Recent L3 results on BEC at LEP, improved description of the tau-model

for the L3 Collaboration

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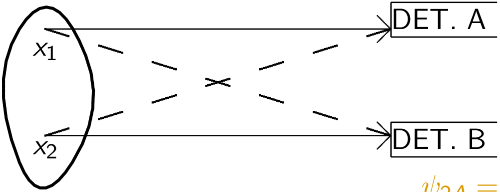
Introduction - Bose-Einstein Correlation

$$\psi_{1A} = e^{-ip_1(x_1-x_A)+i\phi_1}$$



$$\psi_{2A} = e^{-ip_2(x_2-x_B)+i\phi_2}$$

After Bose-Einstein symmetrization the amplitude for two identical pions

$$|p_1p_2\rangle = \frac{1}{\sqrt{2}}\left[e^{ip_1(x_1-x_A)+i\phi_1}e^{p_2(x_2-x_B)+i\phi_2} + e^{ip_1(x_2-x_A)+i\phi_1}e^{p_2(x_1-x_B)+i\phi_2}\right]$$

$$= \frac{1}{\sqrt{2}}e^{-i(p_1x_A+p_2x_B-\phi_1-\phi_2)}\left[e^{ip_1x_1}e^{ip_2x_2} + e^{ip_1x_2}e^{ip_2x_1}\right]$$

Introduction - BEC

2-particle correlation function:

$$R_2(p_1,p_2) = \frac{\rho_2(p_1,p_2)}{\rho_1(p_1)\rho_1(p_2)}$$

To study only BEC, not all correlations, replace $\rho_1(p_1)\rho_1(p_2)$ by $\rho_0(p_1,p_2)$ (reference sample → mixed events).

$$R_2(p_1,p_2) = \frac{\rho_2(p_1,p_2)}{\rho_0(p_1,p_2)}$$

Since BEC only at small $Q^2 = -q^\mu q_\mu = -(p_1 - p_2)^2$ integrate over other variable

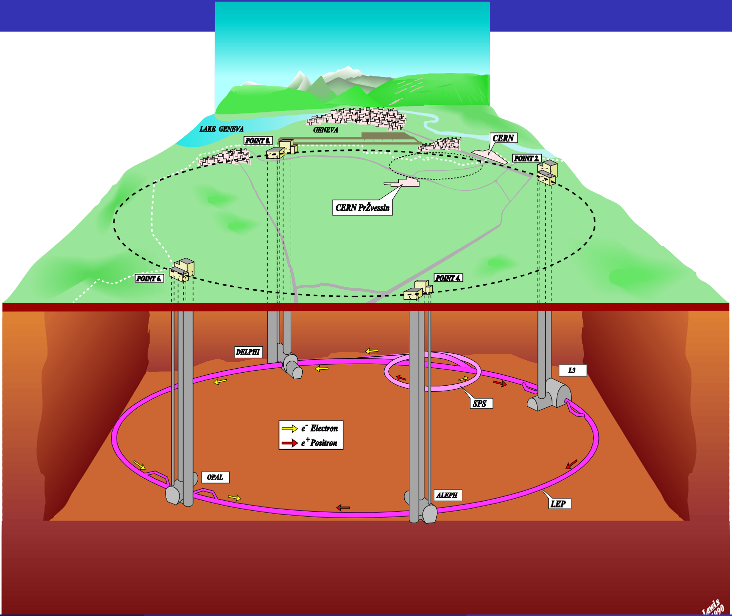
$$R_2(Q) = \rho_2(Q)/\rho_0(Q)$$

Assuming incoherent particle production and spatial source density $f(x)$,

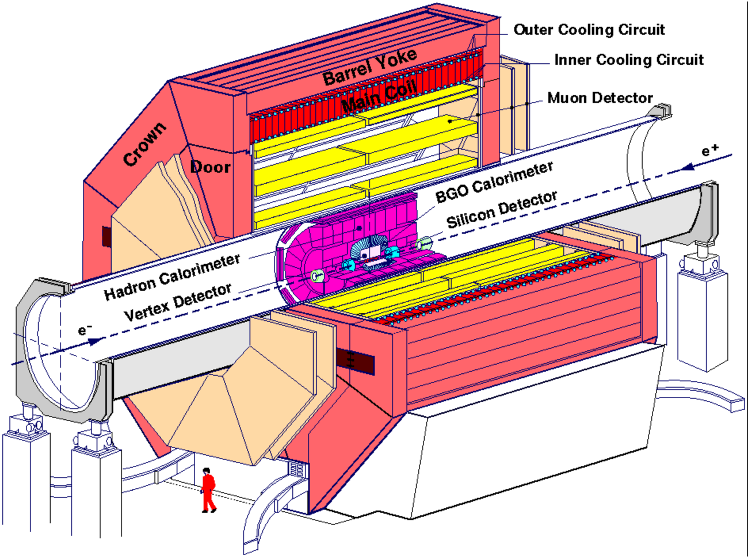
$$R_2(Q) = 1 + \lambda|\tilde{f}(Q)|^2,$$

where $\tilde{f}(Q) = \int \mathrm{d}x \exp(iQx) f(x)$

Large Electron Positron Collider



The L3 Detector



Data Sample

Data Sample:

- Hadronic Z decay from 1994 ($\sqrt{s} = 91.2$ GeV)
- 'Standard' event and track selesction $\approx$ .8 million events $\approx$ 36 million like-sign pairs
- Concentrate on 2-jet and 3-jet events using Dhuram algorithm ($y_{\mathrm{cut}} = 0.006$)
- Correct distribution bin-by-bin by MC
- $R_2(Q) = \frac{\rho_2(Q)}{\rho_0(Q)}$ In practice $\rho_2(Q)$ is measured from the data, but $\rho_0(Q)$ does not exist in nature $\rightarrow$ create it (mixed events)!

Puzzles in the late 80's

- First, the measured correlation functions are consistent with the invariant Q dependence.

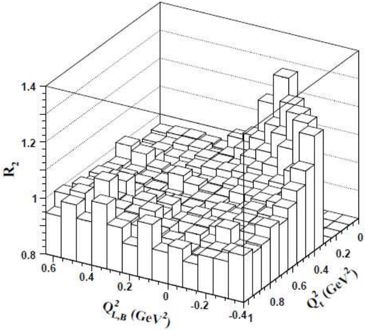
TASSO Collab., Althoff, M, Z. Phys. **C30** (1986) 355–369.

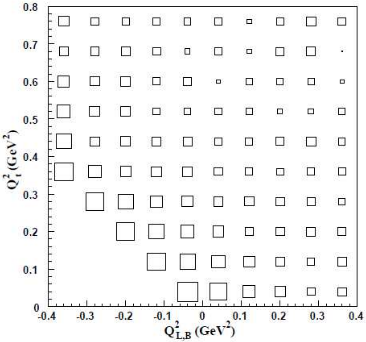
(The statisctics were small.)

- Second, the correlation function is more peaked than a Gaussian.

There was no theoretical model which predicted a mere Q dependence.

Q dependence in e^+e^-

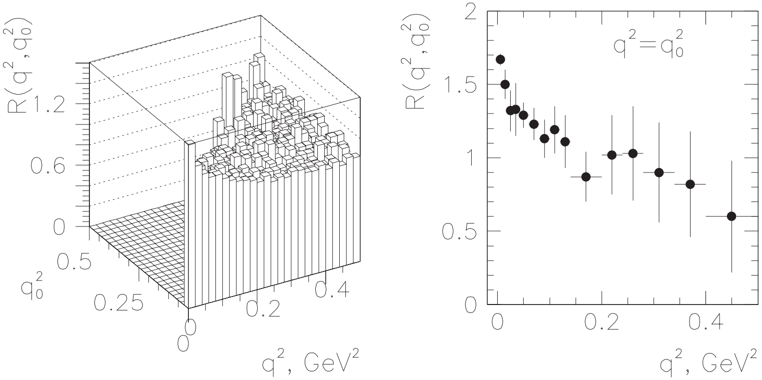




$$Q_{\mathrm{t}}^2 = (p_{\mathrm{t}1}, p_{\mathrm{t}2})^2 \qquad Q_{\mathrm{L,B}}^2 = (p_{\mathrm{l}1} - p_{\mathrm{l}2})^2 - (E_1 - E_2)^2$$

BEC is maximal when the invariant Q is small, although any of the components of Q are large.

Q dependence is not present in h+p



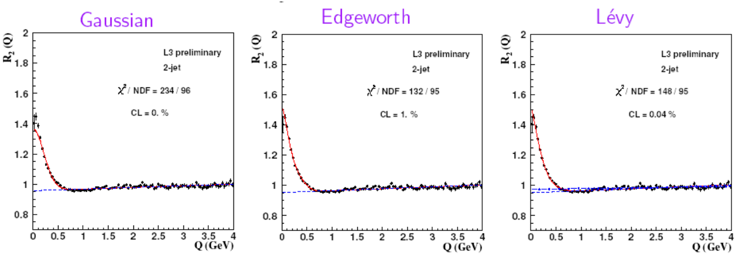
In NA22 data, BEC is NOT maximal when
the invariant Q is small and one of its components is large.

Examples with Q dependence

The correlation function will be investigated as a function of Q, the invariant four-momentum difference.

Source function	Correlation function
Gaussian	$R_2(Q) = 1 + \lambda \exp(-(RQ)^2)$
Edgeworth Expansion	$R_2(Q) = 1 + \lambda \exp(-(RQ)^2) \left[1 + \frac{\kappa}{3!} H_3(RQ)\right]$
Sym. Levy Stable	$R_2(Q) = 1 + \lambda \exp(-(RQ)^\alpha)$

Beyond the Gaussian



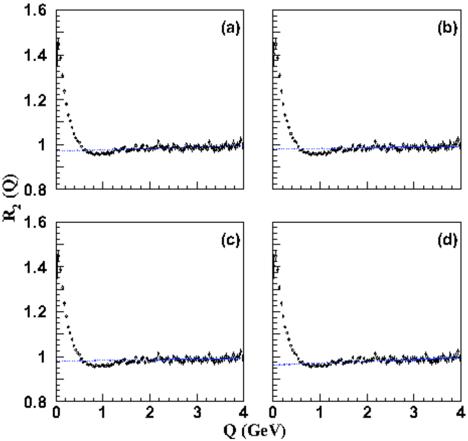
Far from Gaussian: $\kappa = 0.74 \pm 0.07$, $\alpha = 1.44 \pm 0.06$

Poor CIs. Edgeworth and Levy better than Gaussian.

Note that data dip in the region $0.6 < Q < 1.5$ GeV.

Same conclusions for 3-jet events.

$R_2 \neq 1+$ positive definit



Fit $\gamma(1 + \delta Q)$ starting from: 1.5, 1.8, 2.1, 2.5 GeV.

Beyond the Gaussian

Summary:

- We have assumed static source – certainly wrong
- BEC depends (at least approximately) only on Q
- $R_2(Q) \neq 1+$ positive definit form

Let's turn to a model incorporating these points.

The τ -model

Assumptions:

- Assume that the average production point of particles and the four momentum are strongly correlated.
- This correlation is much narrower than the proper-time distribution.
- In the plane-wave approximation, using the Yano-Koonin formula, one gets for 2-jet events:

$$R_2(Q) = 1 + \lambda Re \tilde{H}^2 \left(\frac{Q^2}{2m_t} \right) .$$

The τ-model

Further assumptions:

- Assume a Levy stable distribution for $H(\tau)$.
- Since no particle production before the collision, $H(\tau)$ is one-sided.

Then

$$R_2(Q,\overline{m}_t) = \gamma \left[1 + \lambda \cos\left(\frac{\tau_0 Q^2}{\overline{m}_t} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2\overline{m}_t}\right)^{\alpha}\right) \exp\left(-\left(\frac{\Delta\tau Q^2}{2\overline{m}_t}\right)^{\alpha}\right)\right](1+\delta Q)$$

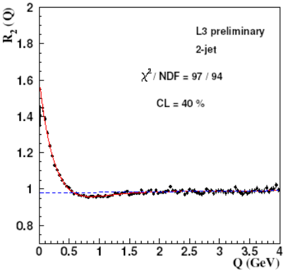
where	α	is the index of stability
	τ_0	is the proper-time of the onset of part. prod.
	$\Delta\tau$	is a measure of the width of the dist.

$$R_2(Q,\overline{m}_t) = \gamma\left[1+\lambda\cos\left(\frac{\tau_0 Q^2}{\overline{m}_t}+\tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2\overline{m}_t}\right)^{\alpha}\right)\exp\left(-\left(\frac{\Delta\tau Q^2}{2\overline{m}_t}\right)^{\alpha}\right)\right](1+\delta Q)$$

Before fitting in two dimensions $(Q,\overline{m}_t)$, assume an 'average' $\overline{m}_t$ dependence by introducing eff. radius $R=\sqrt{\Delta\tau/(2\overline{m}_t)}$. Also assumed $\tau_0 = 0$. Then

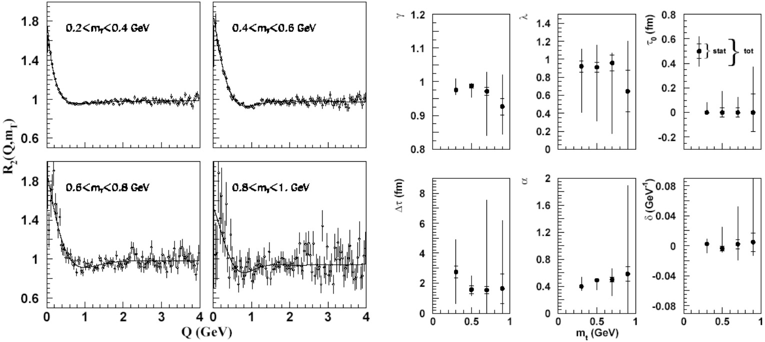
$$R_2(Q) = \gamma\left[1+\lambda\cos\left[(R_a Q)^{2\alpha}\right]\exp\left(-(RQ)^{2\alpha}\right)\right](1+\delta Q)$$
$$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$$

parameter	R_a free	$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$
α	0.42 ± 0.02	0.42 ± 0.01
λ	0.67 ± 0.03	0.67 ± 0.03
R (fm)	0.79 ± 0.04	0.79 ± 0.03
R_a (fm)	0.59 ± 0.03	—
δ	0.003 ± 0.002	0.003 ± 0.001
γ	0.979 ± 0.005	0.979 ± 0.005
χ^2/DoF	97/94	97/95
CL	40%	42%



R_a free or not gives same results. – Good CL

Result for 2-jet events

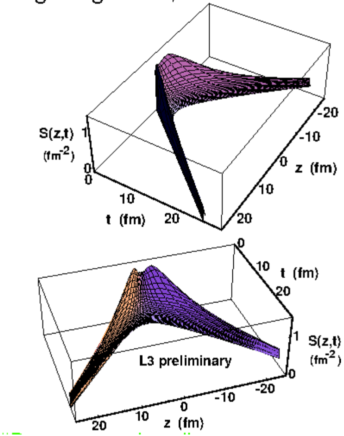


CLs are OK. Parameters are approximately independent of $\overline{m_t}$.

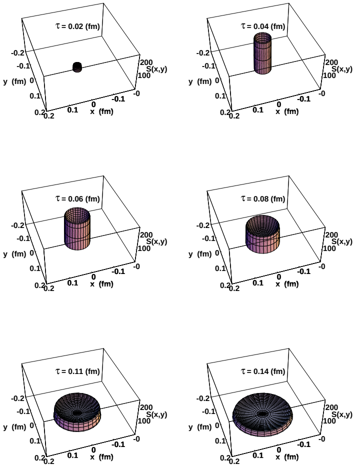
$\tau_0 = 0.0 \pm 0.0$ fm $\alpha = 0.47 \pm 0.01$ $\Delta\tau = 1.16 \pm 0.12$ fm

Reconstruction of the Source Function

Integrating over r,



Integrating over z,



Results of the τ-model

- Time-dependence – OK!
- Mere Q dependence – OK!
- Describe the dip – OK!

Pretty nice, but...

What about the elongation?

Elongation $R_{\text{long}} > R_{\text{side}}, R_{\text{out}}$ is seen by all LEP experiments.

Elongation Results (L3)

parameter	Gaussian	Edgeworth
λ	$0.41 \pm 0.01^{+0.02}_{-0.19}$	$0.54 \pm 0.02^{+0.04}_{-0.26}$
r_{L} (fm)	$0.74 \pm 0.02^{+0.04}_{-0.03}$	$0.69 \pm 0.02^{+0.04}_{-0.03}$
r_{out} (fm)	$0.53 \pm 0.02^{+0.05}_{-0.06}$	$0.44 \pm 0.02^{+0.05}_{-0.06}$
r_{side} (fm)	$0.59 \pm 0.01^{+0.03}_{-0.13}$	$0.56 \pm 0.02^{+0.03}_{-0.12}$
$r_{\text{out}}/r_{\text{L}}$	$0.71 \pm 0.02^{+0.05}_{-0.08}$	$0.65 \pm 0.03^{+0.06}_{-0.09}$
$r_{\text{side}}/r_{\text{L}}$	$0.80 \pm 0.02^{+0.03}_{-0.18}$	$0.81 \pm 0.02^{+0.03}_{-0.19}$
κ_{L}	–	$0.5 \pm 0.1^{+0.1}_{-0.2}$
κ_{out}	–	$0.8 \pm 0.1 \pm 0.3$
κ_{side}	–	$0.1 \pm 0.1 \pm 0.3$
δ	$0.025 \pm 0.005^{+0.014}_{-0.015}$	$0.036 \pm 0.007^{+0.012}_{-0.023}$
ϵ	$0.005 \pm 0.005^{+0.034}_{-0.012}$	$0.011 \pm 0.005^{+0.037}_{-0.012}$
ξ	$-0.035 \pm 0.005^{+0.031}_{-0.024}$	$-0.022 \pm 0.006^{+0.020}_{-0.025}$
χ^2/DoF	2314/2189	2220/2186
C.L. (%)	3.1	30

- $\rho_{\text{L,out}} = 0$ So fix to 0.
- Edgeworth fit significantly better than Gaussian
- $r_{\text{side}}/r_{\text{L}} < 1$ more than 5 std. dev. Elongation along thrust axis
- Models which assume a spherical source are too simple.

Test: allowing for elongation

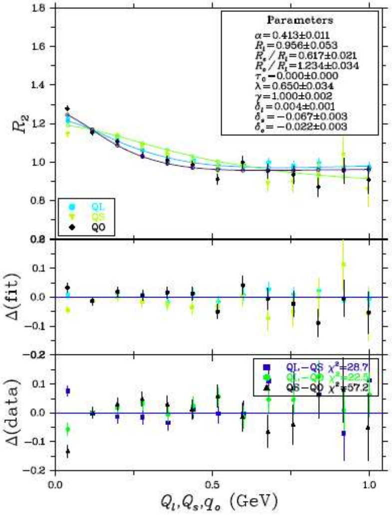
$$a^2 = R_L^2 Q_L^2 + R_{side}^2 Q_{side}^2 + r_{out}^2 q_{out}^2$$

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left(\tan\left(\frac{\alpha\pi}{2}\right) a^{2\alpha}\right) \exp\left(-a^{2\alpha}\right)\right] (1 + \epsilon_L Q_L + \epsilon_{side} Q_{side} + \epsilon_{out} q_{out})$$

λ	0.65 ± 0.03	0.57 ± 0.03
α	0.41 ± 0.01	0.44 ± 0.01
R_L (fm)	0.96 ± 0.05	0.82 ± 0.04
R_{side}/R_L	0.62 ± 0.02	1
r_{out}/R_L	1.23 ± 0.03	1
ϵ_L (GeV^{-1})	0.004 ± 0.001	0.003 ± 0.001
ϵ_{side} (GeV^{-1})	-0.067 ± 0.003	-0.059 ± 0.003
ϵ_{out} (GeV^{-1})	-0.022 ± 0.003	-0.029 ± 0.002
γ	1.000 ± 0.002	1.003 ± 0.002
χ^2/DoF	10966/10647	11430/10649
CL	2%	10^{-7}

Elongation is needed to describe the L3 data.

Test: allowing for elongation



Elongation is needed to describe the L3 data.

Summary

- Symmetric Gaussian, Edgeworth and Levy do not fit well.
- The τ-model with one-sided Levy proper-time distribution leads to $R_2(Q, \overline{m}_t)$, which successfully fits R_2 for 2-jet events.
- Emission function shaped like a boomerang in $z - t$ and an expanding ring in $x - y$.
  Particle production is close to light cone.
- Elongation is not described by the τ-model. It has to be generalized.