nonstatistical fluctuations of the temperature of the hadronizing system.

(*) They can be described by the nonextensive Tsallis statistics with the nonextensivity parameter q, which can be associated with the number of participants N_P (assumed to represent the size of the colliding system).

(*) Data require a generalization of the notion of fluctuating temperature by adding effects of the possible energy transfer between the hadronizing source and the surroundings composed of nucleons not participating directly in the reaction.
This resulted in the introduction of a q-dependent effective temperature T_{eff}.

(*) One has to allow for the fluctuations of the full accessible energy U, otherwise Var(N)/<N> remains constant, independent of N_P/A.

Summary [2]:

(*) The above multiplicity fluctuations describe also fully:

[#] the observed transverse momentum fluctuations;

[#] the observed transverse momenta correlations;

[#] the observed pattern of particle ratios fluctuations.

In all above examples fluctuations in p_T only play minor role .

The End

Thank You