

Recent L3 results on BEC at LEP, improved description of the tau-model for the L3 Collaboration

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16/10/2009

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Introduction - Bose-Einstein Correlation

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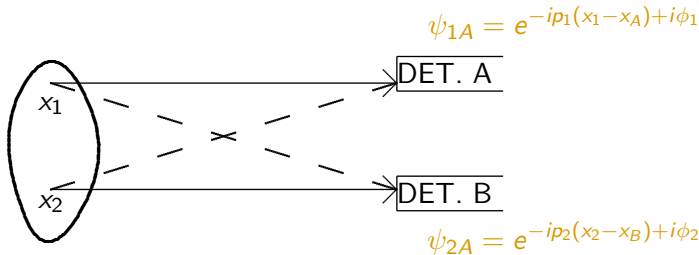
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After **Bose-Einstein symmetrization** the amplitude for two identical pions

$$\begin{aligned}
 |p_1 p_2\rangle &= \frac{1}{\sqrt{2}} \left[e^{ip_1(x_1 - x_A) + i\phi_1} e^{ip_2(x_2 - x_B) + i\phi_2} + e^{ip_1(x_2 - x_A) + i\phi_1} e^{ip_2(x_1 - x_B) + i\phi_2} \right] \\
 &= \frac{1}{\sqrt{2}} e^{-i(p_1 x_A + p_2 x_B - \phi_1 - \phi_2)} \left[e^{ip_1 x_1} e^{ip_2 x_2} + e^{ip_1 x_2} e^{ip_2 x_1} \right]
 \end{aligned}$$

2-particle correlation function:

$$R_2(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)}$$

To study **only** BEC, not all correlations, replace $\rho_1(p_1)\rho_1(p_2)$ by $\rho_0(p_1, p_2)$ (reference sample \rightarrow mixed events).

$$R_2(p_1, p_2) = \frac{\rho_2(p_1, p_2)}{\rho_0(p_1, p_2)}$$

Since BEC only at small $Q^2 = -q^\mu q_\mu = -(p_1 - p_2)^2$ integrate over other variable

$$R_2(Q) = \rho_2(Q)/\rho_0(Q)$$

Assuming incoherent particle production and spatial source density $f(x)$,

$$R_2(Q) = 1 + \lambda |\tilde{f}(Q)|^2,$$

where $\tilde{f}(Q) = \int dx \exp(iQx)f(x)$

Large Electron Positron Collider

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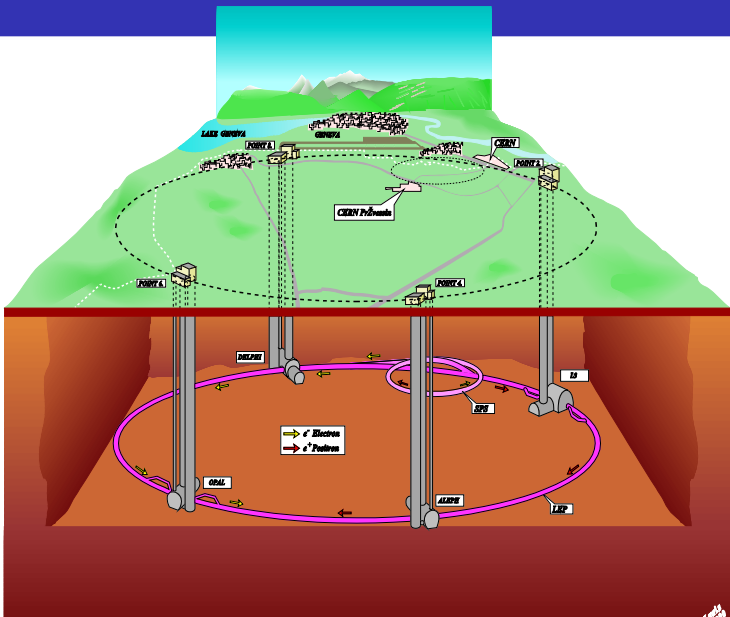
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The L3 Detector

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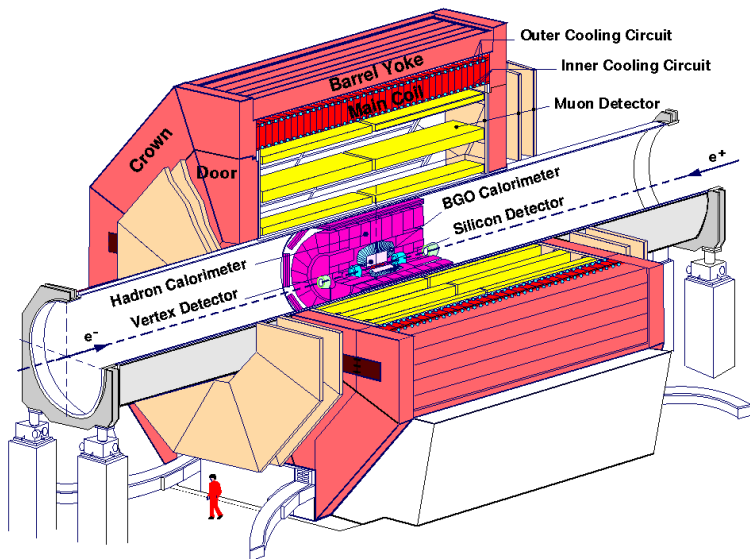
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Data Sample:

- Hadronic Z decay from 1994 ($\sqrt{s} = 91.2$ GeV)
- 'Standard' event and track selection $\approx .8$ million events
 ≈ 36 million like-sign pairs
- Concentrate on 2-jet and 3-jet events using **Dhuram algorithm** ($y_{\text{cut}} = 0.006$)
- Correct distribution **bin-by-bin by MC**
- $R_2(Q) = \frac{\rho_2(Q)}{\rho_0(Q)}$ In practice $\rho_2(Q)$ is **measured** from the **data**, but $\rho_0(Q)$ **does not exist** in nature \rightarrow create it (mixed events)!

Puzzles in the late 80's

- First, the measured correlation functions are consistent with the invariant Q dependence.

TASSO Collab., Althoff, M, Z. Phys. C30 (1986) 355–369.

(The statistics were small.)

- Second, the correlation function is more peaked than a Gaussian.

There was no theoretical model
which predicted a mere Q dependence.

Q dependence in e^+e^-

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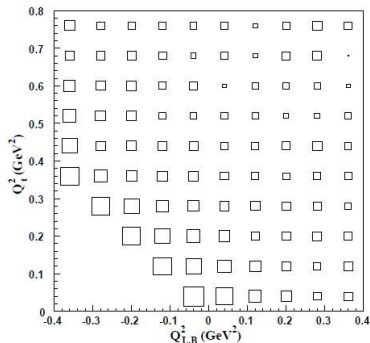
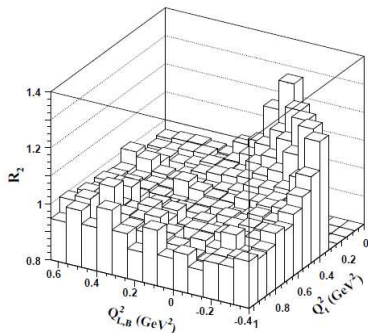
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$$Q_t^2 = (p_{t1}, p_{t2})^2 \quad Q_{L,B}^2 = (p_{l1} - p_{l2})^2 - (E_1 - E_2)^2$$

BEC is maximal when the invariant Q is small,
although any of the components of Q are large.

Q dependence is not present in h+p

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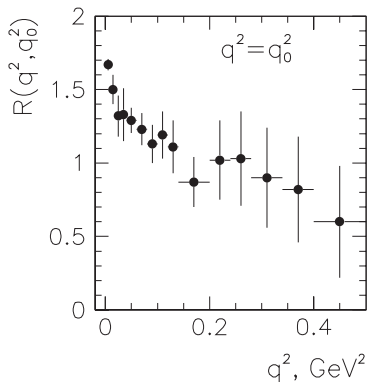
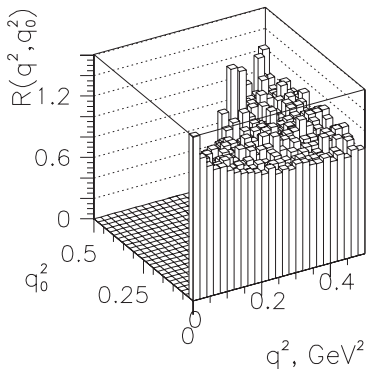
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In NA22 data, BEC is **NOT** maximal when the invariant Q is small and one of its components is large.

Examples with Q dependence

The correlation function will be investigated as a function of Q , the invariant four-momentum difference.

Source function

Gaussian

Edgeworth Expansion

Sym. Levy Stable

Correlation function

$$R_2(Q) = 1 + \lambda \exp(-(RQ)^2)$$

$$R_2(Q) = 1 + \lambda \exp(-(RQ)^2) \left[1 + \frac{\kappa}{3!} H_3(RQ) \right]$$

$$R_2(Q) = 1 + \lambda \exp(-(RQ)^\alpha)$$

Beyond the Gaussian

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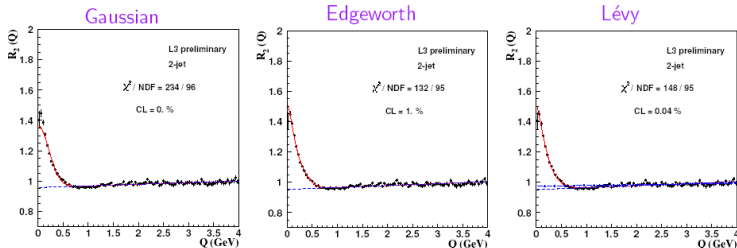
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Far from Gaussian: $\kappa = 0.74 \pm 0.07$, $\alpha = 1.44 \pm 0.06$

Poor Cls. Edgeworth and Levy better than Gaussian.

Note that data dip in the region $0.6 < Q < 1.5$ GeV.

Same conclusions for 3-jet events.

$R_2 \neq 1 +$ positive definit

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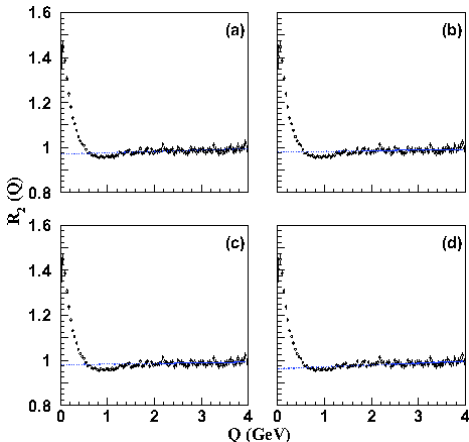
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Fit $\gamma(1 + \delta Q)$ starting from: 1.5, 1.8, 2.1, 2.5 GeV.

Summary:

- We have assumed static source – **certainly wrong**
- BEC depends (at least approximately) only on Q
- $R_2(Q) \neq 1 +$ positive definit form

Let's turn to a model incorporating these points.

Assumptions:

- Assume that the average production point of particles and the four momentum are strongly correlated.
- This correlation is much narrower than the proper-time distribution.
- In the plane-wave approximation, using the Yano-Koonin formula, one gets for 2-jet events:

$$R_2(Q) = 1 + \lambda \text{Re} \tilde{H}^2 \left(\frac{Q^2}{2m_t} \right).$$

The τ -model

Further assumptions:

- Assume a Levy stable distribution for $H(\tau)$.
- Since no particle production before the collision, $H(\tau)$ is one-sided.

Then

$$R_2(Q, \bar{m}_t) = \gamma \left[1 + \lambda \cos \left(\frac{\tau_0 Q^2}{\bar{m}_t} + \tan \left(\frac{\alpha \pi}{2} \right) \left(\frac{\Delta \tau Q^2}{2 \bar{m}_t} \right)^\alpha \right) \exp \left(- \left(\frac{\Delta \tau Q^2}{2 \bar{m}_t} \right)^\alpha \right) \right] (1 + \delta Q)$$

- where
- α is the index of stability
 - τ_0 is the proper-time of the onset of part. prod.
 - $\Delta \tau$ is a measure of the width of the dist.

$$R_2(Q, \bar{m}_t) = \gamma \left[1 + \lambda \cos \left(\frac{\tau_0 Q^2}{\bar{m}_t} + \tan \left(\frac{\alpha \pi}{2} \right) \left(\frac{\Delta \tau Q^2}{2\bar{m}_t} \right)^\alpha \right) \exp \left(- \left(\frac{\Delta \tau Q^2}{2\bar{m}_t} \right)^\alpha \right) \right] (1 + \delta Q)$$

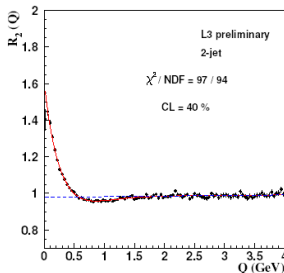
Before fitting in two dimensions (Q, \bar{m}_t) , assume an 'average' \bar{m}_t dependence by introducing eff. radius $R = \sqrt{\Delta \tau / (2\bar{m}_t)}$. Also assumed $\tau_0 = 0$. Then

$$R_2(Q) = \gamma \left[1 + \lambda \cos \left[(R_a Q)^{2\alpha} \right] \exp \left(- (RQ)^{2\alpha} \right) \right] (1 + \delta Q)$$

$$R_a^{2\alpha} = \tan \left(\frac{\alpha \pi}{2} \right) R^{2\alpha}$$

parameter	R_a free	$R_a^{2\alpha} = \tan \left(\frac{\alpha \pi}{2} \right) R^{2\alpha}$
α	0.42 ± 0.02	0.42 ± 0.01
λ	0.67 ± 0.03	0.67 ± 0.03
R (fm)	0.79 ± 0.04	0.79 ± 0.03
R_a (fm)	0.59 ± 0.03	—
δ	0.003 ± 0.002	0.003 ± 0.001
γ	0.979 ± 0.005	0.979 ± 0.005
χ^2/DoF	97/94	97/95
CL	40%	42%

R_a free or not gives same results. – Good CL



Result for 2-jet events

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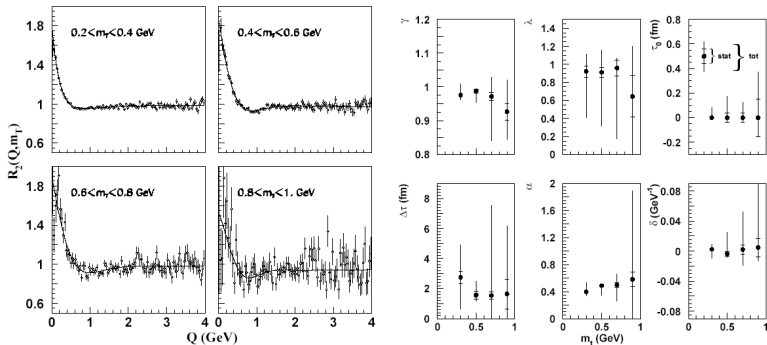
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CLs are OK. Parameters are approximately independent of \overline{m}_t .

$$\tau_0 = 0.0 \pm 0.0 \text{ fm} \quad \alpha = 0.47 \pm 0.01 \quad \Delta\tau = 1.16 \pm 0.12 \text{ fm}$$

Results for 2-jet events

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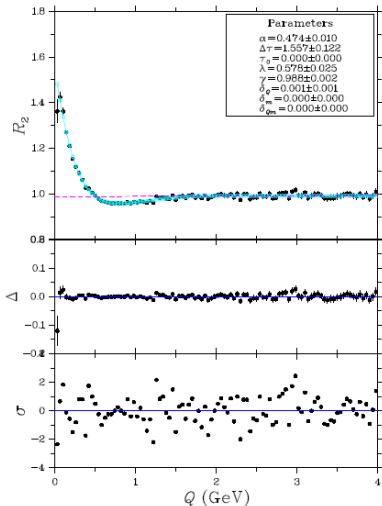
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parameter	
α	$0.47 \pm 0.01^{+0.04}_{-0.02}$
λ	$0.58 \pm 0.03^{+0.08}_{-0.24}$
$\Delta\tau$ (fm)	$1.56 \pm 0.12^{+0.32}_{-0.45}$
ϵ (GeV ⁻¹)	$0.001 \pm 0.001 \pm 0.003$
γ	$0.988 \pm 0.002^{+0.006}_{-0.002}$
χ^2/DoF	90/95
confidence level	62%

Reconstruction of the Source Function

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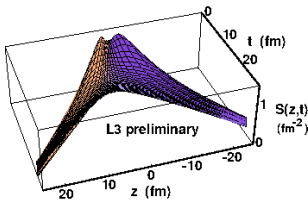
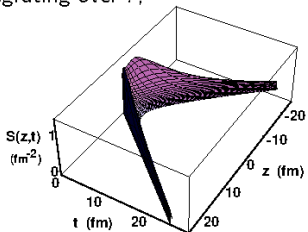
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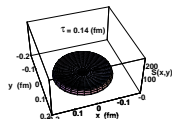
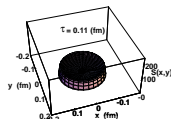
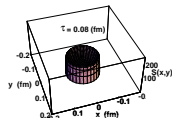
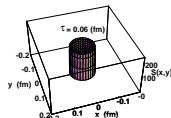
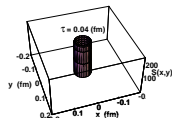
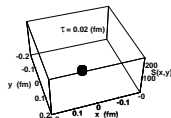
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Integrating over r ,



Integrating over z ,



Results of the τ -model

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- Time-dependence – OK!
- Mere Q dependence – OK!
- Describe the dip – OK!

Pretty nice, but...

What about the elongation?

Elongation $R_{\text{long}} > R_{\text{side}}, R_{\text{out}}$ is seen by all LEP experiments.

Elongation Results (L3)

parameter	Gaussian	Edgeworth
λ	$0.41 \pm 0.01^{+0.02}_{-0.19}$	$0.54 \pm 0.02^{+0.04}_{-0.26}$
r_L (fm)	$0.74 \pm 0.02^{+0.04}_{-0.03}$	$0.69 \pm 0.02^{+0.04}_{-0.03}$
r_{out} (fm)	$0.53 \pm 0.02^{+0.05}_{-0.06}$	$0.44 \pm 0.02^{+0.05}_{-0.06}$
r_{side} (fm)	$0.59 \pm 0.01^{+0.03}_{-0.13}$	$0.56 \pm 0.02^{+0.03}_{-0.12}$
r_{out}/r_L	$0.71 \pm 0.02^{+0.05}_{-0.08}$	$0.65 \pm 0.03^{+0.06}_{-0.09}$
r_{side}/r_L	$0.80 \pm 0.02^{+0.03}_{-0.18}$	$0.81 \pm 0.02^{+0.03}_{-0.19}$
κ_L	–	$0.5 \pm 0.1^{+0.1}_{-0.2}$
κ_{out}	–	$0.8 \pm 0.1 \pm 0.3$
κ_{side}	–	$0.1 \pm 0.1 \pm 0.3$
δ	$0.025 \pm 0.005^{+0.014}_{-0.015}$	$0.036 \pm 0.007^{+0.012}_{-0.023}$
ϵ	$0.005 \pm 0.005^{+0.034}_{-0.012}$	$0.011 \pm 0.005^{+0.037}_{-0.012}$
ξ	$-0.035 \pm 0.005^{+0.031}_{-0.024}$	$-0.022 \pm 0.006^{+0.020}_{-0.025}$
χ^2/DoF	2314/2189	2220/2186
C.L. (%)	3.1	30

- $\rho_{L,\text{out}} = 0$ So fix to 0.

- Edgeworth fit significantly better than Gaussian

- $r_{\text{side}}/r_L < 1$ more than 5 std. dev. Elongation along thrust axis

- Models which assume a spherical source are too simple.

Test: allowing for elongation

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$$a^2 = R_L^2 Q_L^2 + R_{\text{side}}^2 Q_{\text{side}}^2 + r_{\text{out}}^2 q_{\text{out}}^2$$

$$R_2(Q) = \gamma \left[1 + \lambda \cos \left(\tan \left(\frac{\alpha\pi}{2} \right) a^{2\alpha} \right) \exp(-a^{2\alpha}) \right] (1 + \epsilon_L Q_L + \epsilon_{\text{side}} Q_{\text{side}} + \epsilon_{\text{out}} q_{\text{out}})$$

λ	0.65 ± 0.03	0.57 ± 0.03
α	0.41 ± 0.01	0.44 ± 0.01
R_L (fm)	0.96 ± 0.05	0.82 ± 0.04
R_{side}/R_L	0.62 ± 0.02	1
r_{out}/R_L	1.23 ± 0.03	1
ϵ_L (GeV ⁻¹)	0.004 ± 0.001	0.003 ± 0.001
ϵ_{side} (GeV ⁻¹)	-0.067 ± 0.003	-0.059 ± 0.003
ϵ_{out} (GeV ⁻¹)	-0.022 ± 0.003	-0.029 ± 0.002
γ	1.000 ± 0.002	1.003 ± 0.002
χ^2/DoF	10966/10647	11430/10649
CL	2%	10 ⁻⁷

Elongation is needed to describe the L3 data.

Test: allowing for elongation

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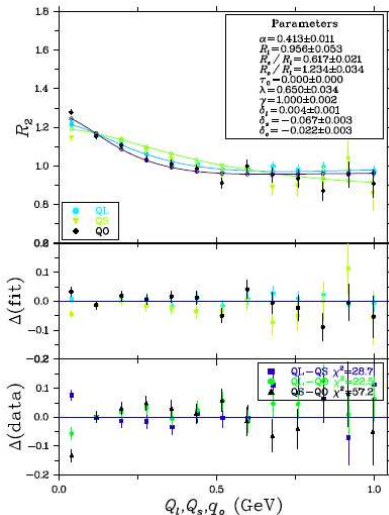
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- Symmetric Gaussian, Edgeworth and Levy do not fit well.
- The τ -model with one-sided Levy proper-time distribution leads to $R_2(Q, \bar{m}_t)$, which successfully fits R_2 for 2-jet events.
- Emission function shaped like a boomerang in $z - t$ and an expanding ring in $x - y$.
Particle production is close to light cone.
- Elongation is not described by the τ -model. It has to be generalized.